

Assumption of Homoscedasticity

Homoscedasticity
(also referred to as homogeneity of variance)
(also referred to as uniformity of variance)

Transformations

Assumption of normality script

Practice problems

Assumption of Homoscedasticity

- Homoscedasticity refers to the assumption that the dependent variable exhibits similar amounts of variance across the range of values for an independent variable.
- While it applies to independent variables at all three measurement levels, the methods that we will use to evaluate homoscedasticity requires that the independent variable be non-metric (nominal or ordinal) and the dependent variable be metric (ordinal or interval). When both variables are metric, the assumption is evaluated as part of the residual analysis in multiple regression.

Evaluating homoscedasticity

- Homoscedasticity is evaluated for pairs of variables.
- There are both graphical and statistical methods for evaluating homoscedasticity .
- The graphical method is called a boxplot.
- The statistical method is the Levene statistic which SPSS computes for the test of homogeneity of variances.
- Neither of the methods is absolutely definitive.

Transformations

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- When the assumption of homoscedasticity is not supported, we can transform the dependent variable and test it for homoscedasticity. If the transformed variable demonstrates homoscedasticity, we can substitute it in our analysis.
- We use the sample three common transformations that we used for normality: the logarithmic transformation, the square root transformation, and the inverse transformation.
- All of these change the measuring scale on the horizontal axis of a histogram to produce a transformed variable that is mathematically equivalent to the original variable.

When transformations do not work

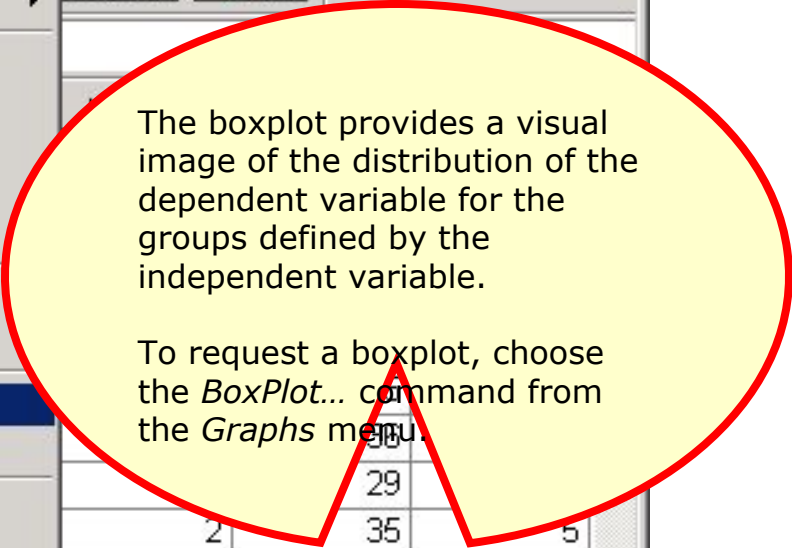
- When none of the transformations results in homoscedasticity for the variables in the relationship, including that variable in the analysis will reduce our effectiveness at identifying statistical relationships, i.e. we lose power.

Problem 1

In the dataset GSS2000.sav, is the following statement true, false, or an incorrect application of a statistic? Use 0.01 as the level of significance.

Based on a diagnostic hypothesis test for homogeneity of variance, the variance in "highest academic degree" is homogeneous for the categories of "marital status."

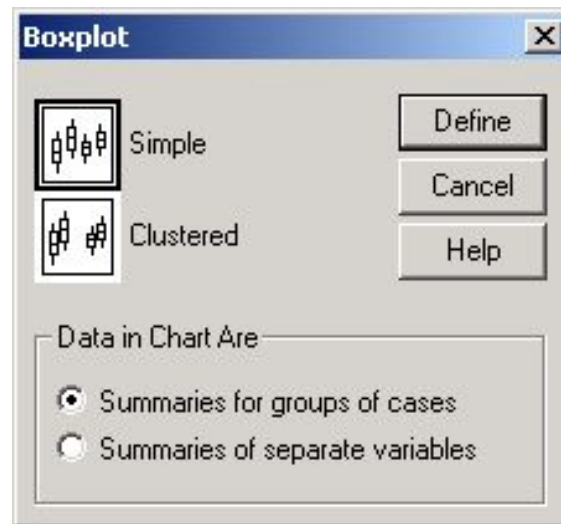
1. True
2. True with caution
3. False
4. Incorrect application of a statistic



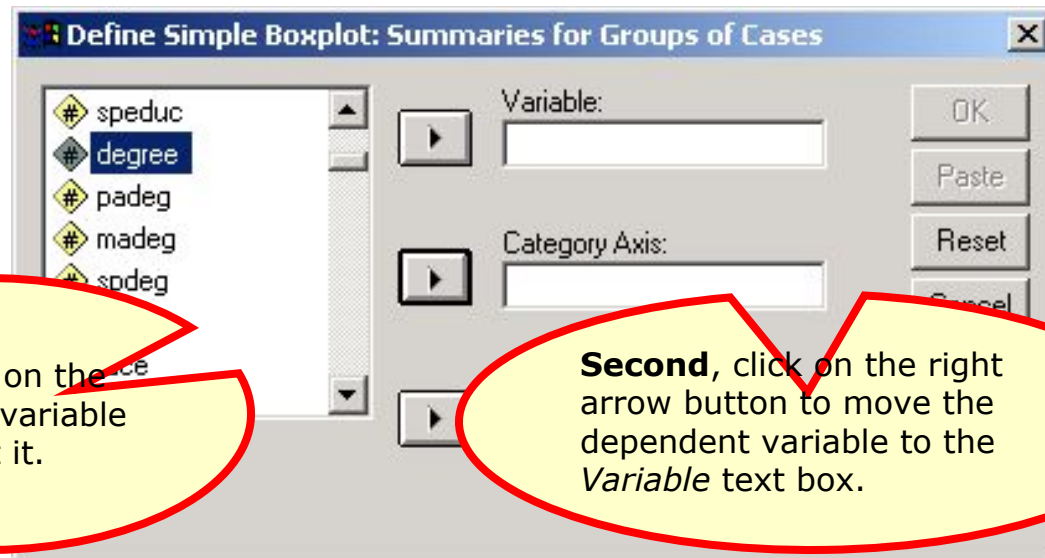
Specify the type of boxplot

First, click on the *Simple* style of boxplot to highlight it with a rectangle around the thumbnail drawing.

Second, click on the *Define* button to specify the variables to be plotted.



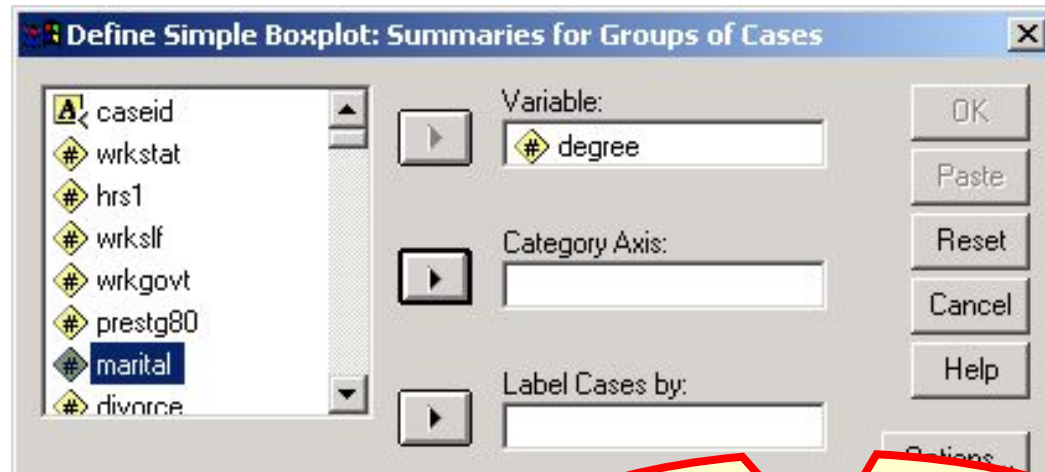
Specify the dependent variable



First, click on the dependent variable to highlight it.

Second, click on the right arrow button to move the dependent variable to the *Variable* text box.

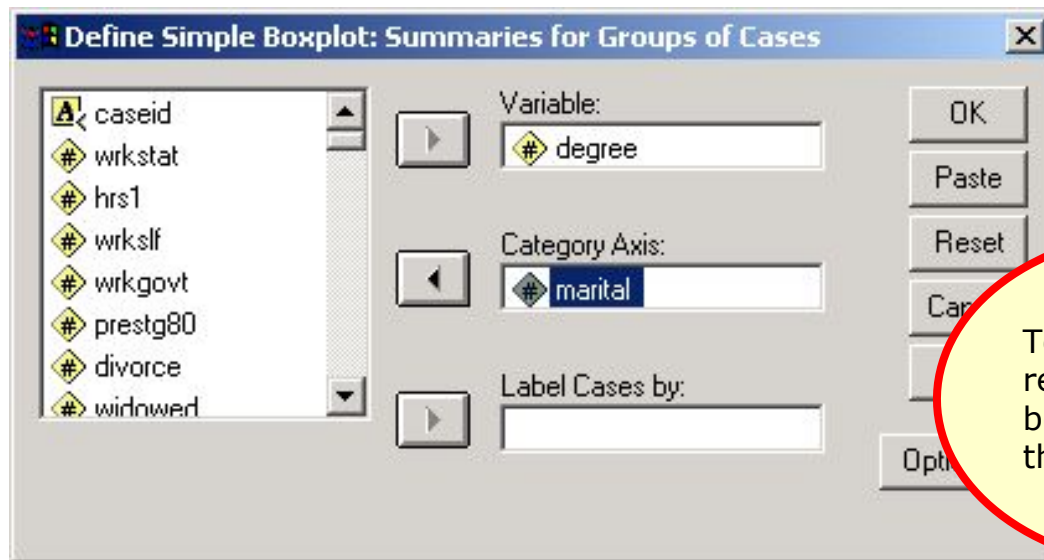
Specify the independent variable



First, click on the independent variable to highlight it.

Second, click on the right arrow button to move the independent variable to the *Category Axis* text box.

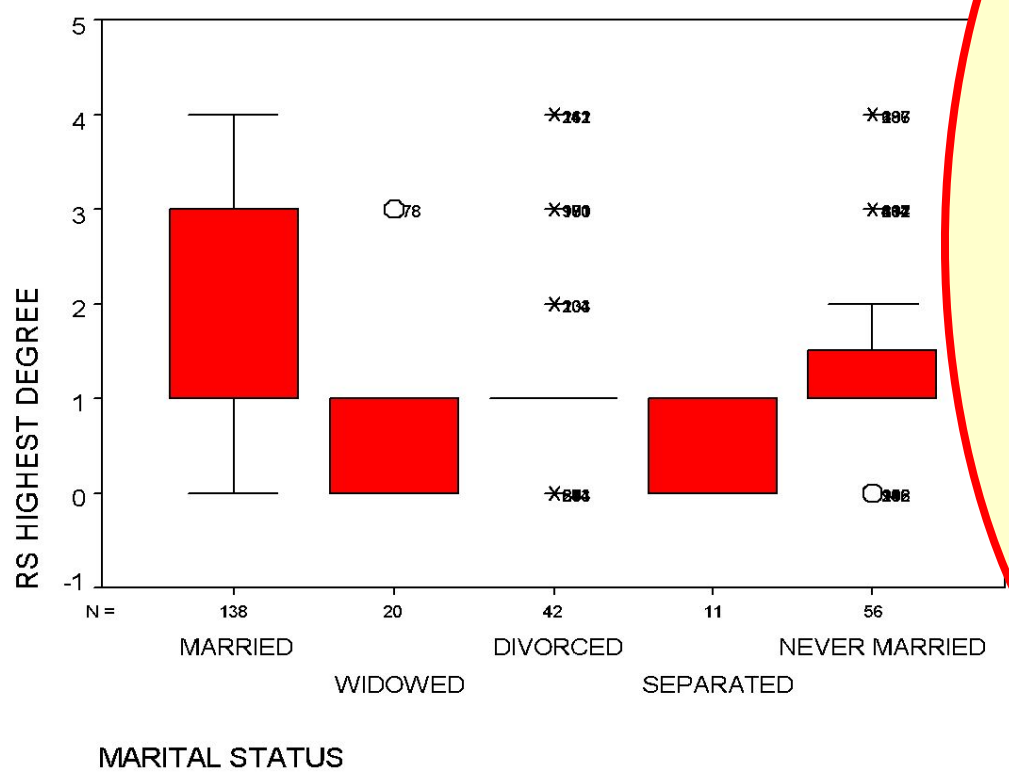
Complete the request for the boxplot



To complete the request for the boxplot, click on the OK button.

The boxplot

Each red box shows the middle 50% of the cases for the group, indicating how spread out the group of scores is.



If the variance across the groups is equal, the height of the red boxes will be similar across the groups.

If the heights of the red boxes are different, the plot suggests that the variance across groups is not homogeneous.

The married group is more spread out than the other groups, suggesting unequal variance.

Request the test for homogeneity of variance

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$$H_1: \mu < 0$$

The screenshot shows the SPSS Data Editor window for 'GSS2000.sav'. The 'Analyze' menu is open, and the path 'Compare Means' > 'One-Way ANOVA...' is highlighted. A red callout bubble points to this path with the following text:

To compute the Levene test for homogeneity of variance, select the *Compare Means* | *One-Way ANOVA...* command from the *Analyze* menu.

The background data table is as follows:

	caseid	wrkstat	marital
1	20000009		
2	20000012		
3	20000020		
4	20000029		
5	20000032		
6	20000034		
7	20000043		
8	20000060		
9	20000070		
10	20000072	5	
11	20000079	1	40
12	20000097	1	40
13	20000117	1	49
14	20000126	1	40
15	20000138	5	

Specify the independent variable



First, click on the independent variable to highlight it.

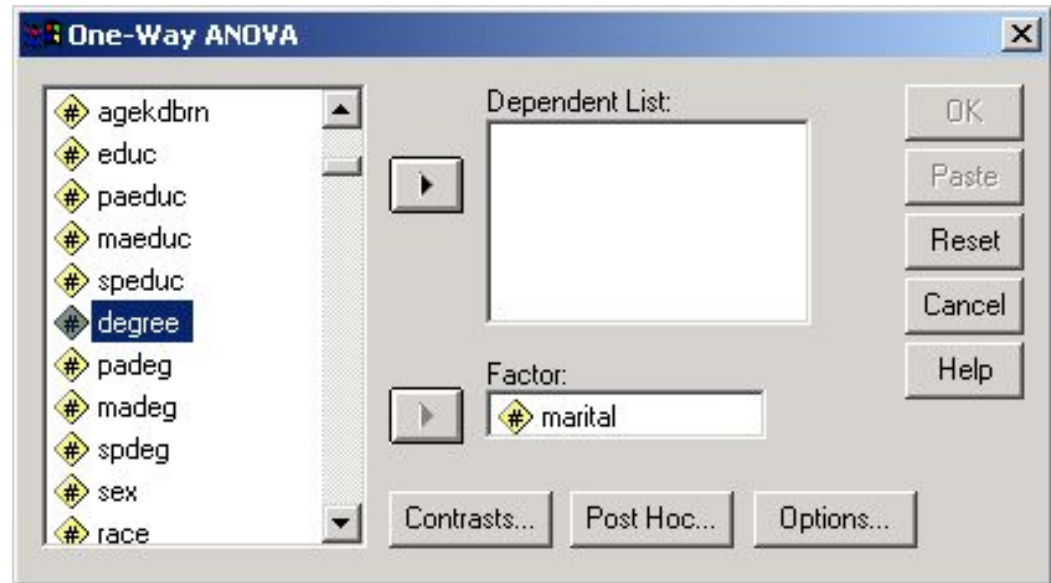
Second, click on the right arrow button to move the independent variable to the *Factor* text box.

Specify the dependent variable

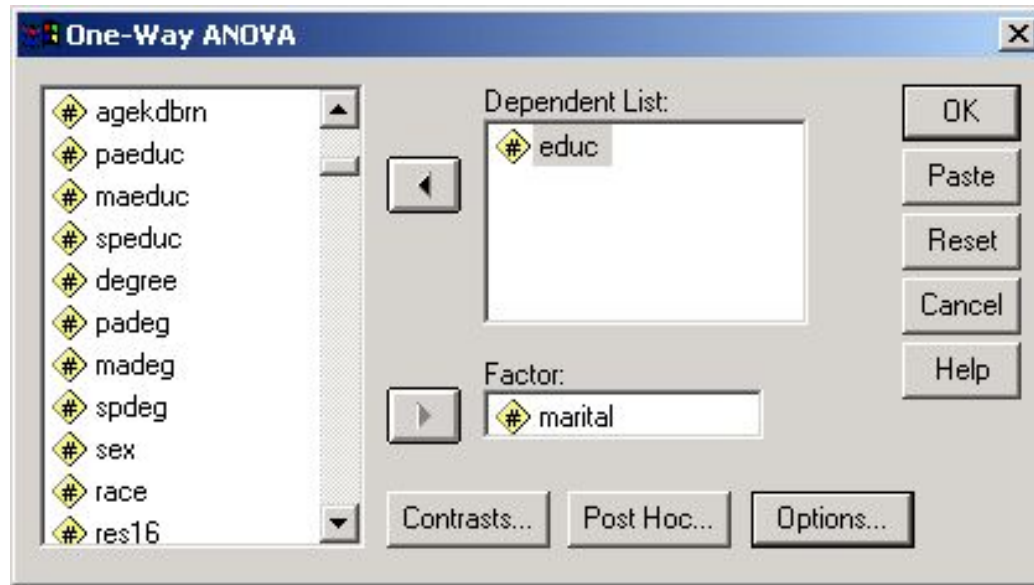
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First, click on the dependent variable to highlight it.

Second, click on the right arrow button to move the dependent variable to the *Dependent List* text box.



The homogeneity of variance test is an option

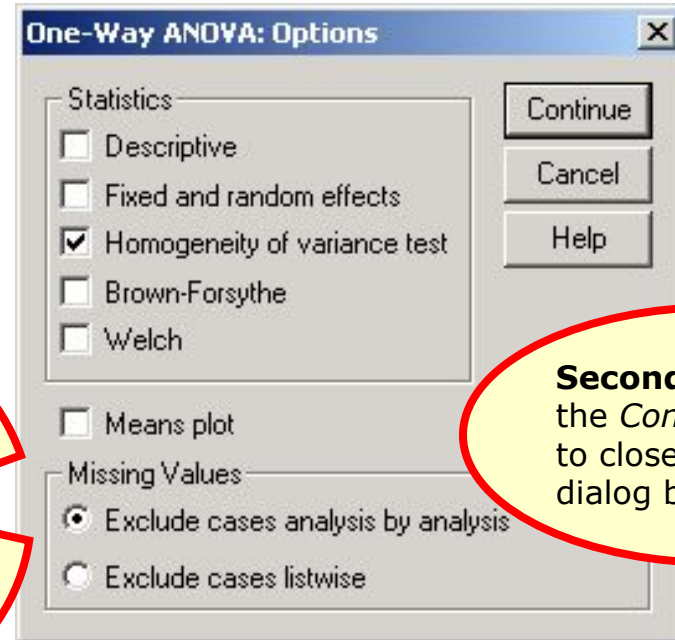


Click on the Options... button to open the options dialog box.

Specify the homogeneity of variance test

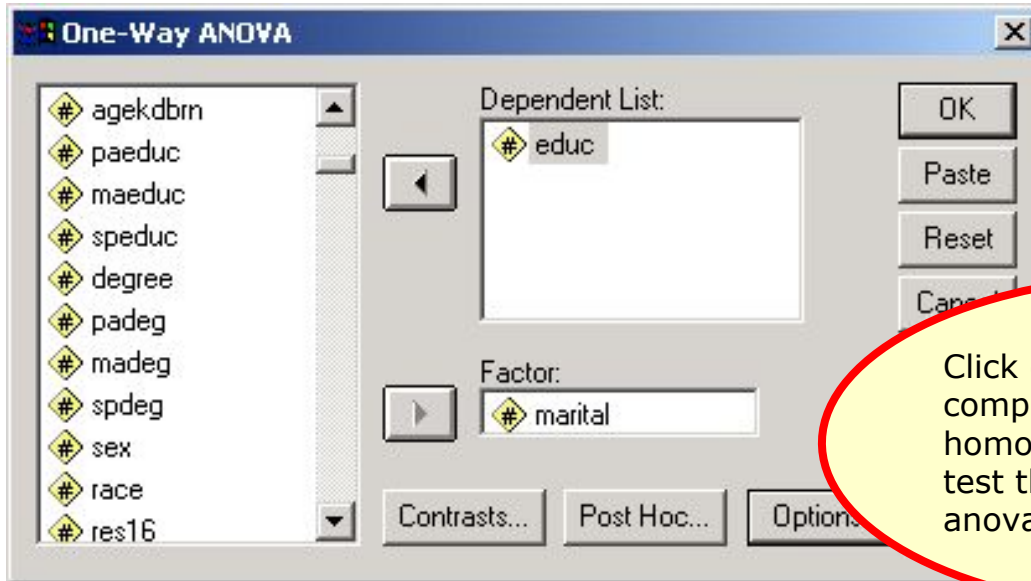
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First, mark the checkbox for the *Homogeneity of variance test*. All of the other checkboxes can be cleared.



Second, click on the *Continue* button to close the options dialog box.

Complete the request for output



Click on the OK button to complete the request for the homogeneity of variance test through the one-way anova procedure.

Interpreting the homogeneity of variance test

Test of Homogeneity of Variances

Statistic	1	505	0.000
Levene	4		

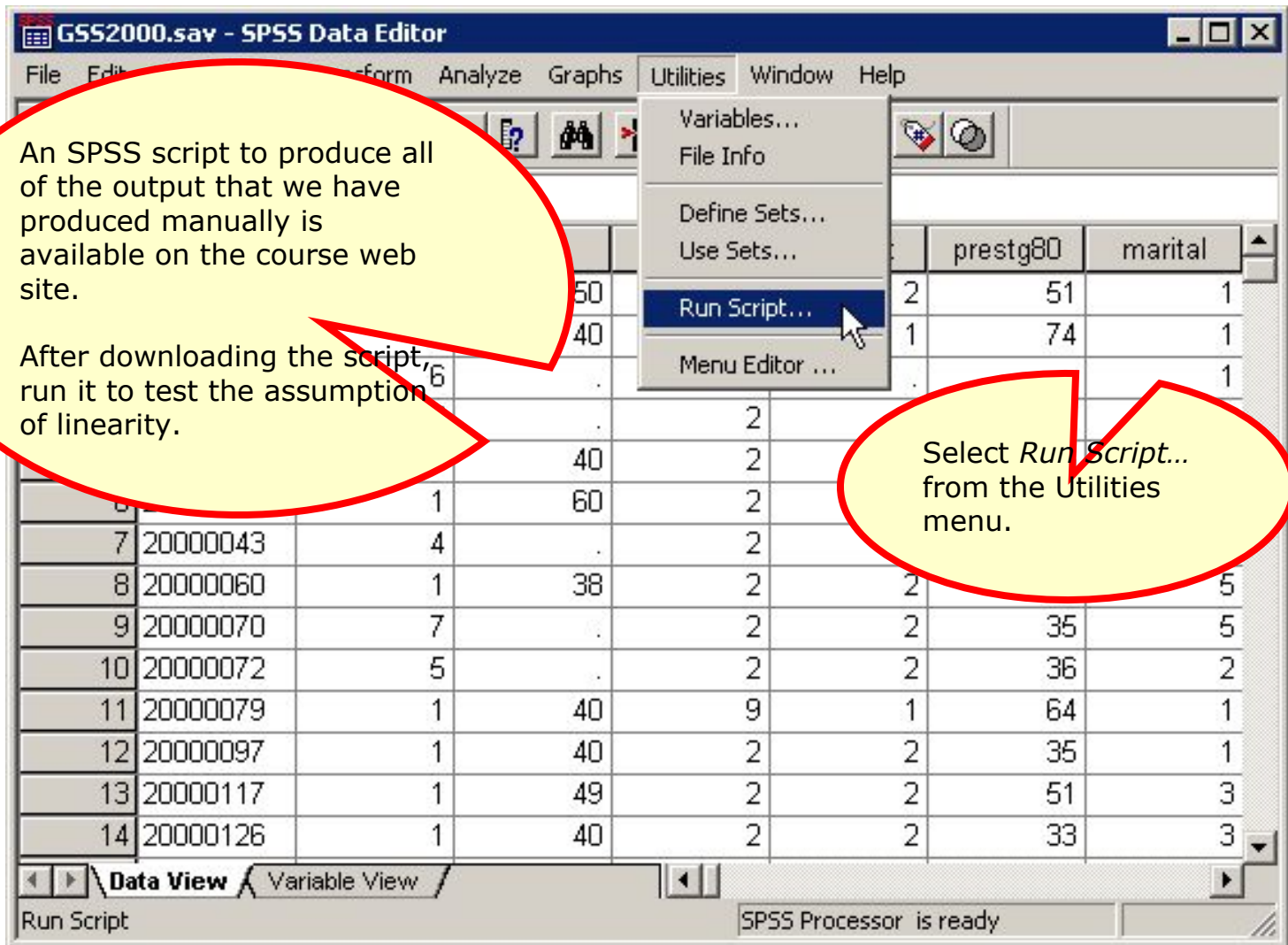
The null hypothesis for the test of homogeneity of variance states that the variance of the dependent variable is equal across groups defined by the independent variable, i.e., the variance is homogeneous.

Since the probability associated with the Levene Statistic (<0.001) is less than or equal to the level of significance, we reject the null hypothesis and conclude that the variance is not homogeneous.

The answer to the question is **false**.

After downloading the script, run it to test the assumption of linearity.

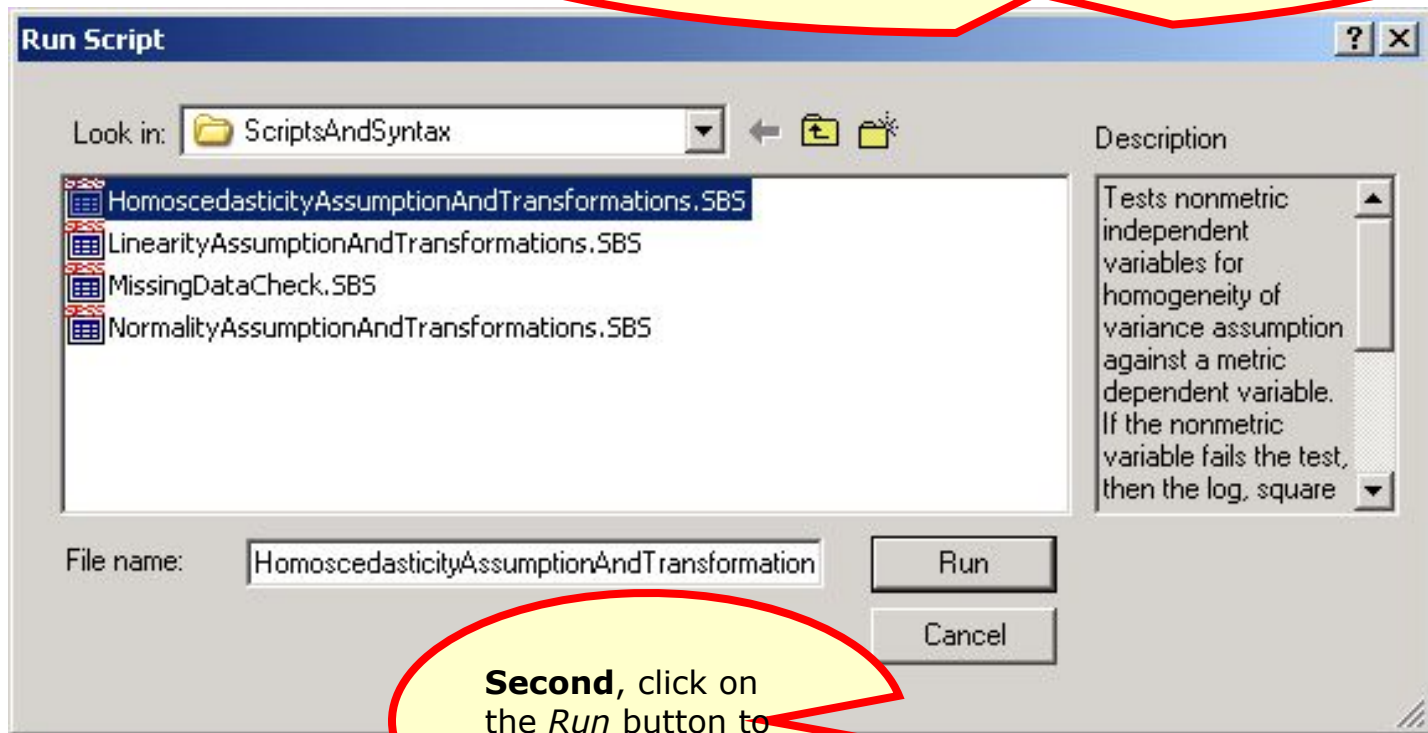
Select *Run Script...* from the Utilities menu.



Selecting the assumption of homoscedasticity script

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First, navigate to the folder containing your scripts and highlight the script:
HomoscedasticityAssumptionAndTransformations.SBS



Second, click on the *Run* button to activate the script.

Specifications for homoscedasticity script

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Test for Assumption of Homogeneity of Variance

Data set: D:\2003_01_SW388R7\SPSSProblems\GSS2000.sav

Variables in the Data Set:

WRKSTAT	LABOR FORCE STATUS
HRS1	NUMBER OF HOURS WORKED LAST WEEK
WRKSLF	R SELF-EMP OR WORKS FOR S
WRKGOVT	GOVT OR PRIVATE EMPLOYE
PRESTG80	RS OCCUPATIONAL PRESTIG
DIVORCE	EVER BEEN DIVORCED OR SEI
WIDOWED	EVER BEEN WIDOWED
SPWRKSTA	SPOUSE LABOR FORCE STA
SPHRS1	NUMBER OF HRS SPOUSE WOF

Dependent (Y) Variable:

DEGREE RS HIGHEST DEGREE

Independent (X) Variables:

MARITAL MARITAL STATUS

Transformations:

- ☒ Logarithmic
- ☒ Square Root
- ☒ Inverse

Options:

- ☒ Delete output from previous commands
- ☒ Delete transformed variables from data

OK

First, move the dependent variable to the *Dependent (Y) Variable* text box.

Second, move the independent variable to the *Independent (X) Variables* text box.

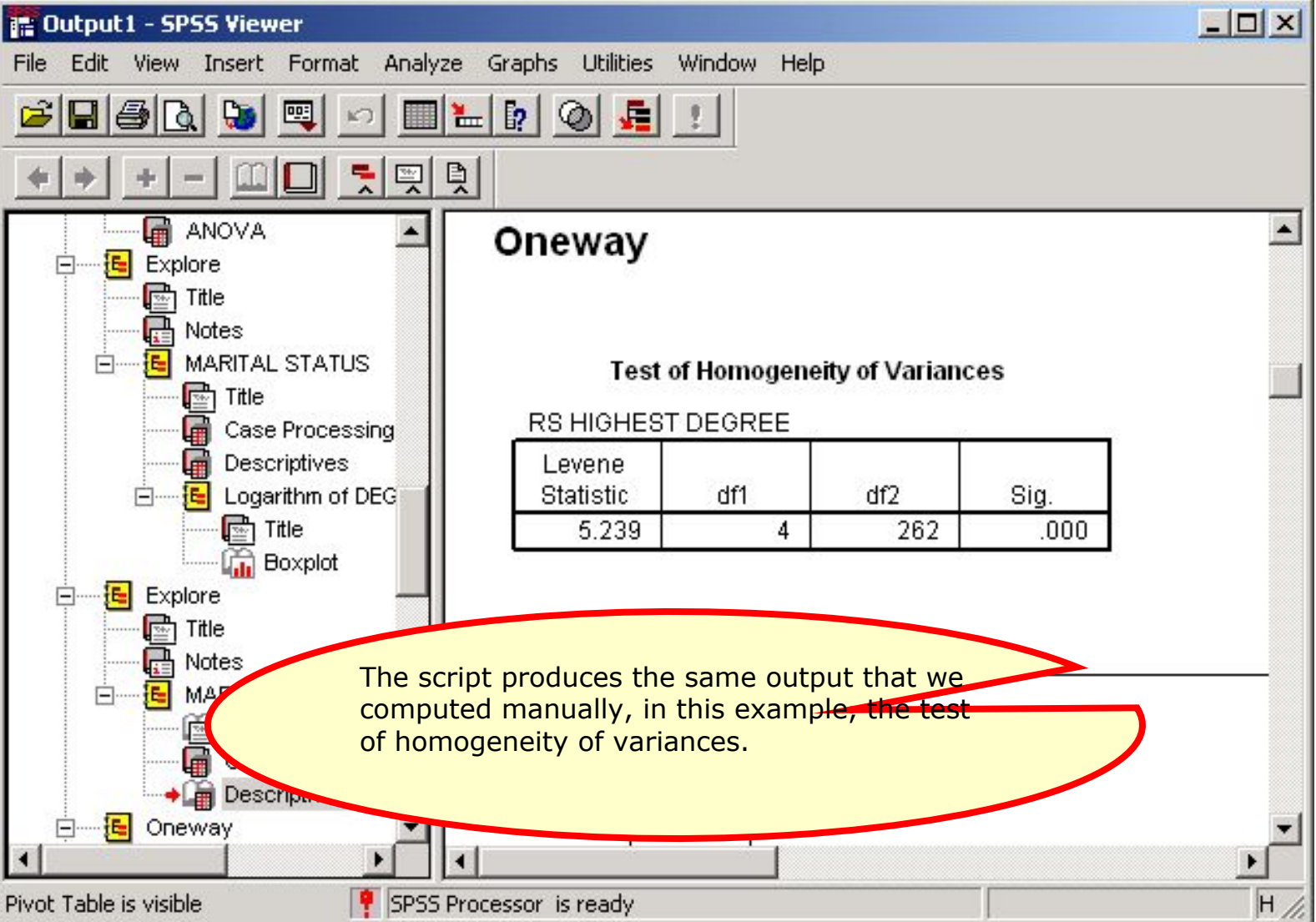
The default output is to do all of the transformations of the variable. To exclude some transformations from the calculations, clear the checkboxes.

Third, click on the OK button to run the script.

The test of homogeneity of variance

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$$H_1: \mu < 0$$



Output1 - SPSS Viewer

File Edit View Insert Format Analyze Graphs Utilities Window Help

Oneway

Test of Homogeneity of Variances

RS HIGHEST DEGREE

Levene Statistic	df1	df2	Sig.
5.239	4	262	.000

The script produces the same output that we computed manually, in this example, the test of homogeneity of variances.

Pivot Table is visible SPSS Processor is ready

Problem 2

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In the dataset GSS2000.sav, is the following statement true, false, or an incorrect application of a statistic?

Based on a diagnostic hypothesis test for homogeneity of variance, the variance in "highest academic degree" is not homogeneous for the categories of "marital status." However, the variance in the logarithmic transformation of "highest academic degree" is homogeneous for the categories of "marital status."

1. True
2. True with caution
3. False
4. Incorrect application of a statistic

Computing the logarithmic transformation

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GSS2000.sav - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities Window Help

Compute...
Random Number Seed...
Count...
Recode
Categorize Variables...
Rank Cases...
Auto...
Cr...
Re...
Run...

1 : logwww

	caseid	wrkslf	wrkgovt	prestg80	marital
1	20000009			51	1
2	20000012			54	1
3	20000020				1
4	20000029				3
5	20000032				1
6	20000034			55	5
7	20000043	4		36	3
8	20000060	1	38	29	5
9	20000070	7		35	5
10	20000072	5		36	2
11	20000079	1	40	64	1
12	20000097	1	40	35	1
13	20000117	1	49	51	3
14	20000126	1	40	33	3
15	20000138	5		33	2

Data View Variable View

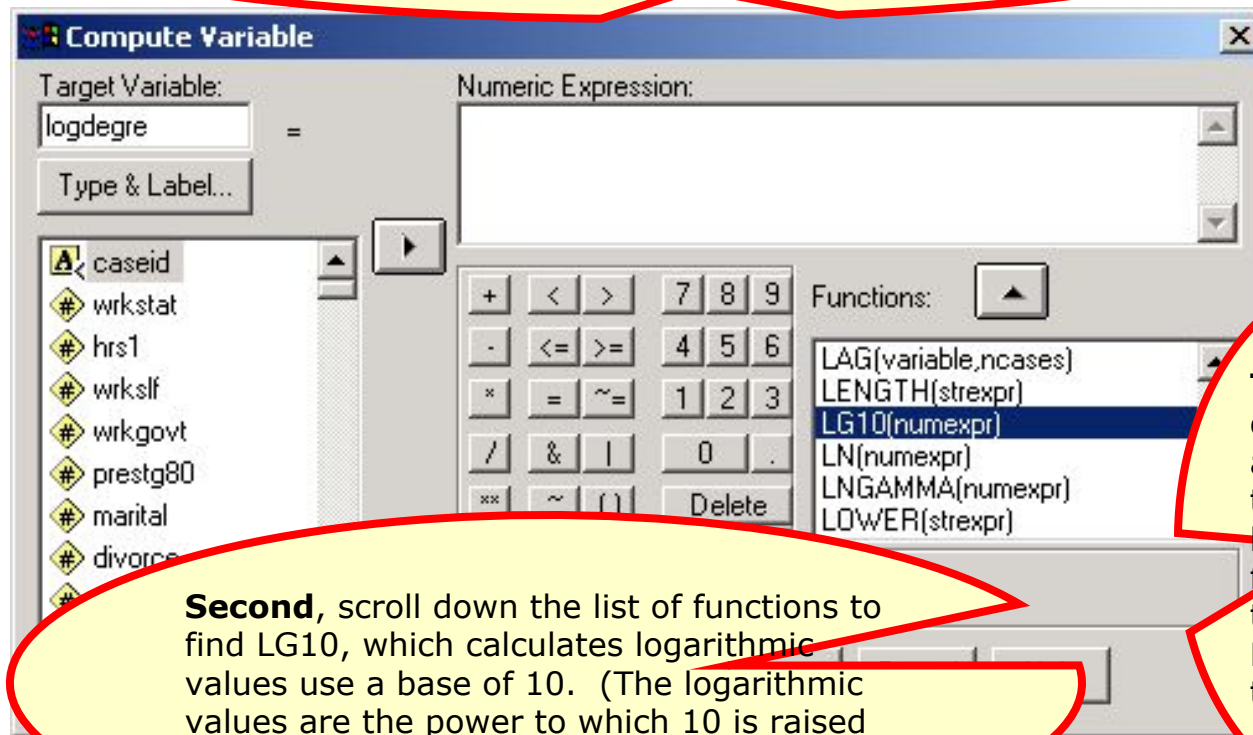
Compute SPSS Processor is ready

To compute the logarithmic transformation for the variable, we select the *Compute...* command from the *Transform* menu.

Specifying the variable name and function

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First, in the target variable text box, type the name for the log transformation variable "logdegre".

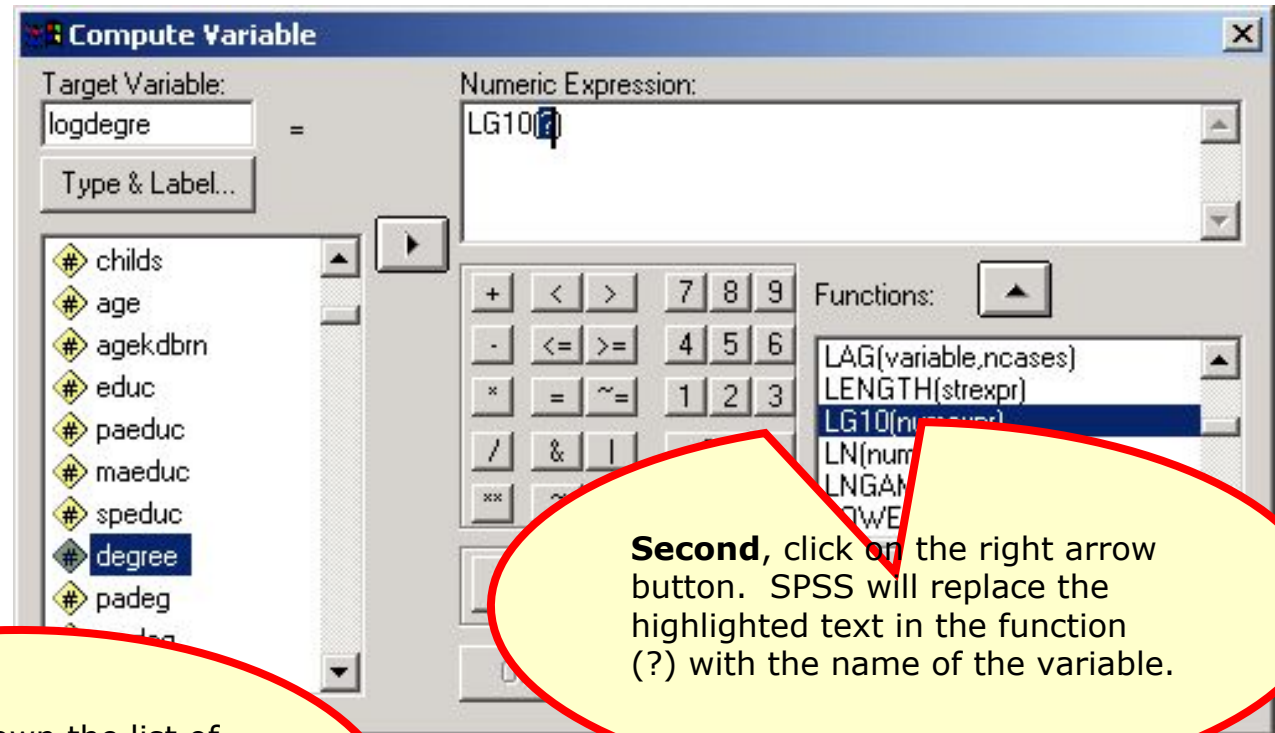


Second, scroll down the list of functions to find LG10, which calculates logarithmic values use a base of 10. (The logarithmic values are the power to which 10 is raised to produce the original number.)

Third, click on the up arrow button to move the highlighted function to the Numeric Expression text box.

Adding the variable name to the function

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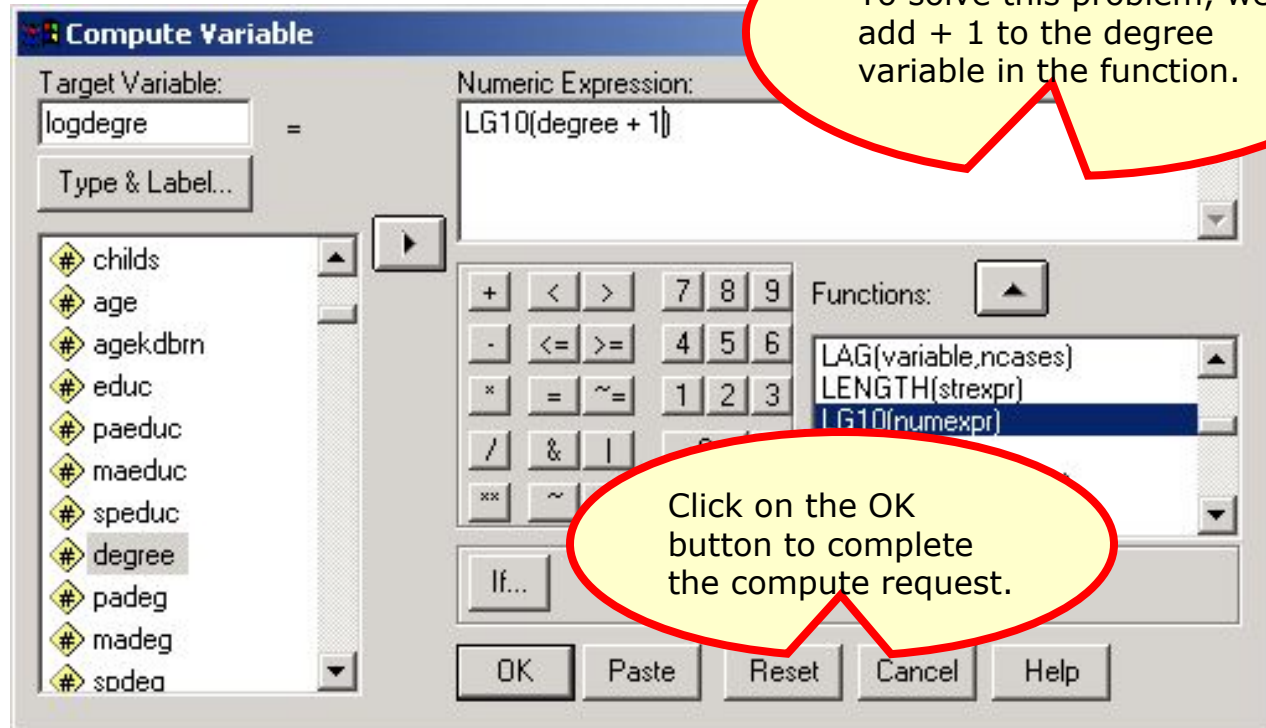
First, scroll down the list of variables to locate the variable we want to transform. Click on its name so that it is highlighted.

Second, click on the right arrow button. SPSS will replace the highlighted text in the function (?) with the name of the variable.

Preventing illegal logarithmic values

The log of zero is not defined mathematically. If we have zeros for the data values of some cases as we do for this variable, we add a constant to all cases so that no case will have a value of zero.

To solve this problem, we add + 1 to the degree variable in the function.



Click on the OK button to complete the compute request.

The transformed variable

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GSS2000.sav - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities Window Help

1 : logdegre 0.698970004336019

	emtime	wwwwtime	chattime	netime	logdegre	var	var
1	3	2	0	5	.699		
				10	.699		
					.301		
					.301		
					.602		
				2	.301		
					.301		
8				1			
9					.000		
10					.000		
11	0						
12							
13	1						
14							
15							

Data View Variable View

SPSS Processor is ready

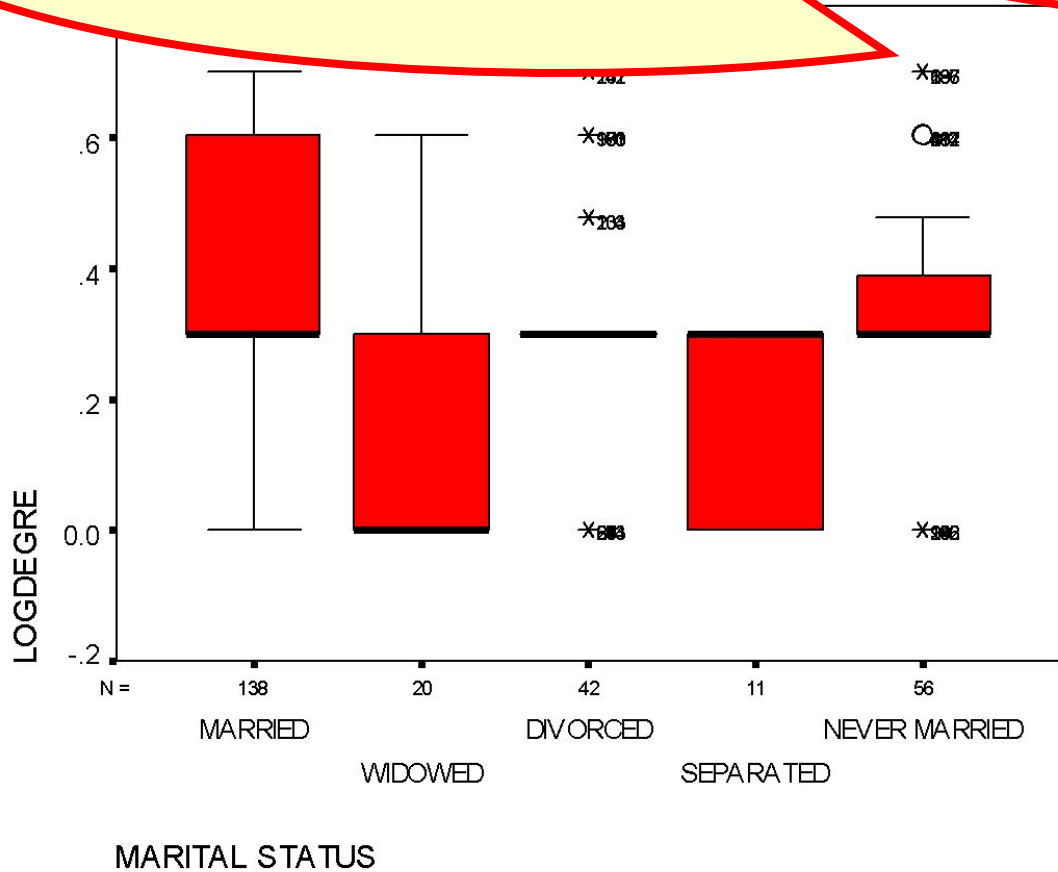
The transformed variable which we requested SPSS compute is shown in the data editor in a column to the right of the other variables in the dataset.

Once we have the transformation variable computed, we repeat the "Boxplot" analysis using this variable.

The boxplot

In this boxplot, the spread is the same for 3 of the 5 groups, which is an improvement over the original boxplot.

However, it is difficult to judge whether or not the problem is solved based solely on the graphic.



The homogeneity of variance test

Test of Homogeneity of Variances

Statistic	4	505	210
Levene Statistic			

The null hypothesis for the test of homogeneity of variance states that the variance of the transformed dependent variable is equal across groups defined by the independent variable, i.e., the variance is homogeneous.

Since the probability associated with the Levene Statistic (0.075) is greater than the level of significance, we fail to reject the null hypothesis and conclude that the variance is homogeneous.

The answer to the question is **true with caution**.

Homogeneity of variance test from the script

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$$H_1: \mu < 0$$

Output1 - SPSS Viewer

File Edit View Insert Format Analyze Graphs Utilities Window Help

Oneway

Test of Homogeneity of Variances

Logarithm of DEGREE [LG10(1+DEGREE)]

Levene Statistic	df1	df2	Sig.
2.151	4	262	.075

The script for homoscedasticity creates the transformed dependent variables and tests them for homogeneity of variance.

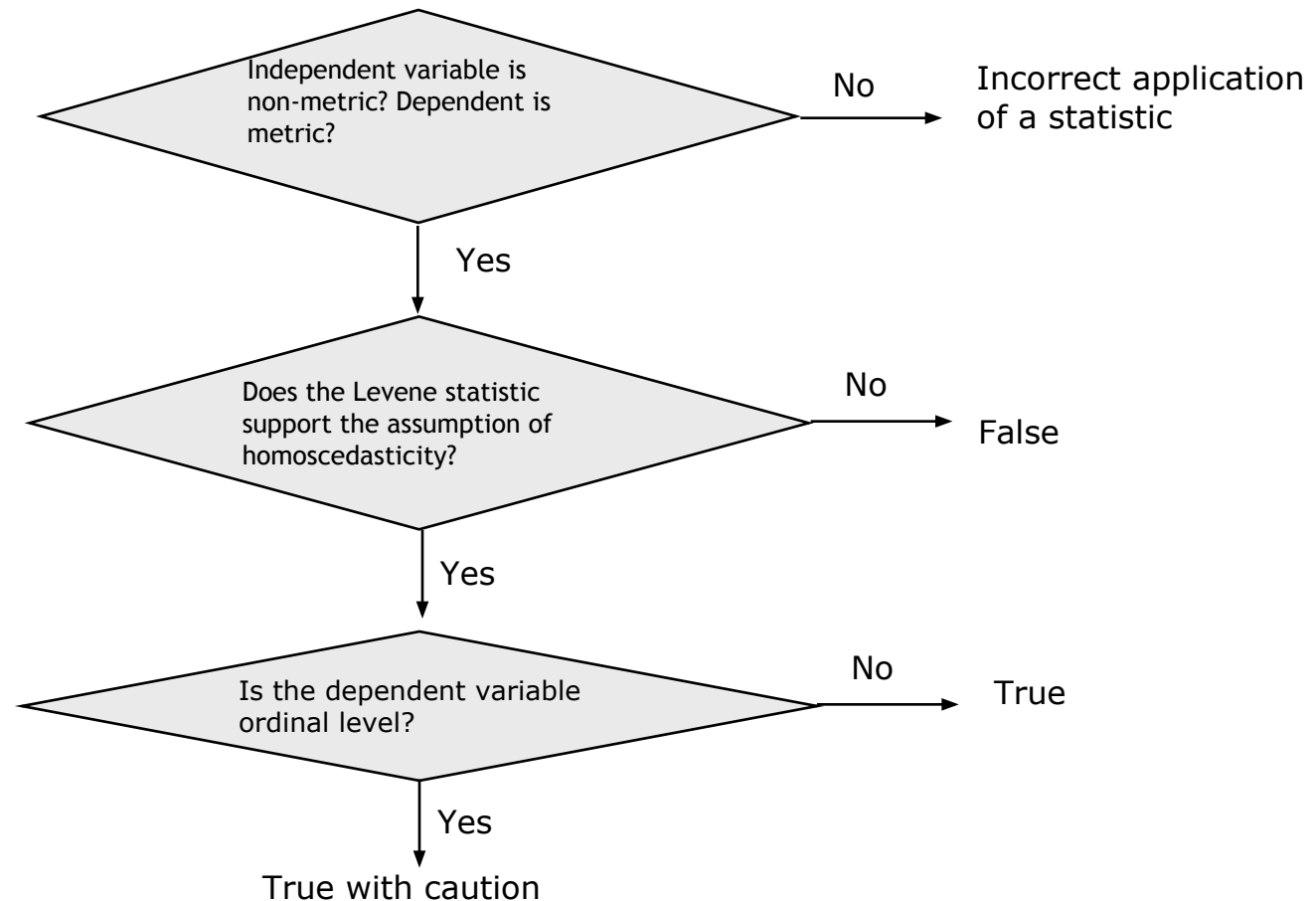
SPSS Processor is ready

Other problems on homoscedasticity assumption

- A problem may ask about the assumption of homoscedasticity for a nominal level dependent variable. The answer will be “An inappropriate application of a statistic” since variance is not computed for a nominal variable. Similarly, an ANOVA cannot be calculated if the independent variable is interval level and the answer will be “An inappropriate application of a statistic.”
- A problem may ask about the assumption of homoscedasticity for an ordinal level dependent variable. If the variable or transformed variable satisfies the assumption of homogeneity of variance, the correct answer to the question is “True with caution” since we may be required to defend treating ordinal variables as metric.

Steps in answering questions about the assumption of homoscedasticity - question 1

The following is a guide to the decision process for answering problems about the homoscedasticity of a variable:



Steps in answering questions about the assumption of homoscedasticity - question 2

The following is a guide to the decision process for answering problems about the homoscedasticity of a transformation:

