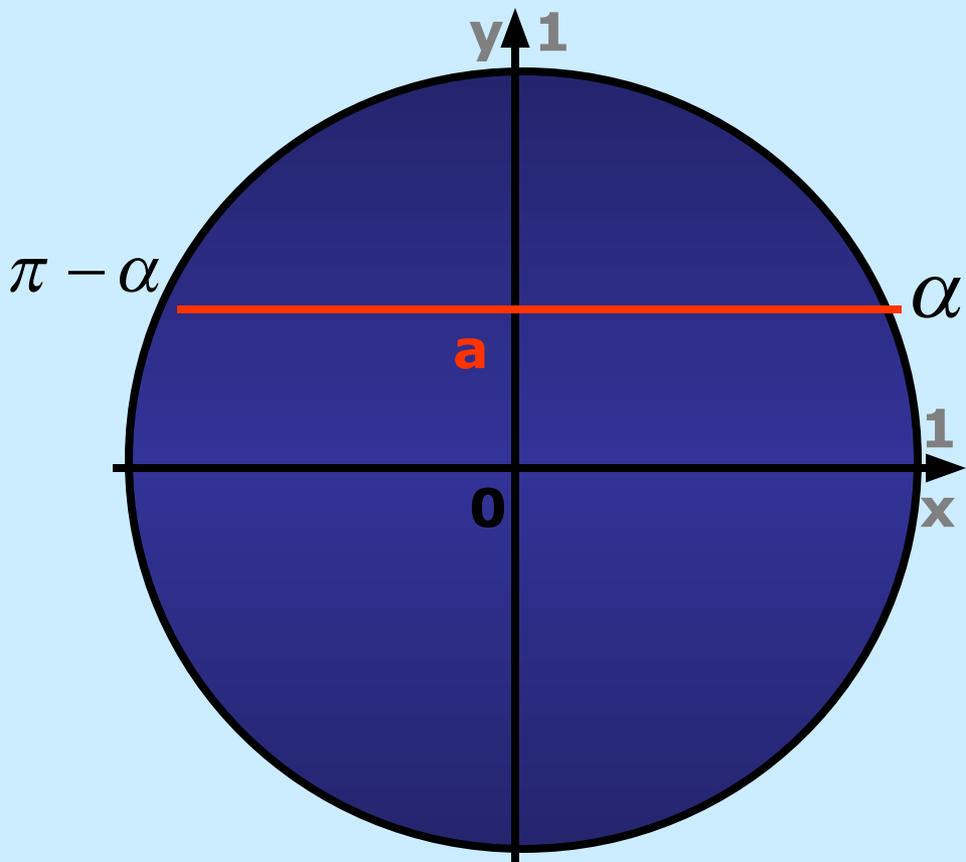


**Решение уравнений $\sin x = a$.
Понятие арксинуса числа.**

Уравнение $\sin x = a$



$$\sin x = a; |a| \leq 1.$$

$$x = \alpha + 2\pi k;$$

$$x = \pi - \alpha + 2\pi k;$$

$$k \in \mathbb{Z}.$$

$$\alpha = \arcsin a$$

АРКСИНУС ЧИСЛА

Определение. Арксинусом числа $a \in [-1;1]$ называется

такое число $\alpha \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$, синус которого равен a

$$\sin(\arcsin a) = a,$$

$$-\frac{\pi}{2} \leq \arcsin a \leq \frac{\pi}{2},$$

$$-1 \leq a \leq 1$$

АРКСИНУС ЧИСЛА

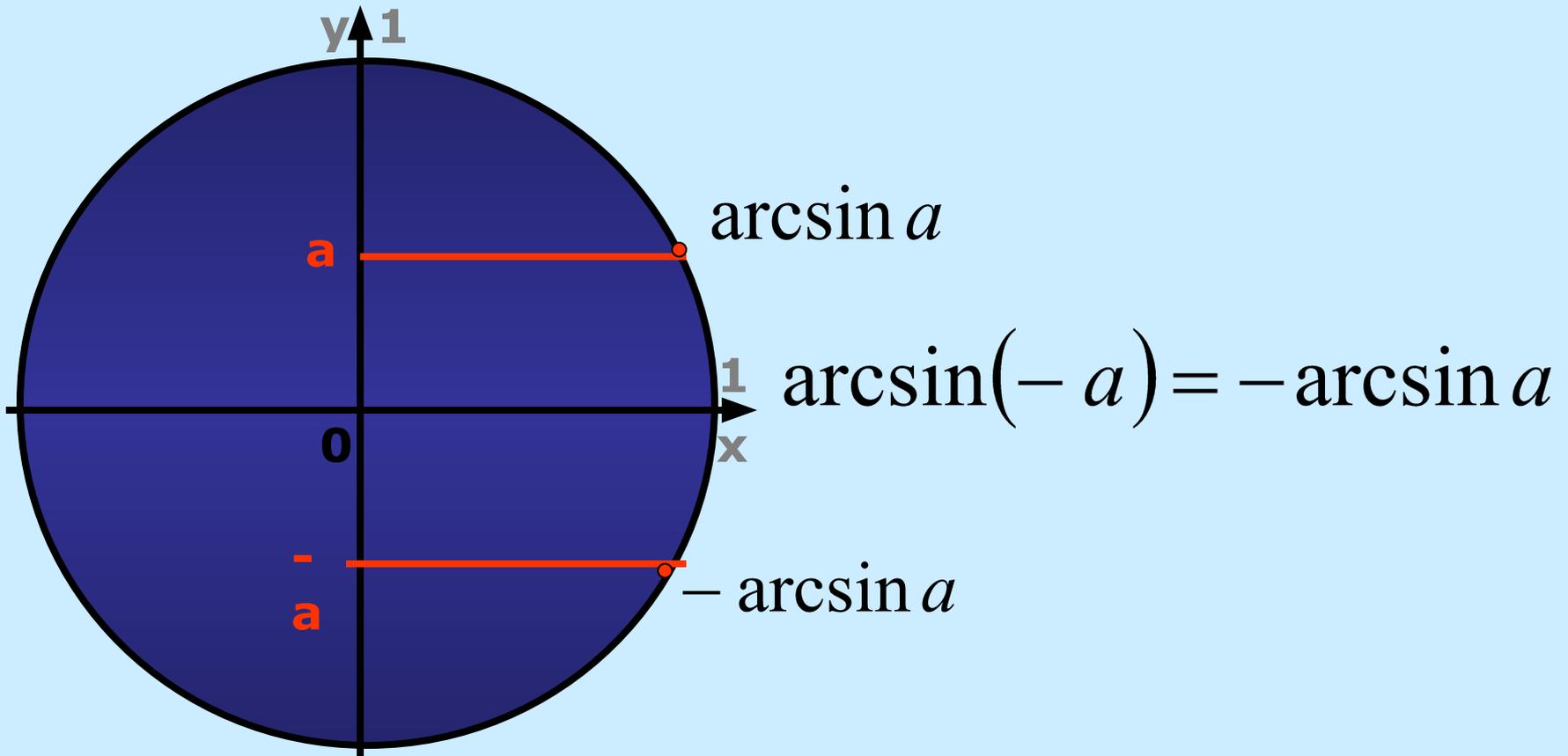
- **Например**

$$\arcsin \frac{\sqrt{2}}{2} = \frac{\pi}{4}; \quad \begin{array}{l} \mathbf{T.} \\ \mathbf{K.} \end{array} \quad -\frac{\pi}{2} \leq \frac{\pi}{4} \leq \frac{\pi}{2}; \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}.$$

$$\arcsin 0 = 0; \quad \begin{array}{l} \mathbf{T.} \\ \mathbf{K.} \end{array} \quad -\frac{\pi}{2} \leq 0 \leq \frac{\pi}{2}; \sin 0 = 0.$$

$$\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}; \quad \begin{array}{l} \mathbf{T.} \\ \mathbf{K.} \end{array} \quad -\frac{\pi}{2} \leq \frac{\pi}{3} \leq \frac{\pi}{2}; \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

АРКСИНУС ЧИСЛА ОСНОВНЫЕ ФОРМУЛЫ



АРКСИНУС ЧИСЛА

ОСНОВНЫЕ ФОРМУЛЫ

• Например

$$1. \quad 3 \arcsin \frac{\sqrt{2}}{2} - 2 \arcsin \left(-\frac{1}{2} \right) = 3 \arcsin \frac{\sqrt{2}}{2} + 2 \arcsin \frac{1}{2} =$$

$$= 3 \cdot \frac{\pi}{4} + 2 \cdot \frac{\pi}{6} = \frac{3\pi}{4} + \frac{\pi}{3} = \frac{13}{12} \pi$$

$$2. \quad \frac{1}{2} \arcsin \left(-\frac{\sqrt{3}}{2} \right) - 2 \arcsin 1 + \sqrt{5} \arcsin 0 =$$

$$= -\frac{1}{2} \arcsin \frac{\sqrt{3}}{2} - 2 \cdot \frac{\pi}{2} + \sqrt{5} \cdot 0 = -\frac{1}{2} \cdot \frac{\pi}{3} - \pi = -\frac{\pi}{6} - \pi = -\frac{7}{6} \pi$$

АРКСИНУС ЧИСЛА

ОСНОВНЫЕ ФОРМУЛЫ

$$\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha}$$

$$\alpha = \arcsin a \quad \arcsin a \in \left[-\frac{\pi}{2}; \frac{\pi}{2} \right]$$

$$\cos(\arcsin a) = \sqrt{1 - \sin^2(\arcsin a)} = \sqrt{1 - a^2}$$

АРКСИНУС ЧИСЛА

ОСНОВНЫЕ ФОРМУЛЫ

$$\sin(\arcsin a) = a$$

• Например

$$3. \sin\left(\arcsin\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2} \quad 4. \sin\left(\arcsin\frac{6}{7}\right) = \frac{6}{7}$$

$$5. \cos\left(\arcsin\frac{1}{2}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$6. \cos\left(\arcsin\left(-\frac{2}{5}\right)\right) = \sqrt{1 - \left(-\frac{2}{5}\right)^2} = \sqrt{1 - \frac{4}{25}} =$$

$$\sqrt{\frac{21}{25}} = \frac{\sqrt{21}}{5}$$

$$\cos(\arcsin a) = \sqrt{1 - a^2}$$

АРКСИНУС ЧИСЛА

ОСНОВНЫЕ ФОРМУЛЫ

$$\sin(\arcsin a) = a, \arcsin a \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right], a \in [-1; 1]$$

$$\arcsin(-a) = -\arcsin a$$

$$\cos(\arcsin a) = \sqrt{1 - a^2}$$

$$\arcsin(\sin \alpha) = \alpha, \alpha \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$$

АРКСИНУС ЧИСЛА

ОСНОВНЫЕ ФОРМУЛЫ

- **Например**

$$7. \quad \operatorname{tg}\left(5 \cdot \arcsin \frac{\sqrt{2}}{2}\right) = \operatorname{tg} \frac{5}{4} \pi = \operatorname{tg}\left(\pi + \frac{\pi}{4}\right) = \operatorname{tg} \frac{\pi}{4} = 1$$

$$8. \quad \operatorname{ctg}\left(\arcsin \frac{1}{3}\right) = \frac{\cos\left(\arcsin \frac{1}{3}\right)}{\sin\left(\arcsin \frac{1}{3}\right)} = \frac{\sqrt{1 - \frac{1}{9}}}{\frac{1}{3}} =$$

$$\sqrt{\frac{8}{9}} \cdot 3 = 2\sqrt{2}$$

АРКСИНУС ЧИСЛА

ОСНОВНЫЕ ФОРМУЛЫ

$$\arcsin(\sin \alpha) = \alpha, \alpha \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$$

• Например

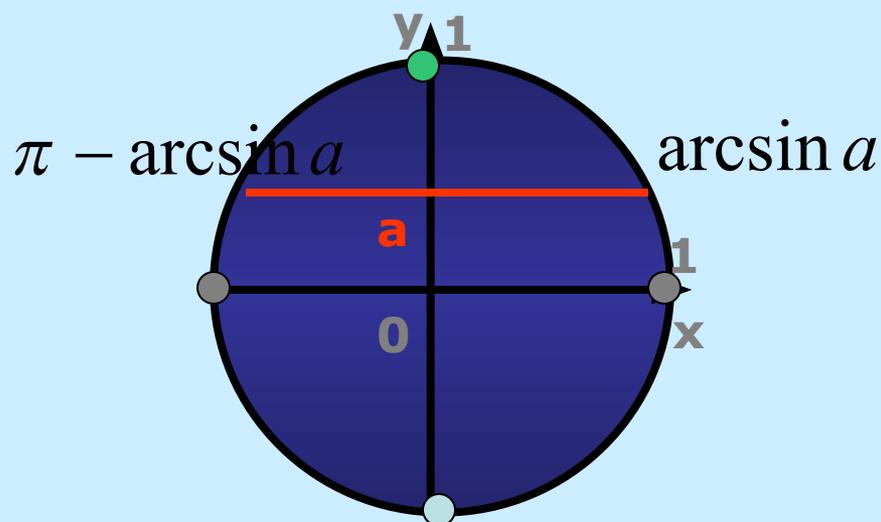
$$9. \arcsin\left(\sin \frac{\pi}{5}\right) = \frac{\pi}{5}$$

$$10. \arcsin\left(\sin \frac{3}{5}\pi\right) = \arcsin\left(\sin\left(\pi - \frac{2}{5}\pi\right)\right) =$$

$$\arcsin\left(\sin \frac{2}{5}\pi\right) = \frac{2}{5}\pi$$

Уравнение $\sin x = a$

$$\sin x = a, |a| \leq 1$$



$$\begin{cases} x = \arcsin a + 2\pi k \\ x = \pi - \arcsin a + 2\pi k \end{cases}, k \in \mathbb{Z}$$

$$x = (-1)^n \arcsin a + \pi n, n \in \mathbb{Z}$$

$$\sin x = 0$$

$$x = \pi k, k \in \mathbb{Z}$$

$$\sin x = 1$$

$$x = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$$

$$\sin x = -1$$

$$x = -\frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$$

Уравнение $\sin x = a$

$$x = (-1)^n \arcsin a + \pi n, n \in \mathbb{Z}$$

Пусть n -чётное число, $n=2k$, тогда

$$x = (-1)^{2k} \cdot \arcsin a + 2\pi k = \arcsin a + 2\pi k, k \in \mathbb{Z}$$

Пусть n -нечётное число, $n=2k+1$, тогда

$$x = (-1)^{2k+1} \cdot \arcsin a + \pi \cdot (2k+1) = \pi - \arcsin a + 2\pi k, k \in \mathbb{Z}$$

**Ита
к**

$$\left[\begin{array}{l} x = \arcsin a + 2\pi k \\ x = \pi - \arcsin a + 2\pi k \end{array} \right], k \in \mathbb{Z}$$



Уравнение $\sin x = a$

• **Пример 1.** $\sin 2x = \frac{1}{2}$

$$\left[\begin{array}{l} 2x = \arcsin \frac{1}{2} + 2\pi k \\ 2x = \pi - \arcsin \frac{1}{2} + 2\pi k \end{array} \right. ; \left[\begin{array}{l} 2x = \frac{\pi}{6} + 2\pi k \\ 2x = \pi - \frac{\pi}{6} + 2\pi k \end{array} \right. ; \left[\begin{array}{l} 2x = \frac{\pi}{6} + 2\pi k \\ 2x = \frac{5\pi}{6} + 2\pi k \end{array} \right. ;$$

$$\left[\begin{array}{l} x = \frac{\pi}{12} + \pi k \\ x = \frac{5\pi}{12} + \pi k \end{array} \right. , k \in Z.$$

ИЛ

$$2x = (-1)^n \arcsin \frac{1}{2} + \pi n;$$

$$2x = (-1)^n \cdot \frac{\pi}{6} + \pi n ;$$

$$x = (-1)^n \cdot \frac{\pi}{12} + \frac{\pi n}{2}, n \in Z.$$

Уравнение $\sin x = a$

Пример 2.

$$\sin\left(x - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\begin{cases} x - \frac{\pi}{4} = \arcsin\left(-\frac{1}{\sqrt{2}}\right) + 2\pi k \\ x - \frac{\pi}{4} = \pi - \arcsin\left(-\frac{1}{\sqrt{2}}\right) + 2\pi k \end{cases} ; \quad \begin{cases} x - \frac{\pi}{4} = -\arcsin\frac{1}{\sqrt{2}} + 2\pi k \\ x - \frac{\pi}{4} = \pi + \arcsin\frac{1}{\sqrt{2}} + 2\pi k \end{cases} ;$$

$$\begin{cases} x - \frac{\pi}{4} = -\frac{\pi}{4} + 2\pi k \\ x - \frac{\pi}{4} = \pi + \frac{\pi}{4} + 2\pi k \end{cases} ; \quad \begin{cases} x = 2\pi k \\ x = \frac{3}{2}\pi + 2\pi k, k \in \mathbb{Z}. \end{cases}$$

Уравнение $\sin x = a$

• Пример 2.

$$\sin\left(x - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} \quad \left[\begin{array}{l} x = 2\pi k \\ x = \frac{3}{2}\pi + 2\pi k, k \in \mathbb{Z}. \end{array} \right.$$

ИЛИ

$$x - \frac{\pi}{4} = (-1)^n \cdot \arcsin\left(-\frac{1}{\sqrt{2}}\right) + \pi n;$$

$$x = (-1)^n \cdot \left(-\arcsin\frac{1}{\sqrt{2}}\right) + \frac{\pi}{4} + \pi n;$$

$$x = (-1)^{n+1} \cdot \frac{\pi}{4} + \frac{\pi}{4} + \pi n, n \in \mathbb{Z}.$$

Уравнение $\sin x = a$

• Пример 3.

$$(3 \sin x - 1) \cdot (2 \sin x + 1) = 0$$

$$3 \sin x - 1 = 0;$$

$$\sin x = \frac{1}{3};$$

$$x = (-1)^n \arcsin \frac{1}{3} + \pi n, n \in \mathbb{Z}.$$

$$2 \sin x + 1 = 0;$$

$$\sin x = -\frac{1}{2};$$

$$\left[\begin{array}{l} x = -\arcsin \frac{1}{2} + 2\pi k \\ x = \pi + \arcsin \frac{1}{2} + 2\pi k \end{array} \right];$$

$$\left[\begin{array}{l} x = -\frac{\pi}{6} + 2\pi k \\ x = \frac{7}{6}\pi + 2\pi k \end{array} \right], k \in \mathbb{Z}.$$

Уравнение $\sin x = a$

• Пример 4.

$$\sin x + \sqrt{2} \cos x + \sin 3x = 0$$

$$(\sin x + \sin 3x) + \sqrt{2} \cos x = 0$$

$$2 \sin 2x \cdot \cos x + \sqrt{2} \cos x = 0$$

$$\cos x \cdot (2 \sin 2x + \sqrt{2}) = 0$$

$$\cos x = 0$$

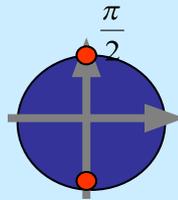
$$2 \sin 2x + \sqrt{2} = 0$$

Уравнение $\sin x = a$

• Пример 4.

$$\sin x + \sqrt{2} \cos x + \sin 3x = 0$$

$$\cos x = 0$$



$$x = \frac{\pi}{2} + \pi k, k \in \mathbb{Z}.$$

$$2 \sin 2x + \sqrt{2} = 0$$

$$\sin 2x = -\frac{\sqrt{2}}{2}$$

$$\left[\begin{array}{l} 2x = -\frac{\pi}{4} + 2\pi n \\ 2x = \pi + \frac{\pi}{4} + 2\pi n \end{array} \right. ;$$

$$\left[\begin{array}{l} x = -\frac{\pi}{8} + \pi n \\ x = \frac{5}{8}\pi + \pi n \end{array} \right. , n \in \mathbb{Z}.$$