Gas Dynamics
(Introduction to Compressible Flow)
Lecture 6a and 6b

Yahia M. Fouda
Mechanical Power Engineering
Mansoura University

Chapter 4
Rayleigh Flow

Effect of upstream Mach number on the flow

	Subsonic Inlet (M<1) **MLT1**	**Supersonic Inlet (M>1)** **MGT1**
q<q* L<L*	MLT1a No problem, exit M<1	MGT1a No problem, exit M>1
q=q* L=L*	MLT1b No problem, exit M=1	MGT1b No problem, exit M=1
q>q* L>L*	MLT1c Problem, what will happen?	MGT1c Problem, what will happen?

Critical added heat is at Ma=1

Energy

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

$$h = C_p T \qquad C_p = \frac{\gamma R}{\gamma - 1} \, .$$

Continuity

$$\rho_1 u_1 = \rho_2 u_2 = \dot{m}/A = G \, ,$$

Energy eq. becomes

$$\frac{\gamma}{\gamma - 1} P_1 v_1 + \frac{1}{2} v_1^2 G^2 + q^* = \frac{\gamma}{\gamma - 1} P v + \frac{1}{2} v^2 G^2$$

State 1 is constant and 2 is an arbitrary point (variable state)

$$\frac{\gamma}{\gamma-1}P_1v_1+\frac{1}{2}v_1^2G^2+q^*=\frac{\gamma}{\gamma-1}Pv+\frac{1}{2}v^2G^2$$

Differentiating this equation with respect to v, we get

$$\frac{\gamma}{\gamma-1}\left(v\frac{dP}{dv}+P\right)+vG^2=0$$

Thus $$\frac{dP}{dv}=-\frac{\gamma-1}{\gamma}G^2-\frac{P}{v}$$

Equating this to $-G^2$, the slope of the Rayleigh line, leads to

$$-\frac{\gamma-1}{\gamma}G^2-\frac{P}{v}=-G^2 \qquad \frac{P}{v}=\frac{G^2}{\gamma}$$

Substituting $G=\rho u$ and $v=1/\rho$, $$u_2=\sqrt{\frac{\gamma P}{\rho}}=a_2$$

At the point of tangency of H-curve and Rayleigh line the Mach number ALWAYS equals one

Effect of upstream Mach number on the flow

	Subsonic Inlet ($M<1$) **MLT1**	**Supersonic Inlet ($M>1$)** **MGT1**
$q<q^*$ $L<L^*$	MLT1a No problem, exit $M<1$	MGT1a No problem, exit $M>1$
$q=q^*$ $L=L^*$	MLT1b No problem, exit $M=1$	MGT1b No problem, exit $M=1$
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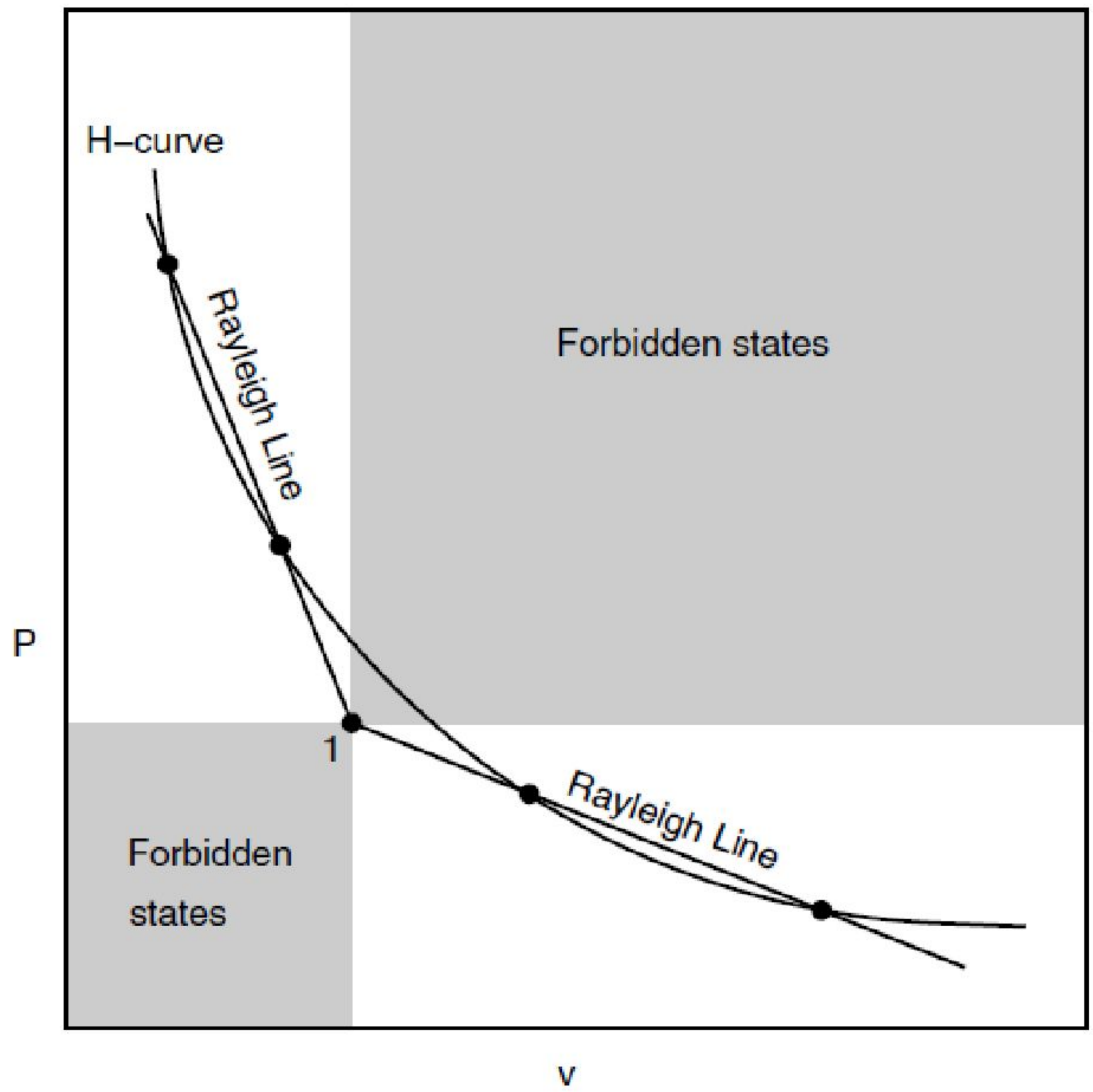
Beyond the tangent H-curve



Subsonic inlet (state 1) with q>q* lower Rayleigh line

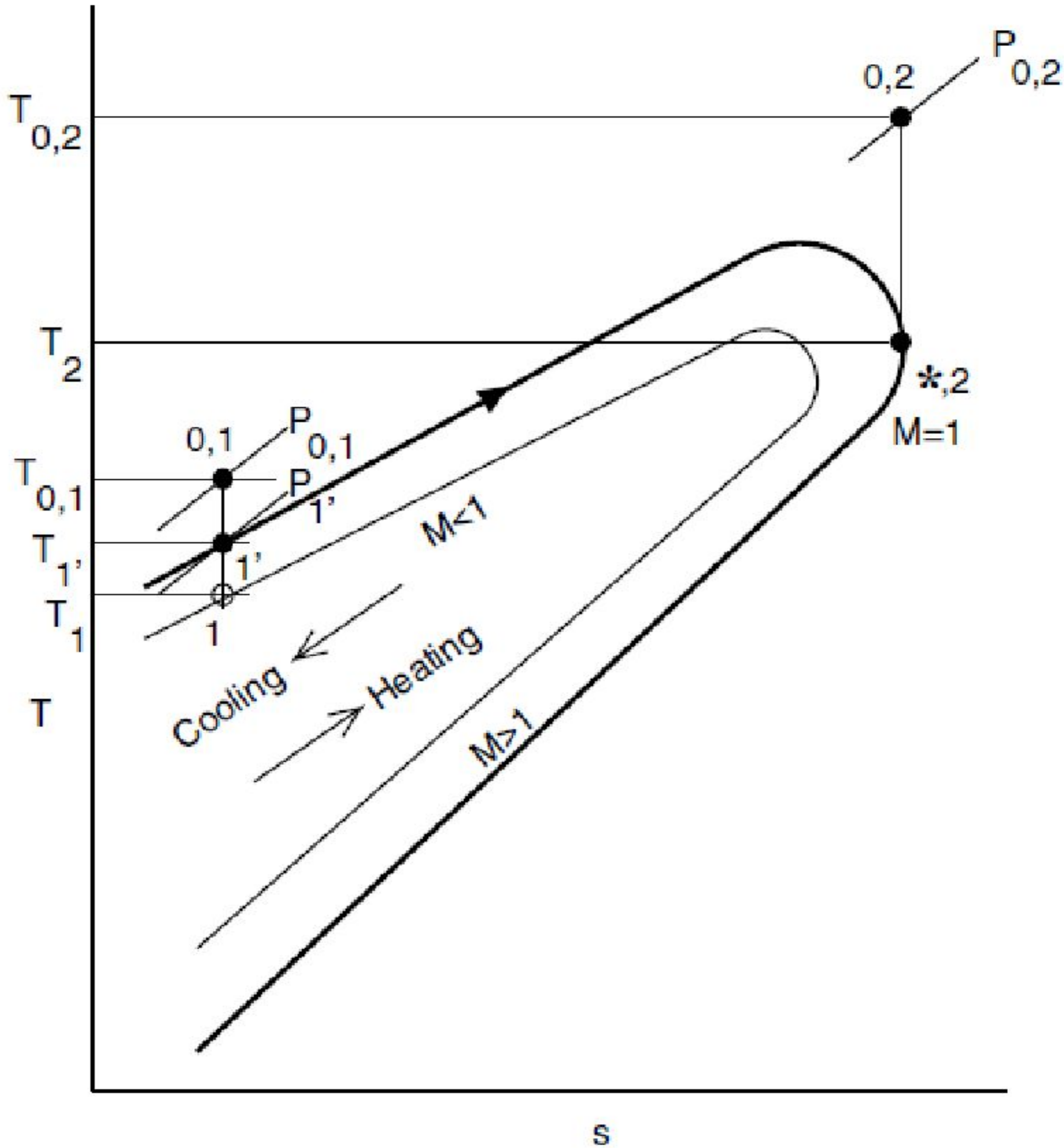


Fig. 4.4: (a) Illustration of heat addition process with $q > q^*$ for $M_1 < 1$.

Supersonic inlet (state 1) with q>q* Shock wave

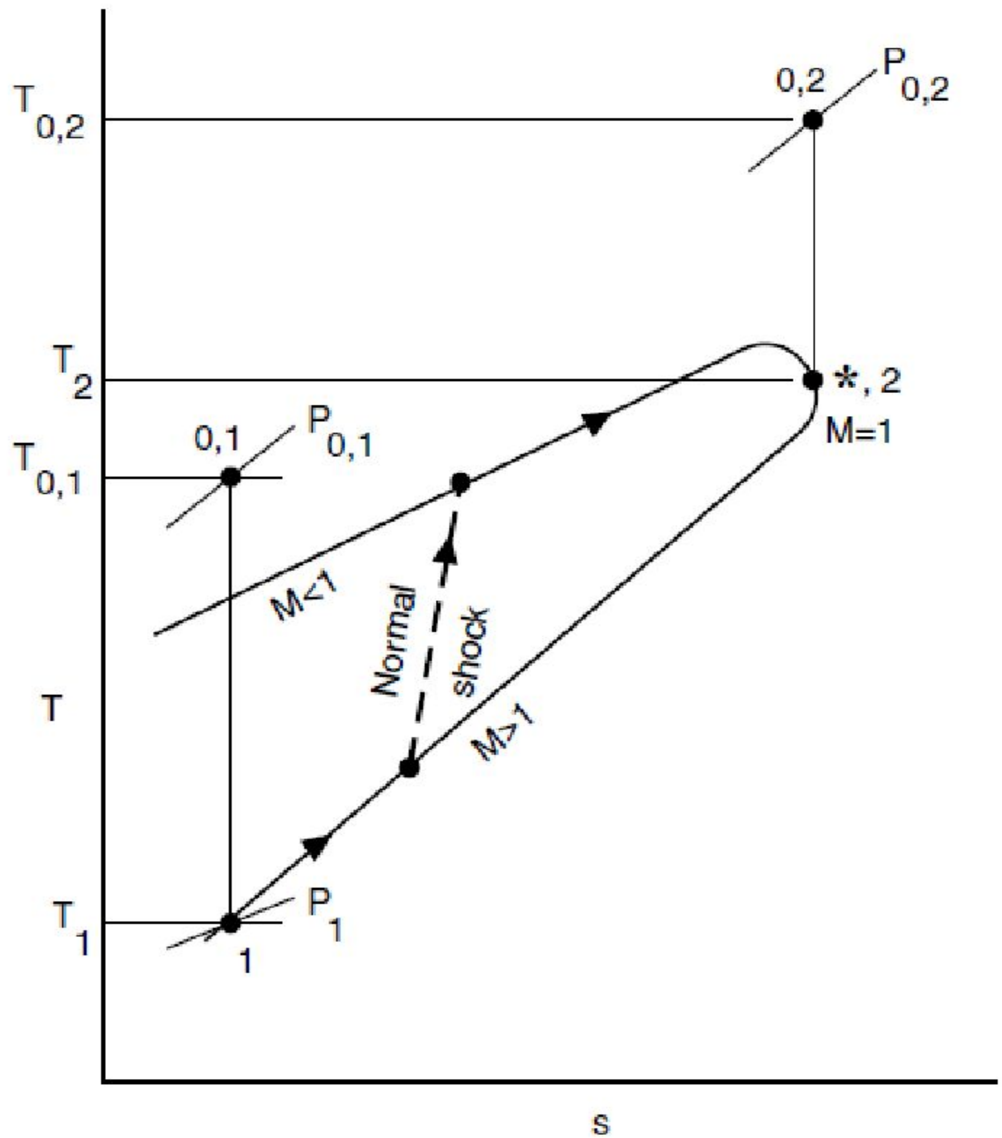


Fig. 4.4: (b) Illustration of heat addition process with $q > q^*$ for $M_1 > 1$.

Chapter 4: Lecture Problems

1. In Rayleigh flow, prove that the point of tangency of H-curve and Rayleigh line represents a sonic flow.
2. Stagnation pressure proof.

End of Chapter 4

Questions???