42nd International Conference on Boundary Elements and other Mesh Reduction Methods

SINGULAR BOUNDARY METHOD IN FREE VIBRATION ANALYSIS OF COMPOUND LIQUID-FILLED SHELLS

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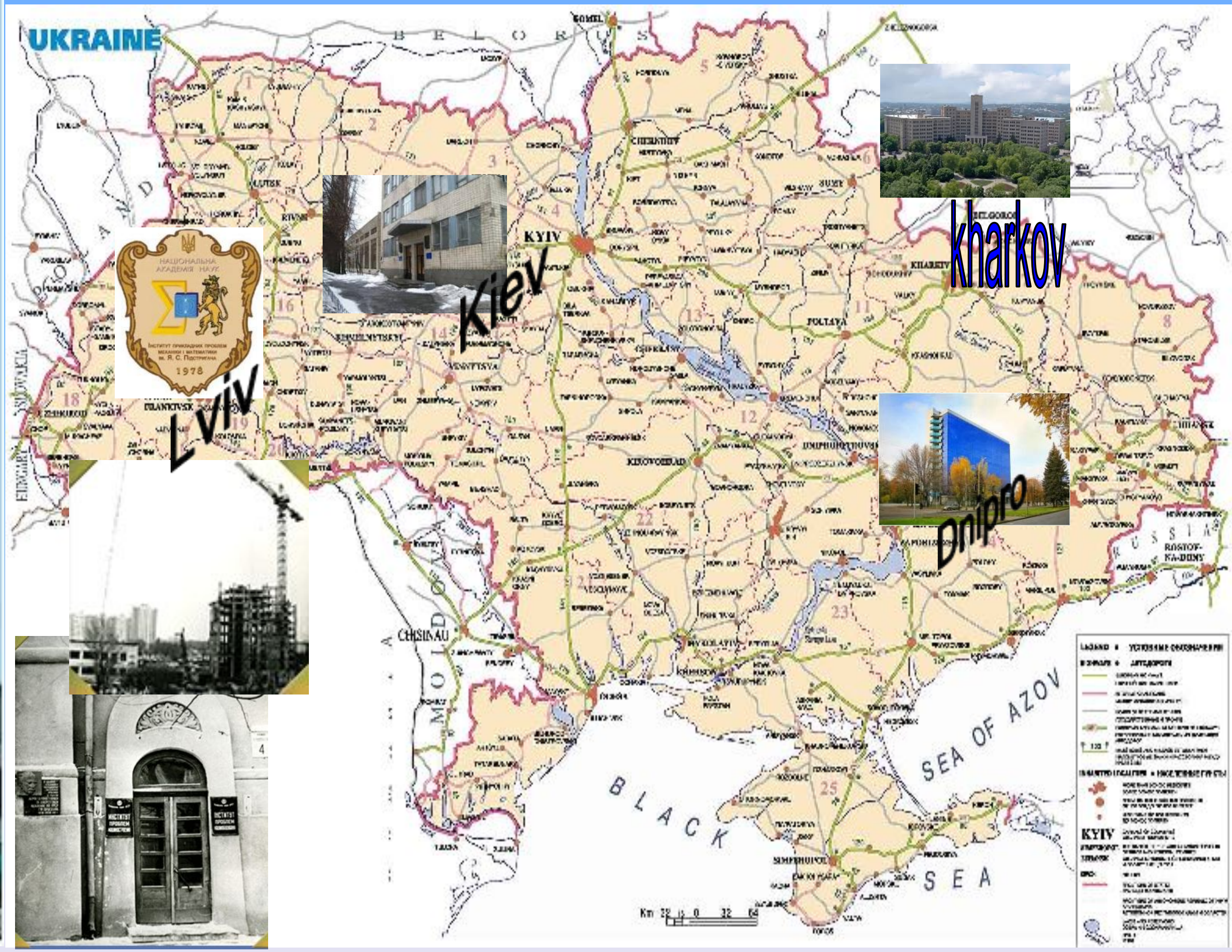

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A. Podgorny Institute of Mechanical Engineering Problems

- The A.N.Podgorny Institute for Mechanical Engineering Problems **of the National Academy of Sciences of Ukraine**
- **(IPMash NAS of Ukraine) is a renown research centre in power and mechanical engineering.**

- **IPMash has 5 research departments with a staff of 346 specialists (133 research workers, including one Academician and five Corresponding Members of NAS of Ukraine; and 32 Doctors and 77 Candidates of Science). The Institute also has a special Design-and-Engineering Bureau, and a pilot production facility.**

- ***Key research areas***
- **optimisation of processes in power machinery**
- **energy saving technologies**
- **predicting the reliability, dynamic strength and life of power equipment;**
- **simulation and computer technologies in power machine building**

LIQUID FILLED SHELLS

PROBLEM STATEMENT

$$\mathbf{L}(\mathbf{U}) + \mathbf{M}(\ddot{\mathbf{U}}) = p\mathbf{n} \qquad \Delta\Phi = 0 \qquad p = -\rho_l\left(\frac{\partial\Phi}{\partial t} + gz\right) + p_0$$

- with the next set of boundary conditions relative to ϕ

$$\left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_1} = \frac{\partial w}{\partial t} \qquad \left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_0} = \frac{\partial\zeta}{\partial t}; \qquad \left.\frac{\partial\Phi}{\partial t} + gz\right|_{S_0} = 0$$

- **w=(U,n)**
- fixation conditions of the shell relative to **U**
- Initial conditions

$$\zeta(x, y, 0) = H; \quad \dot{\zeta}(x, y, 0) = 0$$

BOUNDARY CONDITIONS ON ELASTIC AND RIGID SURFACES

$$\left.(\mathrm{grad}\,\varphi \cdot \mathbf{n})\right|_{s_1} = \frac{\partial (U,\mathbf{n})}{\partial t};\left.(\mathrm{grad}\,\varphi \cdot \mathbf{n})\right|_{s_2} = 0$$

HARMONIC VIBRATIONS

$$\mathrm{U}(\mathrm{x},\mathrm{t}) = \mathrm{e}^{\mathrm{i}\Omega \mathrm{t}}\mathrm{u}(\mathrm{x})$$

MODE DECOMPOSITION METHOD FOR COUPLED DYNAMIC PROBLEMS

- **Displacements are linear combination of structure natural modes without liquid**

$$\mathbf{u} = \sum_{k=1}^{N} c_k \mathbf{u}_k$$

- $\mathbf{u}_k$ are the normal modes of vibrations of the empty shell.

$$\mathbf{L}(\mathbf{u}_k) = \Omega_k^2 \mathbf{M}(\mathbf{u}_k), \ (\mathbf{M}(\mathbf{u}_k), \mathbf{u}_j) = \delta_{kj}$$

the first system of basic functions

Representation for velocity potential

$$\Phi = \Phi_1 + \Phi_2$$

BOUNDARY VALUE PROBLEM FOR POTENTIAL Φ_1

POTENTIAL Φ_1 DEFINES ELASTIC WALL VIBRATIONS second system of basic functions

$$\nabla^2 \Phi_1 = 0$$

$$\frac{\partial \Phi_1}{\partial n} = \frac{\partial w}{\partial t}, \quad P \in S_1$$

$$\frac{\partial \Phi_1}{\partial t} = 0, \quad P \in S_0$$

$$\Phi_1(x, y, z, t) = \sum_{k=1}^{m} \varphi_{1k}(x, y, z) c_k(t)$$

- Boundary value problems for functions φ_{1k}

$$\nabla^2 \varphi_{1k} = 0$$

$$\left.\frac{\partial \varphi_{1k}}{\partial n}\right|_{S_1} = w_k$$

$$\varphi_{1k} = 0, \quad P \in S_0$$

Boundary value problem for velocity potential Φ_2

third system of basic functions

$$\nabla^2\Phi_2 = 0$$

$$\frac{\partial \Phi_2}{\partial n} = 0, \quad P \in S_1 \qquad \frac{\partial \Phi_2}{\partial n} = \dot{\zeta}, \quad P \in S_0 \qquad \left.\frac{\partial \Phi_2}{\partial t} + g\zeta + a_s(t)x\right|_{S_0} = 0$$

- representation for velocity potential Φ_2

$$\Phi_2 = \sum_{k=1}^{M} \dot{d}_k(t)\varphi_{2k}$$

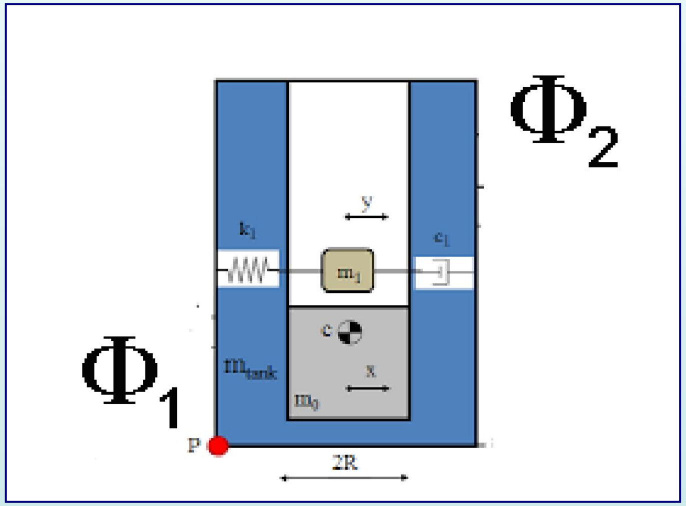
- harmonic vibrations of liquid in rigid shell

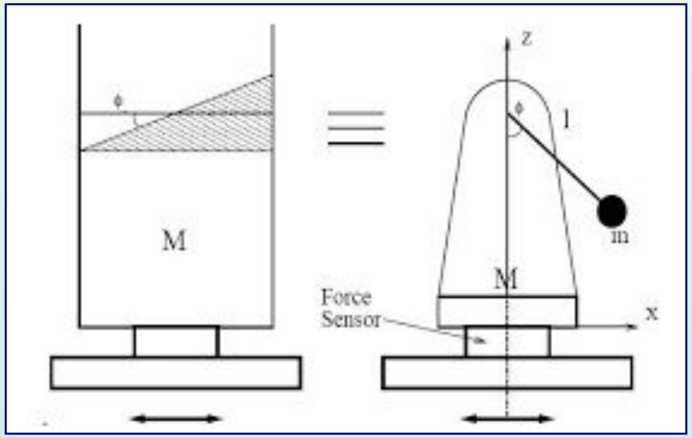
$$\varphi_2(x, y, z, t) = e^{i\kappa t}\psi(x, y, z)$$

- $\nabla^2\psi = 0$

$$\frac{\partial \psi}{\partial n} = 0, \quad P \in S_1 \qquad \frac{\partial \psi}{\partial n} = \frac{\kappa^2}{g}\psi, \quad P \in S_0$$

SPRING- PENDULUM ANALOGY





$$\Phi_1(x,y,z,t) = \sum_{k=1}^{m} \varphi_{1k}(x,y,z)\,\dot{c}_k(t)$$

$$\Phi_2 = \sum_{k=1}^{M} \dot{d}_k(t)\,\varphi_{2k}(x,y,\zeta(x,y))$$

Three systems of basic functions

- **At first we obtain the natural modes and frequencies of structure without liquid – the first system of basic functions**
- **Second, we represent the velocity potential**
- **as a sum** $\Phi = \Phi_1 + \Phi_2$ **and for each component consider the corresponding boundary value problem** for Laplace equation.

- **The potential** Φ_1 **corresponds to the problem of elastic structure** vibrations with the liquid **but without including the force of gravity - the second system of basic functions**
- **The potential** Φ_2 **corresponds to the problem of** rigid structure **vibrations with the liquid including the force of gravity -**
- **the third system of basic functions**

EIGENVALUE PROBLEM

- fWfe **uFOR HARMONIC VIBRATIONS WE HAVE**

se th

$$c_k(t) = C_k \exp(i\omega t); \qquad d_k(t) = D_k \exp(i\omega t);$$

$$a_S(t) = 0$$

We use the direct BEM formulation

direct

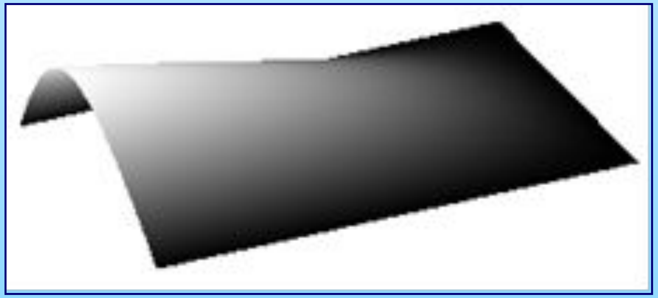
$$2\pi\varphi(P_0) = \iint_S \frac{\partial\varphi}{\partial n} \frac{1}{|P - P_0|} dS - \iint_S \varphi \frac{\partial}{\partial n} \frac{1}{|P - P_0|} dS$$

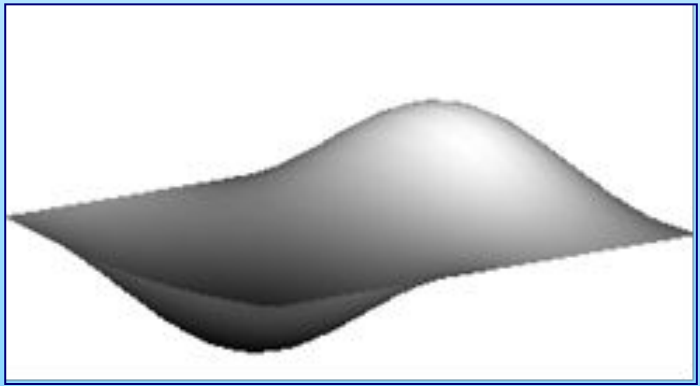
FIRST AND SECOND BASIC FUNCTIONS

VIBRATIONS OF RECTANGULAR PLATES

h, м	ω air. experiment/ calculation	Reduction coefficient	ω_1 Liquid BEM	Reduction coefficient	ω_1 Liquid experiment	Reduction coefficient
0,00215	194/197	1,57	112	1,75	106	1,82
0,003	290/280	1,27	182	1,53	179	1,62
0,005	413/458	1,23	360	1,27	305	1,21

MODES OF VIBRATIONS

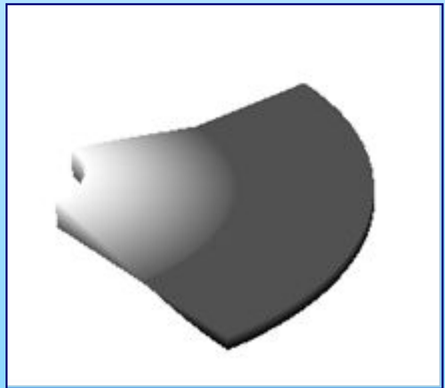


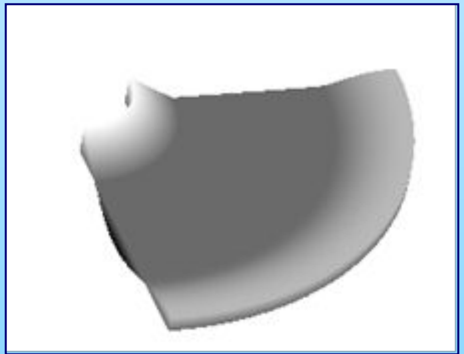


VIBRATIONS OF SECTORIAL PLATES

Number of frequency	frequencies, Hz			
	air		liquid	
	experiment	calculation	experiment	calculation
1	402	398.2	159	161
2	416	425.0	–	242
3	514	549.7	277	300
4	714	791.0	420	460

MODES OF VIBRATIONS





VIBRATIOBS OF FRANSIS TURBINE

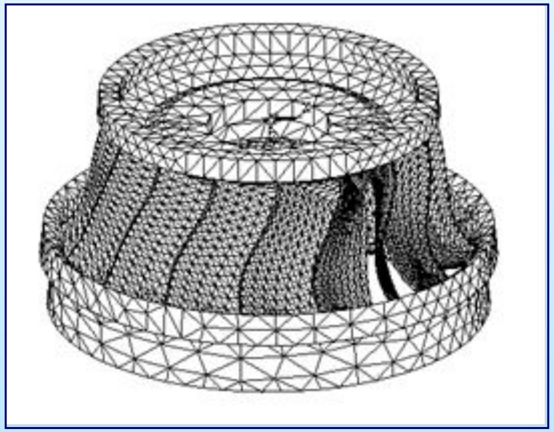
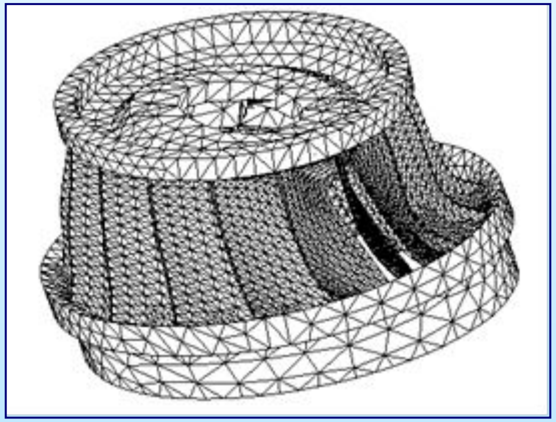
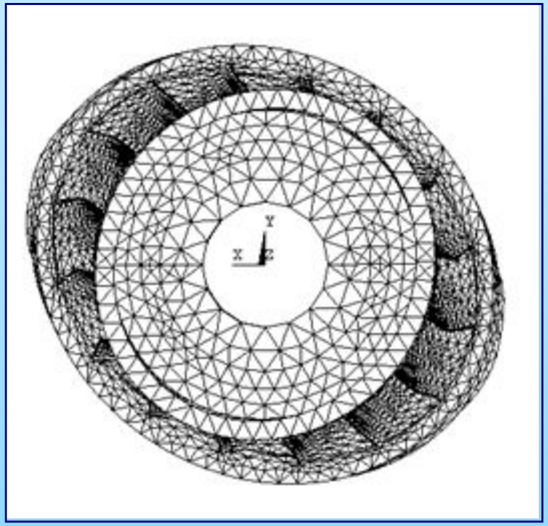
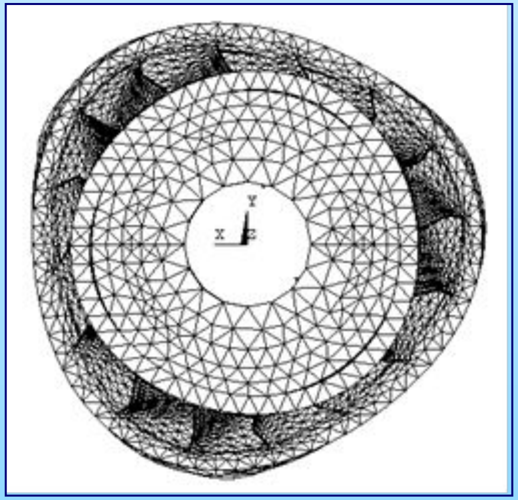
Without added liquid masses

Number of frequency	FEM	Leningrad metal plant Experiment	TURBOATOM experiment
1	33.05	33.68	33.87
2,3	35.80	37.13	35.93
4,5	42.90	39.87	63.06
6,7	71.90	57.89	66.99
8	80.70	80.44	83.83

With added liquid masses

Number of frequency	FEM	Leningrad metal plant Experiment	TURBOATOM experiment
1	24.00	22.5	21.6
2,3	29.20	28.5	28.5
4,5	31.50	31.1	32.7
6,7	37.00	33.3	37.2
8	52.50		40.2

FRANSIS TURBINE MODES OF VIBRATIONS

Free vibrations of liquid in rigid shells. Boundary value problem. Third system of basic functions.

$$\nabla^2 \varphi = 0$$

$$\frac{\partial \varphi}{\partial n} = 0\,, \quad P \in S_1$$

$$\frac{\partial \varphi}{\partial z} = \dot{\zeta}\,, \quad P \in S_0$$

$$\dot{\varphi} + g\zeta = 0\,, \quad P \in S_0$$

$$\varphi(x, y, z, t) = e^{i\kappa t}\psi(x, y, z)$$

Free vibrations

$$\nabla^2 \psi = 0$$

$$\frac{\partial \psi}{\partial n} = 0\,, \quad P \in S_1$$

$$\frac{\partial \psi}{\partial n} = \frac{\kappa^2}{g}\psi\,, \quad P \in S_0$$

Singular boundary method

$$\varphi(\mathbf{r},t)=\psi(\mathbf{r})e^{i\omega t}, j^2=-1.$$

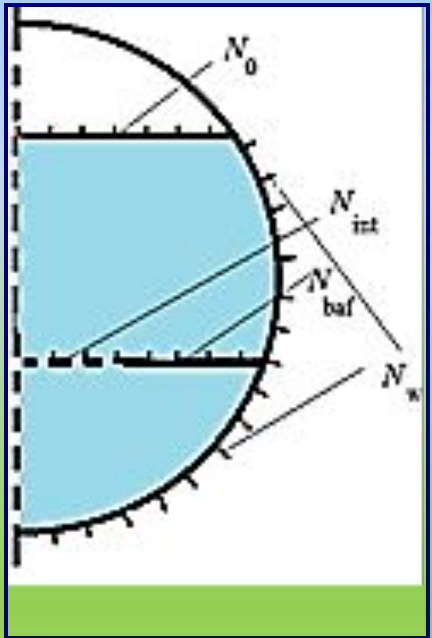
$$\Delta\psi=0, \left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{S_0}=\frac{\omega^2}{g}\psi\Big|_{S_0}, \left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{S_1}=0, \qquad \iint_{S_0}\frac{\partial\psi}{\partial\mathbf{n}}dS_0=0.$$

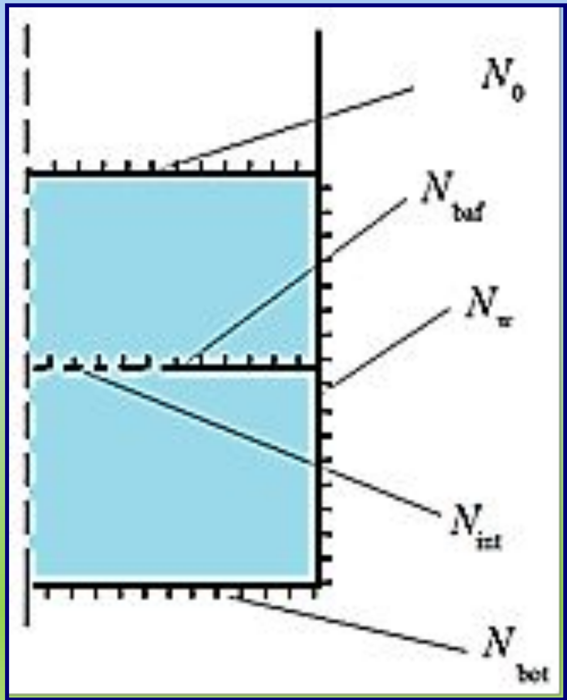
- **THIRD GREEN'S IDENTITY**

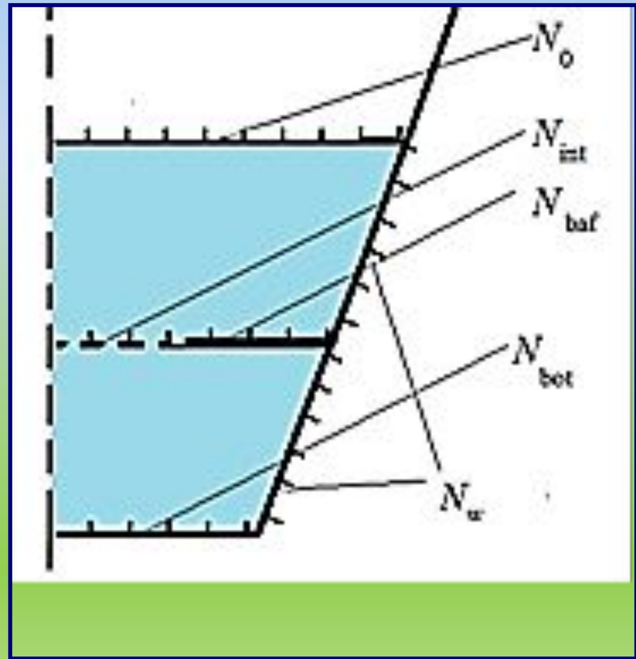
$$\psi(\xi)+\oint_S\psi(\mathbf{r})q^*(\xi,\mathbf{r})dS(\mathbf{r})=\oint_S\frac{\partial\psi(\mathbf{r})}{\partial\mathbf{n}(\mathbf{r})}u^*(\xi,\mathbf{r})dS$$

$$u^*(\xi,\mathbf{r})=\frac{1}{|\xi-\mathbf{r}|}, q^*(\xi,\mathbf{r})=\frac{\partial}{\partial\mathbf{n}(\xi)}\left[\frac{1}{|\xi-\mathbf{r}|}\right]$$

ONE-DIMENSIONAL BOUNDARY ELEMENTS







Singular boundary method

- We divide boundary into **N** boundary elements; **m** elements belong to the free surface and others **N-m** belong to the shell walls. At each boundary element we select the collocation point r_k

$$2\pi\psi(\mathbf{r}_k)+\sum_{i=1}^{N}\int_{S_i}\psi(\mathbf{r})q^*(\mathbf{r}_k,\mathbf{r})dS_i=\sum_{i=1}^{N}\int_{S_i}\frac{\partial\psi(\mathbf{r})}{\partial\mathbf{n}(\mathbf{r})}u^*(\mathbf{r}_k,\mathbf{r})dS_i$$

- Cauchy's average theorem

- $$2\pi\psi(\mathbf{r}_k)+\sum_{i=1}^{N}q^*(\mathbf{r}_k,\mathbf{r}_i)\int_{S_i}\psi(\mathbf{r})dS_i=\sum_{i=1}^{N}u^*(\mathbf{r}_k,\mathbf{r}_i)\int_{S_i}\frac{\partial\psi(\mathbf{r})}{\partial\mathbf{n}(\mathbf{r})}dS_i$$

Origin intensity factors

$$2\pi\psi(\mathbf{r}_k) + \sum_{i=1}^{m} q^*(\mathbf{r}_k, \mathbf{r}_i) \int_{S_i} \psi(\mathbf{r}) dS_i + \sum_{i=m+1}^{N} q^*(\mathbf{r}_k, \mathbf{r}_i) \int_{S_i} \psi(\mathbf{r}) dS_i =$$

$$= \frac{\omega^2}{g} \sum_{i=1}^{m} u^*(\mathbf{r}_k, \mathbf{r}_i) \int_{S_i} \psi(\mathbf{r}) dS_i .$$

$$\beta_{0i} = \int_{S_i} \psi(\mathbf{r}) dS_i$$

$$\beta_{1i} = \int_{S_i} \psi(\mathbf{r}) dS_i$$

$$S_i \in S_0$$

$$S_i \in S_1$$

Origin intensity factors

$$U_{kk}^{*\alpha\beta} = \frac{\int\limits_{S_k} u^*(\mathbf{r}_k, \mathbf{r}) dS_k}{S_k}, Q_{kk}^{*\alpha\beta} = \frac{\int\limits_{S_k} q^*(\mathbf{r}_k, \mathbf{r}) dS_k}{S_k}.$$

$$\iint\limits_{S_k} \frac{1}{|\mathbf{r}_k - \mathbf{r}|} dS_k = \int\limits_{\Gamma_k} \Phi(\mathbf{r}_k, \mathbf{r}) \rho(z) d\Gamma_k,$$

$$\iint\limits_{S_k} \frac{\partial}{\partial \mathbf{n}} \left(\frac{1}{|\mathbf{r}_k - \mathbf{r}|} \right) dS_k = \int\limits_{\Gamma_k} \Theta(\mathbf{r}_k, \mathbf{r}) \rho(z) d\Gamma_k,$$

REDUCING TO ELLIPTICAL INTEGRALS

$$\Phi(r_k, r) = \varphi^1(r_k, r)E(m) + \varphi^2(r_k, r)E(m)$$

$$\Theta(r_k, r) = \theta^1(r_k, r)E(m) + \theta^2(r_k, r)E(m)$$

$$E(m) = \int_0^{\pi/2} \sqrt{1 - m^2 \sin^2\theta}\, d\theta$$

$$K(m) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - m^2 \sin^2\theta}}$$

$$K(m) = \frac{2}{\pi}\ln\frac{1}{m'} + \frac{4}{\pi}\int_0^1 \frac{1}{\sqrt{(1 - x^2)(1 - m'^2 x^2)}}\ln\frac{1}{x}\, dx$$

The system of integral equations

- Notations

$$2\pi\psi_1 + \iint\limits_{S_1} \psi_1 \frac{\partial}{\partial n}\frac{1}{r(P,P_0)}dS_1 = A\psi_1$$

$$B\psi_0 = \iint\limits_{S_0} \psi_0 \frac{1}{r} dS_0; \quad C\psi_0 = \iint\limits_{S_0} \psi_0 \frac{\partial}{\partial z}\left(\frac{1}{r}\right) dS_0$$

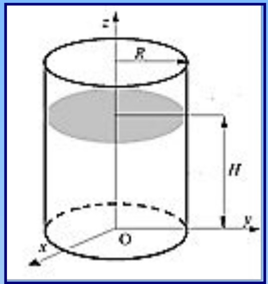
$$-\iint\limits_{S_1} \psi_1 \frac{\partial}{\partial n}\frac{1}{r(P,P_0)}dS_1 = D\psi_1$$

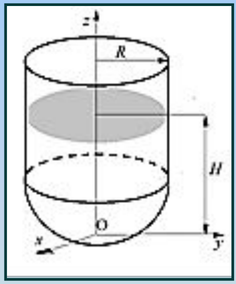
$$2\pi\psi_0 - \frac{\kappa^2}{g}\iint\limits_{S_0} \psi_0 \frac{1}{r} dS_0 = 2\pi E\psi_0 - \frac{\kappa^2}{g}F\psi_0$$

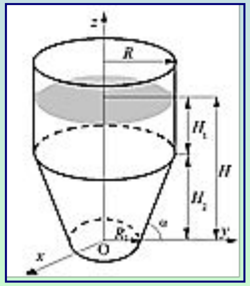
- **The natural modes (third system of basic functions) and eigenvalues problem**

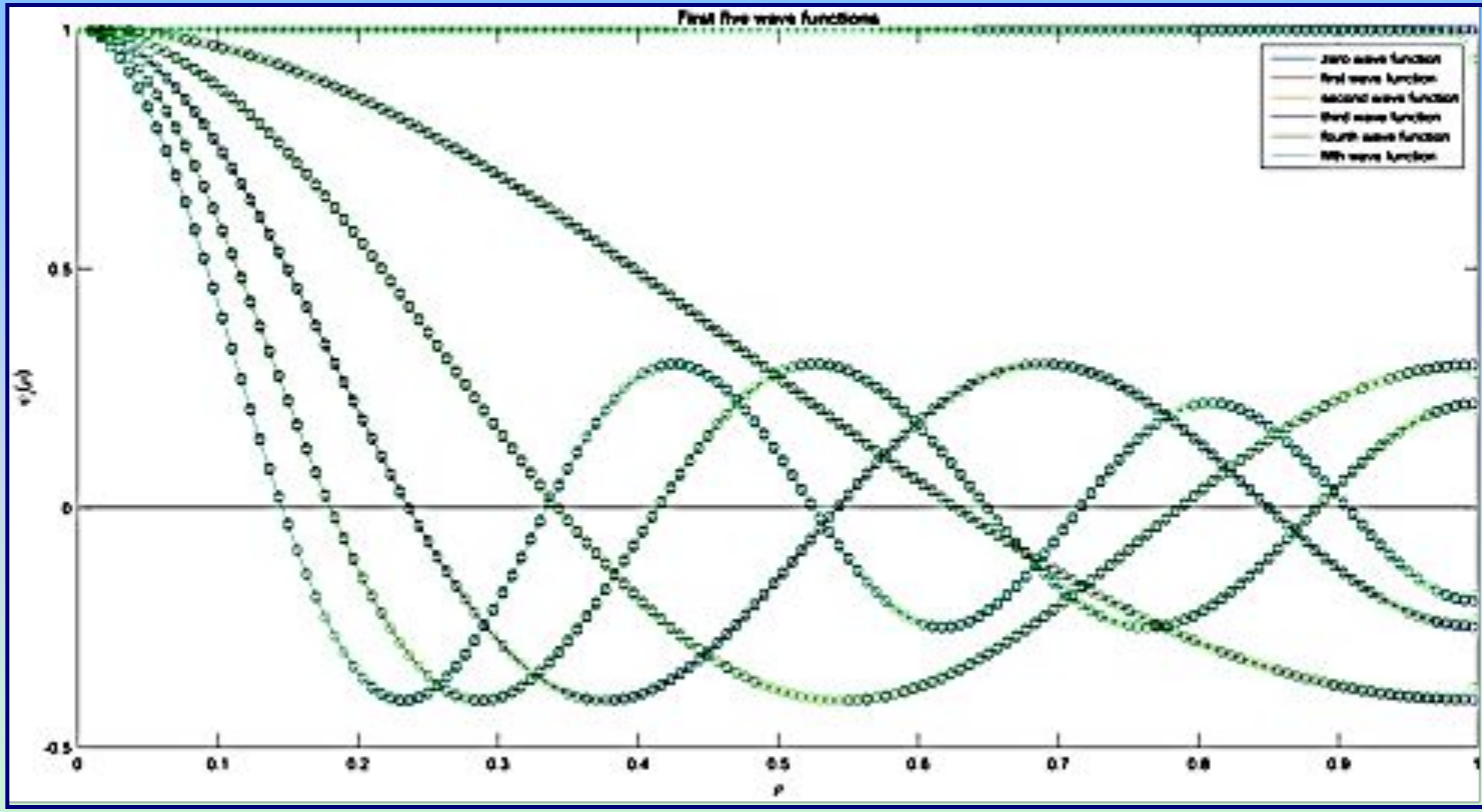
$$\left(\tilde{A} - \lambda E\right)\psi_0 = 0, \qquad \tilde{A} = (DA^{-1}B + F)^{-1}(2\pi E + DA^{-1}C); \qquad \lambda = \frac{\kappa^2}{g}$$

First five eigenmodes for cylindrical shell with different bottoms using BEM and SBM

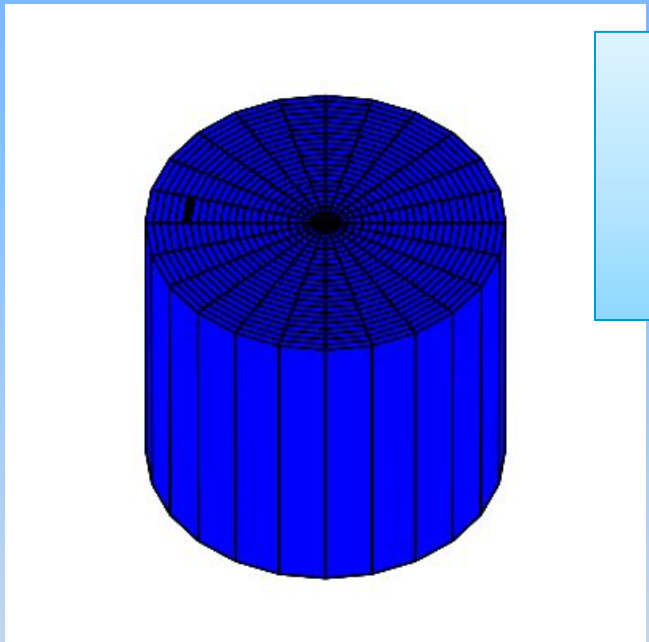




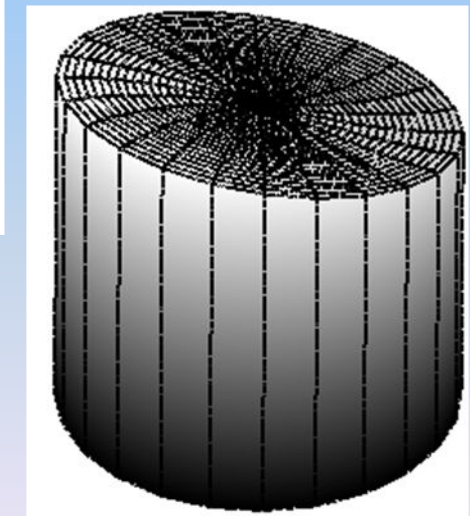


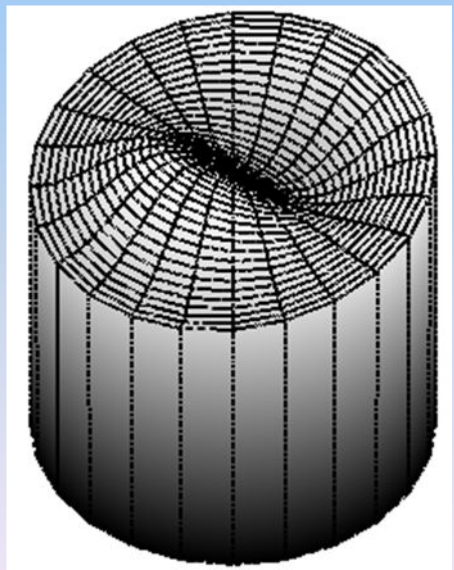


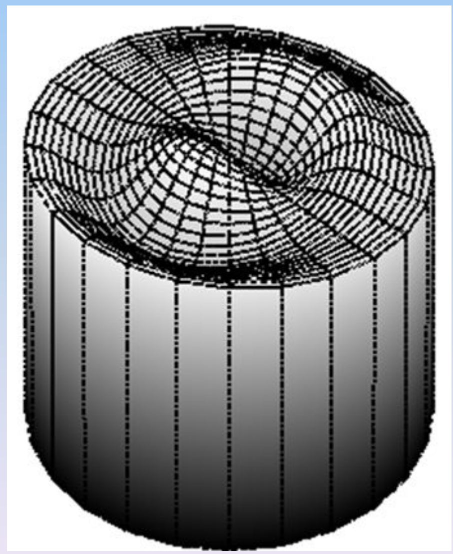
VALIDATION OF SINGULAR BOUNDARY METHOD



Method	Frequency parameter ω^2/g				
	n=1	n=2	n=3	n=4	n=5
BEM	1.833886	5.331447	8.536322	11.706103	14.864072
ANALYTICAL	1.833885	5.331442	8.536316	11.706005	14.863589







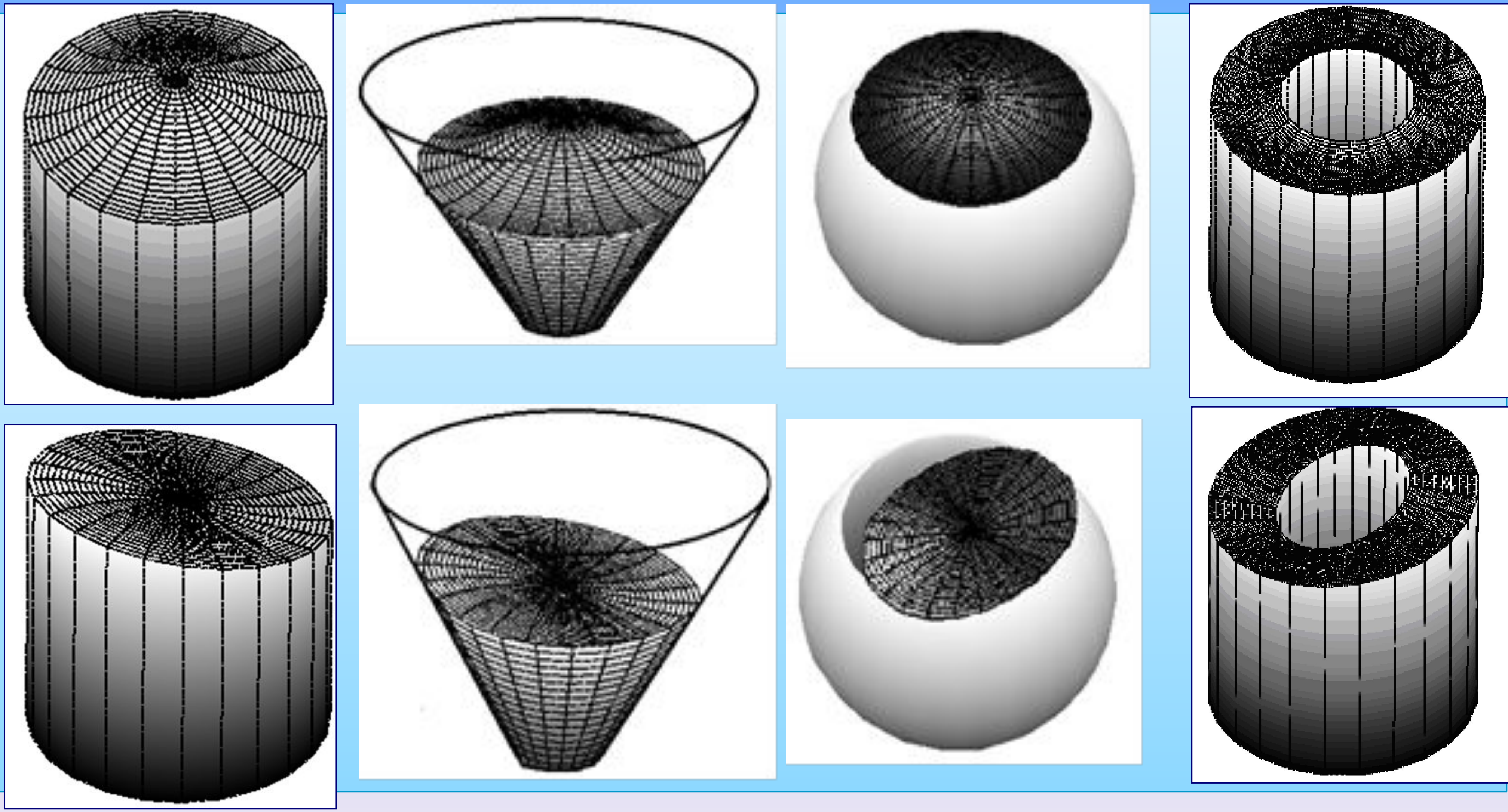
Vibrations of compound cylindrical-spherical elastic shells

- **fluid-filled elastic shell composed of a cylindrical part bounded by a hemispherical edge with thickness h=0.01m, radius R=1m, height L=R+H=2m, elasticity modulus E=2,11•106 MPa, Poisson's ratio ν =0.3, mass density ρ_s=8000 kg/m3, and liquid density ρ_l=1000 kg/m^3**

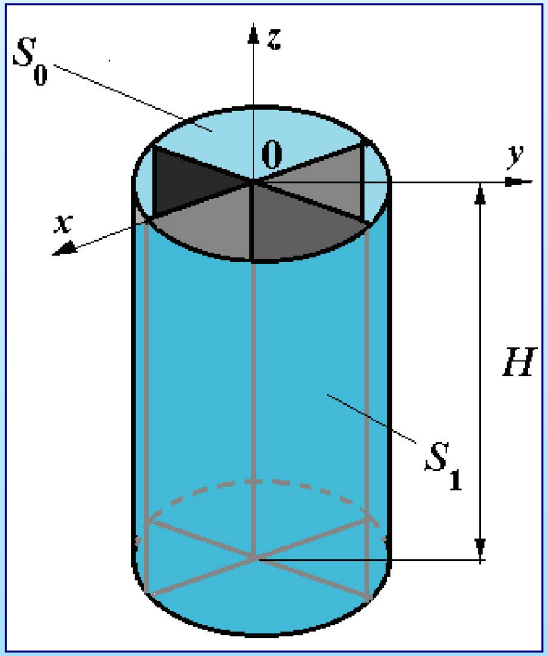
  - **Frequencies of empty and fluid-filled shells, Hz**

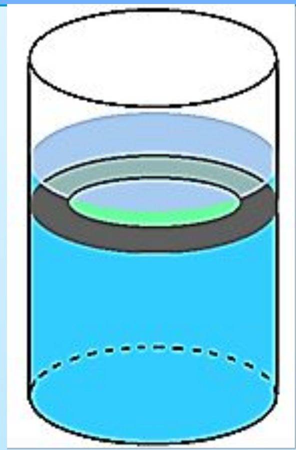
α	*m*	Sloshing CS	Sloshing CSS	Empty CS	Empty CSS	Fluid-filled CS	Fluid-filled CSS
0	1	6.1278	6.1283	398.132	796.263	145.582	221.553
	2	8.1217	8.2929	610.929	799.048	344.468	417.007
	3	9.9849	9.9871	810.703	817.043	398.132	512.361
1	1	4.3494	4.2392	235.485	473.302	77.780	169.845
	2	7.2283	7.2291	606.710	779.754	348.210	389.965
	3	9.1463	9.1479	730.413	811.617	629.659	492.862
2	1	5.4709	5.4708	117.679	290.119	49.489	136.445
	2	8.1076	8.1077	389.150	671.876	184.712	366.504
	3	9.8843	9.8861	619.164	774.231	319.253	483.943
4	1	7.2188	7.2188	54.491	134.018	28.032	73.046
	2	9.5376	9.5383	186.299	426.793	100.414	251.088
	3	11.1482	11.1494	374.973	654.876	213.825	421.393
5	1	7.9292	7.9286	65.136	110.755	35.401	60.490
	2	10.1535	10.1535	148.954	348.935	83.067	204.221
	3	11.7078	11.7082	300.608	592.120	176.017	379.170
6	1	8.5739	8.5724	88.609	112.813	52.074	68.111
	2	10.7239	10.7227	139.468	300.132	83.090	188.162
	3	12.2322	12.3824	255.945	538.350	158.914	359.072

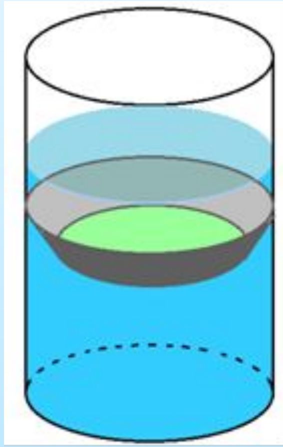
FREE SURFACE VIBRATIONS IN DIFFEREBT SHELLS

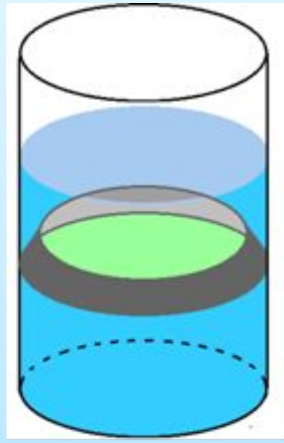


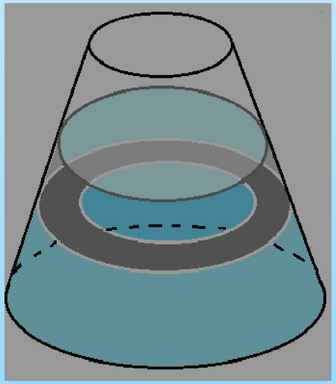
BAFFLED SHELLS

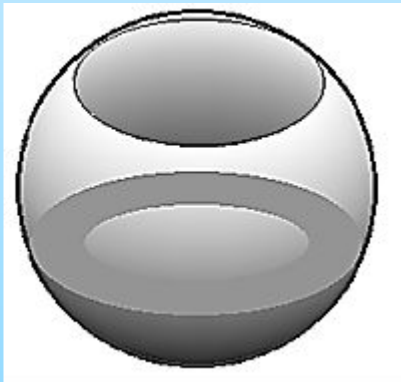


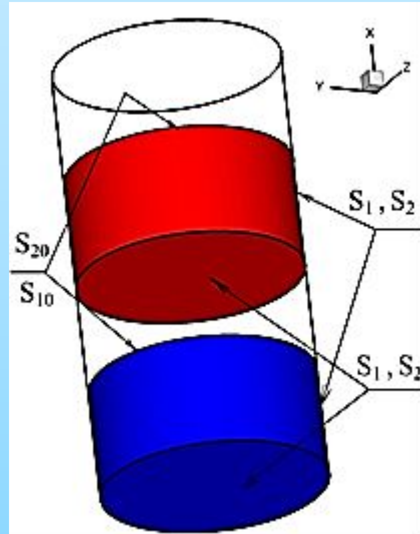




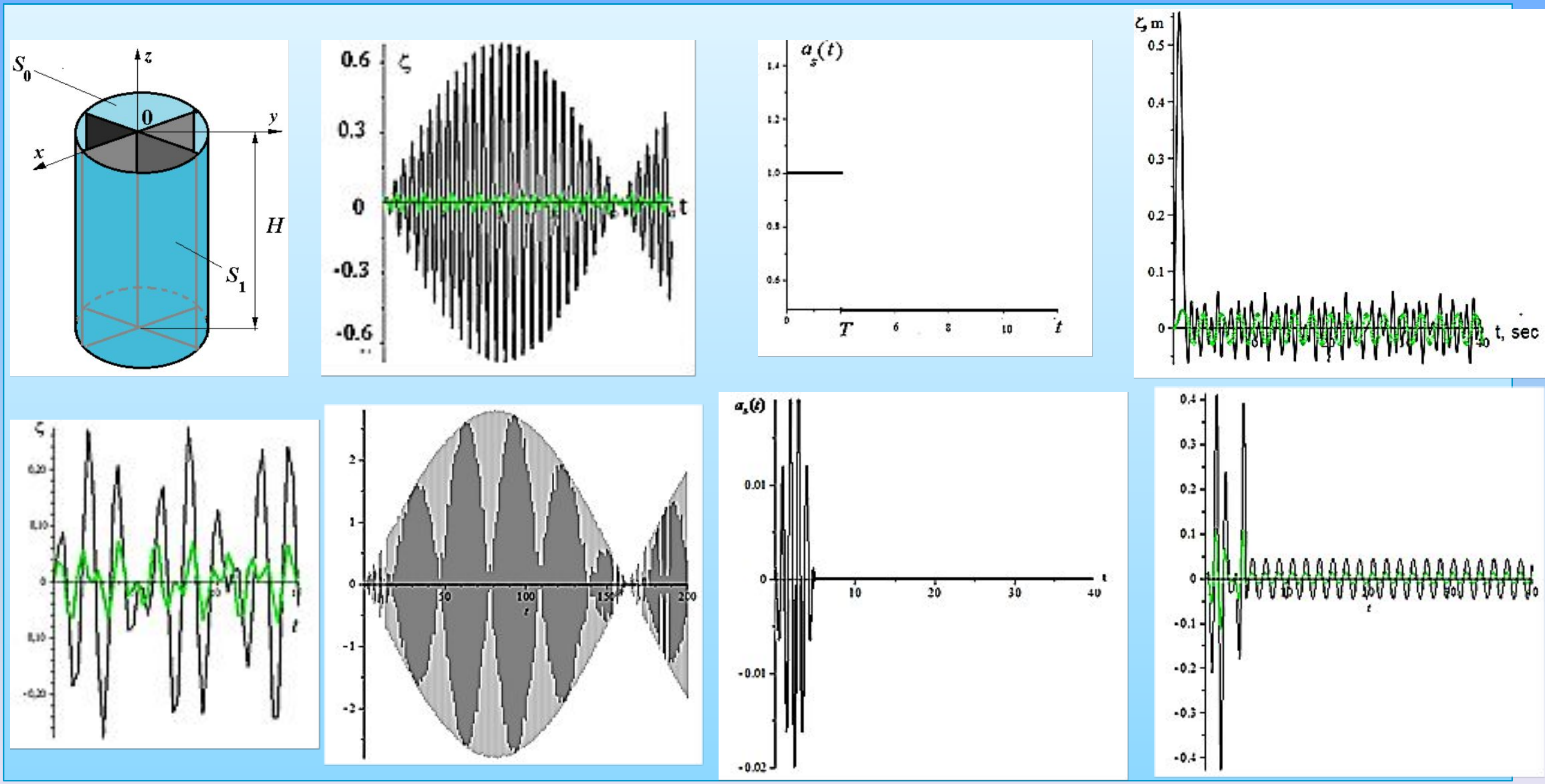








INFLUENCE OF BAFFLES ON SLOSHING AMPLITUDES

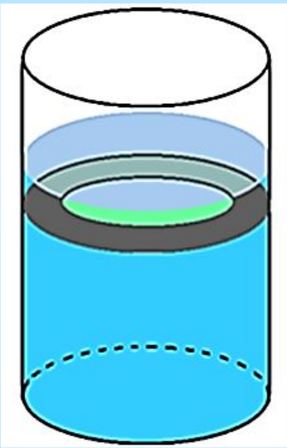


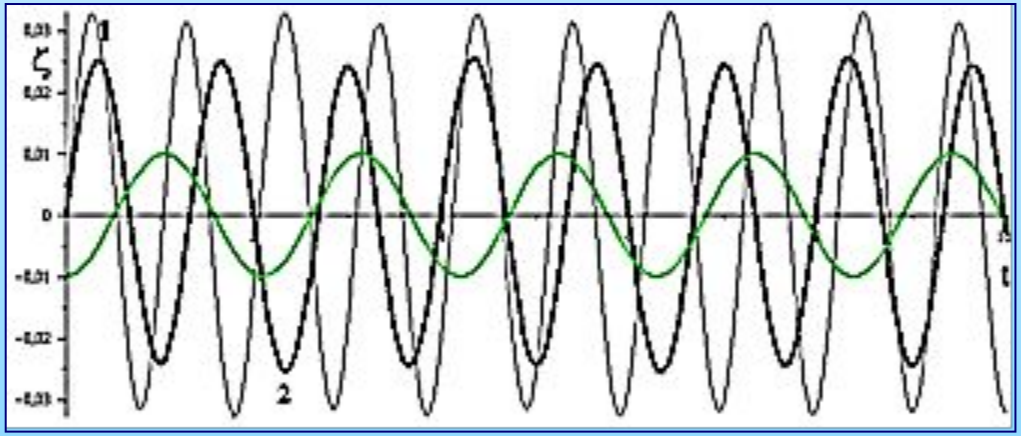
VERTICAL EXCITATIONS

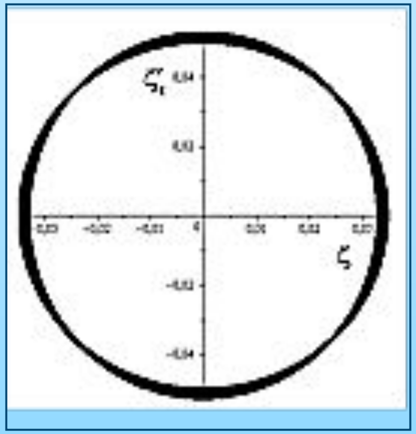
$$\ddot{d}_{0k} + \chi_{0k}^2 \left(1 - \frac{a_2 \cos \omega t}{g}\right) d_{0k} = 0, \quad k = \overline{1, M}.$$

$$\dot{d}_{01}(0) = 0.05, \ \dot{d}_{0k}(0) = 0.0, k = \overline{2, M}, \ d_{0k}(0) = 0, \ k = \overline{1, M}.$$

$\omega = 3\text{Hz}, \ A$

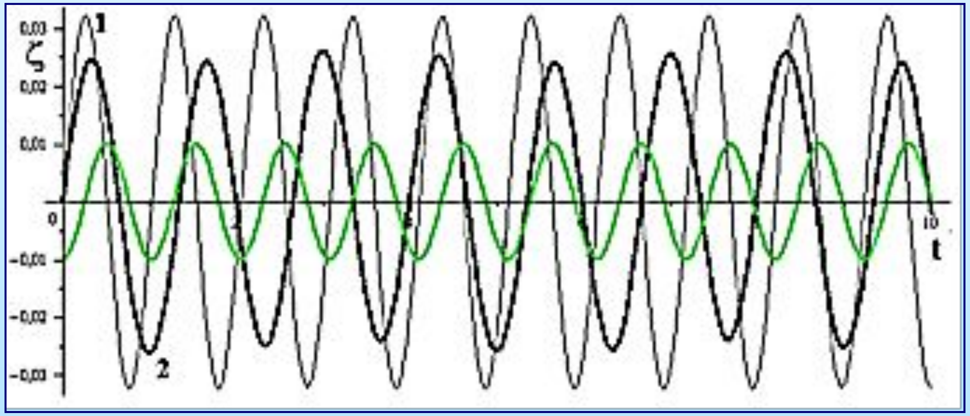


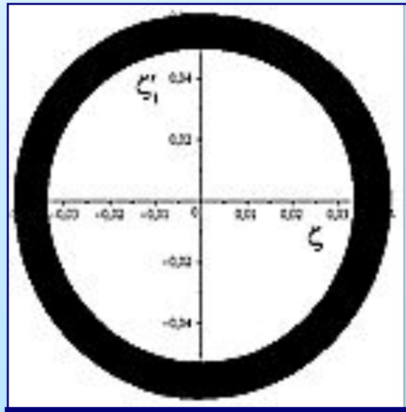




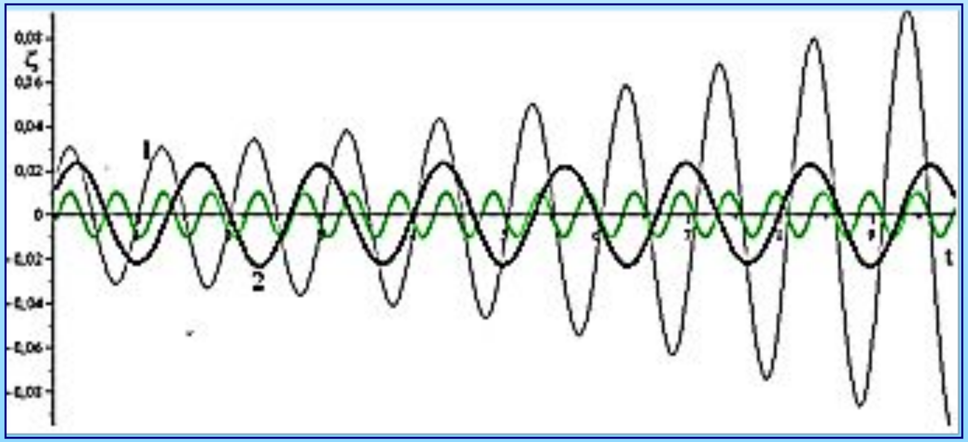
VERTICAL EXCITATIONS

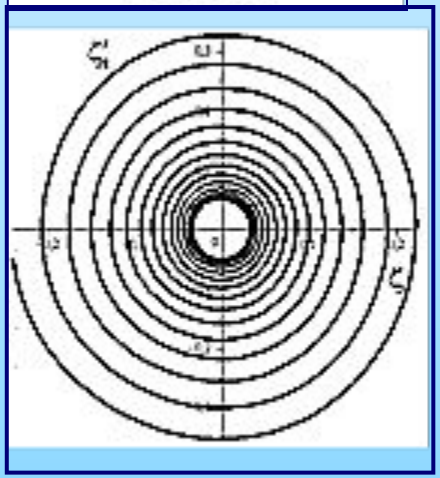
$\omega = 6.125$Hz, B



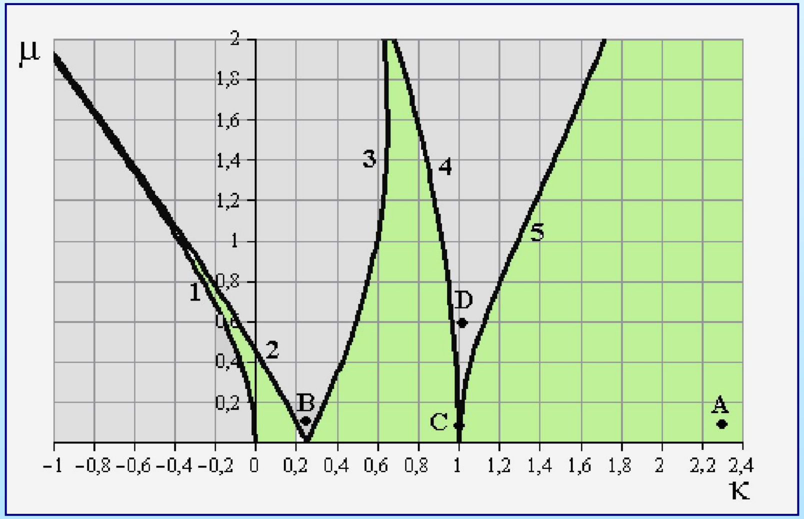


$\omega = 12.25$Hz, C

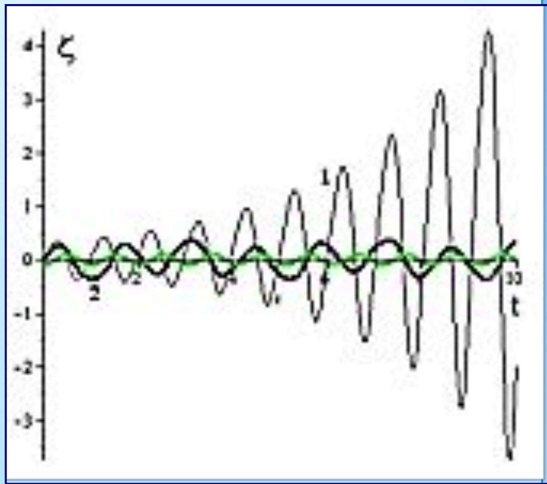




Ince-Strutt diagram



ω = 6.125Hz,
a = 0.6, D



$$\mu_1(\kappa) = 2\sqrt{\kappa(\kappa-1)(\kappa-4)/(3\kappa-8)}, \kappa < 0,$$

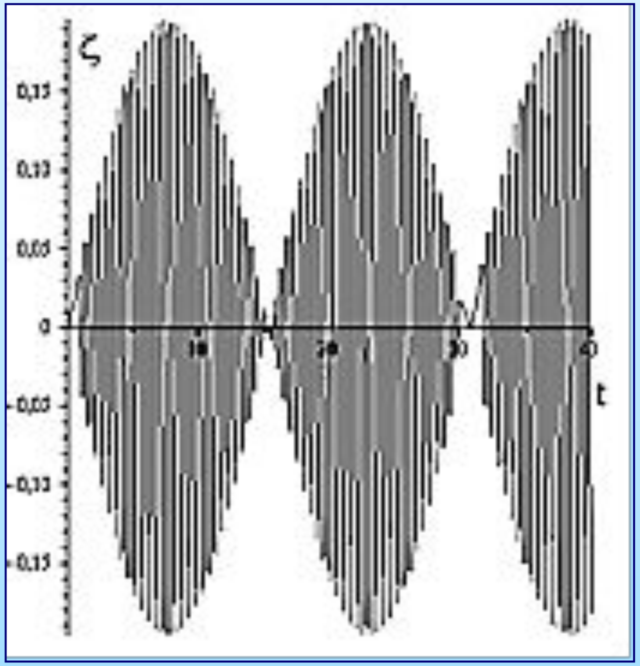
$$\mu_2(\kappa) = \frac{1}{4}\sqrt{(9-4\kappa)(13-29\kappa)} - (9-4\kappa), 0 < \kappa < 1/4,$$

$$\mu_3(\kappa) = \frac{1}{4}\left(9-4\kappa \pm \sqrt{(9-4\kappa)(13-29\kappa)}\right), 1/4 < \kappa < 13/20,$$

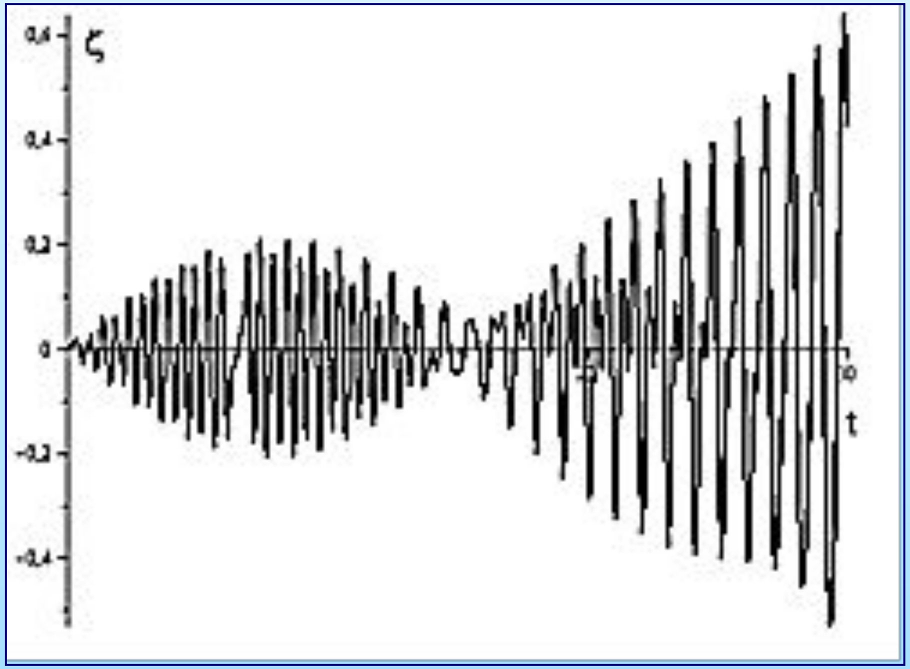
$$\mu_4(\kappa) = \sqrt{2(\kappa-1)(\kappa-4)(\kappa-9)/(\kappa-5)}, 13/20 < \kappa < 1,$$

$$\mu_5(\kappa) = 2\sqrt{\kappa(\kappa-1)(\kappa-4)/(3\kappa-8)}, \kappa > 1.$$

Free surface elevation without (a) and with (b) longitudinal excitations

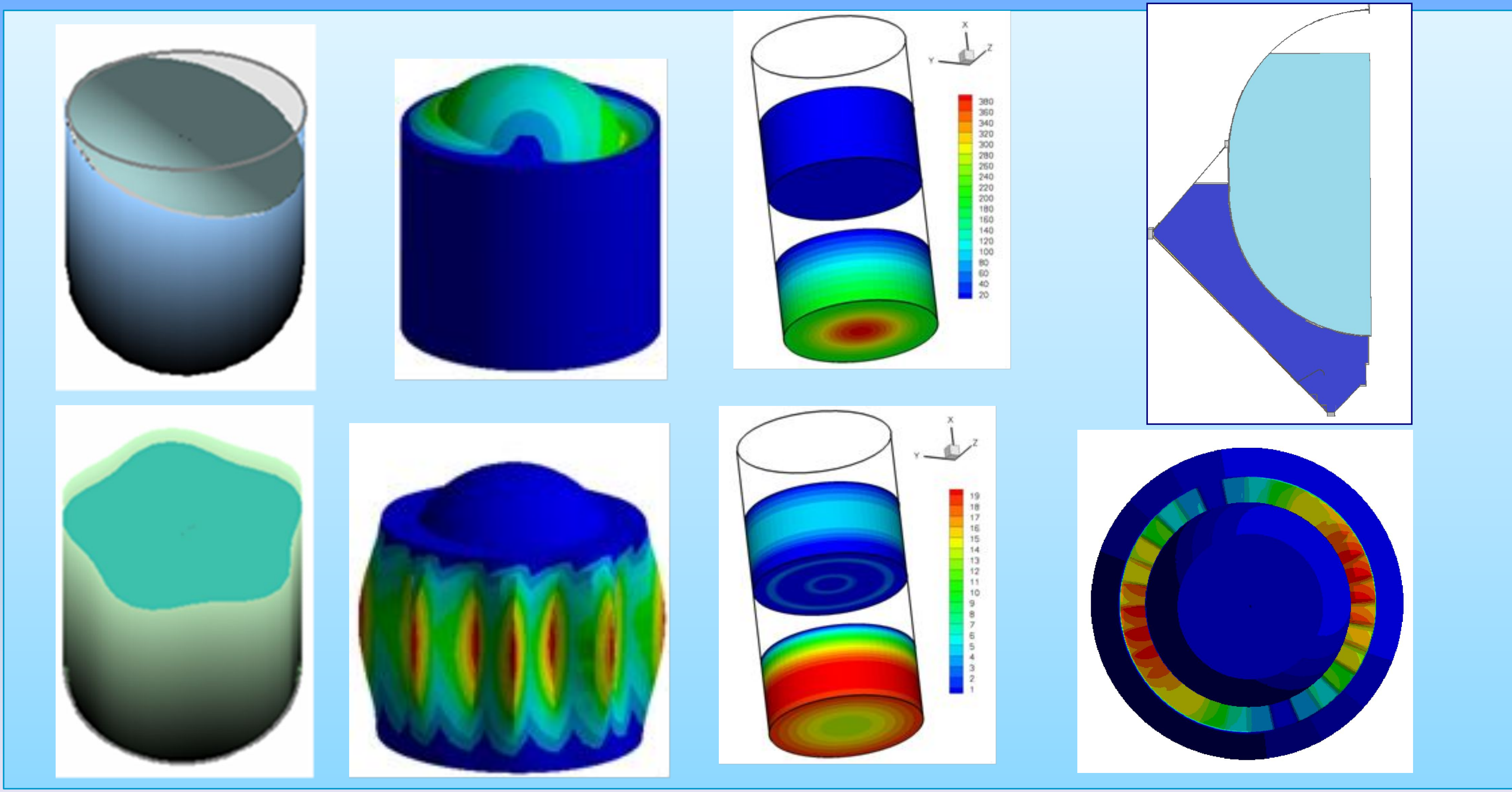


(a)



(b)

LIQUID VIBRATIONS IN DIFFERENT FUEL TANKS



Thank you very much for your attention

Welcome to Kharkov!