

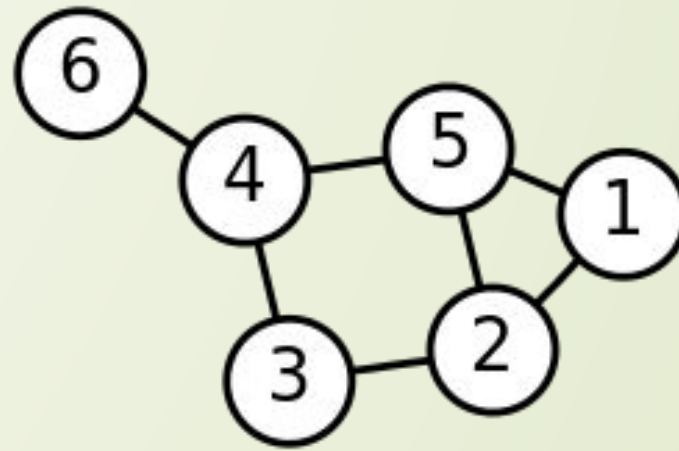


Introduction to graphs

Lyzhin Ivan, 2015

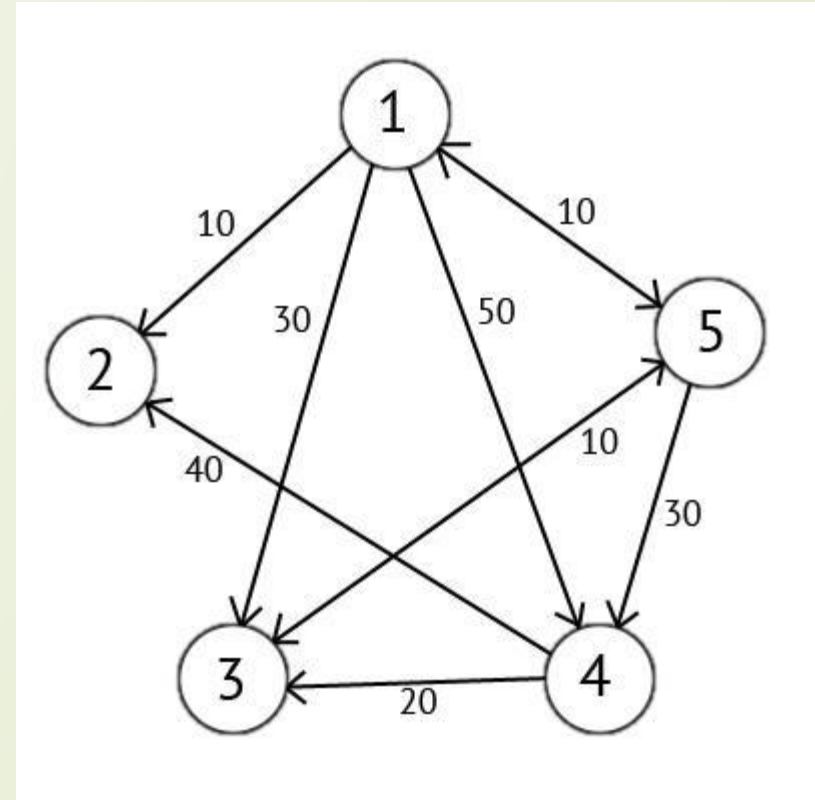
Definition

- $G = (V, E)$
- V – vertexes
- E – edges



Types

- Directed/undirected
- Weighted/unweighted
- Simple graph/multigraph
- Connected/unconnected
- Bipartite
- With cycles/without cycles

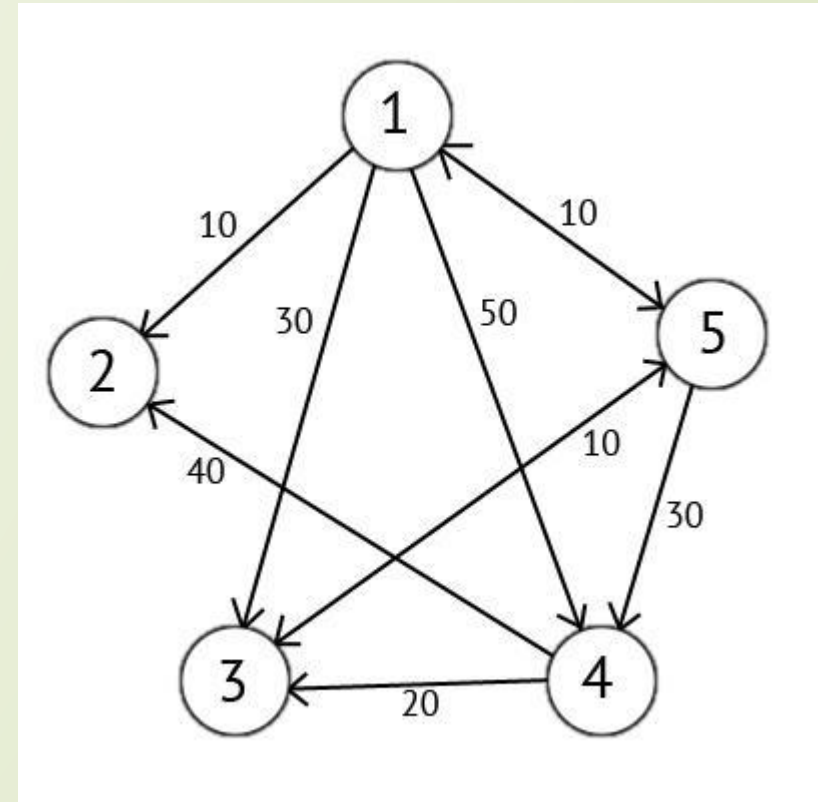


Ways of presenting in memory

Adjacency matrix

	1	2	3	4	5
1	0	10	30	50	10
2	0	0	0	0	0
3	0	0	0	0	10
4	0	40	20	0	0
5	10	0	10	30	0

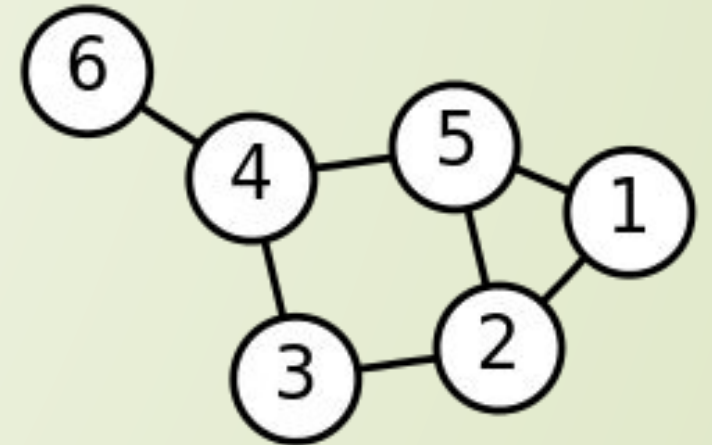
Memory: $O(|V|^2)$



Ways of presenting in memory

Incidence matrix

	1	2	3	4	5	6	7
1	0	0	1	0	0	0	1
2	0	0	0	0	1	1	1
3	0	0	0	1	0	1	0
4	1	1	0	1	0	0	0
5	0	1	1	0	1	0	0
6	1	0	0	0	0	0	0

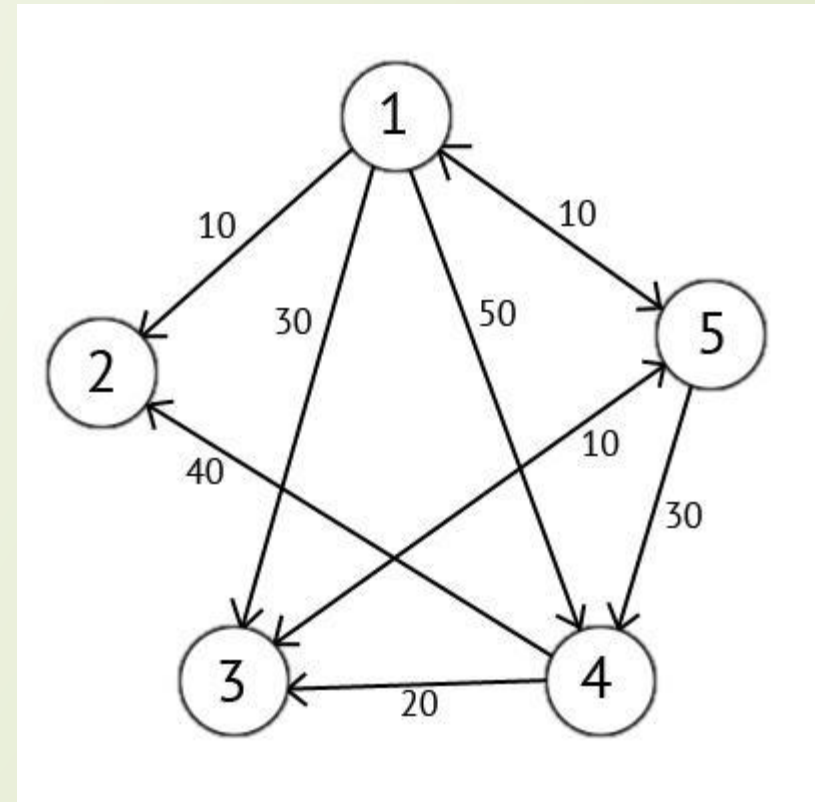


Memory: $O(|V| * |E|)$

Ways of presenting in memory

List of edges

u	v	w
1	2	10
1	3	30
1	4	50
1	5	10
3	5	10
4	2	40
4	3	20
5	1	10
5	3	10
5	4	30



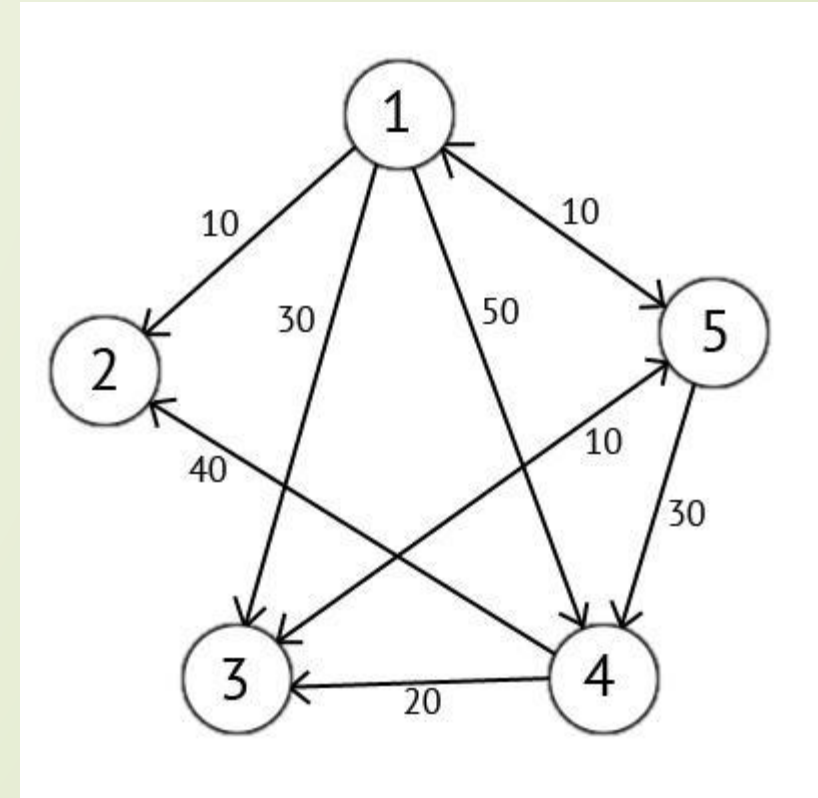
Memory: $O(|E|)$

Ways of presenting in memory

Adjacency list

1	(2, 10)	(3, 30)	(5, 50)	(5, 10)
2				
3	(5, 10)			
4	(2, 40)	(3, 20)		
5	(1, 10)	(3, 10)	(4, 30)	

Memory: $O(|E|)$



Problems without explicit graph

- Labyrinth
- Number of objects

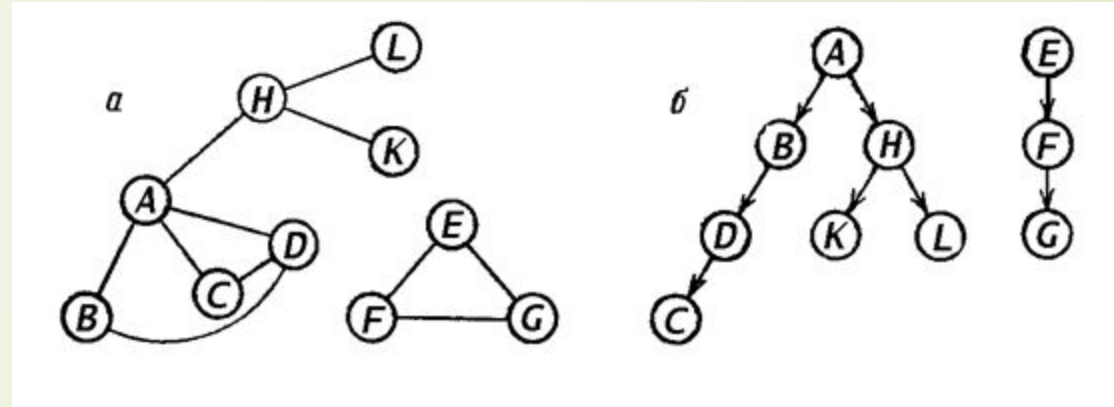
```
S...###...#.
..#...##.#.
#...#.#...#.
..##...#.#.
#.....
##.##.##.#
...#...#..
#####F
```

```
.....##
..#.....#
.#.#...#..
..#...###.
.....#..
#####
#####.##.
.....##.
```


Basic algorithms

Depth-First Search (DFS)

```
void dfs(int u)
{
    if (used[u]) return;
    used[u] = true;
    for (auto v : g[u])
        dfs(v);
}
```

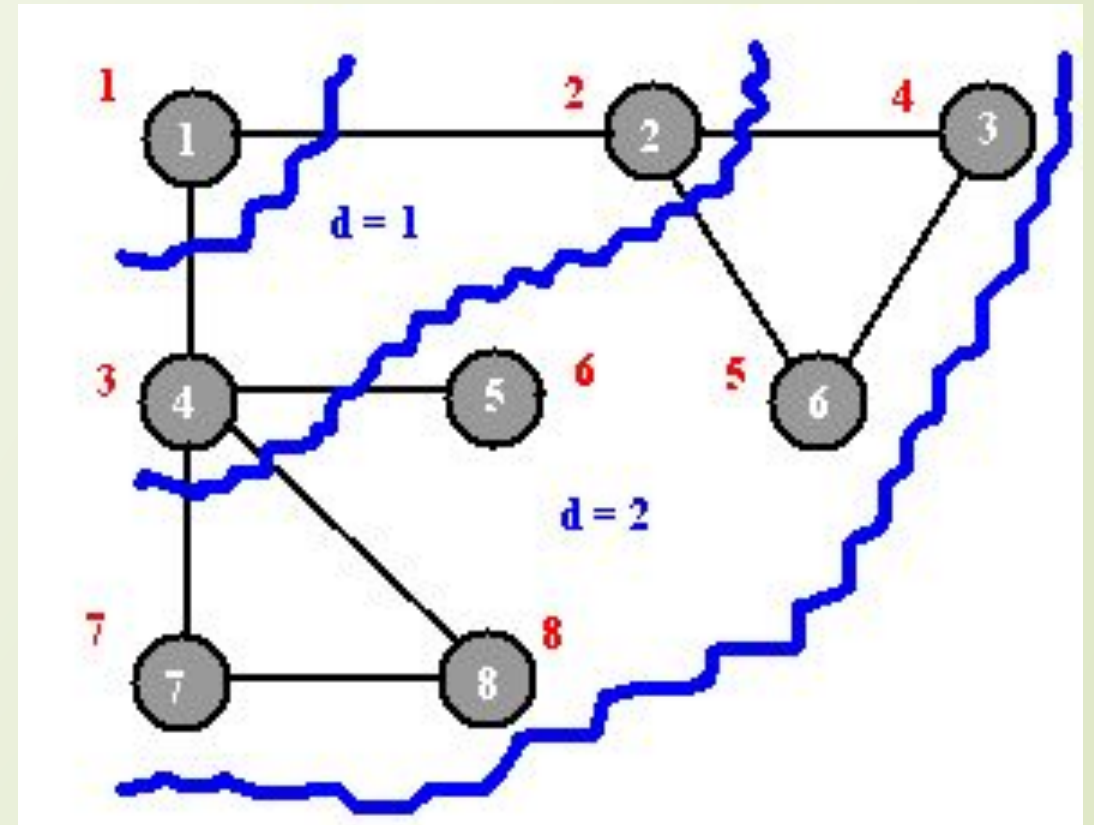


Complexity: $O(|V| + |E|)$

Basic algorithms

Breadth-First Search (BFS)

```
void bfs(int s)
{
    queue<int> q;
    q.push(s);
    used[s] = true;
    while(!q.empty())
    {
        int u = q.front();
        q.pop();
        for(auto v: g[u])
            if(!used[v])
            {
                q.push(v);
                used[v] = true;
            }
    }
}
```



Complexity: $O(|V| + |E|)$



Examples



- Find cycle in graph
- Count number of connected components in graph
- Find distance and path from one vertex to each other in unweighted graph



Home task



- <http://ipc.susu.ac.ru/210-2.html?problem=227>
- <http://ipc.susu.ac.ru/210-2.html?problem=2236>
- <http://ipc.susu.ac.ru/210-2.html?problem=54>
- <http://ipc.susu.ac.ru/210-2.html?problem=1989>
- <http://ipc.susu.ac.ru/210-2.html?problem=55>
- <http://ipc.susu.ac.ru/210-2.html?problem=671>
- <http://codeforces.com/problemset/problem/115/A>
- <http://codeforces.com/problemset/problem/277/A>