Lecture 3. Interest Rates

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QUOTED VS. EFFECTIVE RATES

- Quoted rate -**the annual percentage rate (APR)** - annual rate based on interest being computed once a year.
- **The EAR (Effective Annual Rate)** is the true rate of return to the lender and the true cost of borrowing to the borrower.
- **An EAR**, also known as **the annual percentage yield (APY)** on an investment, is calculated from a given APR and frequency of compounding (m) by using the following equation:

$$EAR = \left(1 + \frac{APR}{m}\right)^{(m)} - 1$$

EXAMPLE: QUOTED VS. EFFECTIVE RATES

Problem: Calculating APY or EAR.

A Bank has advertised one of its loan offerings as follows:

"We will lend you $100,000 for up to 5 years at an APR of 9.5% (interest compounded monthly.)"

If you borrow $100,000 for 1 year and pay it off in one lump sum at the end of the year, how much interest will you have paid and what is the bank's APY?

SOLUTION: QUOTED VS. EFFECTIVE RATES

Nominal annual rate = APR = 9.5%

Frequency of compounding = C/Y = m = 12

Periodic interest rate = APR/m = 0.095/12 = 0.0079167

$$EAR = \left(1 + \frac{APR}{m}\right)^{(m)} - 1$$

APY or EAR = $(1+ 0.0079167)^{12}$ - 1 = $(1.0079167)^{12}$ - 1 = 1.099247 - 1 □<u>9.92%</u>

Payment at the end of the year = 1.099247*100,000 □ <u>$109,924.70</u>

Amount of interest paid = $109, 924.7 - $100,000 □ <u>$9,924.7</u>

Effect of Compounding Periods on the Time Value of Money Equations

- In TVM equations the periodic rate (r%) and the number of periods (n) are taken into account.
- The greater the frequency of payments made per year, the lower the total amount paid.
  - More money goes to principal and less interest is charged.
- The interest rate should be consistent with the frequency of compounding and the number of payments involved.

Example I: Effect of Compounding Periods on the Time Value of Money Equations

Problem: Monthly versus Quarterly Payments

- Patrick needs to borrow $70,000 to start a business expansion project. His bank agrees to lend him the money over a 5-year term at an APR of 9.25% and will accept either monthly or quarterly payments with no change in the quoted APR.
- Calculate the periodic payment under each alternative and compare the total amount paid each year under each option.
- Which payment term should Patrick accept and why?

Solution: Effect of Compounding Periods on the Time Value of Money Equations

$$PV = PMT \times \frac{\left[1 - \left(\frac{1}{(1+r)^n}\right)\right]}{r}$$

OR

$$PV = PMT \times PVIFA_{(r,n)}$$

$$PMT = \frac{PV}{\frac{\left[1 - \left(\frac{1}{(1+r)^n}\right)\right]}{r}}$$

OR

$$PMT = \frac{PV}{PVIFA_{(r,n)}}$$

$$PMT = \frac{PV \times r}{1 - \left(\frac{1}{(1+r)^n}\right)}$$

If it is compounded <u>m</u> times per year, then:

1) To get periodic rate <u>r</u> → divide *APR* in decimal points by *m* →*(APR/m)*

2) To get number of compounding periods during several years <u>n</u> → Multiply number of years (Y) by *m* →*(Y×m)*

Solution: Effect of Compounding Periods on the Time Value of Money Equations

Calculate monthly payment:

n=5 years×12 months = 60;

r = 0.0925/12

PV = 70,000 → PMT= 1,461.59

$$PMT = \frac{70{,}000 \times (0.0925/12)}{1 - \left(\frac{1}{(1 + 0.0925/12)^{60}}\right)} = 1{,}461.59$$

Calculate quarterly payment:

n=5 years×4 quarters =20;

r = 0.0925/4

PV = 70,000 → PMT= 4,411.15

$$PMT = \frac{70{,}000 \times (0.0925/4)}{1 - \left(\frac{1}{(1 + 0.0925/4)^{20}}\right)} = 4{,}411.15$$

Total amount paid per year under each payment type:

With monthly payments = 12× $1,461.59 = $17,539.08

With quarterly payments = 4 × $4,411.15 = $17,644.60

Solution: Effect of Compounding Periods on the Time Value of Money Equations

Total interest paid under monthly compounding:

- Total paid - Amount borrowed

 = 60*$1,461.59 - $70,000

 = $87,695.4 - $70,000

 = $17,695.4

Total interest paid under quarterly compounding:

- 20 *$4,411.15 -$70,000

 = $88,223 - $70,000

 = $18,223

Since less interest is paid over the 5 years with the monthly payment terms, Patrick should accept monthly rather than quarterly payment terms.

Example II: Effect of Compounding Periods on the Time Value of Money Equations

Jill was depositing $3,000 at the end of each year. If she switches to a monthly savings plan and put 1/12 of the $3000 away each month ($250), how much will she have in 10 years at 8% APR?

Solution: Effect of Compounding Periods on the Time Value of Money Equations

$$FV = PMT \times \frac{(1+r)^n - 1}{r} \quad \textbf{OR} \quad FV = PMT \times FVIFA_{r,n}$$

$$FV = 3000 \times \frac{(1+0.08)^{10} - 1}{0.08} = 43459.6874$$

$$FV = 250 \times \frac{(1+0.08/12)^{120} - 1}{0.08/12} = 45736.5087$$

The more frequent the compounding, the larger the cumulative effect.

- **If it is compounded *m* times per year, then:**
- **To get periodic rate *r* → divide *APR* in decimal points by *m* →*(APR/m)***
- **To get number of compounding periods during several years → multiply number of years (Y) by *m* →*(Y×m)***

Nominal interest rate vs Real interest rate

- Nominal interest rates (r) are made up of two primary components: expected inflation rate (h) and the real interest rate (r*)
- The real rate of interest is a reward for waiting
- Nominal rate: $r = r^* + h$ (approximation)
- Fischer Effect: the true nominal rate is made up of three components: the real rate, the inflation rate and the product of real and inflation rates:

$$r = r^* + h + (r^* \times h)$$ (*in decimal points*) or

$$(1+r) = (1+r^*) \times (1+h)$$

Nominal interest rate vs Real interest rate

Example: If you have $ 100 today and lend it to someone for a year at a nominal rate of interest of 11.3%, you will get back $111.3 in 1 year. But if during the year prices of goods and services rise by 5%, it will take $105 at year-end to purchase the same goods and services that $100 purchased at the beginning of the year. What was the real interest rate for year?

REAL INTEREST RATE

- **The quick answer:** 11.3% - 5% = 6.3%.

Approximation:

Nominal interest rate – Inflation = Real interest rate

- To get more precise answer, use **Fisher relation**:

$$1 + h = \frac{1 + r}{1 + r^*}$$

$$r^* = \frac{1 + r}{1 + h} - 1 = \frac{1.113}{1.05} - 1 = 0.06 = 6\%$$

r **- the nominal interest rate;**

r* **- the real rate;** ***h*** **- the inflation rate**

Default Risk Premium, Risk Free Rate & Maturity Risk Premium

The rate of return on investments *(r)* would have to include a default risk premium (*dp*) and a maturity risk premium (*mp*):

$$r = r^* + h + d_p + m_p$$

- **Default risk premium** compensates for a potential losses due to default (bankruptcy) of a borrower (contingent upon existence of collateral; the type of collateral, if any; and upon category of a borrower – certain categories of borrowers default more frequently then others)
- The base rate which has no potential for default is called **a risk free rate**: $r = r^* + h$
- **Maturity risk premium** compensates for additional waiting time it takes to receive repayment in full.

THE END