

Решение уравнения $\cos x = a$.

Понятие арккосинуса числа.

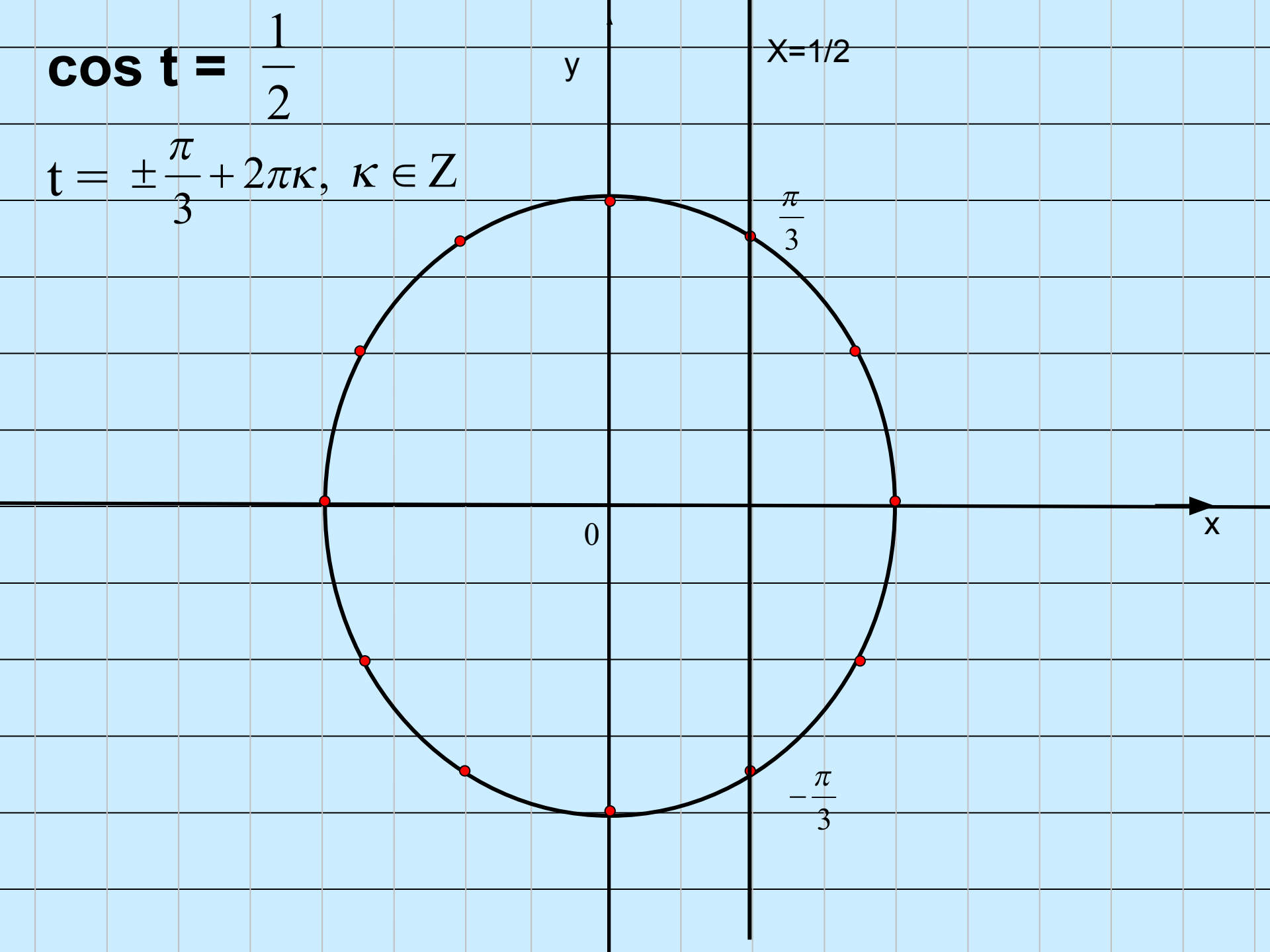
Решить уравнения:

$$1) \cos t = \frac{1}{2};$$

$$2) \cos t = 1.$$

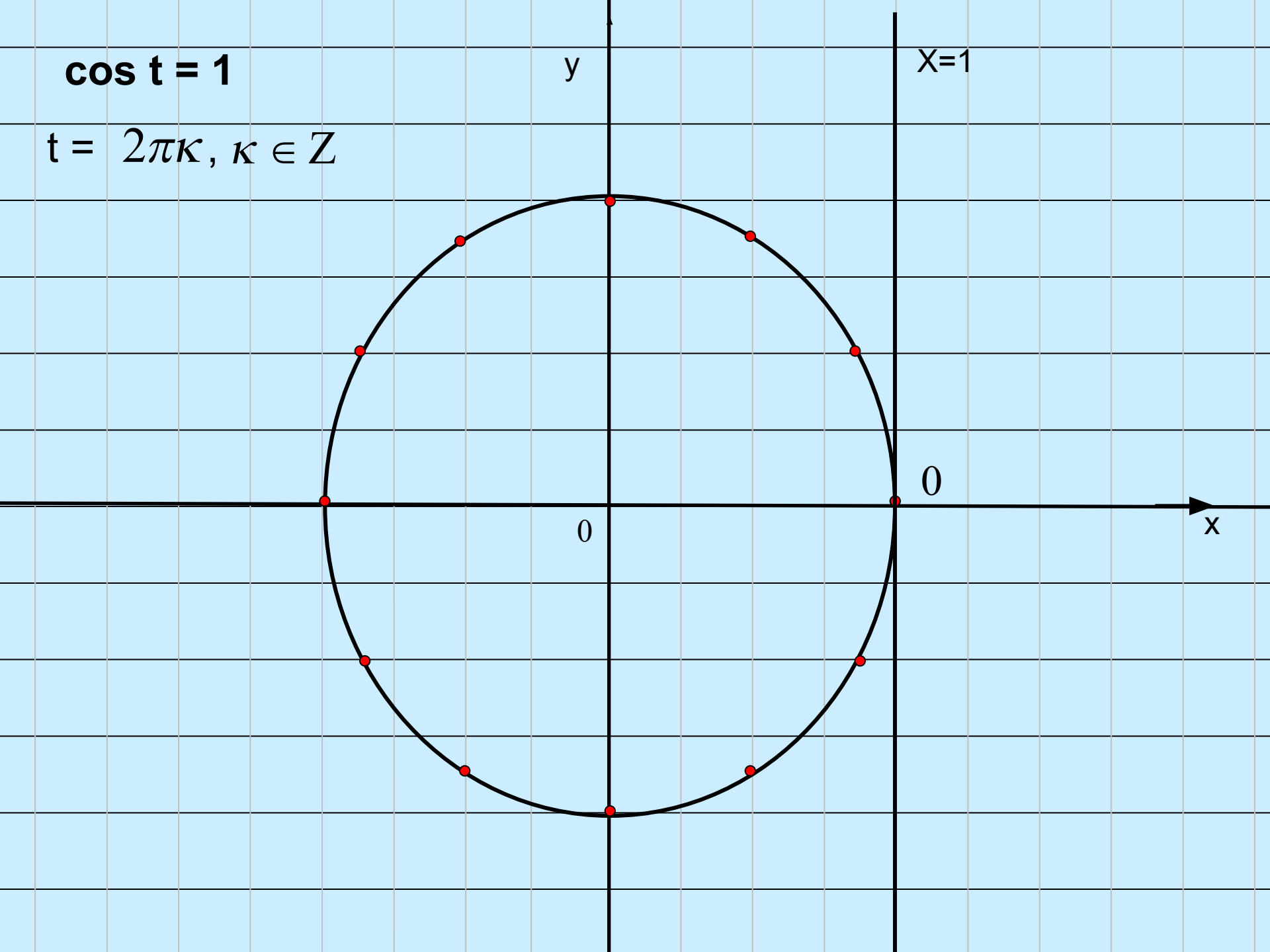
$$\cos t = \frac{1}{2}$$

$$t = \pm \frac{\pi}{3} + 2\pi k, \quad k \in \mathbb{Z}$$



$$\cos t = 1$$

$$t = 2\pi k, k \in \mathbb{Z}$$

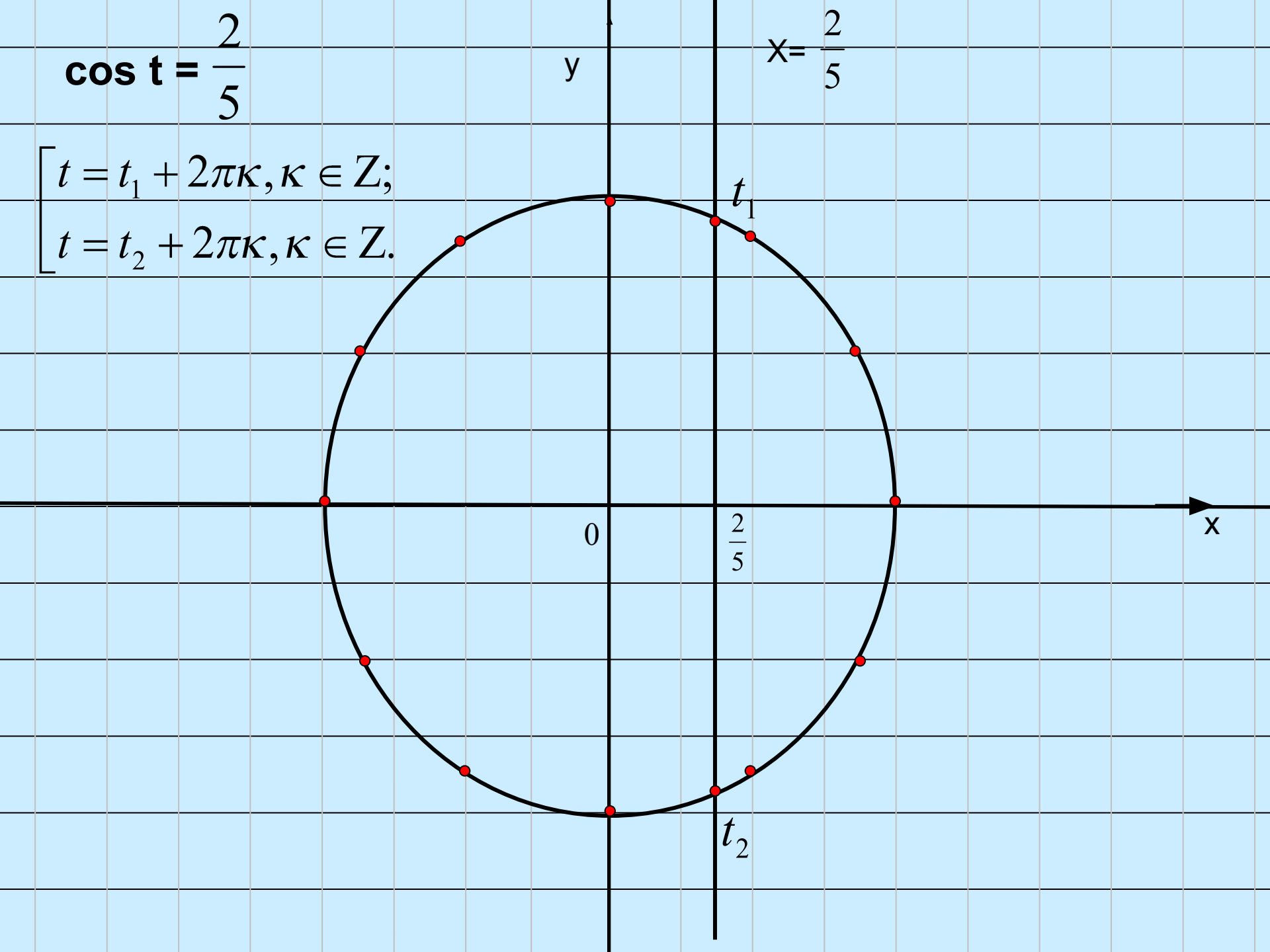


Решить уравнение:

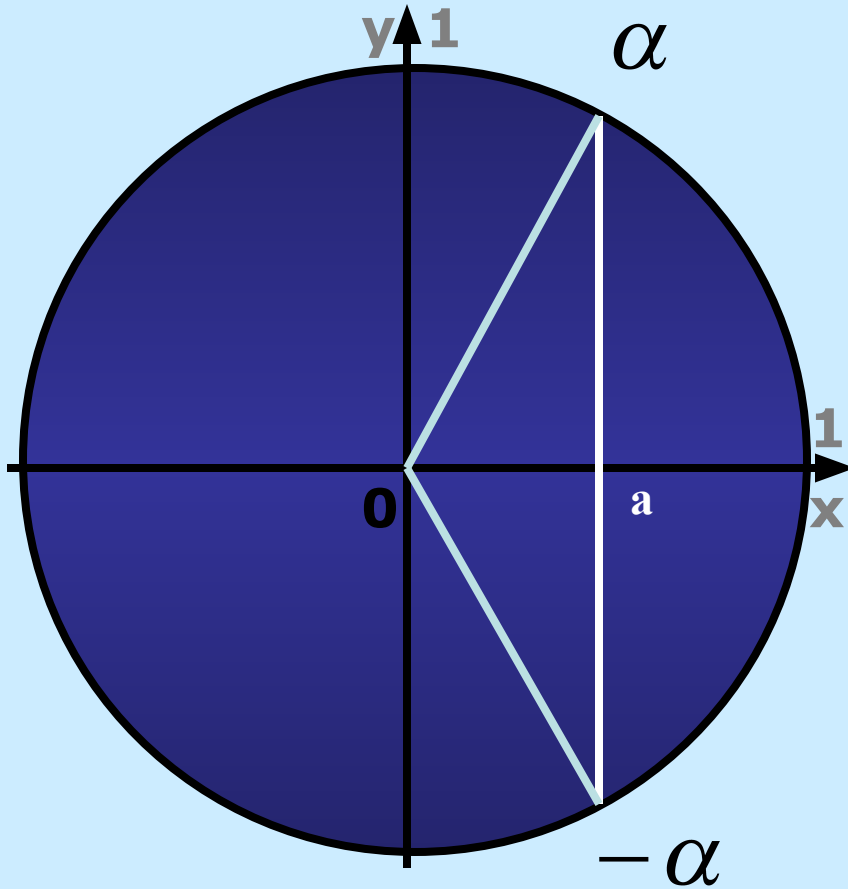
$$\cos t = \frac{2}{5}.$$

$$\cos t = \frac{2}{5}$$

$$\begin{cases} t = t_1 + 2\pi\kappa, \kappa \in \mathbb{Z}; \\ t = t_2 + 2\pi\kappa, \kappa \in \mathbb{Z}. \end{cases}$$



Уравнение $\cos x = a$



$$\cos x = a; |a| \leq 1.$$

$$x = \alpha + 2\pi k;$$

$$x = -\alpha + 2\pi k;$$

$$k \in \mathbb{Z}.$$

$$\alpha = \arccos a$$

АРККОСИНУС ЧИСЛА

Определение. Арккосинусом числа $a \in [-1;1]$ называется такое число $\alpha \in [0;\pi]$, косинус которого **равен** a .

$$\cos(\arccos a) = a,$$

$$0 \leq \arccos a \leq \pi,$$

$$-1 \leq a \leq 1$$

АРККОСИНУС ЧИСЛА

- **Например**

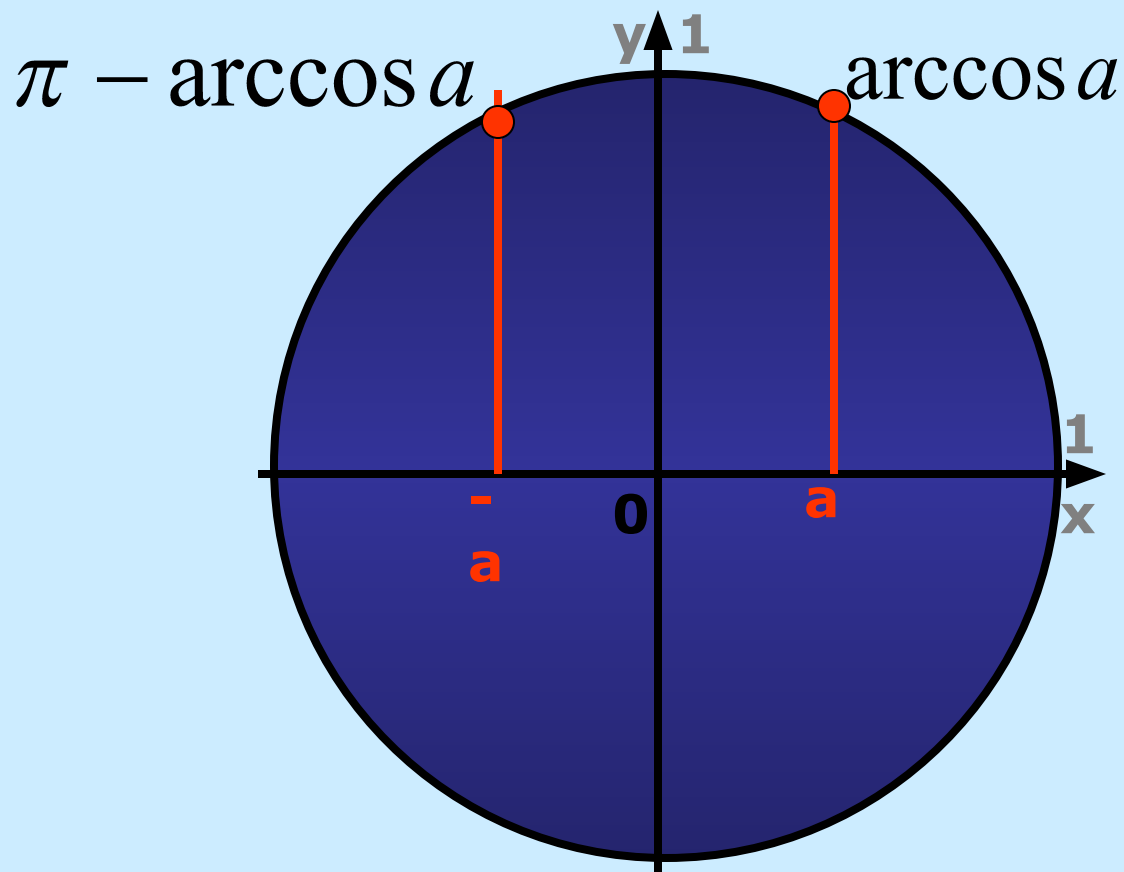
$$\arccos \frac{\sqrt{2}}{2} = \frac{\pi}{4}; \quad \begin{array}{l} \text{т.} \\ \text{к.} \end{array} \quad 0 \leq \frac{\pi}{4} \leq \pi; \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}.$$

$$\arccos 0 = \frac{\pi}{2} \quad \begin{array}{l} \text{т.} \\ \text{к.} \end{array} \quad 0 \leq \frac{\pi}{2} \leq \pi; \cos \frac{\pi}{2} = 0.$$

$$\arccos \frac{\sqrt{3}}{2} = \frac{\pi}{6}; \quad \begin{array}{l} \text{т.} \\ \text{к.} \end{array} \quad 0 \leq \frac{\pi}{6} \leq \pi; \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}.$$

АРККОСИНУС ЧИСЛА ОСНОВНЫЕ ФОРМУЛЫ

$$\arccos(-a) = \pi - \arccos a$$



АРККОСИНУС ЧИСЛА ОСНОВНЫЕ ФОРМУЛЫ

• Например

$$1. \quad 3 \arccos \frac{\sqrt{2}}{2} - 2 \arccos \left(-\frac{1}{2} \right) =$$

$$3 \arccos \frac{\sqrt{2}}{2} - 2 \left(\pi - \arccos \frac{1}{2} \right) = 3 \cdot \frac{\pi}{4} - 2 \cdot \left(\pi - \frac{\pi}{3} \right) =$$

$$\frac{3\pi}{4} - \frac{4\pi}{3} = -\frac{7}{12}\pi$$

$$2. \quad \frac{1}{2} \arccos \left(-\frac{\sqrt{3}}{2} \right) - 2 \arccos(-1) + \frac{1}{3} \arccos 0 =$$

$$\frac{1}{2} \left(\pi - \arccos \frac{\sqrt{3}}{2} \right) - 2 \cdot \pi + \frac{1}{3} \cdot \frac{\pi}{2} = \frac{1}{2} \cdot \frac{5\pi}{6} - 2\pi + \frac{\pi}{6} = -\frac{17}{12}\pi$$

АРККОСИНУС ЧИСЛА ОСНОВНЫЕ ФОРМУЛЫ

$$\sin \alpha = \pm \sqrt{1 - \cos^2 \alpha}$$

$$\alpha = \arccos a$$

$$\arccos a \in [0; \pi]$$

$$\sin(\arccos a) = \sqrt{1 - \cos^2(\arccos a)} = \sqrt{1 - a^2}$$

$$\operatorname{tg}(\arccos a) = \frac{\sin(\arccos a)}{\cos(\arccos a)} = \frac{\sqrt{1 - a^2}}{a}$$

АРККОСИНУС ЧИСЛА ОСНОВНЫЕ ФОРМУЛЫ

$$\cos(\arccos a) = a, \arccos a \in [0; \pi], a \in [-1; 1]$$

$$\arccos(-a) = \pi - \arccos a$$

$$\arccos(\cos \alpha) = \alpha, \alpha \in [0; \pi]$$

$$\sin(\arccos a) = \sqrt{1 - a^2}$$

$$\operatorname{tg}(\arccos a) = \frac{\sqrt{1 - a^2}}{a}$$

АРККОСИНУС ЧИСЛА ОСНОВНЫЕ ФОРМУЛЫ

$$\cos(\arccos a) = a$$

• Например

$$1. \quad \cos\left(\arccos\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$$

$$2. \quad \cos\left(\arccos\frac{5}{9}\right) = \frac{5}{9}$$

$$3. \quad \sin\left(\arccos\frac{\sqrt{3}}{2}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

$$4. \quad \sin\left(\arccos\left(-\frac{3}{7}\right)\right) = \sqrt{1 - \left(-\frac{3}{7}\right)^2} = \sqrt{1 - \frac{9}{49}} =$$

$$\sqrt{\frac{40}{49}} = \frac{2\sqrt{10}}{7}$$

$$\sin(\arccos a) = \sqrt{1 - a^2}$$

АРККОСИНУС ЧИСЛА ОСНОВНЫЕ ФОРМУЛЫ

- **Например**

5.
$$\operatorname{ctg}\left(7 \cdot \arccos\left(-\frac{\sqrt{2}}{2}\right)\right) = \operatorname{ctg}\left(7 \cdot \left(\pi - \frac{\pi}{4}\right)\right) = \operatorname{ctg}\frac{21}{4}\pi =$$
$$\operatorname{ctg}5\frac{1}{4}\pi = \operatorname{ctg}\frac{\pi}{4} = 1$$

6.
$$\operatorname{tg}\left(\arccos\frac{1}{4}\right) = \frac{\sin\left(\arccos\frac{1}{4}\right)}{\cos\left(\arccos\frac{1}{4}\right)} = \frac{\sqrt{1 - \frac{1}{16}}}{\frac{1}{4}} = \sqrt{\frac{15}{16}} \cdot 4 = \sqrt{15}$$

АРККОСИНУС ЧИСЛА ОСНОВНЫЕ ФОРМУЛЫ

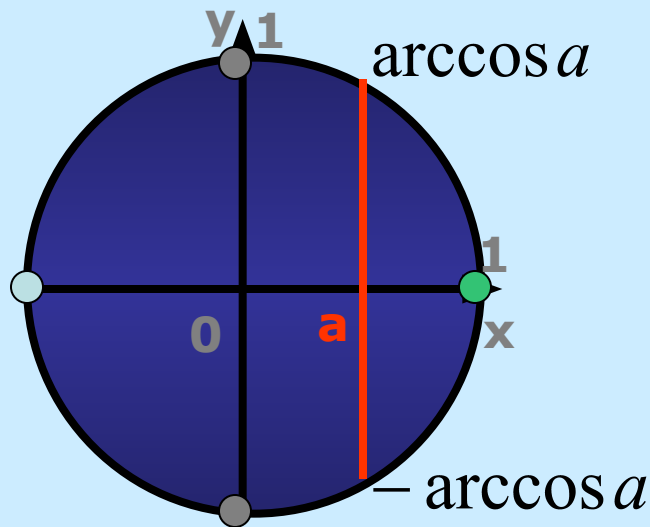
$$\arccos(\cos \alpha) = \alpha, \alpha \in [0; \pi]$$

• Например

$$7. \arccos\left(\cos \frac{2\pi}{5}\right) = \frac{2\pi}{5}$$

$$8. \arccos\left(\cos \frac{8}{5}\pi\right) = \arccos\left(\cos\left(\pi + \frac{3}{5}\pi\right)\right) =$$
$$\arccos\left(-\cos \frac{3}{5}\pi\right) = \pi - \arccos\left(\cos \frac{3}{5}\pi\right) = \pi - \frac{3}{5}\pi =$$
$$= \frac{2}{5}\pi$$

Уравнение $\cos x = a$



$$\cos x = a, |a| \leq 1$$

$$x = \pm \arccos a + 2\pi n, n \in \mathbb{Z}$$

$$\cos x = 0$$

$$x = \frac{\pi}{2} + \pi k, k \in \mathbb{Z}$$

$$\cos x = 1$$

$$x = 2\pi k, k \in \mathbb{Z}$$

$$\cos x = -1$$

$$x = \pi + 2\pi k, k \in \mathbb{Z}$$

Уравнение $\cos x = a$

- **Пример 1.**

$$\cos 3x = \frac{1}{2};$$

$$3x = \pm \arccos \frac{1}{2} + 2\pi k;$$

$$3x = \pm \frac{\pi}{3} + 2\pi k;$$

$$x = \pm \frac{\pi}{9} + \frac{2\pi k}{3}, k \in \mathbb{Z}.$$

Уравнение $\cos x = a$

Пример 2.

$$\cos \frac{x}{4} = -\frac{\sqrt{3}}{2};$$

$$\frac{x}{4} = \pm \arccos\left(-\frac{\sqrt{3}}{2}\right) + 2\pi k;$$

$$\frac{x}{4} = \pm \left(\pi - \arccos \frac{\sqrt{3}}{2} \right) + 2\pi k;$$

$$\frac{x}{4} = \pm \left(\pi - \frac{\pi}{6} \right) + 2\pi k;$$

$$\frac{x}{4} = \pm \frac{5}{6} \pi + 2\pi k;$$

$$x = \pm \frac{10}{3} \pi + 8\pi k, k \in \mathbb{Z}.$$

Уравнение $\cos x = a$

- **Пример 3.**

$$\cos\left(x - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$x - \frac{\pi}{4} = \pm \arccos\left(-\frac{1}{\sqrt{2}}\right) + 2\pi k;$$

$$x - \frac{\pi}{4} = \pm\left(\pi - \arccos\frac{1}{\sqrt{2}}\right) + 2\pi k;$$

$$x = \pm\left(\pi - \frac{\pi}{4}\right) + \frac{\pi}{4} + 2\pi k;$$

$$x = \pm\frac{3}{4}\pi + \frac{\pi}{4} + 2\pi k;$$

$$\begin{cases} x = \pi + 2\pi k \\ x = -\frac{\pi}{2} + 2\pi k \end{cases}, k \in \mathbb{Z}.$$

Уравнение $\cos x = a$

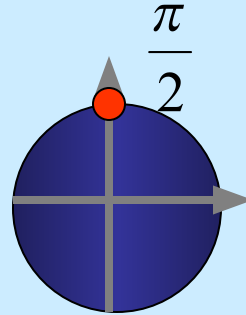
• **Пример 4.** $(3 \sin 2x - 3) \cdot (2 \cos x + 1) = 0$

$$3 \sin 2x - 3 = 0;$$

$$\sin 2x = 1;$$

$$2x = \frac{\pi}{2} + 2\pi k;$$

$$x = \frac{\pi}{4} + \pi k, k \in \mathbb{Z}.$$



$$2 \cos x + 1 = 0;$$

$$\cos x = -\frac{1}{2};$$

$$x = \pm \arccos\left(-\frac{1}{2}\right) + 2\pi n;$$

$$x = \pm\left(\pi - \frac{\pi}{3}\right) + 2\pi n;$$

$$x = \pm\frac{2}{3}\pi + 2\pi n, n \in \mathbb{Z}.$$

Уравнение $\cos x = a$

• **Пример 5.** $\cos 5x + \cos 7x = \cos(\pi + 6x)$

$$\cos 5x + \cos 7x = -\cos 6x$$

$$2 \cos 6x \cdot \cos x + \cos 6x = 0$$

$$\cos 6x \cdot (2 \cos x + 1) = 0$$

$$\cos 6x = 0$$

$$2 \cos x + 1 = 0$$

$$\cos x = -\frac{1}{2}$$