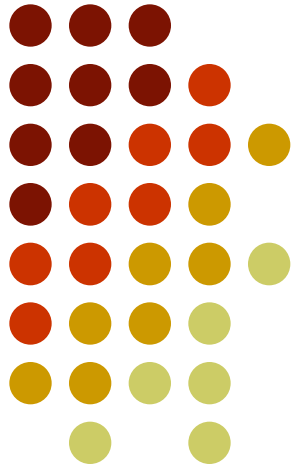
Value at Risk



The Question Being Asked in VaR

"What loss level is such that we are X% confident it will not be exceeded in N business days?"

VaR and Regulatory Capital

(Business Snapshot 18.1, page 436)

- Regulators base the capital they require banks to keep on VaR
- The market-risk capital is k times the 10-day 99% VaR where k is at least 3.0

VaR vs. C-VaR
(See Figures 18.1 and 18.2)

- VaR is the loss level that will not be exceeded with a specified probability
- C-VaR (or expected shortfall) is the expected loss given that the loss is greater than the VaR level
- Although C-VaR is theoretically more appealing, it is not widely used

Advantages of VaR

- It captures an important aspect of risk in a single number
- It is easy to understand
- It asks the simple question: “How bad can things get?”

Time Horizon

- Instead of calculating the 10-day, 99% VaR directly analysts usually calculate a 1-day 99% VaR and assume

$$10\text{-day VaR} = \sqrt{10} \times 1\text{-day VaR}$$

- This is exactly true when portfolio changes on successive days come from independent identically distributed normal distributions

Historical Simulation

(See Tables 18.1 and 18.2, page 438-439))

- Create a database of the daily movements in all market variables.
- The first simulation trial assumes that the percentage changes in all market variables are as on the first day
- The second simulation trial assumes that the percentage changes in all market variables are as on the second day
- and so on

Historical Simulation continued

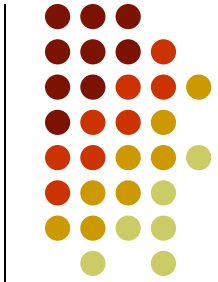
- Suppose we use m days of historical data
- Let v_i be the value of a variable on day i
- There are m-1 simulation trials
- The ith trial assumes that the value of the market variable tomorrow (i.e., on day m+1) is

$$v_m \frac{v_i}{v_{i-1}}$$

The Model-Building Approach

- The main alternative to historical simulation is to make assumptions about the probability distributions of return on the market variables and calculate the probability distribution of the change in the value of the portfolio analytically
- This is known as the model building approach or the variance-covariance approach

Daily Volatilities



- In option pricing we measure volatility "per year"
- In VaR calculations we measure volatility "per day"

$$\sigma_{\text{day}} = \frac{\sigma_{\text{year}}}{\sqrt{252}}$$

Daily Volatility continued

- Strictly speaking we should define σ_{day} as the standard deviation of the continuously compounded return in one day
- In practice we assume that it is the standard deviation of the percentage change in one day

Microsoft Example (page 440)

- We have a position worth $10 million in Microsoft shares
- The volatility of Microsoft is 2% per day (about 32% per year)
- We use N=10 and X=99

Microsoft Example continued

- The standard deviation of the change in the portfolio in 1 day is $200,000
- The standard deviation of the change in 10 days is

$$200{,}000\sqrt{10} = \$632{,}456$$

Microsoft Example continued

- We assume that the expected change in the value of the portfolio is zero (This is OK for short time periods)
- We assume that the change in the value of the portfolio is normally distributed
- Since $N(-2.33)=0.01$, the VaR is

$$2.33 \times 632{,}456 = \$1{,}473{,}621$$

AT&T Example (page 441)

- Consider a position of $5 million in AT&T
- The daily volatility of AT&T is 1% (approx 16% per year)
- The S.D per 10 days is

$$50{,}000\sqrt{10} = \$158{,}144$$

- The VaR is

$$158{,}114 \times 2.33 = \$368{,}405$$

Portfolio

- Now consider a portfolio consisting of both Microsoft and AT&T
- Suppose that the correlation between the returns is 0.3

S.D. of Portfolio

- A standard result in statistics states that

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

- In this case σ_X = 200,000 and σ_Y = 50,000 and ρ = 0.3. The standard deviation of the change in the portfolio value in one day is therefore 220,227

VaR for Portfolio

- The 10-day 99% VaR for the portfolio is

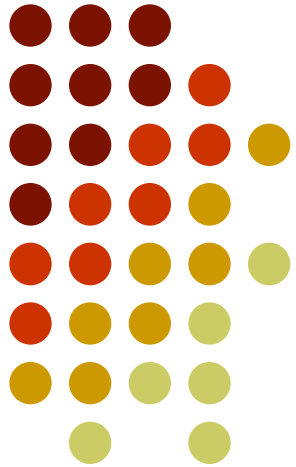
$$220{,}227 \times \sqrt{10} \times 2.33 = \$1{,}622{,}657$$

- The benefits of diversification are

(1,473,621+368,405)–1,622,657=$219,369

- What is the incremental effect of the AT&T holding on VaR?

Value at Risk



Overview

- Concepts
- Components
- Calculations
- Corporate perspective
- Comments

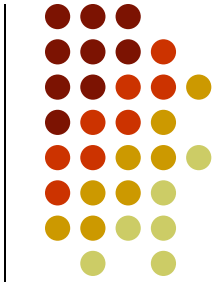
- I VALUE AT RISK - CONCEPTS

Risk

- Financial Risks - Market Risk, Credit Risk, Liquidity Risk, Operational Risk
- Risk is the variability of returns.
- Risk is Defined as “Bad” Outcomes
  - Volatility Inappropriate Measure
  - What Matters is Downside Risk

VAR measures

- Market risk
- Credit risk of late



Value at Risk (VAR)

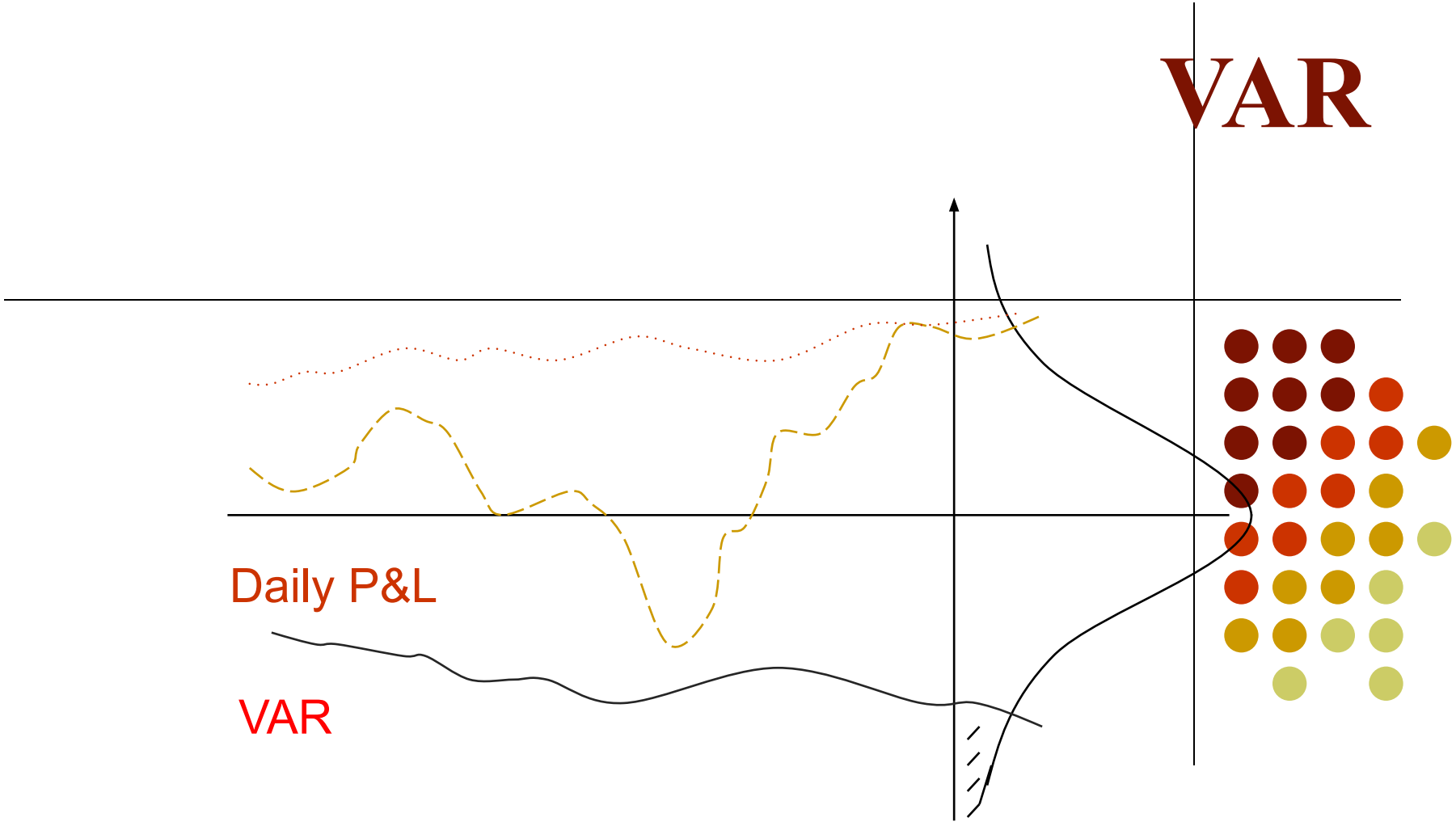
- VAR is an estimate of the adverse impact on P&L in a conservative scenario.
- It is defined as the loss that can be sustained on a specified position over a specified period with a specified degree of confidence.

Value at Risk (VAR)

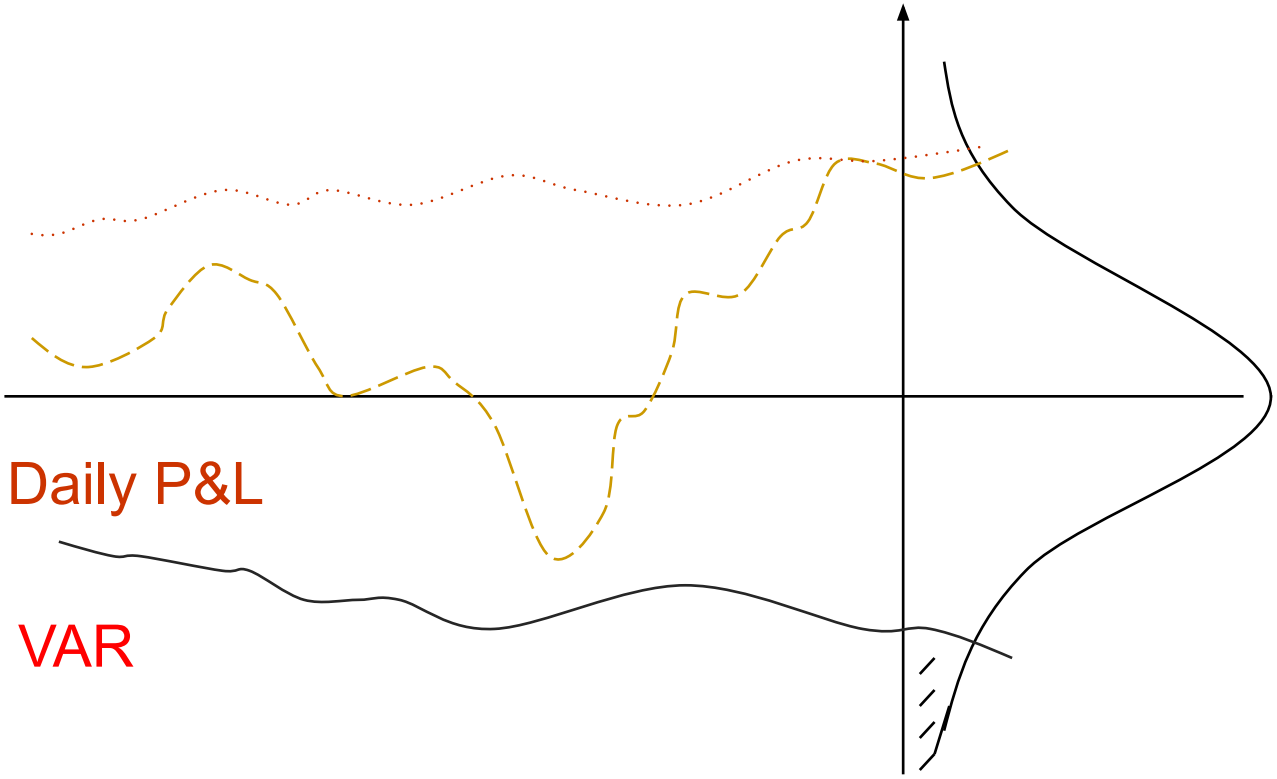
- Ingredients -
  - Exposure to market variable
  - Sensitivity
  - Probability of adverse market movement

- Probability distribution of market variable - key assumption
- Normal, Log-normal distribution

VAR



VAR



- II VALUE AT RISK - COMPONENTS

Key components of VAR

- Market Factors (MF)
- Factor Sensitivity (FS)
- Defeasance Period (DP)
- Volatility

Market Factors (MF)

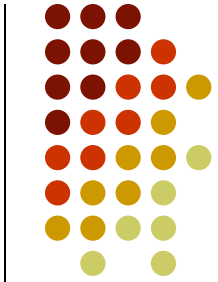
- A market variable that causes the price of an instrument to change
- A market factors group (MFG) is a group of market factors with significant correlation. The major MFGs are:
  - Interest rates,
  - Foreign exchange rates
  - Equity prices
  - Commodity prices
  - Implied volatilities (only in options)
- Complex positions can be sensitive to several MFG (e.g. FX forwards or options)

Factor Sensitivity (FS)

- FS is the change in the value of a position due to a unit change in an independent market factor, all other market factors, if applicable, remaining constant.

- Other names - PVBP

Factor Sensitivity - Zero Coupon Bond



- What is the 1 BP FS of a $2,100 1-year zero coupon bond? (assume market rate is 5%)

- MTM Value = $2,100 / (1.05) = $2,000.00

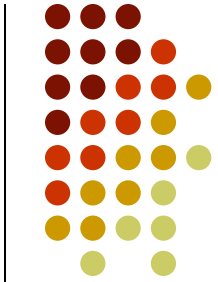
MTM Value = $2,100 / (1.0501) = $1,999.81

FS = $1,999.81 - $2,000.00 = -$0.19

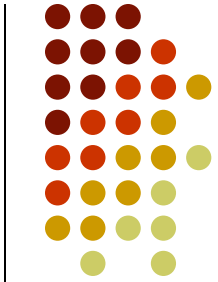
Market Volatility

- Volatility is a measure of the dispersion of a market variable against its mean or average. This dispersion is called Standard Deviation.
  - Variance := average deviation of the mean for a historical sample size
  - Standard deviation : Square Root of the variance
- The market expresses volatility in terms of annualized Standard Deviation (1SD)

Estimating Volatility



- 1. Historical data analysis
- 2. Judgmental
- 3. Implied (from options prices)



Defeasance period

- This is defined as the time elapsed (normally expressed in days) before a position can be neutralized either by hedging or liquidating
- Defeasance period incorporates liquidity risk (for trading) in risk measurement

- Other names - Holding Period, Time horizon

Defeasance Factor (DF)

- DF is the total volatility over the defeasance period

- On the assumption that daily price changes are independent variables (~ correlation zero), volatility is scaled by the square root of time

DF = Daily 2.326 SD * sqrt (DP), or

DF = Market Volatility * 2.326 *sqrt (DP / 260)

DF = Annual 1SD * 2.326 * sqrt (DP/260)

VAR formula

$$VAR = Z_{\alpha} \sigma_p \sqrt{\Delta t} * FS$$

Where:

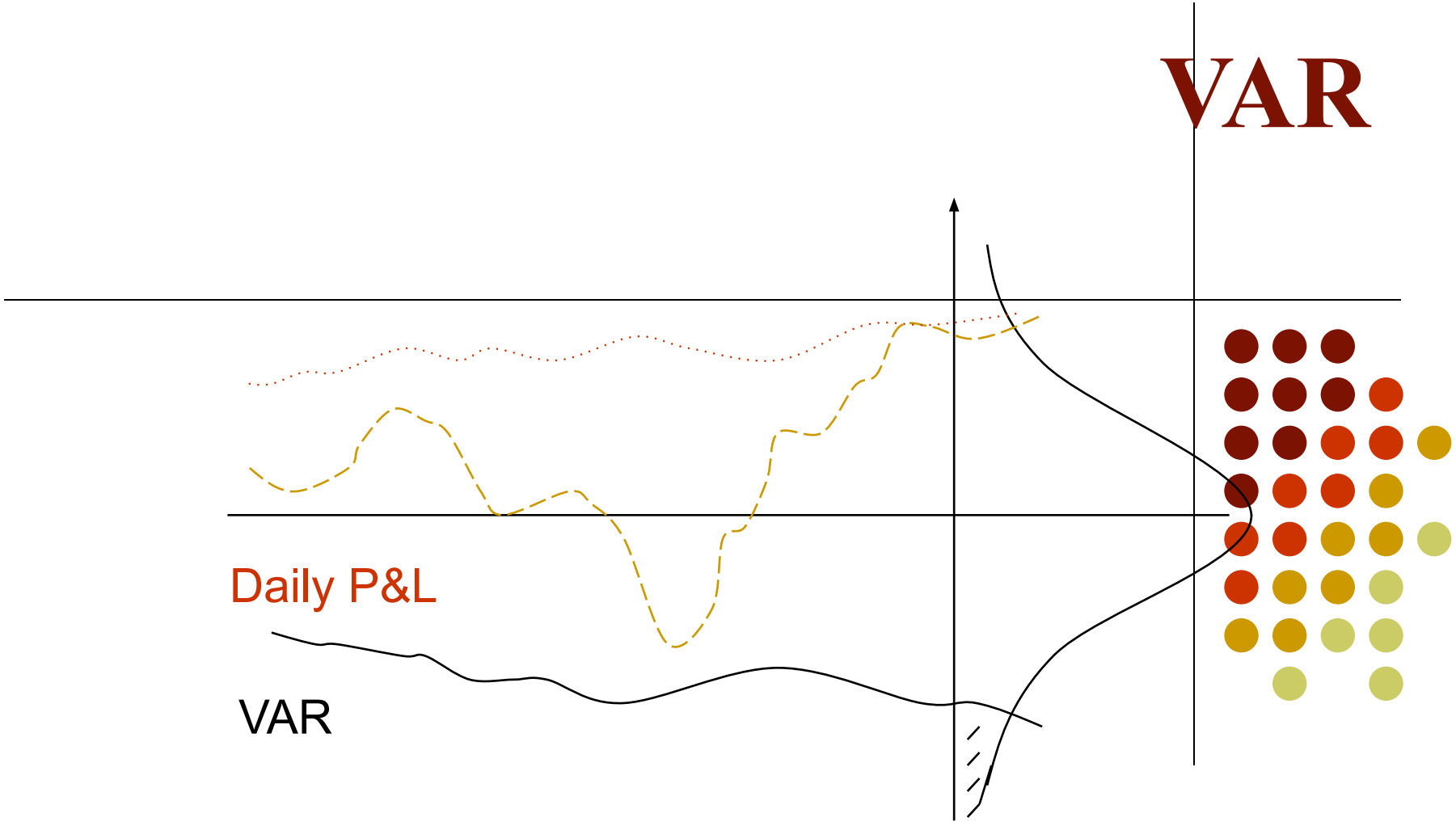
Z_{α} **is the constant giving the appropriate one-tailed Confidence Interval.**

σ_p **is the annualized standard deviation of the portfolio's return**

Δt **is the holding period horizon**

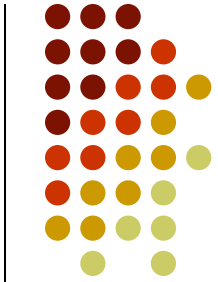
FS Factor Sensitivity

VAR



- III VALUE AT RISK - CALCULATIONS

Sample VAR Calculations



- Let us consider the following positions:
- Long EUR against the USD : $ 1 MM
- Long JPY against the USD : $ 1 MM
- Each of these positions has a factor sensitivity of +10,000

Sample VAR Calculations

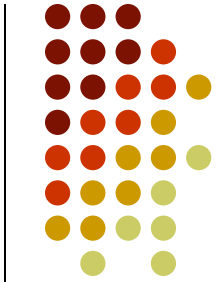
- Annual volatility of DEM is 9%
- Volatility for N days = annual volatility x SQRT(N/ T)
- where T is the total number of trading days in a year (260)
- Therefore, 1 day volatility of DEM= 9 x SQRT (1/260)
- = 0.56%
- This is 1σ,
- so, 2.326σ = 2.326 x 0.56% = 1.30%

Sample VAR Calculations

- Now, a 1% change has an impact of 10,000 (FS)
- So, a 1.30% change will have an impact of
- 1.30 x 10,000 = 13,000
- This represents the impact of a 2.326 SD change in the market factor over a 1 day period

- Thus, in 1 out of 100 days we may cross actual loss of
- $ 13,000. Our Value at Risk (VAR) is $13,000 on this position

Sample VAR Calculations



- Similarly, for JPY, the annual volatility is 12%
- The 1 day volatility = 12 x SQRT (1/260) = 0.74%
- 2.326 SD = 2.326 x 0.74 = 1.73%
- Impact of a 1% change = 10,000 (FS)
- So, impact of a 1.73% change = 17,310
- Our VAR on this position is $ 17,310

- IV VALUE AT RISK FOR CORPORATIONS

VAR FOR CORPORATIONS

- Trading portfolios
- Longer time horizons for close outs
- Business risk as opposed to trading risk
- Holding period, business time horizon
- VAR as a percentage of Capital

VAR FOR CORPORATIONS

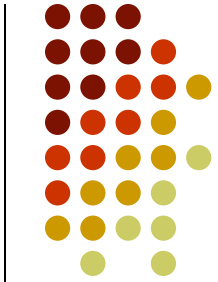
- Identify market variables impacting business
- Map income sensitivity to market variables - Scenario analysis
- Based on volatilities of market factors and their correlations, arrive at a worst case scenario given the degree of confidence
- Worst case income projection - acceptable or not?
- Hedge to reduce VAR

VAR FOR CORPORATIONS

- Hedging tools
- Forward FX
- Currency swaps
- Interest Rate swaps
- Options on non-INR market variables
- Commodity futures
- Commodity derivatives

- V VALUE AT RISK- A FEW COMMENTS

Significance of VAR



- Applicable mainly to trading portfolios
- Regulatory capital requirements
- Provides senior executives with a simple and effective way to monitor risk.
- VAR incorporates portfolio effects.
- Uses history to predict near term future.

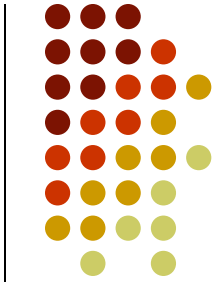
VAR : A Few Comments

- VAR does not represent the maximum loss
- VAR does not represent the actual loss
- It represents the potential loss associated with a specified level of confidence. In this case, 99% over 1 day
- Increased VAR represents increased risk, decrease in VAR represents decrease in risk
- VAR limit is related to revenue potential

Where to use VAR?

- Macro measure. High level monitoring, managing, eg. Regional level
- Currently used mainly for trading limits.
- Strategic planning - Allocation of resources
- However..
- Not an efficient day to day tool.
- Components - FS, Market volatility, Defeasance period, Correlations are all integral parts of trading strategy.

How to use Var



- Stress Testing : * “worst case” scenario
- * Multiple Stress Scenarios
- * Should include not only price moves
- In excess of 2SD, but also other
- market events likely to adversely
- affect business
- Back Testing : Compares actual daily P&L movements predicted variance of P&L

General Market Risk Issues

- Integrity - Rate Reasonability
- - At Inception
- - Revaluation
- Model Certification
- Control Mechanisms / Checks and Balances
- Corporate Culture!

Review

- Loss occurs only if rates move adversely to the position
- The loss is proportional to the sensitivity of the position
- The loss is proportional to size of the adverse movement
- Loss = FS multiplied by the adverse rate movement
- We cannot limit adverse rate movements in the marketplace
- We can limit our sensitivity (P&L impact) with FSL
- FSL should be set against potential adverse movement
- Potential adverse movements estimated through volatility