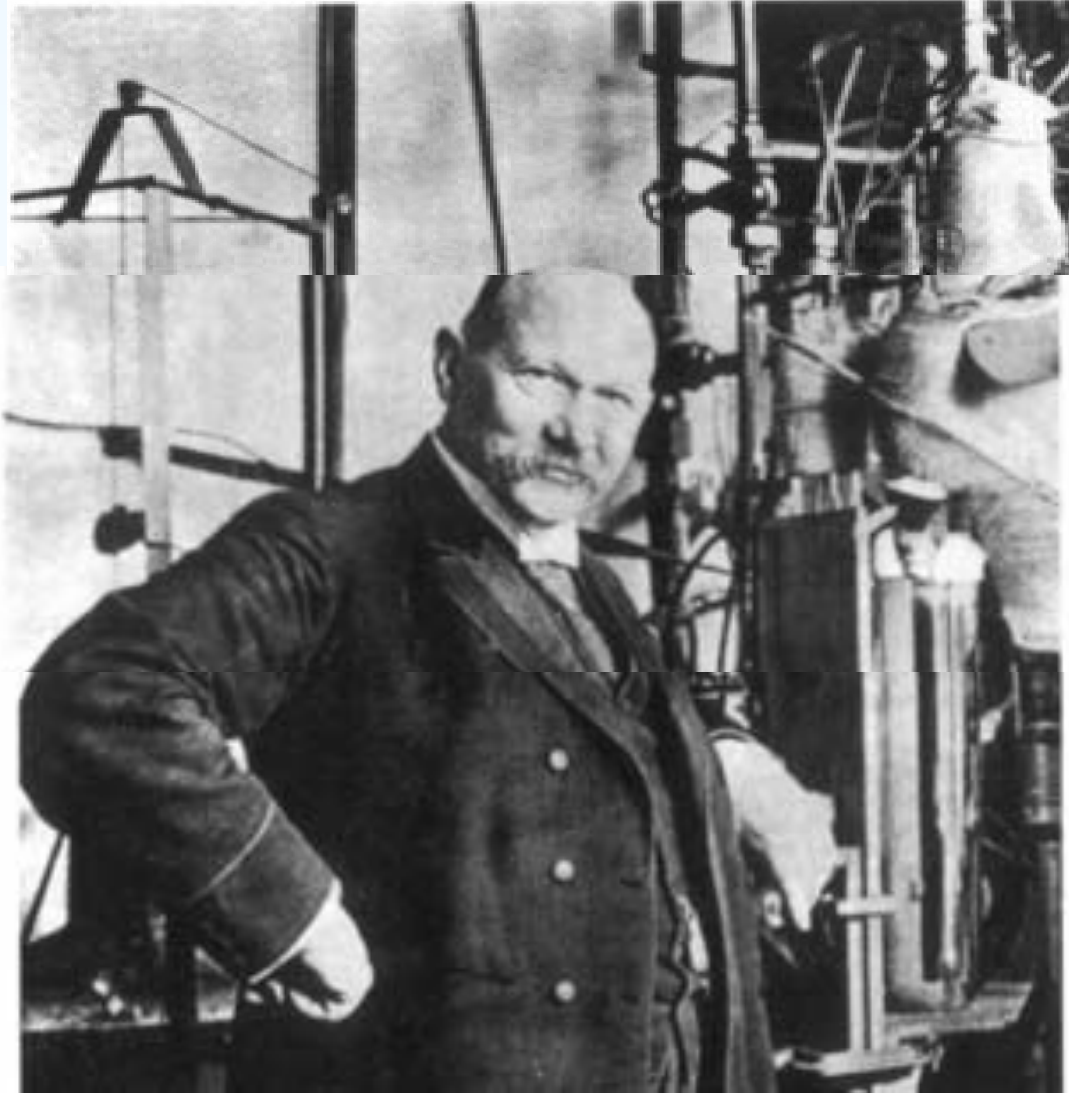# 3
SUPERCONDUCTIVITY

The fascinating phenomenon of superconductivity and its potential applications have attracted the attention of scientists, engineers and businessmen.
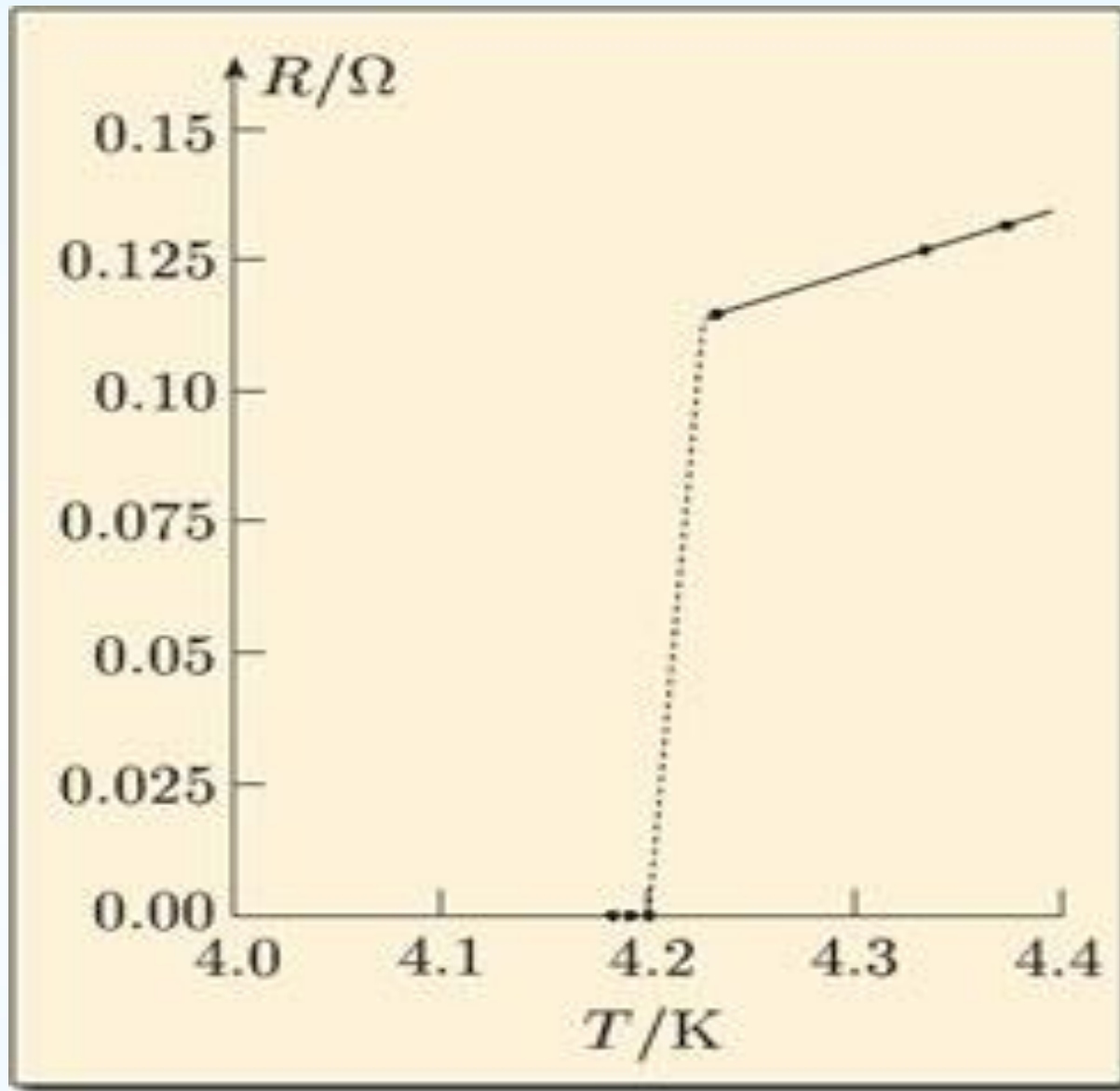
Superconductivity was discovered in 1911 by Heike Kamerlingh Onnes, as he studied the properties of metals at low temperatures.

Heike Kamerlingh Onnes (1908)

- **A few years earlier he had become the first person to liquefy helium, which has a boiling point of 4.2 K at atmospheric pressure, and this had opened up a new range of temperature to experimental investigation.**

- **On measuring the resistance of a small tube filled with mercury, he was astonished to observe that its resistance fell from ~0.1 Ω at a temperature of 4.3 K to less than 3×10^{-6} Ω at 4.1 K.**

- **Below 4.1 K, mercury is said to be a superconductor, and no experiment has yet detected any resistance to steady current flow in a superconducting material.**

- **The temperature below which the mercury becomes superconducting is known as its *critical temperature Tc.***

- **Intense research has taken place to discover new superconductors, to understand the physics that underlies the properties of superconductors, and to develop new applications for these materials.**

- **Superconducting electromagnets produce the large magnetic fields required in the world's largest particle accelerators, in MRI machines used for diagnostic imaging of the human body, in magnetically levitated trains and in superconducting magnetic energy storage systems.**

- **But at the other extreme superconductors are used in SQUID (superconducting quantum interference device) magnetometers, which can measure the tiny magnetic fields (10^{-13}T) associated with electrical activity in the brain, and**

- **There is great interest in their potential as extremely fast switches for a new generation of very powerful computers.**

Graph showing the resistance of a specimen of mercury versus absolute temperature.

The Periodic Table showing all known elemental superconductors and their critical temperatures.
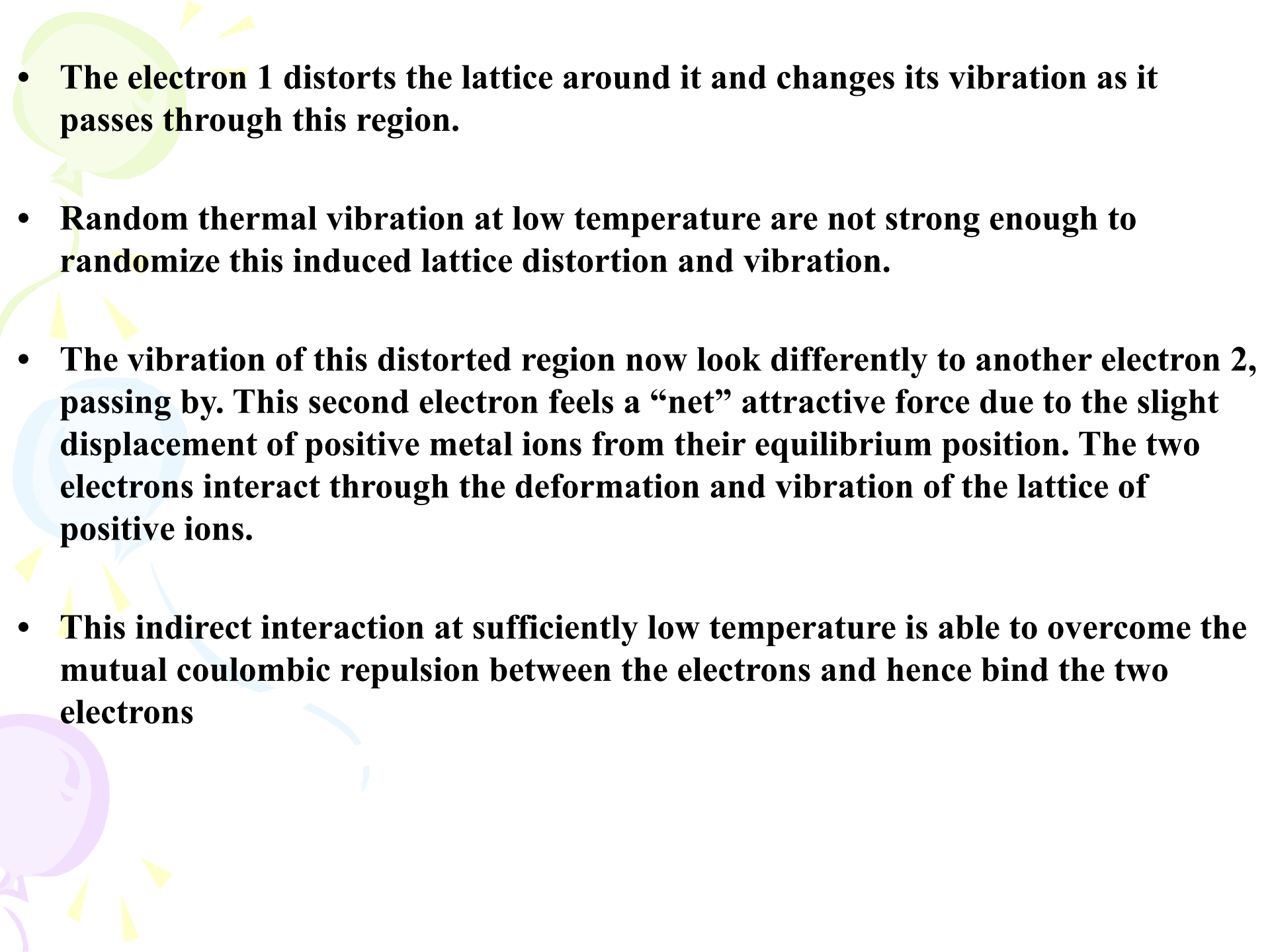
- Since this initial discovery, many more elements have been discovered to be superconductors.
- The dark pink cells indicate elements that become superconducting at atmospheric pressure, and the numbers at the bottoms of the cells are their critical temperatures, which range from 9.3 K for niobium (Nb, $Z = 41$) down to 3×10^{-4} K for rhodium (Rh, $Z = 45$).

- The orange cells are elements that become superconductors only under high pressures.

- The four pale pink cells are elements that are superconducting in particular forms: carbon (C, $Z = 6$) in the form of nanotubes, chromium (Cr, $Z = 24$) as thin films, palladium (Pd, $Z = 46$) after irradiation with alpha particles, and platinum (Pt, $Z = 78$) as a compacted powder.

- It is worth noting that copper (Cu, $Z = 29$), silver (Ag, $Z = 47$) and gold (Au, $Z = 79$), three elements that are excellent conductors at room temperature, do not become superconductors even at the lowest temperatures that are attainable.

- Besides temperature, the superconducting state depends upon other variables of which magnetic field density (B) and current density (J) are the most important.

- **The Quantum theory of superconductivity**

- In 1952 the three US physicists, John Bardeen, Leon Cooper and John Schrieffer, jointly developed theory of superconductivity, usually called the BCS theory.

- According to their theory, in the superconducting state there is an attractive interaction between electrons that is mediated by the vibrations of the ion lattice.

- **At sufficiently low temperature, two oppositely spinning and oppositely travelling electrons can attract each other indirectly through the deformation of the crystal lattice of positive metal ions.**
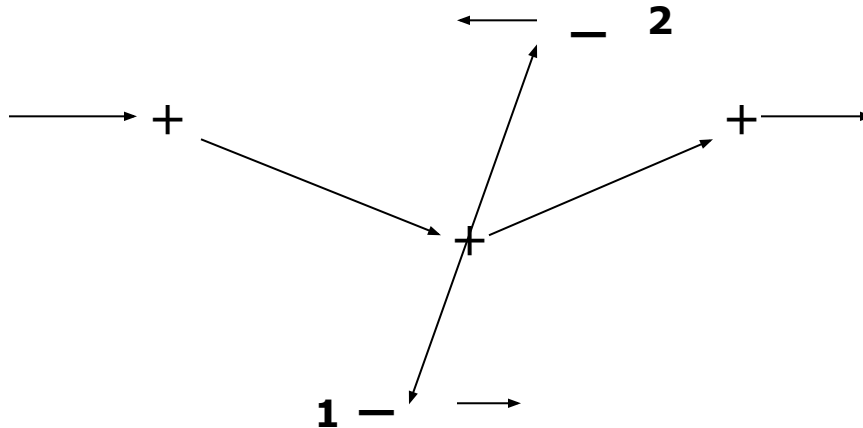
A pictorial & intuitive view of
An indirect attraction between
 two Oppositely travelling
Electrons via lattice distortion &
Vibration.

- The electron 1 distorts the lattice around it and changes its vibration as it passes through this region.

- Random thermal vibration at low temperature are not strong enough to randomize this induced lattice distortion and vibration.

- The vibration of this distorted region now look differently to another electron 2, passing by. This second electron feels a "net" attractive force due to the slight displacement of positive metal ions from their equilibrium position. The two electrons interact through the deformation and vibration of the lattice of positive ions.

- This indirect interaction at sufficiently low temperature is able to overcome the mutual coulombic repulsion between the electrons and hence bind the two electrons

- **A consequence of this interaction is that pairs of electrons are coupled together, and all of the pairs of electrons condense into a macroscopic quantum state, called the condensate (Cooper pairs), that extends through the superconductor.**

- **Not all of the free electrons in a superconductor are in the condensate; those that are in this state are called superconducting electrons, and the others are referred to as normal electrons which are assumed to be few at a low temperature of interest.**

- **The net spin of the Cooper pair is zero and their net linear momentum is also zero. Hence they do not obey the Fermi-Dirac statistics and they can condense to the lowest energy level. They possess one single wave function that can describe the whole collection of Cooper pairs.**

- **Because the superconducting electrons are linked in a macroscopic state, they behave coherently, and a consequence of this is that there is a characteristic distance over which their number density can change, known as the coherence length ξ (the Greek lower-case xi, pronounced 'ksye').**

- It takes a significant amount of energy to scatter an electron from the condensate – more than the thermal energy available to an electron below the critical temperature – so the superconducting electrons can flow without being scattered, that is, without any resistance.

- Thus, The BCS theory successfully explained many of the known properties of superconductors,
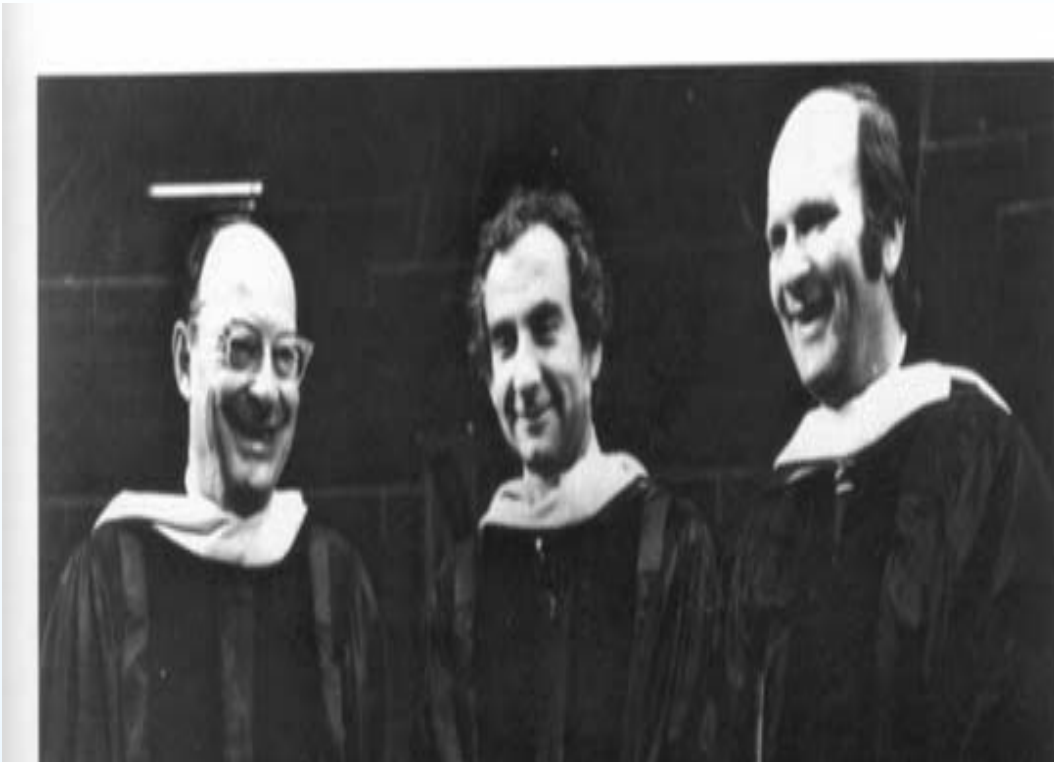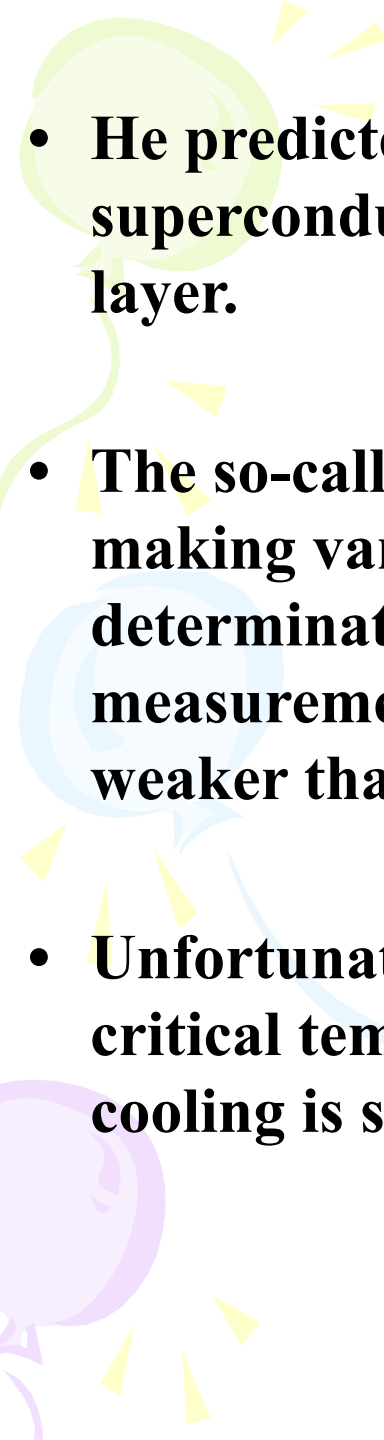

From left ⟶

John Bardeen (Nobel 1956/1972),

Leon Cooper (Nobel 1972) and

John Robert Schrieffer (Nobel 1972)

- By the early 1960s there had been major advances in superconductor technology, with the discovery of alloys that were superconducting at temperatures higher than the critical temperatures of the elemental superconductors.

- In particular, alloys of niobium and titanium (NbTi, T_c = 9.8 K) and niobium and tin (Nb_3Sn, T_c = 18.1 K) were becoming widely used to produce high-field magnets, and a major impetus for this development was the requirement for powerful magnets for particle accelerators.

- At about the same time, Brian Josephson made an important theoretical prediction that was to have major consequences for the application of superconductivity on a very small scale.

- He predicted that a current could flow between two superconductors that were separated by a very thin insulating layer.

- The so-called Josephson tunnelling effect has been widely used for making various sensitive measurements, including the determination of fundamental physical constants and the measurement of magnetic fields that are a billion (10^{-9}) times weaker than the Earth's field.

- Unfortunately, no superconductors have yet been found with critical temperatures above room temperature, so cryogenic cooling is still a vital part of any superconducting application.
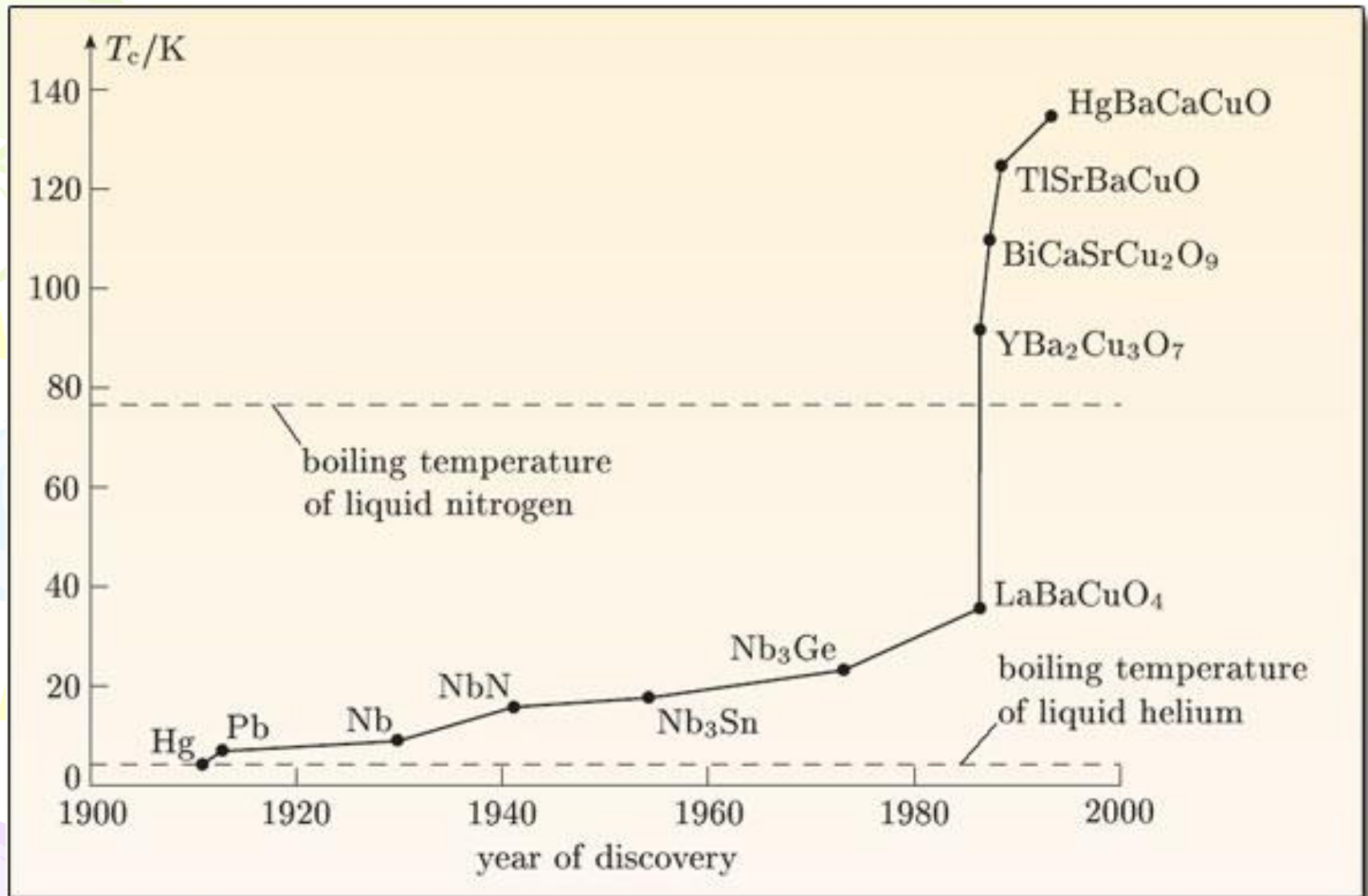
The critical temperature T_c of various superconductors plotted against their discovery date.

Properties of superconductors

a) **Zero electrical resistance** - The most obvious characteristic of a superconductor is the complete disappearance of its electrical resistance below a temperature that is known as its critical temperature.

b) **Persistent currents lead to constant magnetic flux** - An important consequence of the persistent currents that flow in materials with zero resistance is that the magnetic flux that passes through a continuous loop of such a material remains constant.

c) The Meissner effect - when a magnetic field is applied to a sample of tin, say, in the superconducting state, the applied field is excluded, so that $B = 0$ throughout its interior. This property of the superconducting state is known as the **Meissner effect.**

- The exclusion of the magnetic field from a superconductor takes place regardless of whether the sample becomes superconducting before or after the external magnetic field is applied.

- In the steady state, the external magnetic field is cancelled in the interior of the superconductor by opposing magnetic fields produced by a steady screening current that flows on the surface of the superconductor.
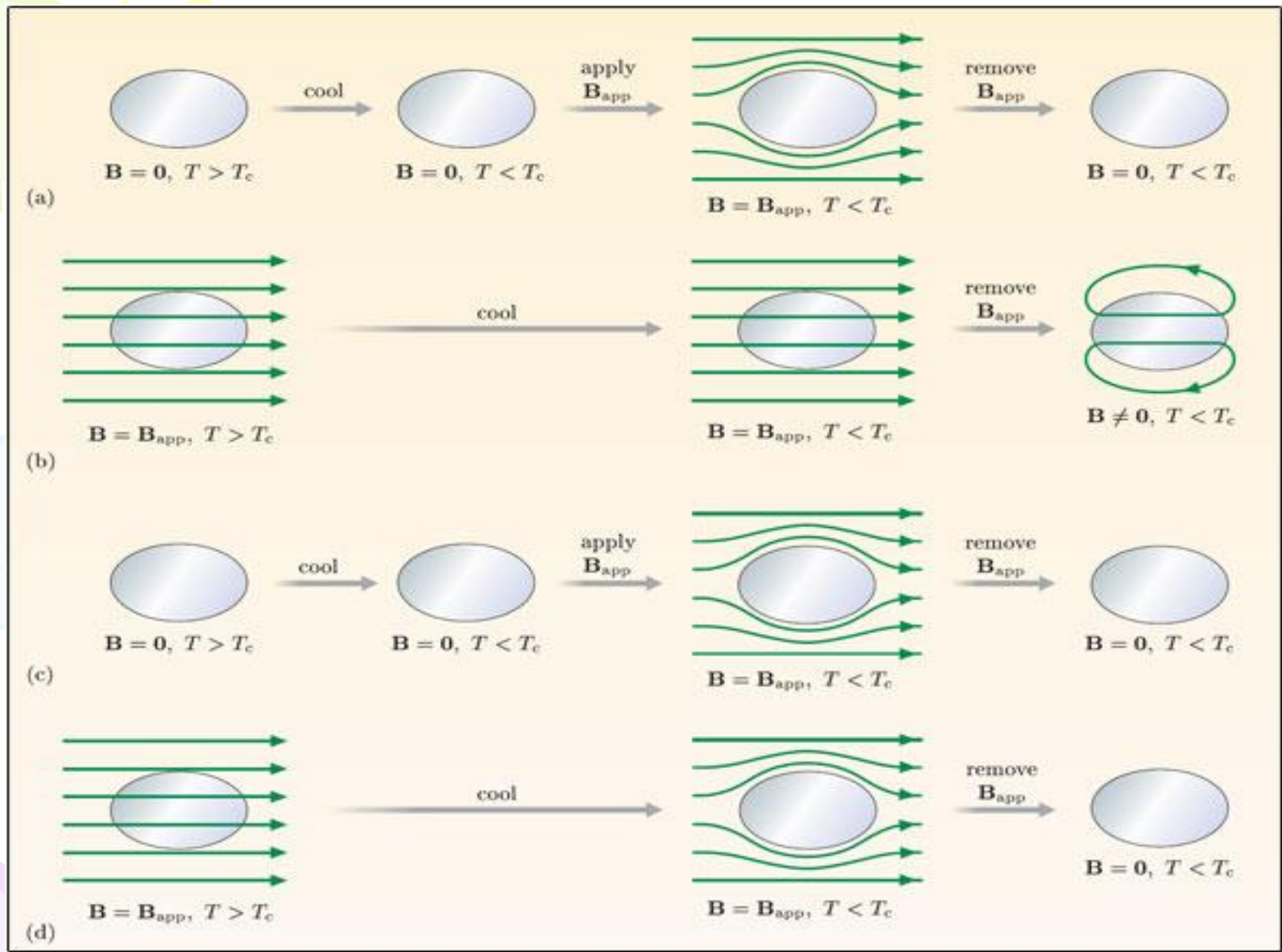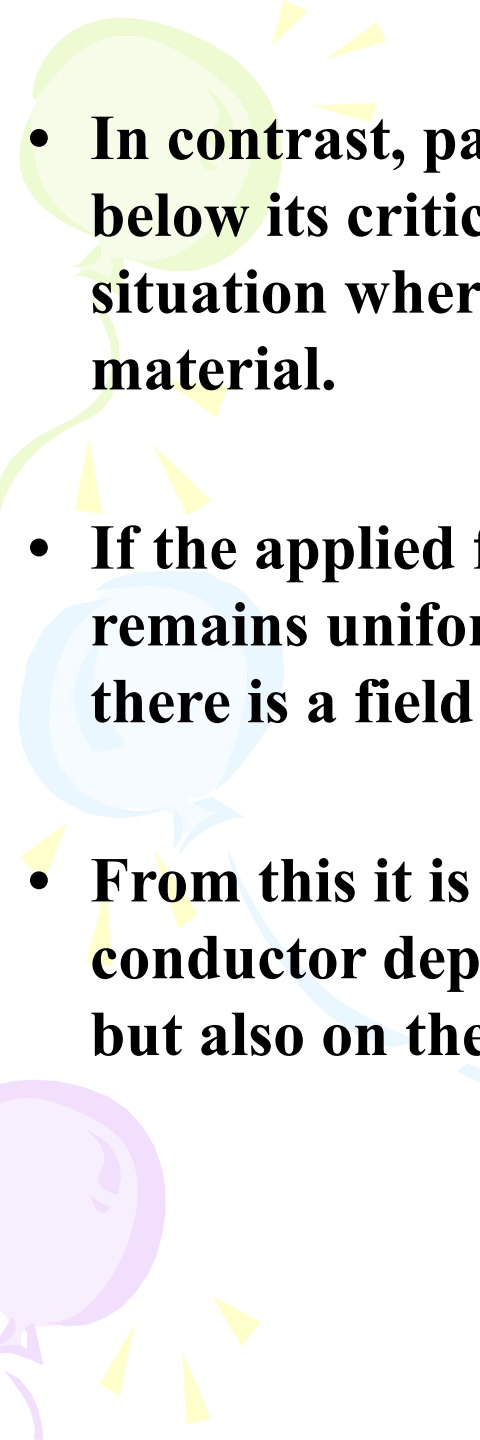
A comparison of the response of a perfect conductor, (a) and (b), and a superconductor, (c) and (d), to an applied magnetic field.
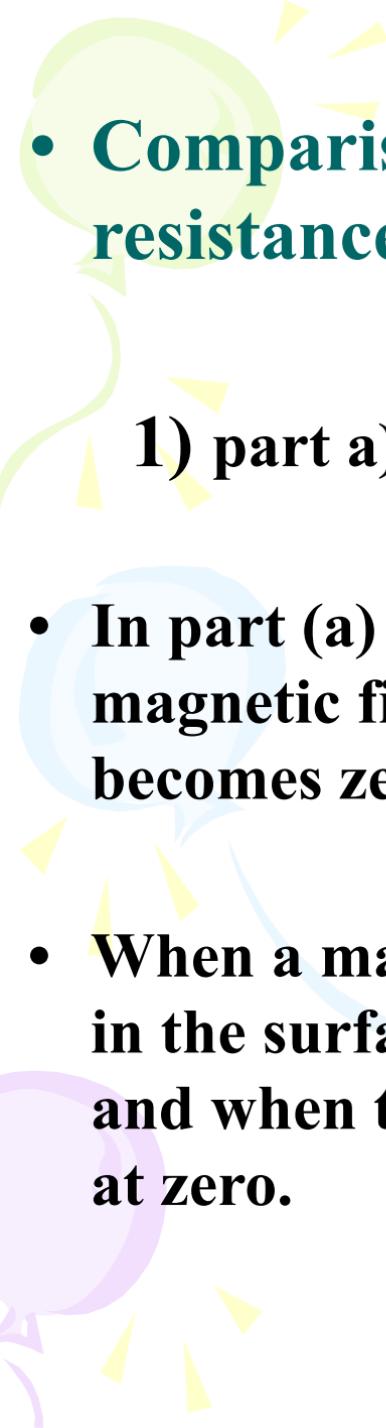
- **Comparison between perfect conductor (zero resistance) and superconductor.**

   1) **part a) and b) of the figure above; (perfect conductor)**

- **In part (a) of this figure, a perfect conductor is cooled in zero magnetic field to below the temperature at which its resistance becomes zero.**

- **When a magnetic field is applied, screening currents are induced in the surface to maintain the field at zero within the material, and when the field is removed, the field within the material stays at zero.**

- **In contrast, part (b) shows that cooling a perfect conductor to below its critical temperature in a uniform magnetic field lead to a situation where the uniform field is maintained within the material.**

- **If the applied field is then removed, the field within the conductor remains uniform, and continuity of magnetic field lines means there is a field in the region around the perfect conductor.**

- **From this it is clear that, the magnetisation state of the perfect conductor depends not just on temperature and magnetic field, but also on the previous history of the material.**

2) part c) and d) (superconductor)

- **Whether a material is cooled below its superconducting critical temperature in zero field, (c), or in a finite field, (d), the magnetic field within a superconducting material is always zero.**

- **The magnetic field is always expelled from a superconductor. This is achieved spontaneously by producing currents on the surface of the superconductor. The direction of the currents is such as to create a magnetic field that exactly cancels the applied field in the superconductor.**

- **It is this active exclusion of magnetic field – the Meissner effect – that distinguishes a superconductor from a perfect conductor, a material that merely has zero resistance. Thus we can regard zero resistance and zero magnetic field as the two key characteristics of superconductivity.**

d) Perfect diamagnetism

- **Diamagnetism is due to currents induced in atomic orbitals by an applied magnetic field. In diamagnetic material, $B = \mu_r \mu_0 H$, with the relative permeability μ slightly less than unity.**

- **Superconductors take the diamagnetic effect to the extreme, since in a superconductor the field B is zero – the field is completely screened from the interior of the material. Thus the relative permeability of a superconductor is zero.**

e) Critical magnetic field

- An important characteristic of a superconductor is that its normal resistance is restored if a sufficiently large magnetic field is applied.

- The nature of this transition to the normal state depends on the shape of the superconductor and the orientation of the magnetic field, and it is also different for pure elements and for alloys.

- If a sufficiently strong magnetic field is applied to a superconductor at any temperature below T_c, it will return to the normal state.

- The field at which superconductivity is destroyed is called critical magnetic field strength (B_c). In some materials if the field is reduced keeping the temperature constant, it returns to superconducting state.

    example; Tin.

- **Experiments indicate that the critical magnetic field strength depends on temperature, and the form of this temperature dependence is shown in the figure below for several elements.**

- **At very low temperatures, the critical field strength is essentially independent of temperature, but as the temperature increases, the critical field strength drops, and becomes zero at the critical temperature.**

- **At temperatures just below the critical temperature it requires only a very small magnetic field to destroy the superconductivity.**

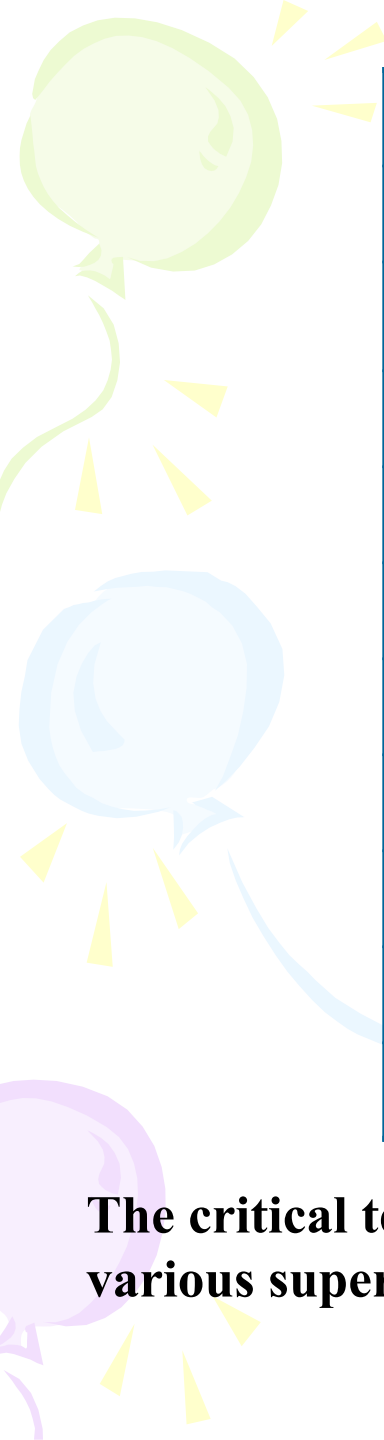- **The temperature dependence of the critical field strength is approximately parabolic:**

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Where, $B_c(0)$ is the extrapolated value of the critical field strength at absolute zero and

T_c is the critical temperature.
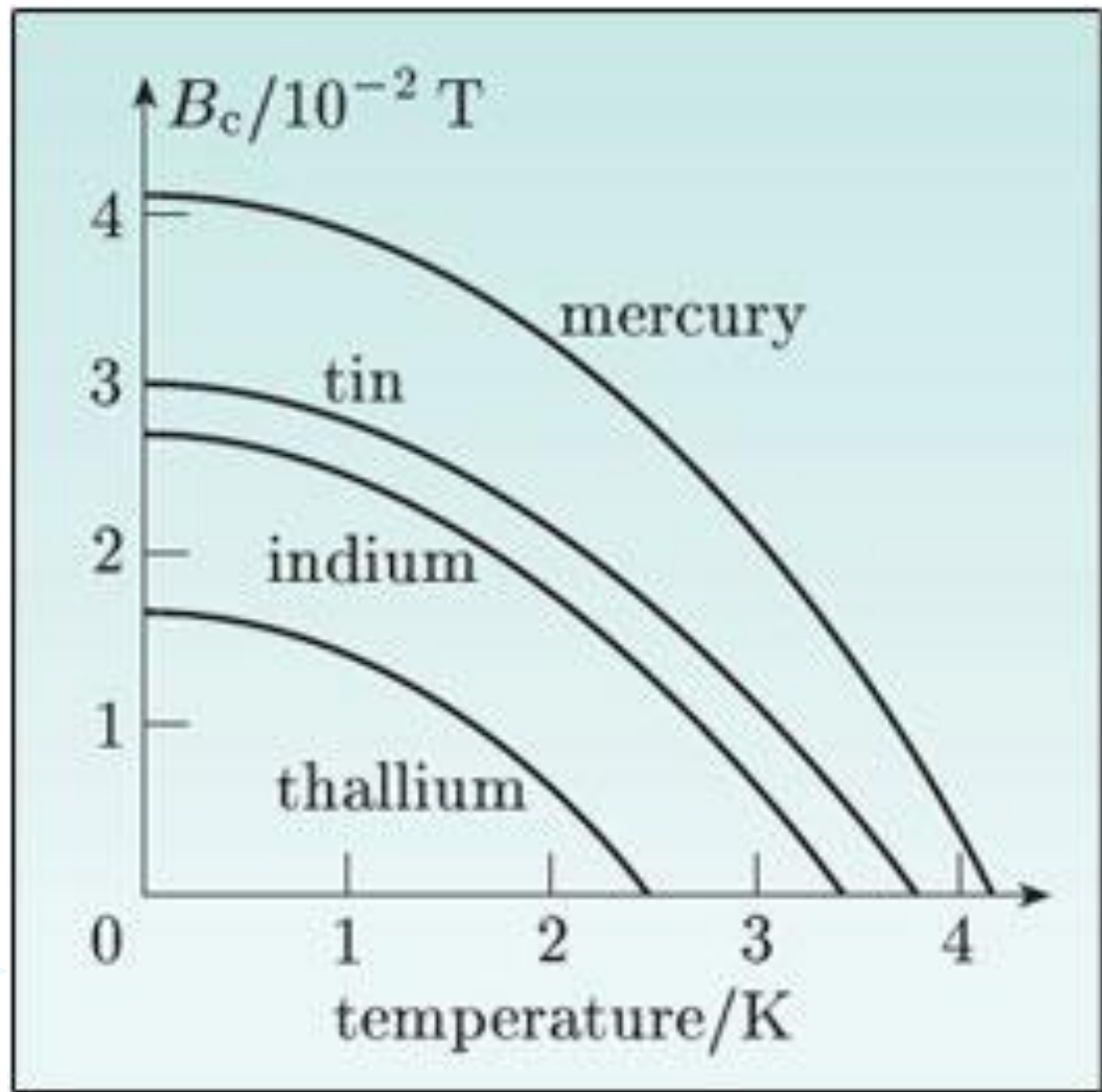
The temperature dependences of the critical magnetic field strengths of some materials
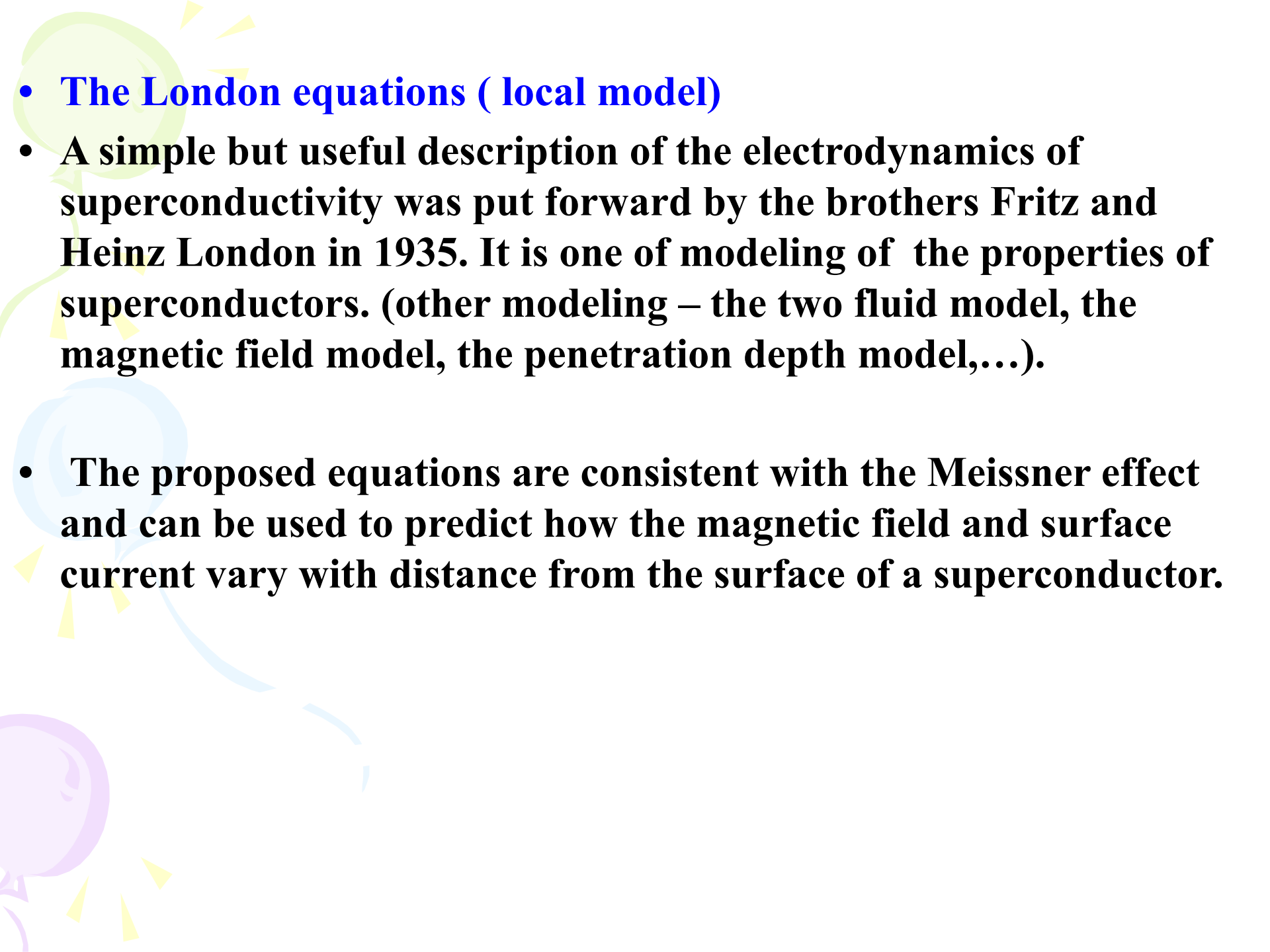
elements	Tc/K	Bc(0)/mT
Aluminium	1.2	10
Cadmium	o.52	2.8
Indium	3.4	28
Lead	7.2	80
Mercury	4.2	41
Titanium	4.5	83
Thalium	2.4	18
Tin	3.7	31
Titanium	0.40	5.6
Zinc	0.85	5.4

The critical temperatures Tc and critical magnetic field strengths Bc(0) for various superconducting elements.

f) Critical current

- The current density for a steady current flowing along a wire in its normal state is essentially uniform over its cross-section. A consequence of this is that the magnetic field strength B within a wire of radius a, carrying current I, increases linearly with distance from the centre of the wire, and reaches a maximum value of $\mu_0 I / 2\Pi a$ at the surface of the wire.

- The magnetic field strength B just outside the surface of the wire is $\mu_0 I / 2\Pi a$. It follows that, if the current flowing in a superconducting wire is increased, eventually the field strength at the surface of the wire will exceed B_c and the sample will revert to its normal state. The maximum current that a wire can carry with zero resistance is known as its critical current.

- **The London equations (local model)**
- **A simple but useful description of the electrodynamics of superconductivity was put forward by the brothers Fritz and Heinz London in 1935. It is one of modeling of the properties of superconductors. (other modeling – the two fluid model, the magnetic field model, the penetration depth model,…).**

- **The proposed equations are consistent with the Meissner effect and can be used to predict how the magnetic field and surface current vary with distance from the surface of a superconductor.**

. **London brothers proposed the following equation which relates current and magnetic field in a superconductor.**

$$\frac{\partial J_s}{\partial t} = \frac{n_s e^2}{m} E.$$

$$\text{curl } J_s = -\frac{n_s e^2}{m} B.$$

They were introduced as a restriction on Maxwell's equations so that the behavior of superconductors deduced from the equations was consistent with experimental observations, and in particular with the Meissner effect.

The London equations lead to the prediction of an exponential decay of the magnetic field within the superconductor.

- **Penetration depth**
- The characteristic length, λ, associated with the decay of the magnetic field at the surface of a superconductor is known as the **penetration depth**, and it depends on the number density n_s of superconducting electrons.
- We can estimate a value for λ by assuming that all of the free electrons are superconducting. If we set $n_s = 10^{29}\ \text{m}^{-3}$, a typical free electron density in a metal, then we find that

$$\lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{1/2} = 1.7 \times 10^{-8}\ \text{m} \simeq 20\ \text{nm}.$$

The small size of λ indicates that the magnetic field is effectively excluded from the interior of macroscopic specimens of superconductors, in agreement with the experimentally observed Meissner effect.

$$B_x(z) = B_0e^{-z/\lambda}$$
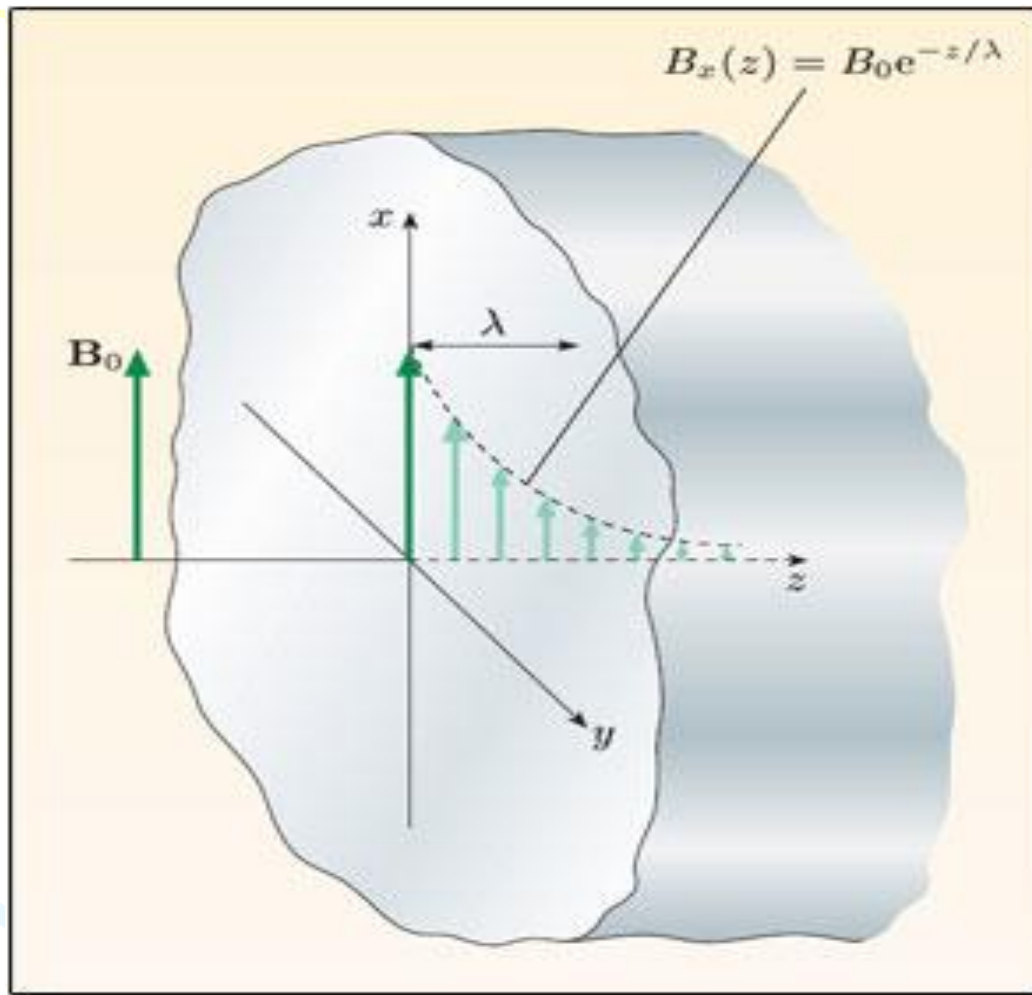
The penetration of a magnetic field into a superconducting material, showing the penetration depth.

- **The number density of superconducting electrons depends on temperature, so the penetration depth is temperature dependent. For $T \ll T_c$, all of the free electrons are superconducting, but the number density falls steadily with increasing temperature until it reaches zero at the critical temperature.**

- **Since $\lambda \propto n_s^{-1/2}$ according to the London model, the penetration depth increases as the temperature approaches the critical temperature, becoming effectively infinite – corresponding to a uniform field in the material – at and above the critical temperature. The following figure shows this temperature dependence for tin, which is well represented by the expression**

$$\lambda(T) = \frac{\lambda(0)}{\left[1 - (T/T_c)^4\right]^{1/2}},$$

where λ(0) is the value of the penetration depth at $T = 0$ K.
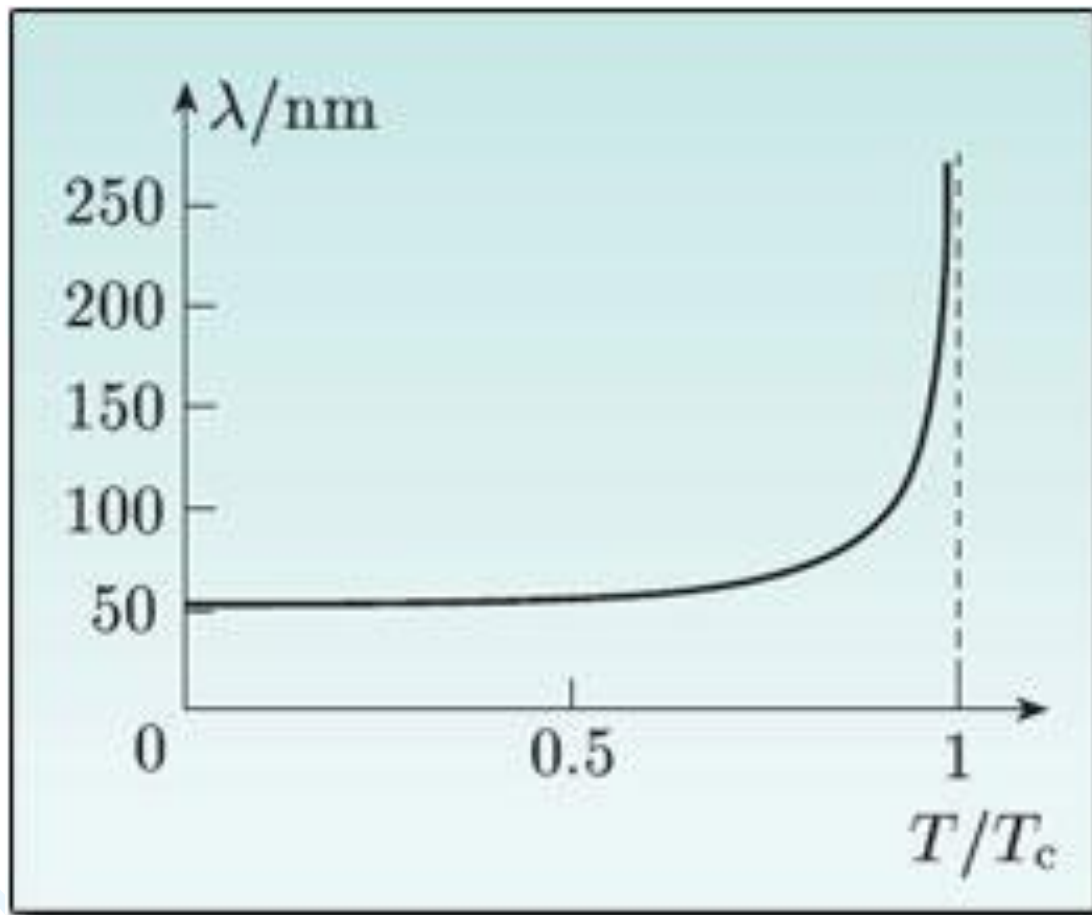
**The penetration depth λ as a function of temperature for tin.**

- **Classifications of superconductors.**

- Based on their behaviour in an applied field, superconductors are classified into two types.

- Type I - At room temperature, in this type superconductors, the applied magnetic field penetrates the sample. However, if the temperature is below T_c and the magnetic field is below Bc, then it expels the magnetic field and behaves like diamagnetic material .

    example: - Lead(Pb), Tin (Sn)

- Type II - This type elements (like niobium, vanadium and technetium) and alloys are highly diamagnetic like type I up to a critical applied magnetic field B_{c1}. Above B_{c1}, the field starts to penetrate and continues to do so until the upper critical field B_{c2} is reached.

- **For simplicity, let us consider a long cylindrical specimen of type-II material, and apply a field parallel to its axis.**

- **In between Bc1 and B$_{c2}$, the superconductor is in a mixed state, and above B$_{c2}$ it returns to the normal conducting state.**
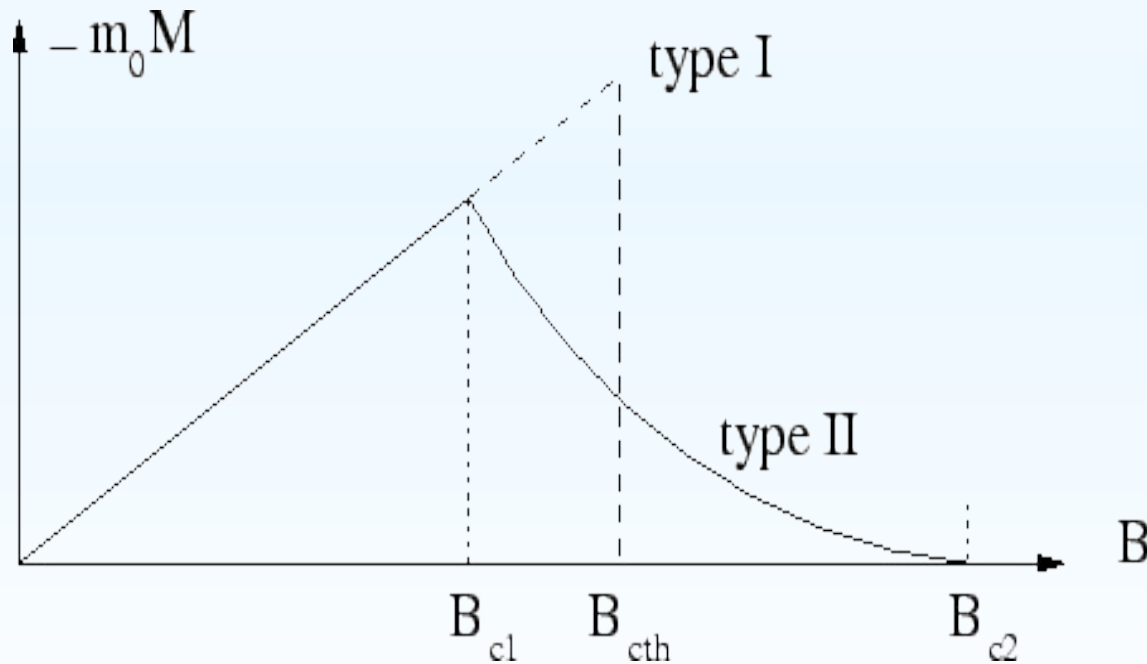
The dependence of the magnetisation of a type II superconductor as function of the applied magnetic field.

- Below a certain critical field strength, known as the lower critical field strength and denoted by the symbol B_{c1}, the applied magnetic field is excluded from the bulk of the material, penetrating into only a thin layer at the surface, just as for type-I materials.

- But above B_{c1}, the material does not make a sudden transition to the normal state. Instead, very thin cylindrical regions of normal material appear, passing through the specimen parallel to its axis. We shall refer to such a normal region as a normal core.

- The normal cores are arranged on a triangular lattice, and as the applied field is increased, more normal cores appear and they become more and more closely packed together.

- **Eventually, a second critical field strength, the upper critical field strength B_{c2}, is reached, above which the material reverts to the normal state.**

- **The state that exists between the lower and upper critical field strengths, in which a type-II superconductor is threaded by normal cores, is known as the mixed state.**

- **Both the upper and lower critical field strengths depend on temperature in a similar way to the critical field strength for a type-I material.**
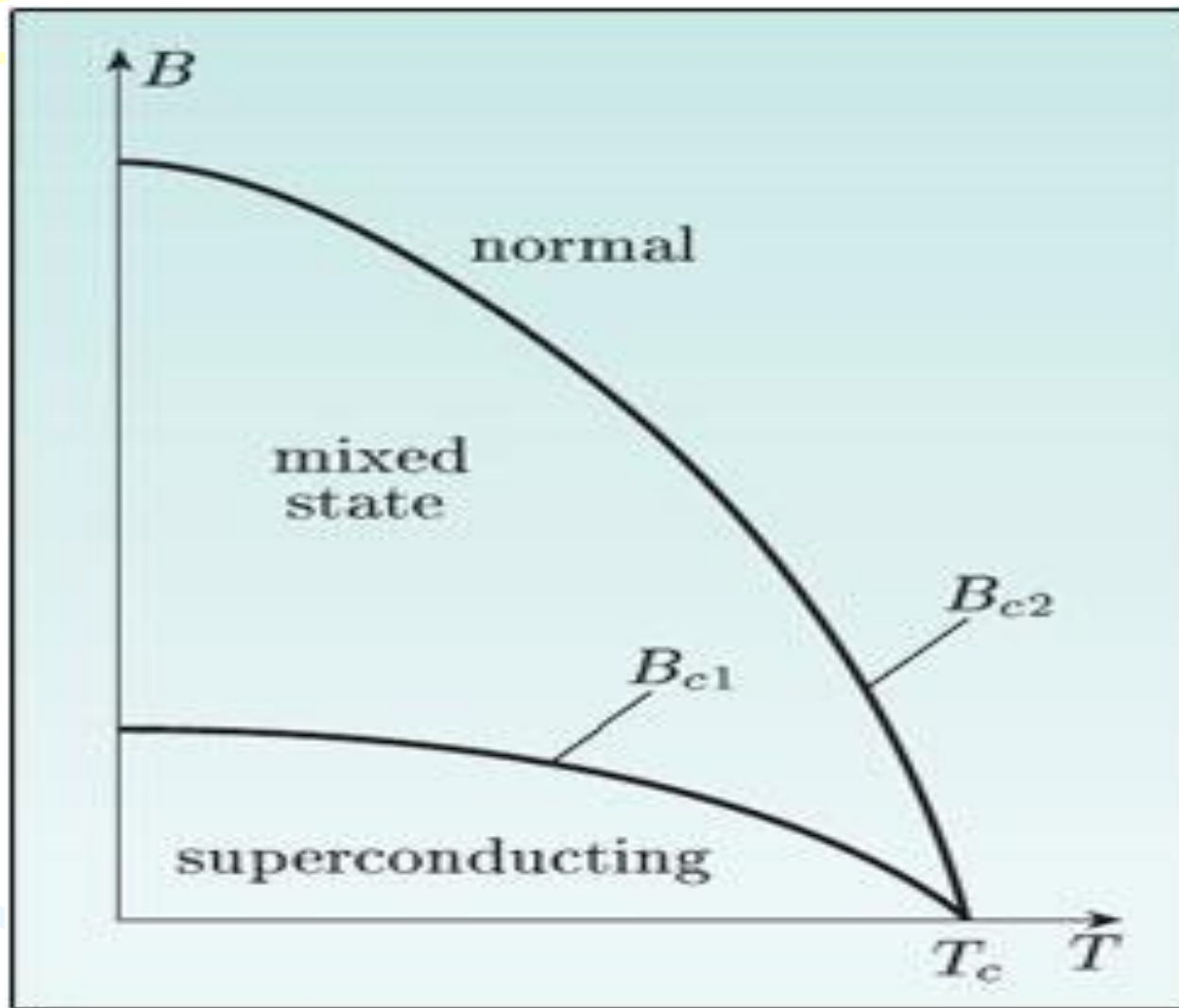
Temperature dependence of the lower critical field strength (B_{c1}) and upper critical field strength (B_{c2}) for a type-II superconductor.

- **The normal cores that exist in type-II superconductors in the mixed state are not sharply delineated.**

- **The following figure shows how the number density of super electrons and the magnetic field strength vary along a line passing through the axes of three neighboring cores.**

- **The value of n_s is zero at the centers of the cores and rises over a characteristic distance ξ, the coherence length.**

- **The magnetic field associated with each normal core is spread over a region with a diameter of 2λ, and each normal core is surrounded by a vortex of circulating current.**
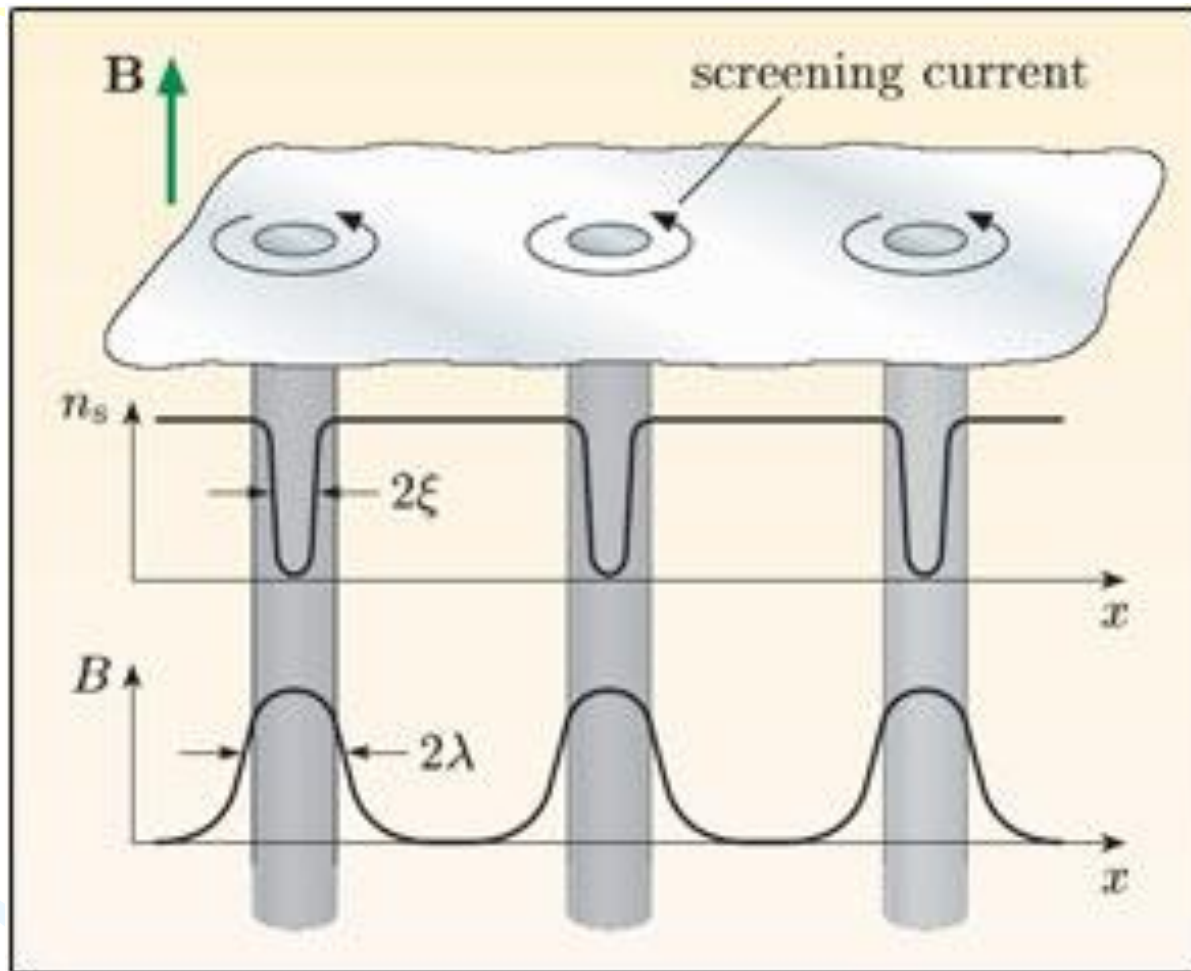
Number density of super electrons n_s and magnetic field strength B around normal cores in a type-II superconductor.

- You can see from the figure that the coherence length ξ, the characteristic distance for changes in n_s, is shorter than the penetration depth λ, the characteristic distance for changes in the magnetic field in a superconductor.

- This is generally true for type-II superconductors, whereas for type-I superconductors, $\xi > \lambda$. For a pure type-I superconductor, typical values of the characteristic lengths are $\xi \sim 1\ \mu$m and $\lambda = 50$ nm. Contrast this with the values for a widely-used type-II alloy of niobium and tin, Nb_3Sn, for which $\xi \sim 3.5$ nm and $\lambda = 80$ nm.

- The reason that the relative magnitude of the coherence length and the penetration depth is so important is that when $\xi > \lambda$, the surface energy associated with the boundary between superconducting and normal regions is positive, whereas when $\xi < \lambda$, this surface energy is negative.

- **Critical currents in type-II superconductors**

- The high values of the upper critical field strength Bc_2 of many type-II superconducting alloys make them very attractive for winding coils for generating high magnetic fields.

- For example, alloys of niobium and titanium ($NbTi_2$) and of niobium and tin (Nb_3Sn) have values of Bc_2 of about 10 T and 20 T, respectively, compared with 0.08 T for lead, a type-I superconductor.

- However, for type-II materials to be usable for this purpose, they must also have high critical currents at high field strengths, and this requires some help from metallurgists to overcome a significant problem.

-

- **This problem is related to the interaction between the current flowing through a type-II superconductor in the mixed state and the 'tubes' of magnetic flux that thread through the normal cores.**

- **The electrons will experience a Lorentz force, perpendicular to both the current density and the magnetic field.**

- **We can regard this as a mutual interaction between the electrons and the flux in the normal cores, as a result of which each normal core experiences a force that is in the opposite direction to the Lorentz force on the electrons.**
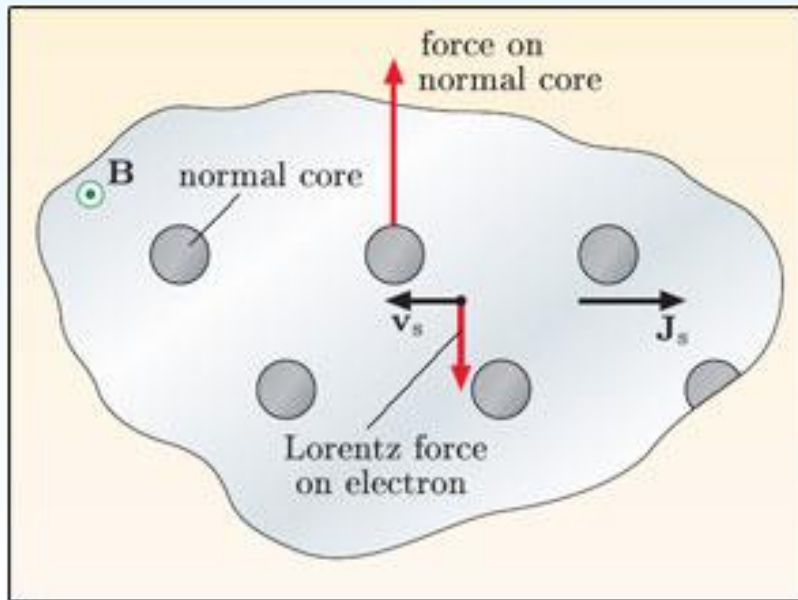
Electrons and normal cores experience forces perpendicular to the current and to the magnetic field, but in opposite directions.

- **This Lorentz force can cause the cores and their associated magnetic flux to move, and the flux motion will induce an emf that drives a current through the normal cores, somewhat like an eddy current.**

- **Energy is therefore dissipated in the normal cores, and this energy must come from the power supply. The energy dissipation means that the flow of electrons is impeded, and therefore there is a resistance to the flow of the current.**

- **Flux motion is therefore undesirable in type-II superconductors, and the aim of the metallurgists who develop processes for manufacturing wire for magnets is to make flux motion as difficult as possible.**

- This is done by introducing defects into the crystalline structure, particularly by preparing the material in such a way that it comprises many small crystalline grains with different orientations and small precipitates of different composition.

- These defects effectively pin the normal cores in position – they provide a potential barrier to motion of the cores, so that the force on the cores must exceed a certain value before the cores can move.

- The more of these flux pinning centres that are present, and the greater the potential barrier they provide, the greater will be the current required to set them in motion, i.e. the greater the critical current.

- So, unlike a normal conductor, for which improving the purity and reducing imperfections in the crystal structure lead to better conductivity, with type-II superconductors the inclusion of impurities and defects in the crystal structure can improve the critical current and make the material more suitable for use in electromagnets.
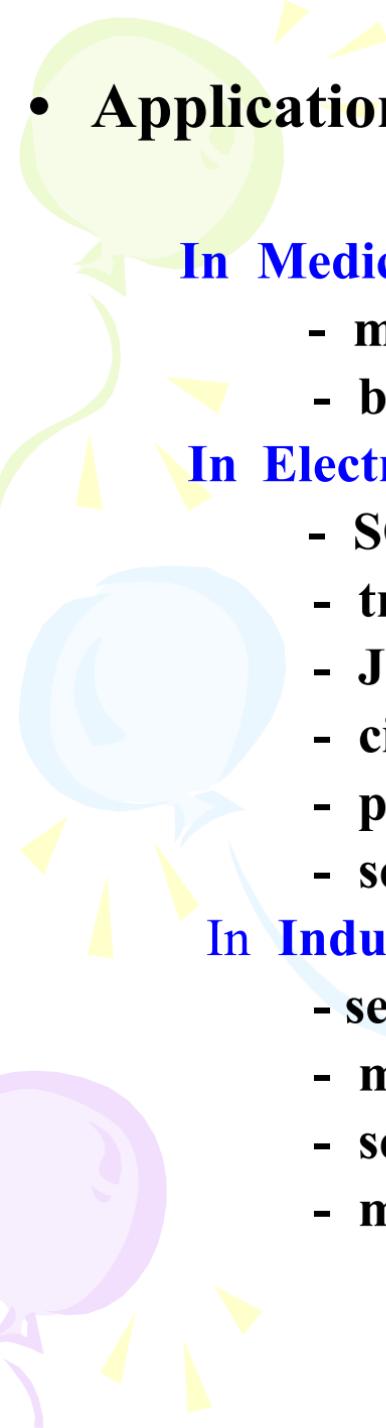
- **Application areas of superconductors**

    In Medical areas
        - **magnetic resonance imaging**
        - **biotechnical engineering.**
    In Electronics
        - **SQUIDs**
        - **transistors**
        - **Josephson Junction devices**
        - **circuitry connections**
        - **particle accelerators**
        - **sensors**
    In Industrial
        - **separation**
        - **magnets**
        - **sensors and**
        - **magnetic shielding**
            e.t.c.

Generic MRI Scanner

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THANK YOU !!