Lesson 18

CONSTRAINTS APPLICATION. SOLUTION AND, DISPLACEMENTS, ROTATIONS, STRESSES AND STRAINS

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What are constraints? [edit]

- The particles could be restricted to travel along a curve or surface. Specifically one could have some function of the coordinates of each particle and time vanish. These restrictions are either kinematical or geometrical in nature.

$$f\left(\vec{r}_1, \vec{r}_2, \vec{r}_3, \cdots, t\right) = 0$$

This is called a **holonomic** constraint. For example we could have

$$\left(\vec{r}_i - \vec{r}_j\right)^2 - c_{ij}^2 = 0$$

which expresses that the distances between two particles that make up a rigid body are fixed.

- There are **non-holonomic** constraints. For example, one could have

$r^2 - a^2 \geq 0$ for a particle travelling outside the surface of a sphere or constraints that depend on velocities as well,

$$f\left(\vec{r}_1, \vec{r}_2, \vec{r}_3, \cdots, \vec{v}_1, \vec{v}_2, \vec{v}_3, \cdots, t\right) = 0.$$

A familar example of the latter is a ball rolling on a surface.

In continuum mechanics, **stress** is a physical quantity that expresses the internal forces that neighbouring particles of a continuous material exert on each other, while strain is the measure of the deformation of the material. For example, when a solid vertical bar is supporting an overhead weight, each particle in the bar pushes on the particles immediately below it. When a liquid is in a closed container under pressure, each particle gets pushed against by all the surrounding particles. The container walls and the pressure-inducing surface (such as a piston) push against them in (Newtonian) reaction. These macroscopic forces are actually the net result of a very large number of intermolecular forces and collisions between the particles in those molecules. Stress is frequently represented by a lowercase Greek letter sigma (σ).

Strain inside a material may arise by various mechanisms, such as *stress* as applied by external forces to the bulk material (like gravity) or to its surface (like contact forces, external pressure, or friction). Any strain (deformation) of a solid material generates an internal *elastic stress*, analogous to the reaction force of a spring, that tends to restore the material to its original non-deformed state. In liquids and gases, only deformations that change the volume generate persistent elastic stress. However, if the deformation changes gradually with time, even in fluids there will usually be some *viscous stress*, opposing that change. Elastic and viscous stresses are usually combined under the name *mechanical stress*.

Beams are structural elements with various engineering applications like roofs, bridges, mechanical assemblies, etc. In general, a beam is slender, straight, rigid, built from isotropic materials, and most important, subjected to loads perpendicular to their longitudinal axis. If instead of perpendicular loads the same structural member would be subjected to longitudinal loads it would be called column or post. If the same member would be subjected to a torque, it would be called and treated as a shaft. Therefore, when identifying mechanical or structural components, consideration of the manner of loading is very important.

A **rotation** is a circular movement of an object around a center (or point) of rotation. A three-dimensional object can always be rotated about an infinite number of imaginary lines called *rotation axes* . If the axis passes through the body's center of mass, the body is said to rotate upon itself, or spin. A rotation around an external point, e.g. the planet Earth around the Sun, is called a *revolution* or *orbital revolution*, typically when it is produced by gravity. The axis is called a **pole**.

Rotation around a fixed axis or **about a fixed axis of revolution** or **motion with respect to a fixed axis of rotation** is a special case of rotational motion. The fixed axis hypothesis excludes the possibility of an axis changing its orientation, and cannot describe such phenomena as wobbling or precession. According to Euler's rotation theorem, simultaneous rotation along a number of stationary axes at the same time is impossible. If two rotations are forced at the same time, a new axis of rotation will appear.

Thanks!

Any questions?

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