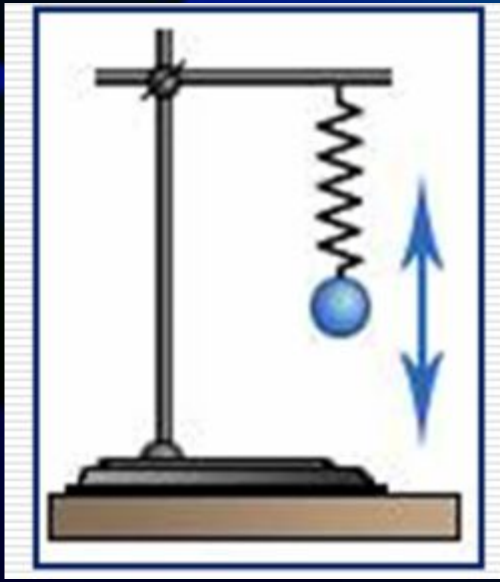
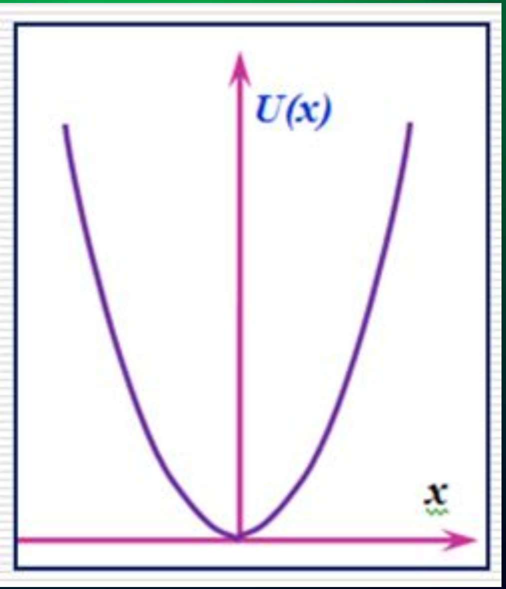
# Harmonic oscillator

# Lecture № 10

- Linear harmonic oscillator — system that performs a one-dimensional oscillatory motion under the action of a quasi-elastic force. It is a model for studying of oscillatory motion.
- In classical physics, it is spring (пружинный), physical, mathematical pendulums.
- In quantum physics- quantum (quantum mechanical) oscillator. But the model is the same.
- The classical harmonic oscillator is a ball of mass m suspended on a spring.





# Harmonic Oscillator

In physics the model of a harmonic oscillator plays an important role, especially at small oscillations of systems around a stable equilibrium position.

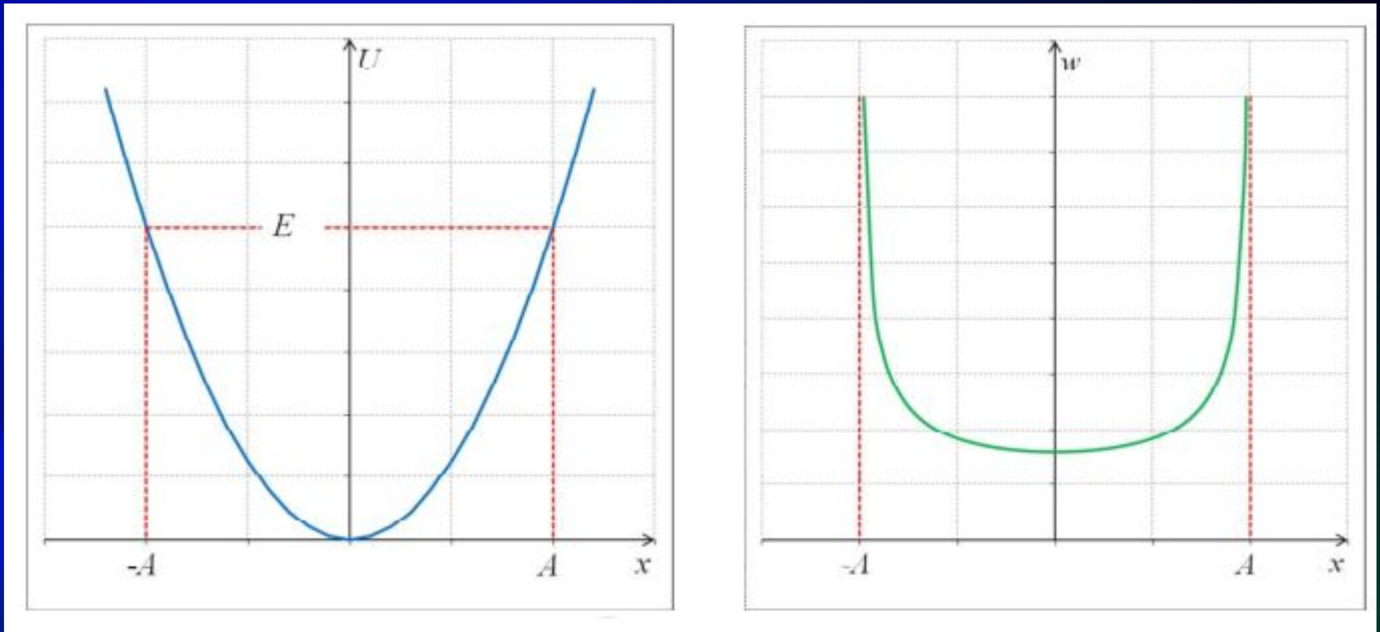
If a material point is affected by a quasielastic force

$$F = -kx$$

then oscillations are made on its own frequency $\omega^2 = k/m$.

The potential energy of the ball is $U = kx^2/2$.

The task of a harmonic oscillator is a task of the behavior of particles in a potential well of a parabolic shape.




Classical oscillator makes movements on a distance (-*A*, *A*). The total energy of the oscillator remains constant and equal $kA^2/2$ . In points $x = \pm A$

kinetic energy is equal zero. The potential energy is equal to total energy:

$$E = T + U = \frac{p^2}{2m} + \frac{kx^2}{2} = \frac{kA^2}{2}$$

The minimum value of the total energy of the classical oscillator is zero. The probability $\omega(x)dx$ of detecting an oscillator in the interval from x to x + dx is proportional to the time of the oscillator travels this interval. If $T = 2\pi/\omega$ oscillation period then

$$w(x)\,dx = \frac{dt}{T} = \frac{\omega}{2\pi}\frac{dx}{V}$$

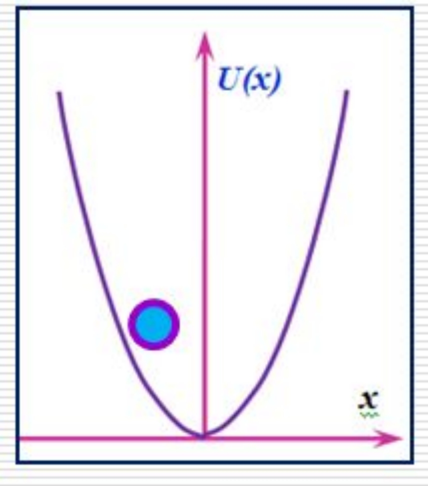
x=Asinωt, then speed

$$V = A\omega\cos\omega t = A\omega\sqrt{1-(x/A)^2}$$

then

$$w(x)\,dx = \frac{1}{2\pi A}\frac{dx}{\sqrt{1-(x/A)^2}}$$

- To solve the problem of a quantum mechanical oscillator, it is necessary to find a finite, unique, continuous and smooth (конечное, однозначное, непрерывное и гладкое) solution of the Schrödinger equation at $U = -kx^2/2$




$$\Delta\psi + \frac{2m}{\hbar}(E - U)\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - U)\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E + \frac{kx^2}{2}\right)\psi = 0$$

$$\frac{\hbar^2}{m}\frac{d^2\psi}{dx^2} + \frac{k}{2}x^2\psi = E\psi$$

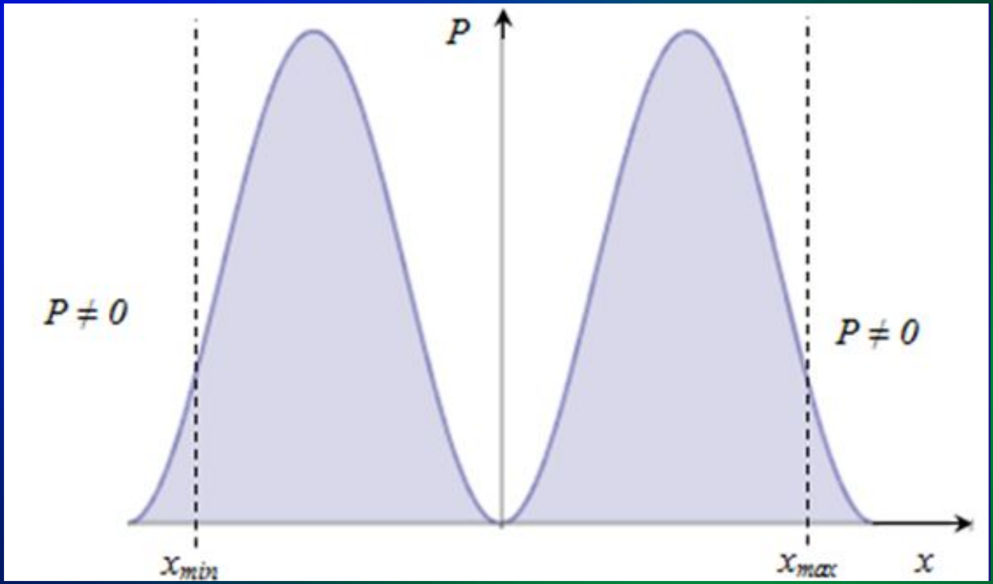
The exact solution of the equation leads to the following expression for the spectrum of possible values of the oscillator energy:

$$E_n = \frac{h}{2\pi}\left(n+\frac{1}{2}\right)\sqrt{\frac{k}{m}} \Longleftrightarrow E_n = \left(n+\frac{1}{2}\right)\hbar\omega \qquad (n = 1, 2, 3, \ldots)$$

$$E_0 = \frac{1}{2}\frac{h}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2}\hbar\omega$$

This shows that the smallest value of the oscillator energy is not zero and is called “zero energy”.

- The minimum value of $E_0$ (zero-point energy) is a consequence of the state of uncertainty, just as in the case of a particle in a "potential well". The presence of zero oscillations means that the particles cannot fall to the bottom of the well, since in this case the momentum p=0, Δp=0, Δx=∞ would be precisely determined, which does not correspond to the Heisenberg uncertainty relation.




$$\Delta E = E_{n+1} - E_n = (n+1+\frac{1}{2})\hbar\omega - (n+\frac{1}{2})\hbar\omega = \hbar\omega$$

The presence of zero-point energy: 1) contradicts the classical ideas, according to which Emin = 0.

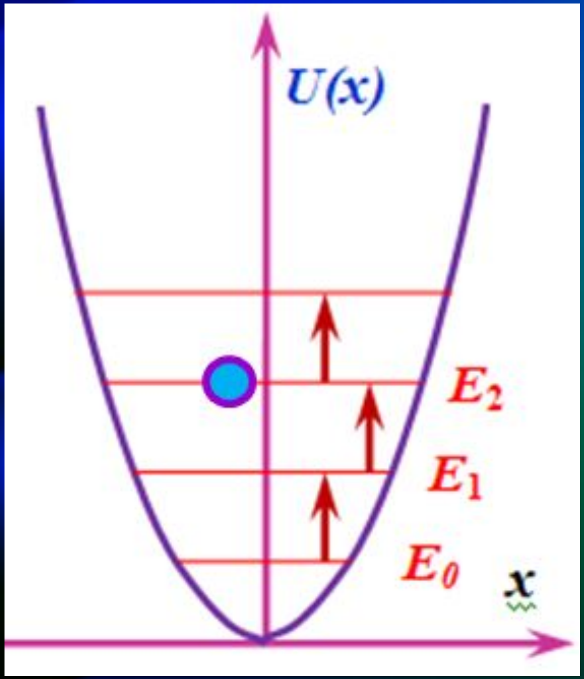
- 2) contradicting the quantization of energy levels and their location at equal distances from each other:

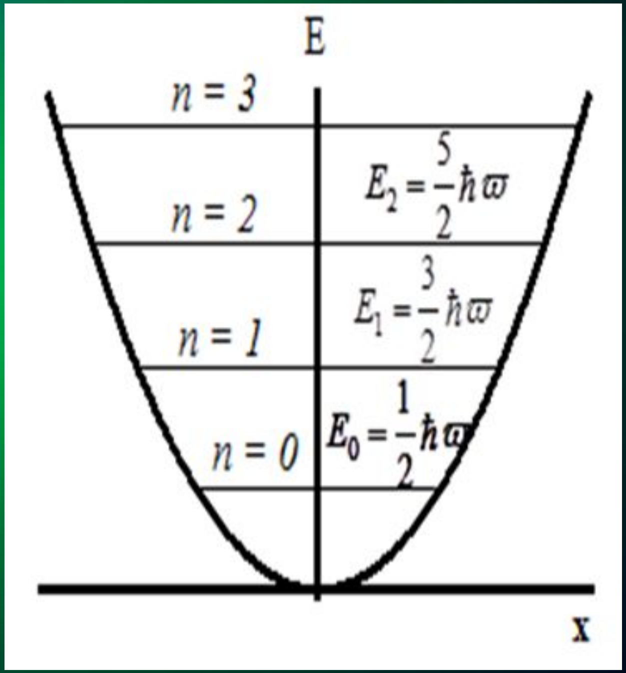
- 3) contradicts the possibility of finding a particle outside the region of potential well

$$-x_{min} \leq x \leq x_{max}$$

$$\Delta E_n = \hbar\omega$$

- A quantum mechanical particle cannot "lie" at the bottom of a parabolic potential well, it cannot lie at the bottom of a rectangular or any other potential well of finite width.
- The energy of the oscillator is proportional to the first degree n, so the energy levels are equidirectional from one another (equidistant)



The figures of the wave functions, which are solutions of the equation for n = 0, 1, 2 and 6: along the x axis are distances equal to twice the amplitude of oscillations of the classical oscillator at energies equal to En.

In the lower figures, the solid curves show the probability density distribution curves for the same states of a quantum oscillator, and the dotted line shows the probability density of the classical oscillator in the vicinity of point x.