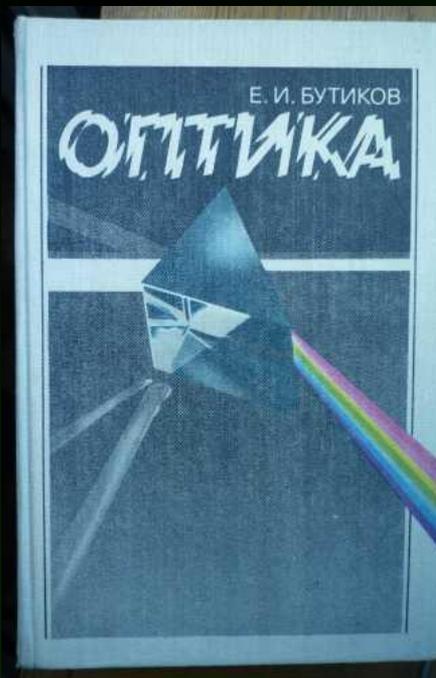
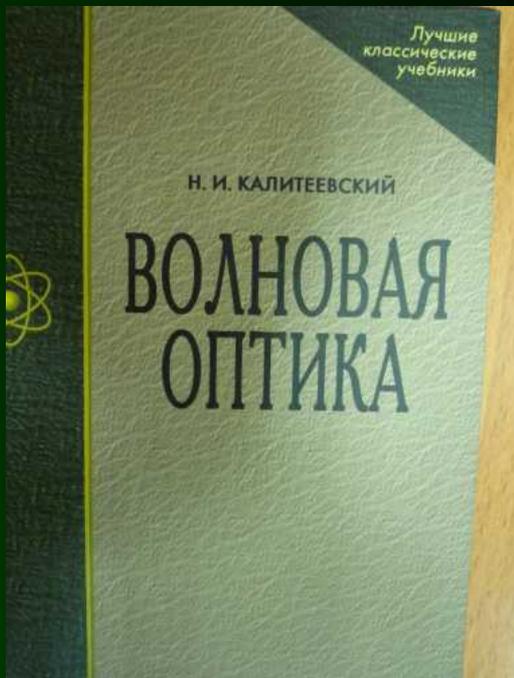
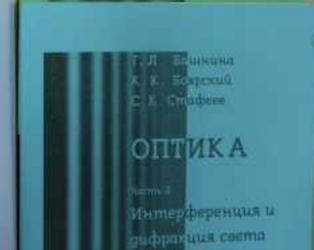
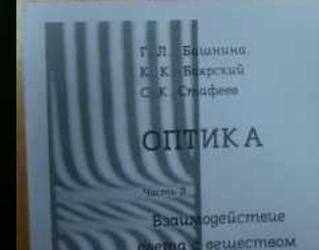
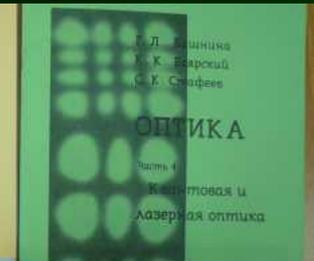
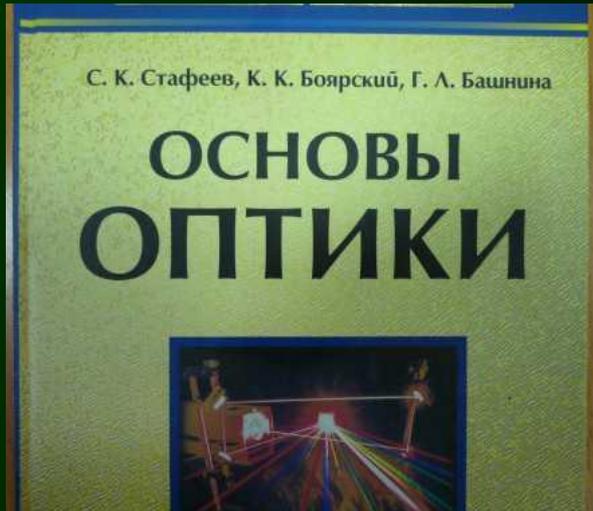


# Литература



# Уравнения Максвелла

1873 г.

$$\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\operatorname{rot} \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

$$\operatorname{div} \vec{D} = \rho$$

$$\operatorname{div} \vec{B} = 0$$

Материальные уравнения

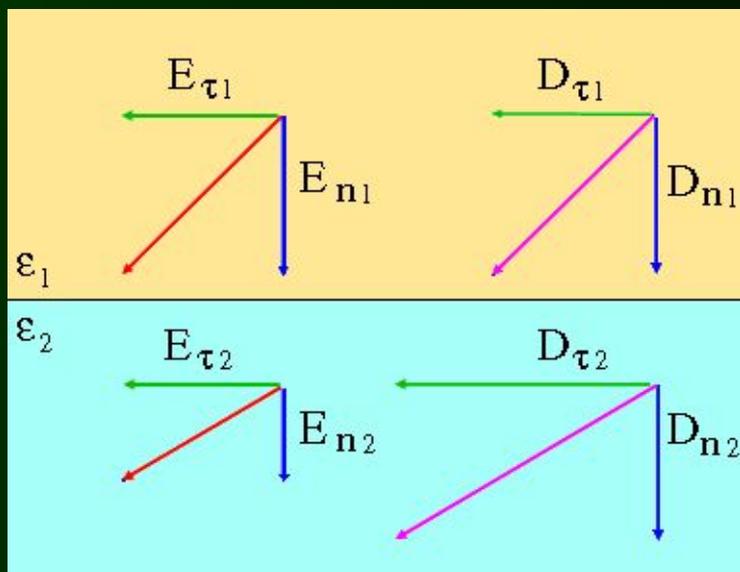
$$\vec{D} = \varepsilon \varepsilon_0 \vec{E}$$

$$\vec{B} = \mu \mu_0 \vec{H}$$

$$\vec{j} = \sigma \vec{E}$$

Параметр	Тип среды
$\sigma = 0$	Диэлектрическая
$\sigma \neq 0$	Проводящая
$\varepsilon = \text{const}$	Однородная, изотропная
$\varepsilon = \varepsilon(x, y, z)$	Неоднородная
$\varepsilon = \varepsilon(\text{направления})$	Анизотропная
$\varepsilon = \varepsilon(\vec{E})$	Нелинейная

# Граничные условия



$$E_{\tau 1} = E_{\tau 2}$$

$$H_{\tau 1} = H_{\tau 2}$$

$$D_{n1} = D_{n2}$$

$$B_{n1} = B_{n2}$$

# Волновое уравнение

Однородный изотропный  
диэлектрик

$$\text{rot } \vec{E} = -\mu\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\frac{\partial}{\partial t} \text{rot } \vec{H} = \varepsilon\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\text{div } \vec{E} = 0$$

$$\text{div } \vec{H} = 0$$

$$\text{rot rot } \vec{E} = -\varepsilon\varepsilon_0\mu\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\text{rot rot } \vec{E} = \text{grad div } \vec{E} - \Delta \vec{E}$$

Оператор Лапласа

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\Delta \vec{E} = \varepsilon\mu\varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

# Скорость э/м волн

$$\Delta E = \varepsilon\mu\varepsilon_0\mu_0 \frac{\partial^2 E}{\partial t^2} \quad \Delta E = \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2} \quad v = \frac{1}{\sqrt{\varepsilon\mu\varepsilon_0\mu_0}}$$

Вакуум:  $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \quad c = 299792456 \text{ м/с}$

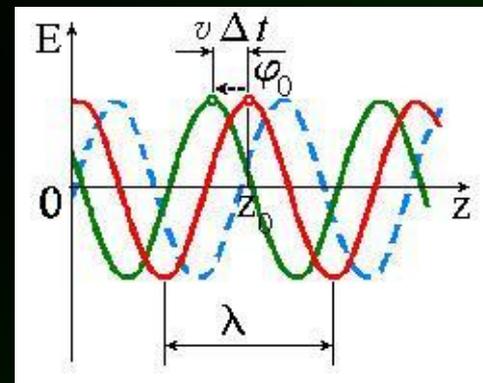
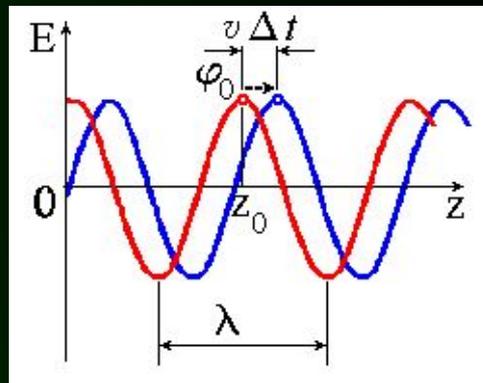
Показатель преломления  $n = \frac{c}{v} = \sqrt{\varepsilon\mu} \quad n = \sqrt{\varepsilon}$

Вещество	n	$\sqrt{\varepsilon}$	Вещество	n	$\sqrt{\varepsilon}$
водород	1,000139	1,000139	бензол	1,501	1,511
воздух	1,000292	1,000302	спирт	1,36	5,1
углекислота	1,000499	1,000485	вода	1,33	9

# Плоская монохроматическая волна

$$\vec{E} = \vec{E}(z) \quad \frac{\partial^2 \vec{E}}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{E}(z, t) = \vec{A}_1 \cos \left[ \omega \left( t - \frac{z}{v} \right) \right] + \vec{A}_2 \cos \left[ \omega \left( t + \frac{z}{v} \right) \right]$$

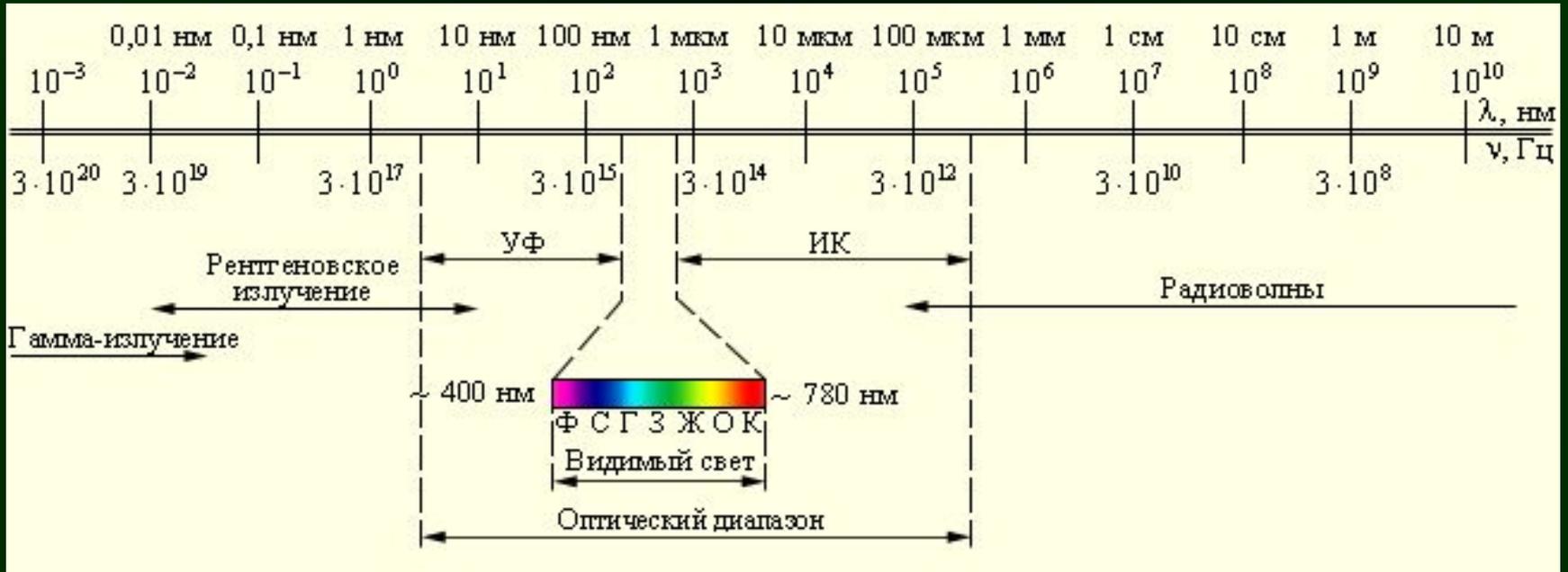


$$\lambda = vT = \frac{2\pi v}{\omega}$$

Плоская волна:

волновой фронт  $z = \text{const}$

# Шкала э/м волн



# Формы уравнения плоской волны

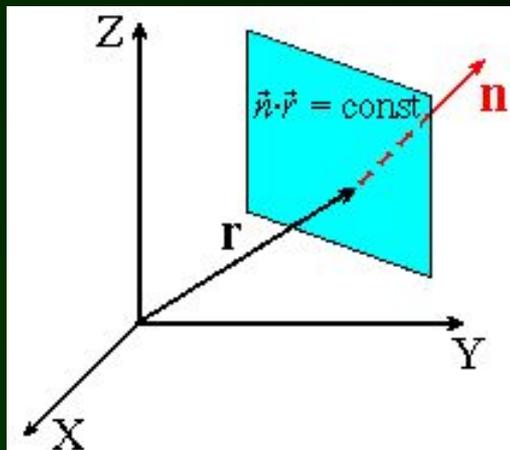
$$\vec{E}(z, t) = A \cos \left[ \omega \left( t - \frac{z}{v} \right) \right]$$

$$\vec{E}(z, t) = A \cos(\omega t - kz)$$

$$k = \frac{\omega}{v} = \frac{2\pi}{\lambda}$$

ВОЛНОВОЕ ЧИСЛО

$$[k] = \text{см}^{-1}$$



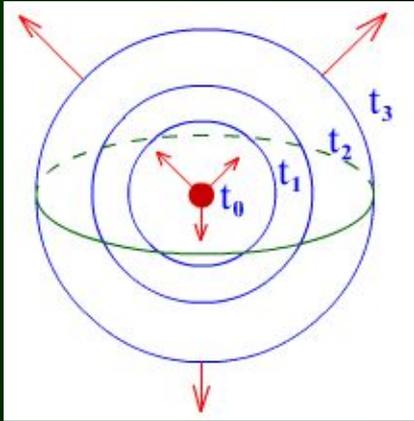
$$\vec{E}(\vec{r}, t) = A \cos(\omega t - \vec{k} \cdot \vec{r})$$

$$\vec{k} = nk$$

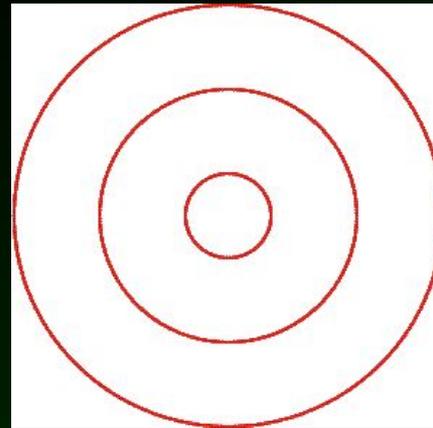
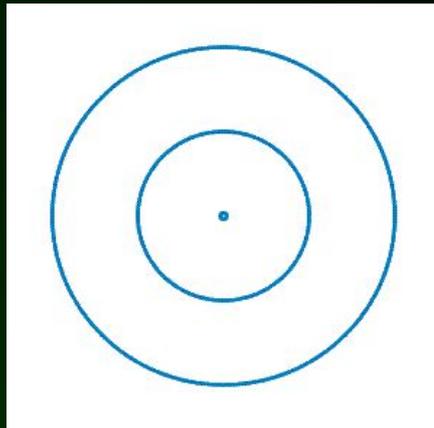
ВОЛНОВОЙ ВЕКТОР

$$\vec{E}(\vec{r}, t) = A e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

# Сферические волны



$$\vec{E}(r, t) = \frac{\vec{A}_1}{r} \cos(\omega t - kr) \quad \text{или} \quad \cos(\omega t + kr)$$

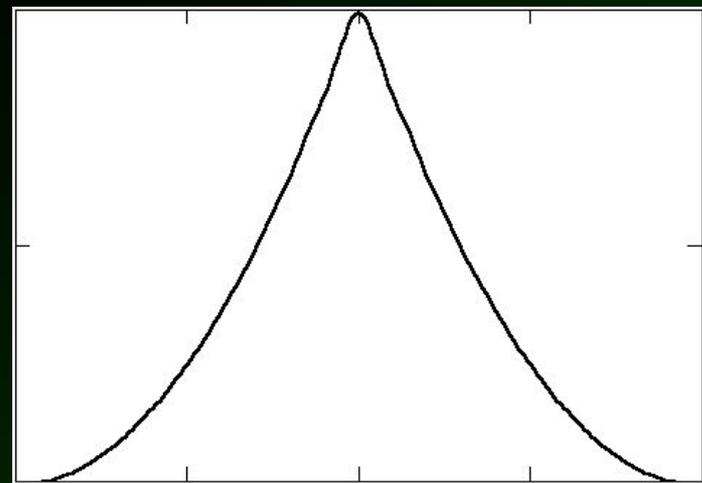
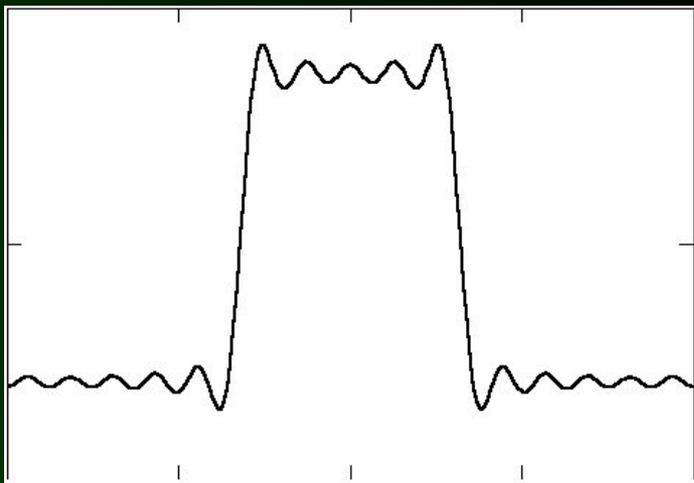
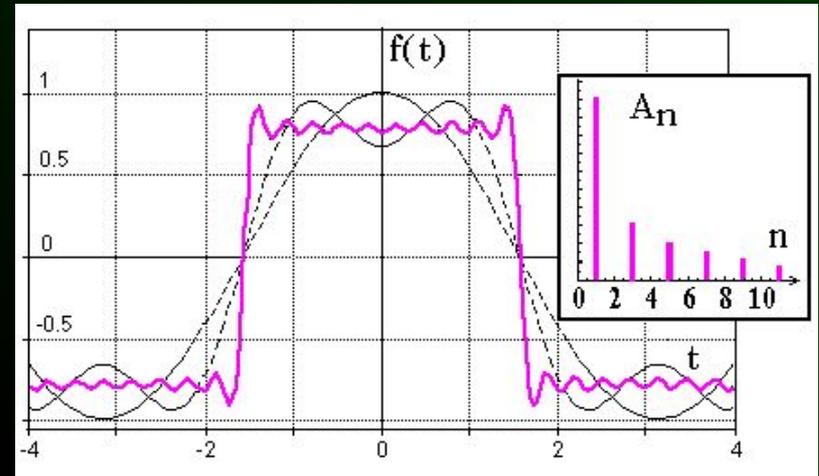


# Спектральное представление

## Преобразование Фурье

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$



# Свойства э/м волн

$$\vec{E}(x, y, z, t) = A e^{i(\omega t - k r)} = A e^{i(\omega t - k_x x - k_y y - k_z z)}$$

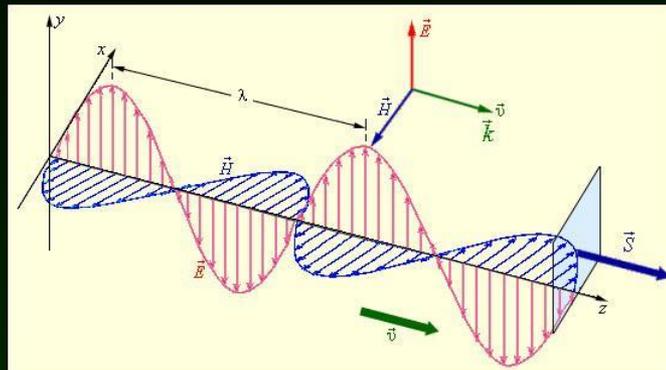
$$\frac{\partial \vec{E}}{\partial t} = i\omega \vec{E} \quad \frac{\partial \vec{E}}{\partial x} = -ik_x \vec{E} \quad \nabla \times \vec{E} = -i\vec{k} \times \vec{E}$$

$$\text{rot } \vec{E} = -\mu\mu_0 \frac{\partial \vec{H}}{\partial t} \quad -i\vec{k} \times \vec{E} = -\mu\mu_0 i\omega \vec{H}$$

$$\text{rot } \vec{H} = \varepsilon\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad -i\vec{k} \times \vec{H} = \varepsilon\varepsilon_0 i\omega \vec{E}$$

$$\vec{E} \perp \vec{k}, \quad \vec{H} \perp \vec{k}$$

Поперечность



$\vec{E}, \vec{H}, \vec{k}$

Правая  
тройка  
векторов

# Интенсивность света

$$kE = \mu\mu_0\omega H \Rightarrow \sqrt{\varepsilon\varepsilon_0}E = \sqrt{\mu\mu_0}H$$

**Вектор Пойнтинга** – вектор плотности потока энергии

$$\vec{S} = \vec{E} \times \vec{H}$$

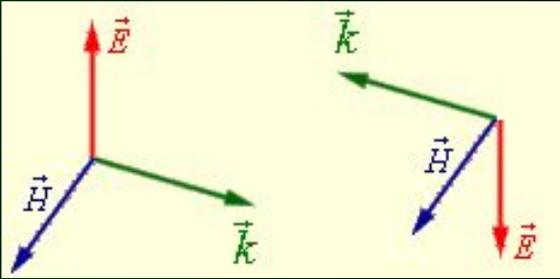
$$\vec{E} = E_0 \cos(\omega t - kr)$$

$$\vec{H} = H_0 \cos(\omega t - kr)$$

$$S = \sqrt{\frac{\varepsilon_0}{\mu_0}} n E_0^2 \cos^2(\omega t - kr)$$

$$I = \langle S \rangle \propto E_0^2 \quad [I] = \text{Вт/м}^2$$

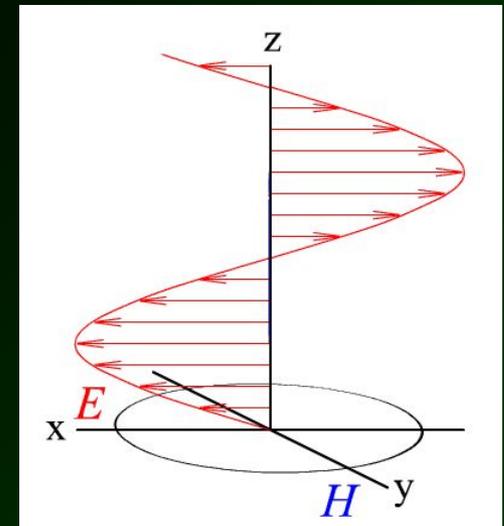
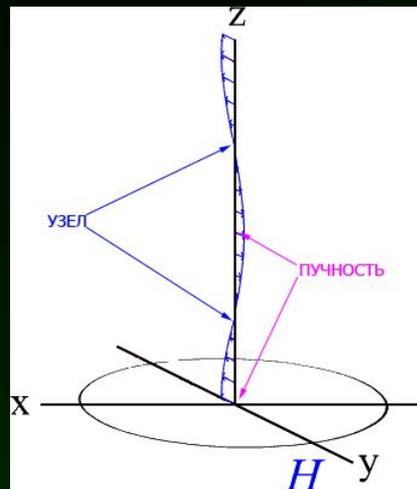
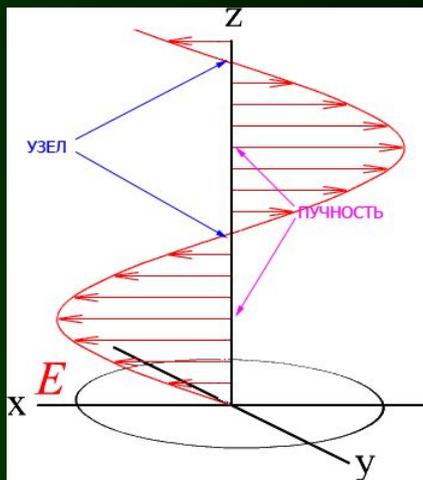
# Стоячие волны



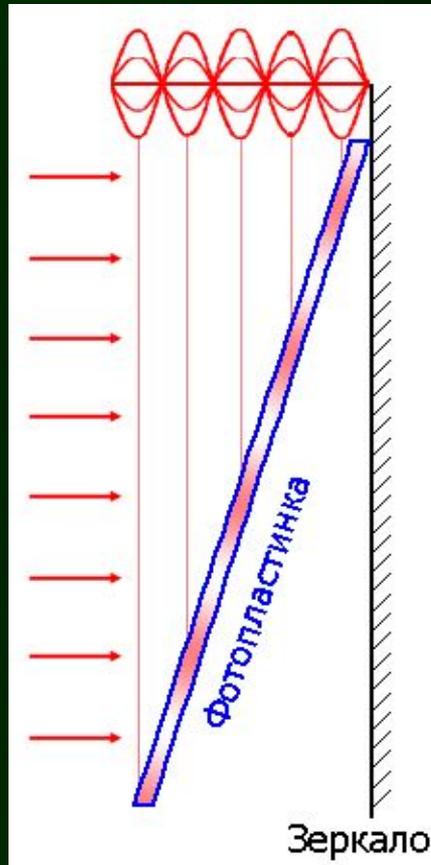
$$E_1 = E_0 \cos(\omega t - kz) \quad H_1 = H_0 \cos(\omega t - kz)$$

$$E_2 = -E_0 \cos(\omega t + kz) \quad H_2 = H_0 \cos(\omega t + kz)$$

$$E = 2E_0 \sin \omega t \sin kz \quad H = 2H_0 \cos \omega t \cos kz$$



# Опыт Винера



$$E = cB$$

$$\frac{F_{\text{ради}}}{F_{\text{фот}}} = \frac{evB}{eE} \approx \frac{v}{c}$$

1890 г.