Кафедра аеронавігаційних систем

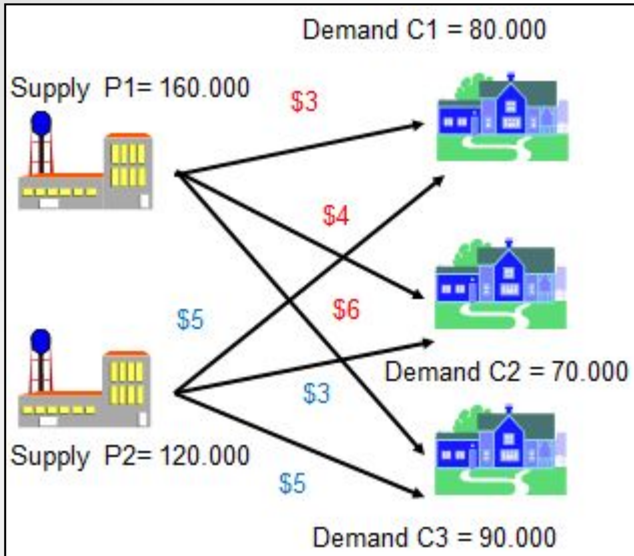
National Aviation University
Department of Airnavigation system

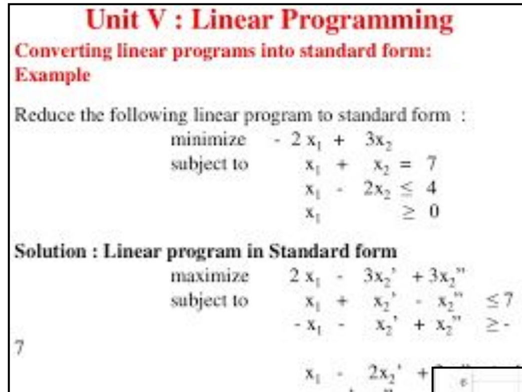
Lecture 5: The Linear Programming

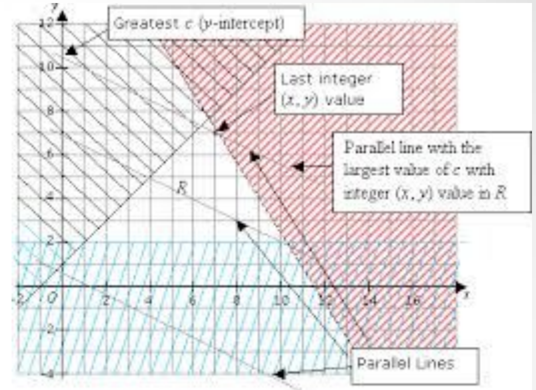
1. *What Is Operations Research? (OR)*
2. *The Linear Programming tasks*
3. *Tasks: Computers Purchase and Diet Problem*
4. *Determine the Linear Programming tasks using MS Excel*

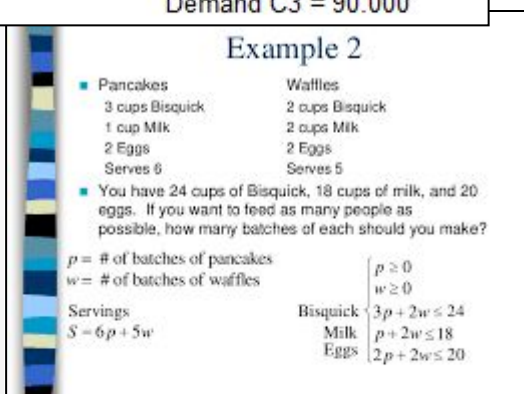
Professor Shmelova T.

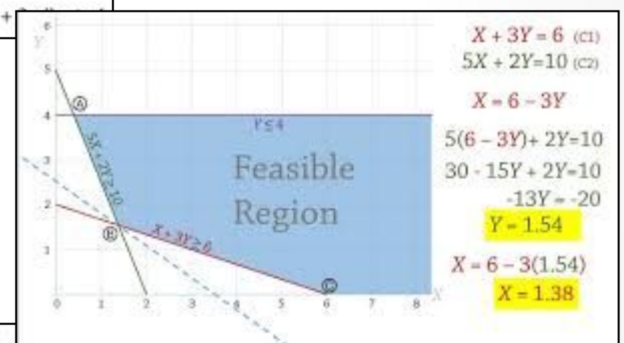
The Linear Program, examples

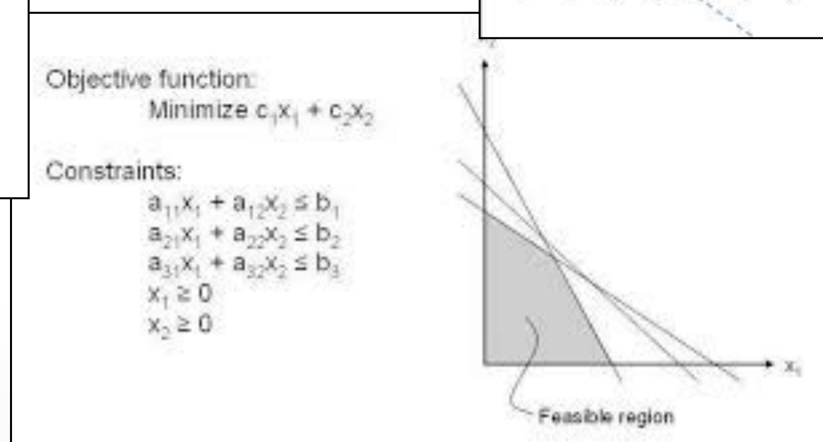












The Linear Program, demo-example

[Questions: How many computers need to buy to get the maximum profit?]

The company wants to buy personal computers: types A and B

The price:

- ✔ The PC of type A is 1000 € for one PC
- ✔ The PC of type B is 1500 € for one PC.

The expected profit from the exploitation of computers:

- ✔ type A - is 2500 € for one year,
- ✔ type B - is 3000 € for one year.

The capacity:

- ✔ The maximum quantity of workstation (automated workstation) - are 25 WS.
- ✔ The amount of money to buy WSs is 30 000 €.

ANSWER ???????

Computers Purchase Task

Initial date:

The company wants to buy personal computers: types A and B

The price:

- ✔The PC of type A is 1000 € for one PC
- ✔The PC of type B is 1500 € for one PC.

The expected profit from the exploitation of computers:

- ✔type A - is 2500 € for one year,
- ✔type B - is 3000 € for one year.

The capacity: The maximum quantity of workstation (automated workstation) - are 25 PC.

The amount of money to buy PC is 30 000 €.

Questions: How many computers need to buy to get the maximum profit?

Initial date in table

	Computers		Constraints
	Type A	Type B	
The capacity	1	1	≤ 25
The price	1	1,5	≤ 30
The profit	2,5	3	***maximum***

	Computers		Constraints
	Type A	Type B	
The capacity	1	1	≤ 25
The price	1	1,5	≤ 30
The profit	2,5	3	***maximum***

Building of LP model (3 steps):

1. Variables

x_1 - quantity of computers of types A

x_2 - quantity of computers of types B

2. Constraints:

- quantity of workstation:

$$x_1 + x_2 \leq 25$$

-the capital:

$$x_1 + 1,5x_2 \leq 30$$

3. Objective (goal) (maximize or minimize).

Maximum profit of computer's exploitation should be:

$$2,5x_1 + 3x_2 \rightarrow max$$

Mathematical model:

$$Maximize\ F = 2,5x_1 + 3x_2 \rightarrow max$$

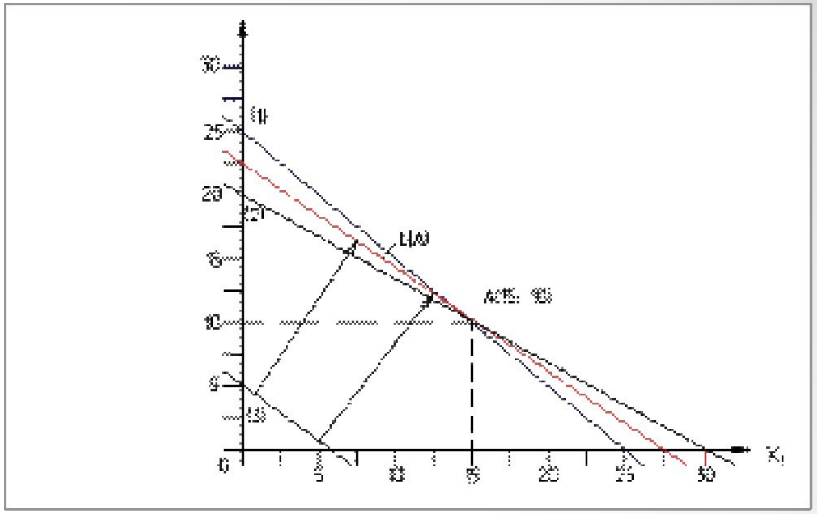
$$x_1 + x_2 \leq 25$$

$$x_1 + 1,5x_2 \leq 30$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

The solution: The simplex method



<u>The graphic calculation of the task.</u>

We build constraints (inequality) on the coordinate plane x0y:

First equality: $x_1 + x_2 = 25$

$x_1 = 0$		$x_1 = 25$
$x_2 = 25$	and	$x_2 = 0$
B (0; 25)		C (25; 0)

Left draw hatching inequality BC

Analogically: $x_1 + 1{,}5x_2 \leq 30$

$x_1 = 0$		$x_1 = 30$
$x_2 = 20$	and	$x_2 = 0$
N (0; 20)		M (30; 0)

Left draw hatching inequality NM

Goal function may be equals any value, for example

$L = 2{,}5x_1 + 3x_2 = 15$

$x_1 = 0,\ x_2 = 5$

and

$x_1 = 6,\ x_2 = 0$

1 approach. Moving mentally L = 15 in the direction of function increasing L, $L \to \max$, (pict.1), to maximal value – point A (15;10).

2 approach. To find the values of the objective function at the corner points N, B, A and search maximum value

$L(N) = 2{,}5x_1 + 3x_2 = 2{,}5*0 + 3*20 = 60$ €

$L(B) = 2{,}5x_1 + 3x_2 = 2{,}5*25 + 3*0 = 62{,}5$ €

$L(A) = 2{,}5x_1 + 3x_2 = 2{,}5*15 + 3*10 = 67{,}5$ €

were point A (15,10) - define coordinates of point A (pic.1) by solving the system of equalities:

$$\begin{cases} x_1 + x_2 = 25 \\ x_1 + 1{,}5x_2 = 30 \end{cases}$$

$$0{,}5x_2 = 5$$

$$x_2 = 10$$

ANSWER :

Maximum value of in point A of the objective function:
L(A) = 67,5
Optimal solution:
$x_1 = 15$ – PC of type A
$x_2 = 10$ – PC of type B

Check:

The amount of money to buy PC is 30 000 €:
1000*15+1500*10 = 30000
The capacity - quantity of workstation:
15 + 10 = 25 PC

THE RESULT: The optimal quantity of computers type A that used to be bought is 15, type B is 10 computers. At the same time the maximal profit of both types computer's exploitation will be 67,5 €

1. What Is Operations Research? (OR)

The first formal activities of Operations Research (OR) were initiated in England, USA, USSR, other countries during World War II, when scientists set out to make scientifically based decisions regarding the best utilization of war materiel.

After the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector.

Operations research, or operational research, is a discipline that deals with the application of advanced analytical methods to help make better decisions.

Mathematical model of linear program (LP) tasks:

Objective function: $L = \sum_{i=1}^{n} c_i x_i \rightarrow \text{ext (min/max)}$

Constraints: $\sum_{i=1}^{n} a_{ij} x_i = (\le, \ge)\ b_j,\ j = 1, \dots m;\ i = 1, \dots n$

$$x_i \ge 0$$

The simplex method – main method of decisions LP tasks

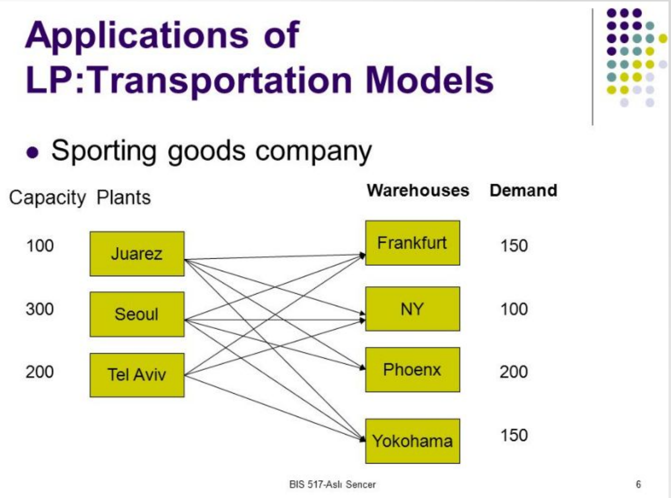
General phases (stages) of construction of a mathematical model (OR):

<u>The principal phases for implementing OR in practice include:</u>

1. Definition of the problem (alternatives, feasible variables, constrains, goal,..)
2. Construction of the model.
3. Solution of the task.
4. Validation of the model.
5. Implementation of the solution in practice.

<u>The LP mathematical model, as in any OR model, has three stages of construction:</u>

1.To find the variables x1, x2, x3, …
2.To find constraints
3.To find an objective function (goal) that we need to optimize (maximum or minimum) – L



2. Applications LP models:

✔the problem of the diet,
✔the problem of cutting materials,
✔problem of the distribution of aircraft on routes,
✔urban planning,
✔currency arbitrage,
✔investment,
✔production planning and inventory control,
✔gasoline blending,
✔manpower planning, and scheduling, etc

- *Задача о рационе питания*
- *Задача о распределении ресурсов*
- *Задача о планировании производства*
- *Задача о загрузке оборудования (раскрое материала)*
- *Задача о снабжении сырьем*
- *Задача о сменно-суточном планировании работы*
- *автобусного парка*
- *Задача о назначениях*

TASK 2. Initial date:

The company can advertise its product on radio and TV and have 500 € on advertisement.

The price:

- 1 minute of radio cost 10 €
- 1 minute of TV cost 25 €

The expected profit from advertisement:

- On radio - 25 € for one day,
- On TV - 40 € for one day.

The company wants to advertise its product on radio and TV – 35 minutes per day.

The amount of money to buy advertisement is 500 €.

Questions: How many R and TV minutes per day need to advertise product to get the maximum profit?

TASK 3*.

Show & Sell can advertise its products on local radio and television (TV). The advertising budget is limited to $10,000 a month. Each minute of radio advertising costs $15 and each minute of TV commercials $300. Show & Sell likes to advertise on radio at least twice as much as on TV. In the meantime, it is not practical to use more than 400 minutes of radio advertising a month. From past experience, advertising on TV is estimated to be 25 times as effective as on radio. Determine the optimum allocation of the budget to radio and TV advertising.

Determine the Linear Programming tasks using MS Excel

Main steps

.Make the task form

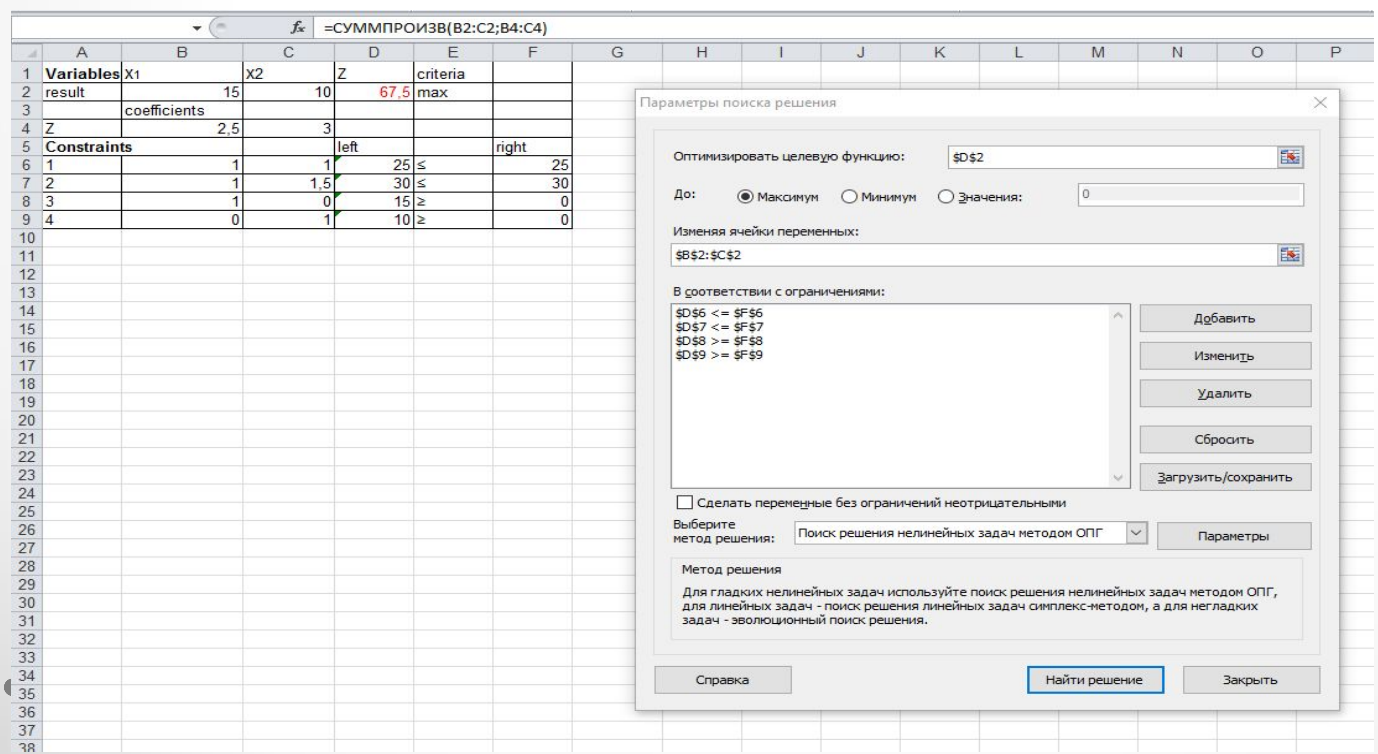
.Enter basic data of the task to the form:

- Enter the dependence for the criterion function ("Function Master" f_x ; "СУММПРОИЗВ" (category: mathematical))
- Enter the dependence for the left part of constrains

3. Working in dialogue box **Solution search:**

- Enter the direction of criterion function
- Inscribe the constrains in area "The limitation"

4. The shunt in " Characteristic "



The Diet Problem

The goal of the diet problem is to select a set of foods that will satisfy a set of daily nutritional requirement at minimum cost.

The problem is formulated as a linear program where the **objective** is to minimize cost and the **constraints** are to satisfy the specified nutritional requirements.

The diet problem constraints typically regulate the *number of calories and the amount of vitamins, minerals, fats, sodium, and cholesterol in the diet.*

Consider the following simple example

Suppose there are 2 foods:

corn and milk, and there are restrictions:

- ✔on the number of calories (between 400 and 800)
- ✔on the amount of Vitamin A (between 200 and 300)

The first table lists, for each food, the cost per serving, the amount of Vitamin A per serving, and the number of calories per serving.

Food	Cost per serving	Vitamin A	Calories
Corn	$0.18	100	50
2% Milk	$0.23	50	200
Capacity		between 200 and 300	between 400 and 800

The LP model, as in any OR model, has three basic components.

1. Decision variables that we seek to determine:

x_i - number of servings of food *i* to purchase/consume

x_1 - Corn

x_2 - 2% Milk

2. Constraints that the solution must satisfy:

- the capacity of Vitamin A:

$$100x_1 + 50x_2 \leq 300$$
$$100x_1 + 50x_2 \geq 200$$

-the capacity of Calories:

$$50x_1 + 200x_2 \leq 800$$
$$50x_1 + 200x_2 \geq 400$$

3. Objective (goal) that we need to optimize (maximize or minimize).

Secondly the cost of food should be minimum:

$$F = 0{,}18x_1 + 0{,}23x_2 \rightarrow \min$$

Solution

Mathematical model:

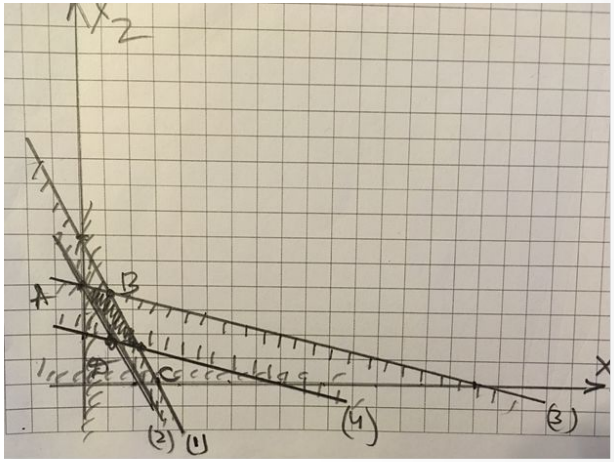
Objective

$$F = 0{,}18x_1 + 0{,}23x_2 \rightarrow min$$

restrictions:

$$107x_1 + 500x_2 \leq 50000$$
$$107x_1 + 500x_2 \geq 5000$$
$$72x_1 + 121x_2 \leq 2250$$
$$72x_1 + 121x_2 \geq 2000$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

		x1	x2	
1	100x1 +50 x2 ≤ 300	0	6	0; 6
		3	0	3; 0
2	100x1 +50 x2 ≥ 200	0	4	0; 4
		2	0	2; 0
3	50x1+200x2 ≤ 800	0	4	0; 4
		16	0	16; 0
4	50x1+200x2 ≥ 400	0	2	0; 2
		8	0	8; 0
	x1 ³ 0			
	x2 ³ 0			
	F = 0,18x1+0,23x2			
A(0;4)	0	4	0,92	
B(1;3,8)	1,3	8	2,074	
C(2,5;1,7)	2,50	1,7	0,841	
D(1;1,8)	1	1,8	0,594	



Example 2

Suppose there are three foods:

corn, milk, and bread, and there are restrictions:

- on the number of calories (between 2000 and 2250) and
- the amount of Vitamin A (between 5000 and 50,000).

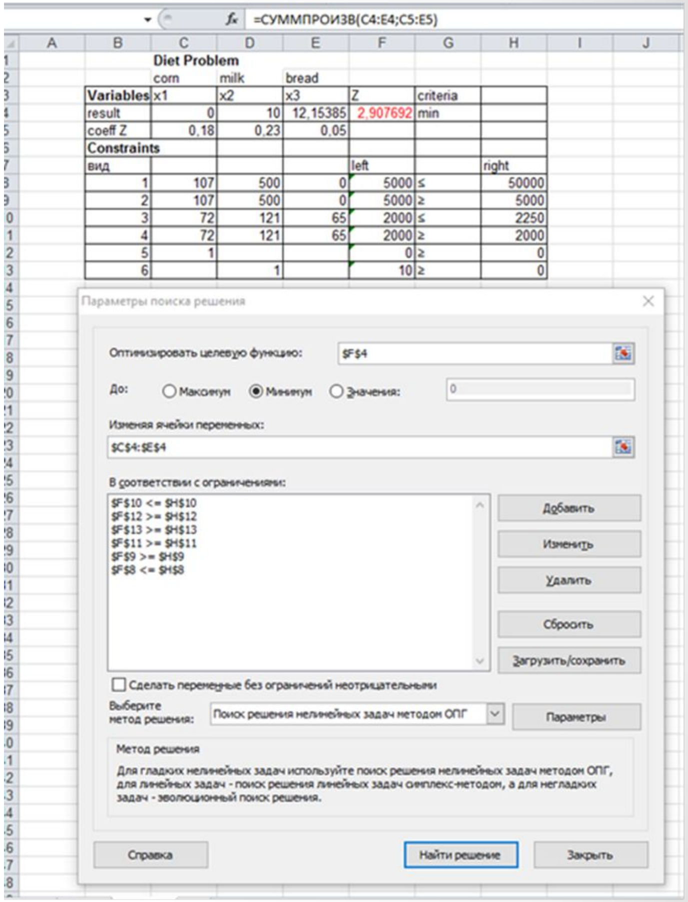
The first table lists, for each food, the cost per serving, the amount of Vitamin A per serving, and the number of calories per serving.

Food	**Cost per serving**	**Vitamin A**	**Calories**
Corn	$0.18	107	72
2% Milk	$0.23	500	121
Wheat Bread	$0.05	0	65

Suppose that the maximum number of servings is 10. Then, the optimal solution for the problem is 1.94 servings of corn, 10 servings of milk, and 10 servings of bread with a total cost of $3.15. The total amount of Vitamin A is 5208 and the total number of calories is 2000.

Solution example 2

Variables	x1	x2	x3	Z	criteria	
result	0	10	12,15385	2,907692	min	
coeff Z	0,18	0,23	0,05			
Constraints						
вид				left		right
1	107	500	0	5000	≤	50000
2	107	500	0	5000	≥	5000
3	72	121	65	2000	≤	2250
4	72	121	65	2000	≥	2000
5	1			0	≥	0
6		1		10	≥	0



4. Determine the Linear Programming tasks using MS Excel

***Operation algorithm for the sum solving*:**

1. Make the task form (pic.2.5).

2. Enter basic data of the task /2.3/ - /2.4/ to the form(pic.2.6).

3. Enter the dependence from the mathematical simulator /2.3/ - /2.4/ to the form:

3.1. Enter the dependence for the criterion function /2.3/:

- The shunt to the cell F6
- The shunt to the button”Function Master” f_x
- At the screen: dialogue box " Function Master–step1 from 2"
- The shunt to the function box “СУММПРОИЗВ” (category :mathematical)
- “OK”
- At the screen: dialogue box “СУММПРОИЗВ”
- In array1 enter B3:C3 (allocate by mouse)
- In array 2 enter B6:C6
- "OK"
- At the screen: in F6 was entered criterion function data “=СУММПРОИЗВ(B3:C3;B6:C6)”

3.2. Enter the dependence for the left part of limitations:

- The shunt to the cell F9
-
- The shunt to the button"Function Master" $\mathbf{f_x}$
- At the screen: dialogue box " Function Master–step1 from 2"
- The shunt to the function box "СУММПРОИЗВ"
- "OK"
- At the screen: dialogue box "СУММПРОИЗВ"
- In array1 enter B3:C3 (allocate by mouse)
- In array 2 enter B9:C9
- "OK"

(the same way for the F10)

- At the screen:

In F9 "=СУММПРОИЗВ(B3:C3;B9:C9)";
in F10 "=СУММПРОИЗВ(B3:C3;B10:C10)".
Basic data entering is over.

1. Working in dialogue box **Solution search**:

- The shunt in menu "Service"
- The command"Solution search"
- At the screen: dialogue box "Solution search"
- The shunt in area"Set target box", enter the address criterion function F6
- Enter the direction of criterion function: " Maximum magnitude "
- The shunt in area "Box changing ",enter the address: B3:C3
- Inscribe the limitations in area" The limitation "
- The shunt in "Add"
- At the screen: dialogue box "Limitation adding"
- In area " linking to box "enter the address: B3, in area" The limitation " insert the sign>=, to the right area- B4.the limitation have received B3 >= B4
- "Add"
- Analogically enter following limitations(after every limitation- "Add"):

C3 >= C4;
F9 < = H9;
F10 <= H10;
B3 = whole;
C3 = whole

- At the end of the last limitation"Add" enter"OK"
- At the screen: dialogue box "Solution search" concerning entered conditions

5. The sum solving according LP:

- The shunt in " Characteristic "
- At the screen: dialogue box "Solution search characteristic "
- Enter the present characteristic of the task ("Linear model", appraisal " Linear ")
- “OK”
- The shunt in " Run "
- At the screen: dialogue box "The result of solution search"
- Save the solution
- “OK”.

The result is on the picture 2.7. The value of criterion function (maximum profit) will be 67500 conventional units (area F6)if the variables are the following x_1=15 (B3); x_2=10 (C3).

	A	B	C	D	E	F	G	H
1				variables				
2	the name	X_1	X_2					
3	magnitude							
4	bottom							
5	high limit					base formula		
6	base formula coefficient	2.5	3					
7				the limitation				
8	sight					left part.	sign	right part
9	capacity	1	1				<=	25
10	capital	1	1.5				<=	30

	A	B	C	D	E	F	G
1				variables			
2	the name	X_1	X_2				
3	magnitude	15	10				
4	bottom						
5	high limit					base formula	
6	base formula coefficient	2.5	3				
7				the limitation			
8	sight					left part.	sign
9	capacity	1	1			25	<=
10	capital	1	1.5			30	<=

Home work

Numerical value of coefficients: a_{11}, a_{12}, a_{21}, a_{22}, в_1, в_2, c_1, c_2 for the computer purchase task

$$L = c_1x_1 + c_2x_2 \rightarrow \max$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 \leq \text{в}_1 \\ a_{21}x_1 + a_{22}x_2 \leq \text{в}_2 \\ x_1, x_2 \geq 0 \end{cases}$$

Variant №	coefficients in mathematical model							
	c_1	c_2	a_{11}	a_{12}	a_{21}	a_{22}	$в_1$	$в_2$
1	2,7	1,4	1	1	1,5	2,1	24	30
2	2,8	1,2	1	1	1,6	2,3	25	34
3	1,5	1,6	1	1	1,3	2,4	30	32
4	1,6	1,5	1	1	1,2	2,2	28	30
5	1,8	1,4	1	1	1,4	2,1	29	32
6	1,7	1,6	1	1	1,1	2,2	30	36
7	1,4	1,8	1	1	1,0	2,0	26	30
8	1,8	1,1	1	1	1,2	2,2	24	34
9	2,0	1,0	1	1	1,5	2,5	26	32
10	2,2	1,5	1	1	1,3	2,8	28	34
11	2,1	1,2	1	1	1,4	2,7	26	35
12	2,3	1,4	1	1	1,3	2,6	24	36
13	2,4	1,8	1	1	1,2	2,3	28	30
14	2,6	2,1	1	1	1,1	2,4	30	32
15	2,5	2,3	1	1	1,2	2,1	32	34
16	1,2	2,5	1	1	0,9	2,9	22	38
17	1,9	2,0	1	1	1,8	3,3	25	28
18	2,9	3,1	1	1	2,1	3,1	20	26
19	3,8	2,2	1	1	2,5	3,5	34	40
20	3,1	2,4	1	1	1,9	3,0	36	30
21	3,7	3,3	1	1	2,0	3,6	40	42
22	3,2	2,6	1	1	2,3	1,9	38	26
23	3,0	3,5	1	1	2,7	3,4	42	44
24	3,6	2,9	1	1	2,6	3,9	44	40
25	3,4	3,7	1	1	3,0	3,7	36	26

Numerical value of coefficients: a_{11}, a_{12}, a_{21}, a_{22}, $в_1$, $в_2$, c_1, c_2 for the computer purchase task

$$L = c_1x_1 + c_2x_2 \rightarrow \max$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 \le в_1 \\ a_{21}x_1 + a_{22}x_2 \le в_2 \\ x_1, x_2 \ge 0 \end{cases}$$

Thank you for your attention