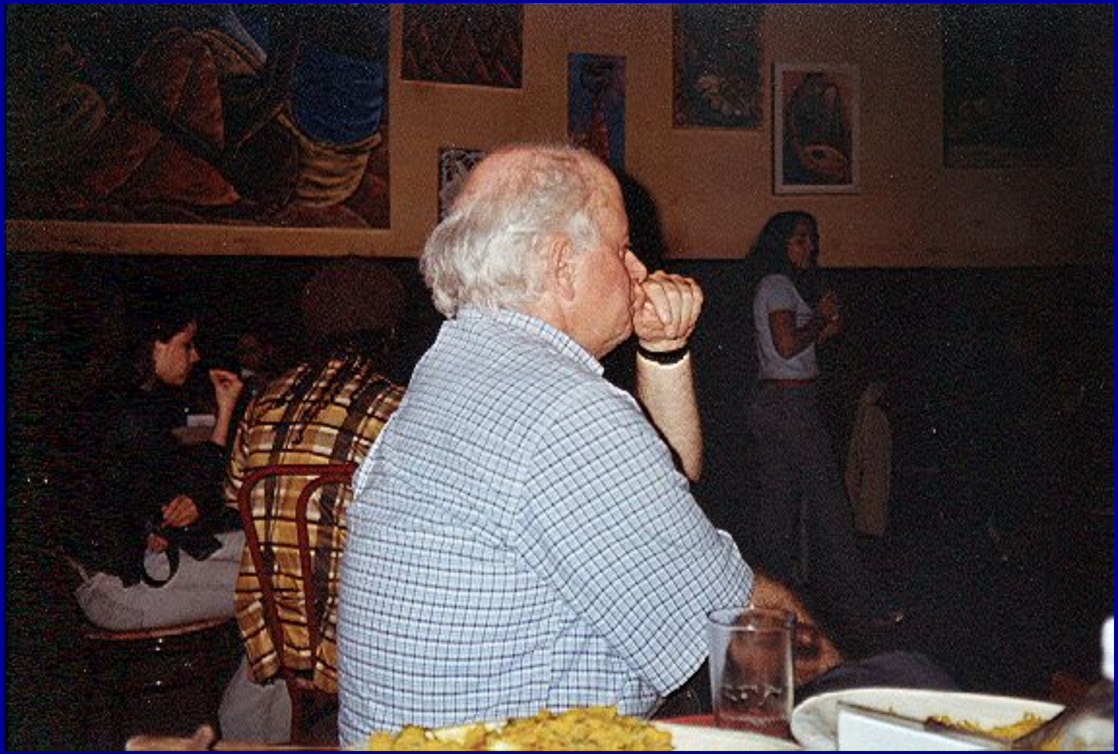
Strings, Gauge Fields and Duality



A conference to mark the retirement of

David Olive, CBE FRS

University of Wales Swansea 24th -27th March 2004

The analytic S-matrix

D.I. Olive, ***Unitarity and the Evolution of Discontinuities***
Nuovo Cimento 26, 73 (1962)

D.I. Olive, ***Exploration of S-Matrix Theory***
Phys. Rev. 135, B745 (1964)

D.I. Olive, et al ....

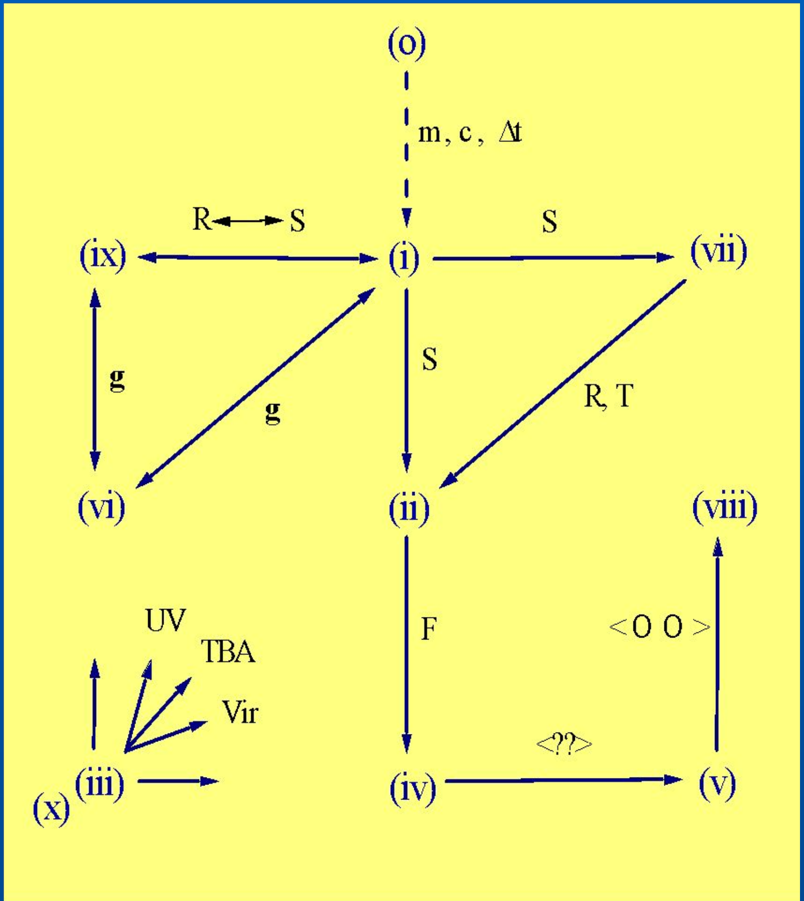
R. Eden, P. Landshoff, D.I. Olive and J. Polkinghorne,
***The analytic S-Matrix* (CUP, 1964)**

Notion of hermitian analyticity opened new door in integrable quantum field theories in 1+1 dimensions

- **theories with unstable particles in their spectra**

- **O.A. Castro-Alvaredo, J. Dreißig and A. Fring, *Integrable scattering theories with unstable particles*, to appear J. of Euro. Phys. C (2004)**
- **O.A. Castro-Alvaredo, A. Fring, *On vacuum energies and renormalizability in integrable quantum field theories*, hepth/0401075**

The bootstrap program (1978-...)



o) classical foreplay

i) determination of the S-matrix

ii) construction of the form factors

iii) consistency checks

iv) correlation (Wightman) functions

v) classification of (local) operators

vi) organise the zoo of models

vii) add boundaries and impurities

viii) compute measurable quantities

ix) relate to lattice statistical models

x) interrelation of the program to other areas (condensed matter)

Scattering theory in 1+1 dim

- in general: $Z_{\mu_m}(\theta'_m)\ldots Z_{\mu_1}(\theta'_1)|0\rangle_{\text{out}} = S^{\mu_1\mu_2\ldots\mu_n}_{\mu_1\mu_2\ldots\mu_m}(\theta'_1,\ldots\theta_n) Z_{\mu_1}(\theta_1)\ldots Z_{\mu_n}(\theta_n)|0\rangle_{\text{in}}$

  - creation operator for a particle of type μ: $Z_\mu(p)$
  - vacuum: $|0\rangle$
  - momentum p /rapidity θ: $p = m(\cosh\theta, \sinh\theta)$

- integrability ≡ ∃ at least one non-trivial conserved charge
  - no particle production
  - incoming and outgoing momenta coincide $\{\theta'_1, \theta'_2, \ldots \theta'_m\} = \{\theta_1, \theta_2, \ldots \theta_n\}$ with $n = m$
  - factorization of the S-matrix:

$$S^{\mu_1\mu_2\ldots\mu_n}_{\mu_1\mu_2\ldots\mu_m}(\theta'_1,\ldots\theta_n) = \prod_{1\le i<j\le n} S_{\mu_i\mu_j}(\theta_i,\theta_j)$$

□ How does one construct S?

- in general use perturbation theory
- in 1+1 dim IQFT: solve consistency equations

i) Lorentz invariance

- S depends on Mandelstam variables

$$s_{ab} = (p_a + p_b)^2 = m_a^2 + m_b^2 + 2m_a m_b \cosh(\theta_a - \theta_b)$$

$$S_{ab}(p_a, p_b) = S_{ab}(\theta_a, \theta_b) = S_{ab}(s_{ab}) = S_{ab}(\theta_{ab})$$

with $\theta_{ab} := \theta_a - \theta_b$

ii) Hermitian analyticity

- central assumption:

S can be continued to the complex plane, it depends on:

$$s_{ab}, \theta_{ab} \in \mathbb{C}$$

- **physical S-matrix:**

$$S_{ab}^{\text{physical}} = \lim_{\varepsilon\to 0} S_{ab}(s + i\varepsilon) = S_{ab}(\theta) \qquad s \in \mathbb{R}, \varepsilon, \theta \in \mathbb{R}^{+}$$

- **S depends on** $\sqrt{s_{ab}}$
  - **no more single valuedness**
  - **remedy: branch cuts** $s_{ab} \geq (m_a + m_b)^2$, $s_{ab} \leq (m_a - m_b)^2$

- **hermitian analyticity:**

$$\lim_{\varepsilon\to 0} S_{ab}(s + i\varepsilon) = \lim_{\varepsilon\to 0} S_{ba}^{*}(s - i\varepsilon)$$
$$S_{ab}(\theta) = [S_{ba}(-\theta^{*})]^{*}$$

D.I. Olive, Nuovo Cimento 26, 73 (1962)

- coincides with real analyticity $S_{ab}(\theta) = [S_{ab}(-\theta^{*})]^{*}$
only for parity invariant theories, that is $S_{ab} = S_{ba}$

- **Examples for theories with real analytic S-matrices:**
  - **sine-Gordon:**

    Al. B. Zamolodchikov, A. B. Zamolodchikov, Annals Phys. 120, 253 (1979)
  - **affine Toda field theories:**

    R. Köberle, J. A. Swieca , Phys. Lett. B86, 209 (1979)

    A. Arinshtein, V. Fateev, A.B. Zamolodchikov, Phys. Lett. B87, 389 (1979)

    many papers in the early 90s, Corrigan et al, Mussardo et al, Freund et. al...

    A.Fring, D.I. Olive; Nucl. Phys. B379, 429 (1992)

    (possibly more on this in the next talk by Ed Corrigan),....

- **Examples for theories with hermitian analytic S-matrices:**
  - **homogeneous sine-Gordon:**

    J.L. Miramontes; Phys. Lett. B455, 231 (1999)

    J.L. Miramontes, C. R. Fernandez-Pousa; Phys. Lett. B472, 392 (2000)

    A. Fring, C. Korff, Phys. Lett. B477, 380 (2000)

    C. Korff, Phys. Lett. B501, 289 (2001)

iii) Unitarity

- completeness and orthogonality • $SS^{\dagger} = S^{\dagger}S = 1$

iv) Crossing symmetry

- Lehmann-Symanzik-Zimmermann formalism
  - change from $s_{ab} = (p_a + p_b)^2$ to $t_{ab} = (p_a - p_b)^2$

$$\lim_{\varepsilon\to 0} S_{ab}(s + i\varepsilon) = \lim_{\varepsilon\to 0} S_{b\bar{a}}(t - i\varepsilon)$$

$$S_{b\bar{a}}(\theta) = S_{ab}(i\pi - \theta)$$

  - here $\bar{a}$ denotes the anti-particle

v) Yang-Baxter equation

- factorization •

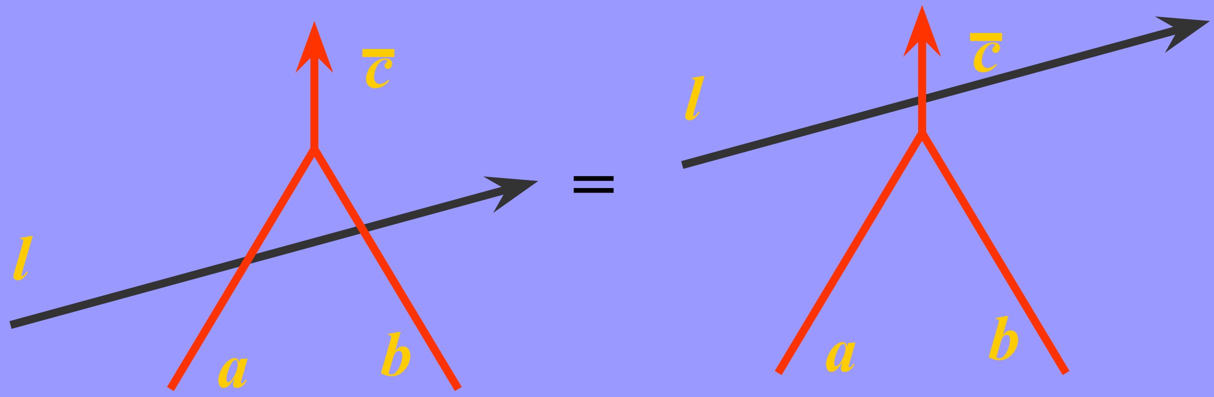
$$S(\theta_{12}) \otimes S(\theta_{13}) \otimes S(\theta_{23}) = S(\theta_{23}) \otimes S(\theta_{13}) \otimes S(\theta_{12})$$

  - no backscattering • diagonal S-matrix

$$S_{ab}^{cd}(\theta) \to S_{ab}(\theta)$$

vi) Fusing bootstrap equation

- suppose you have a fusing process: a +b → c̄

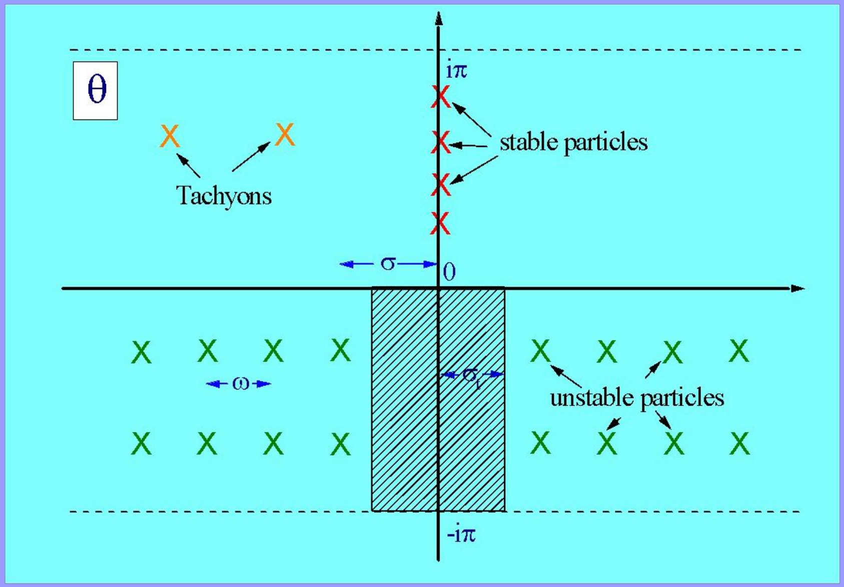


fusing angles

$$\bar{\eta}_{ac}^{b} \in \mathbb{R}^{+}$$

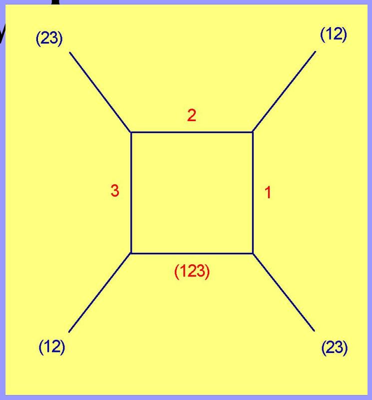
$$S_{la}(\theta + i\bar{\eta}_{ac}^{b})S_{lb}(\theta - i\bar{\eta}_{bc}^{a}) = S_{l\bar{c}}(\theta)$$

vii) Account for all poles



- first order poles in S at
fusing process: $i+j \to$ (ij)

- 2nd order pole Coleman-Thun mechanism
Breit-W



- higher order: generalised CT-mechanism

i) - vii) determine S-exactly !

From form factors to correlation functions

- **Wightman's reconstruction theorem:**

  a QFT is solved once all n-point functions are known

form factor:

form factor consistency equations:

i) CPT invariance:

ii) crossing:

iii) relativistic invariance:

iv) kinematic residue equation:

v) bound state residue equation:

The form factor consistency equations select out solutions corresponding to operators which are mutually local, i.e. they (anti)-commute for space-like separation.

Example: SU(3) homogeneous sine-Gordon (HSG) model (two particles + and -)

[Castro-Alvaredo, Fring, Korff, Phys. Lett. B484 (2001) 167]
[Castro-Alvaredo, Fring, Nucl. Phys. B604 (2001) 367]
[Castro-Alvaredo, Fring, Phys. Rev. D63 (2001) 021701]

Theories with unstable particles generalities

simple Lie algebra of rank with subalgebra

Dynkin diagram

the are free parameters of the theory and label the unstable particles

Example:

associate stable particles to simple roots of

associate unstable particles to **all** positive non-simple roots

D.I. Olive, N. Turok, *The Symmetries of Dynkin diagrams and the reduction of Toda field equations* Nucl. Phys. B215 470 (1983)

For conformal field theory see review:

P. Goddard, D.I. Olive, *Kac-Moody and Virasoro algebras in relation to quantum physics* Int. J Mod. Phys. A1, 303 (1986)

Q-H. Park, Phys. Lett. B328 (1994) 329 (cl.)
T.J. Hollowood, J.L. Miramontes and Q-H. Park, Nucl. Phys. B445 (1995) 451 (cl.)
C.R. Fernández-Pousa, M.V. Gallas, T.J. Hollowood and J.L. Miramontes, Nucl. Phys. B484 (1997) 609 (cl.)
J.L. Miramontes and C.R. Fernández-Pousa, Phys. Lett. B472 (2000) 392 (S)
O.A. Castro-Alvaredo, A. Fring, C. Korff and J.L. Miramontes, Nucl. Phys. B575 (2000) 535 (TBA)
A. Fring and C. Korff, Phys. Lett. B477 (2000) 3[illegible] (S)
C. Korff, Phys. Lett. B501 (2001) 289 (S)
O.A. Castro-Alvaredo, A. Fring and C. Korff, Phys. Lett. B484 (2000) 167 (form factors)
O.A. Castro-Alvaredo and A. Fring, Nucl. Phys. B604 (2001) 367 (correlation functions)
O.A. Castro-Alvaredo and A. Fring, Phys. Rev. D63 (2001) 021701 (RG flow)
O.A. Castro-Alvaredo and A. Fring, Phys. Rev. D64 (2001) 085007 (form factors)

Decoupling rule

decoupling rule:

Virasoro central charge:

Example: $SU(4)_2$-homogeneous sine-Gordon model

P. Goddard, A. Kent, D.I. Olive, *Unitary representations of the Virasoro algebra and Super-Virasoro Algebras* CMP. 103, 105 (1986)

How to detect unstable particles?

TBA equation:

• solve for pseudo-energy

scaling function:

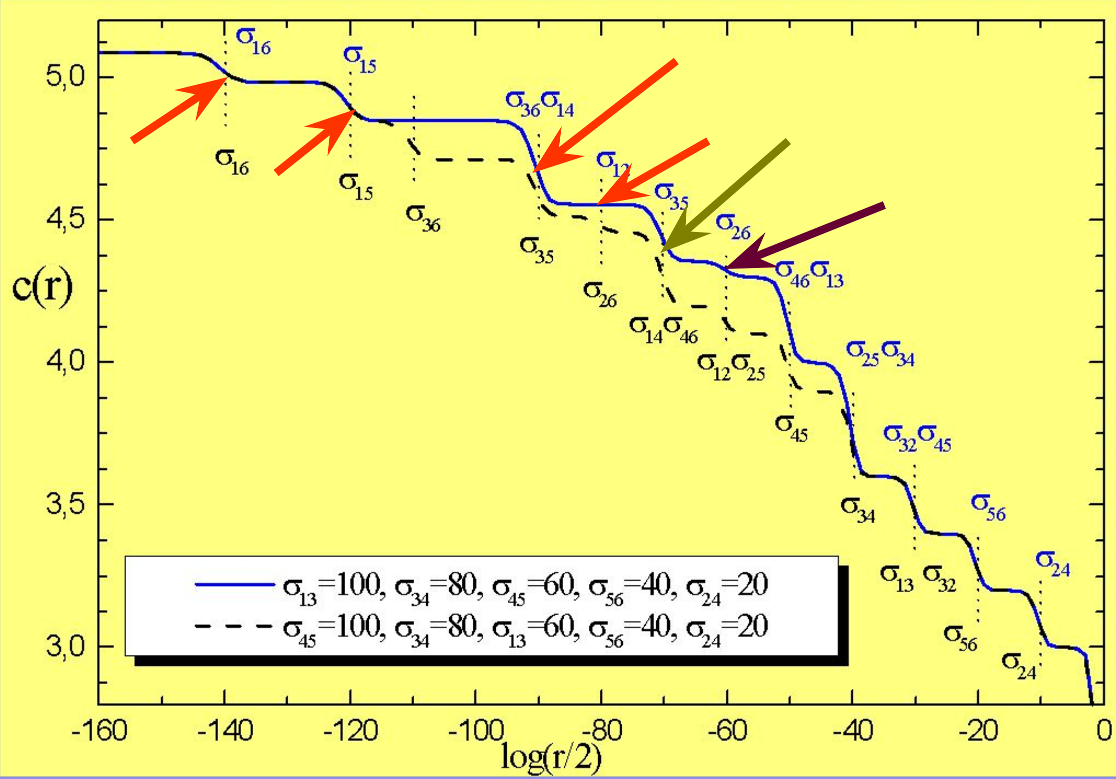
adapted c-theorem:

trace of the energy momentum tensor

adapted Δ-sum rule:

primary field in conformal limit

[Castro-Alvaredo, Fring, Phys. Rev. D63 021701 (2001), Phys. Rev. D64 085007 (2001)]



E6-homogeneous sine-Gordon model:

onset predicted by Breit-Wigner formula:

height of plateaux predicted by decoupling rule:

Particle spectrum:

6 stable particles

30 unstable particles

• TBA confirms all predictions •

15 different masses

+ degeneracy depending on the choices of the sigmas

Thank you very much for your attention.

All the best to David.