
Analytic Hierarchy Process

AHP-General Idea

- Develop an **hierarchy of decision criteria** and define the **alternative courses of actions**.
- AHP algorithm is basically composed of two steps:
 1. Determine the relative **weights** of the decision criteria
 2. Determine the relative **rankings** (priorities) of alternatives
- ! Both **qualitative and quantitative** information can be compared by using informed judgments to derive weights and priorities.

Example: Mobile Selection

- Objective
 - Selecting a mobile
- Criteria
 - Price, storage space, Camera, Looks?
- Alternatives
 - Mobile1, Mobile2, Mobile3, Mobile4

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	Attribute Or Criteria 	Price or Cost	Storage Space	Camera	Looks
Alternative	Mobile 1	250 \$	16 GB	12 MP	5
	Mobile 2	200 \$	16 GB	8 MP	3
	Mobile 3	300 \$	32 GB	16MP	4
	Mobile 4	275 \$	32 GB	8MP	4
	Mobile 5	225 \$	16 GB	16 MP	2

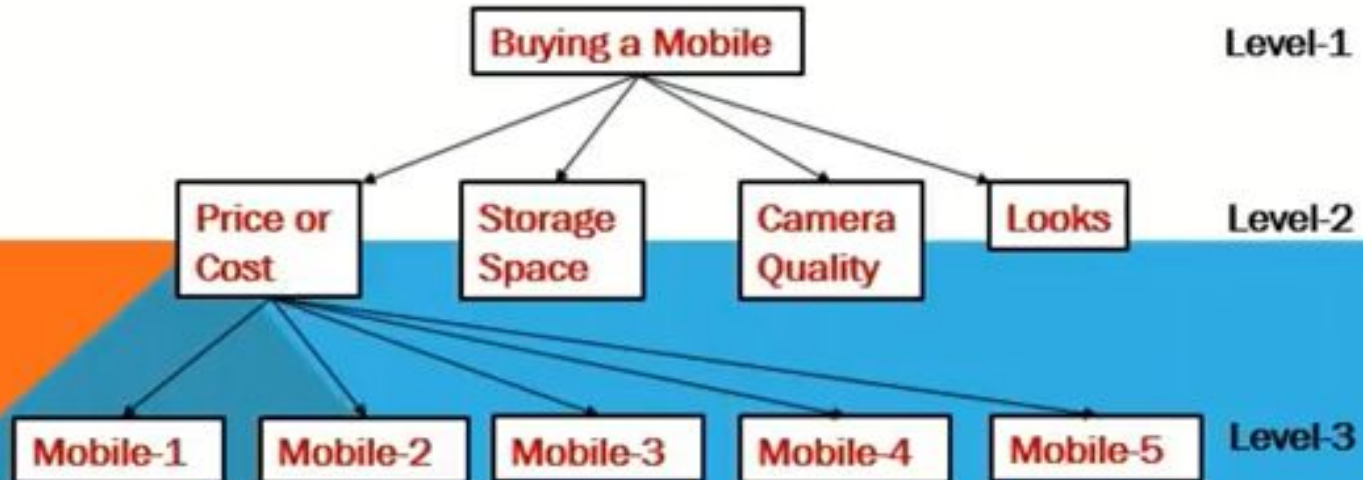
Ranking Scale for Criteria and Alternatives

Intensity of importance	Definition	Explanation
1	Equal importance	Two factors contribute equally to the objective
3	Somewhat more important	Experience and judgement slightly favour one over the other.
5	Much more important	Experience and judgement strongly favour one over the other.
7	Very much more important	Experience and judgement very strongly favour one over the other. Its importance is demonstrated in practice.
9	Absolutely more important.	The evidence favouring one over the other is of the highest possible validity.
2,4,6,8	Intermediate values	When compromise is needed

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	1	2	3	4
Attribute Or Criteria	Price or Cost	Storage Space	Camera	Looks

Step 1) Developing a hierarchical structure with a goal at the top level, the attributes/criteria at the second level and the alternatives at the third level.



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	1	2	3	4
Attribute Or Criteria	Price or Cost	Storage Space	Camera	Looks

Step 2) Determine the relative importance of different attributes or Criteria with respect to the goal.



Pair-wise comparison matrix

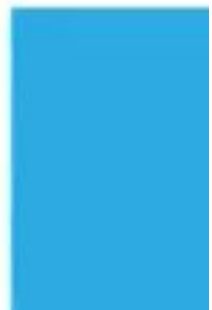
AHP

	1	2	3	4
Attribute Or Criteria →	Price or Cost	Storage Space	Camera	Looks

Pair-wise comparison matrix is created with the help of scale of relative importance



1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2, 4, 6, 8	Intermediate values
$1/3, 1/5, 1/7, 1/9$	Values for inverse comparison



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Pair-wise comparison matrix

	Price or Cost	Storage Space	Camera	Looks
Price or Cost		$\frac{5x}{x} = 5$		
Storage Space	$\frac{x}{5x} = \frac{1}{5}$			
Camera				
Looks				

How important is Price/cost with respect to storage space

Price/Cost is of a strong importance than Storage space

Storage space - x Value
Price/cost - $5x$ Value

- 1
- 3
- 5
- 7
- 9

- Equal importance
- Moderate importance
- Strong importance
- Very strong importance
- Extreme importance

2, 4, 6, 8

Intermediate values

$1/3, 1/5, 1/7, 1/9$

Values for inverse comparison

Manoj Mathew

Row element

Column element

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Pair-wise comparison matrix

	Price or Cost	Storage Space	Camera	Looks
Price or Cost		5	4	
Storage Space	$\frac{1}{5}$			
Camera	$\frac{1}{4}$	I		
Looks				

How important is Price/cost with respect to camera

Price/Cost is of moderate to strong importance than camera

Camera - x Value
Price/cost - 4x Value

1

Equal importance

3

Moderate importance

5

Strong importance

7

Very strong importance

9

Extreme importance

2, 4, 6, 8

Intermediate values

$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$

Values for inverse comparison

AHP

Pair-wise comparison matrix

	Price or Cost	Storage Space	Camera	Looks
Price or Cost		5	4	
Storage Space	$\frac{1}{5}$		$\frac{1}{2}$	
Camera	$\frac{1}{4}$	2		
Looks				

How important is Camera with respect to Storage space

Camera is of equal to moderate importance than camera

Camera - 2x Value
Storage space - x Value

- 1 Equal importance
- 3 Moderate importance
- 5 Strong importance
- 7 Very strong importance
- 9 Extreme importance

2, 4, 6, 8 Intermediate values

$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$ Values for inverse comparison

AHP

Pair-wise comparison matrix

	Price or Cost	Storage Space	Camera	Looks
Price or Cost	1	5	4	7
Storage Space	$\frac{1}{5}$	1	$\frac{1}{2}$	3
Camera	$\frac{1}{4}$	2	1	3
Looks	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{3}$	1

1

Equal importance

3

Moderate importance

5

Strong importance

7

Very strong importance

9

Extreme importance

2, 4, 6, 8

Intermediate values

$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$

Values for inverse comparison

AHP

Pair-wise comparison matrix

	Price or Cost	Storage Space	Camera	Looks
Price or Cost	1	5	4	7
Storage Space	0.2	1	0.5	3
Camera	0.25	2	1	3
Looks	0.14	0.33	0.33	1
Sum	1.59	8.33	5.83	14

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Pair-wise comparison matrix

	Price or Cost	Storage Space	Camera	Looks
Price or Cost	$\frac{1}{1.59}$	$\frac{5}{8.33}$	$\frac{4}{7}$	$\frac{7}{14}$
Storage Space	$\frac{0.2}{1.59}$	$\frac{1}{8.33}$	$\frac{0.5}{7}$	$\frac{3}{14}$
Camera	$\frac{0.25}{1.59}$	$\frac{2}{8.33}$	$\frac{1}{7}$	$\frac{3}{14}$
Looks	$\frac{0.14}{1.59}$	$\frac{0.33}{8.33}$	$\frac{0.33}{7}$	$\frac{1}{14}$
Sum	1.59	8.33	5.83	14

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Normalised Pair-wise comparison matrix

	Price or Cost	Storage Space	Camera	Looks
Price or Cost	0.6289	0.6002	0.6861	0.5000
Storage Space	0.1258	0.1200	0.0858	0.2143
Camera	0.1572	0.2401	0.1715	0.2143
Looks	0.0898	0.0400	0.0572	0.0714

AHP

Normalised Pair-wise comparison matrix

	Price or Cost	Storage Space	Camera	Looks	Criteria weights
Price or Cost	0.6289	0.6002	0.6861	0.5000	0.6038
Storage Space	0.1258	0.1200	0.0858	0.2143	0.1365
Camera	0.1572	0.2401	0.1715	0.2143	0.1958
Looks	0.0898	0.0400	0.0572	0.0714	0.0646

$$\frac{0.6289+0.6002+0.6861+0.5000}{4} = 0.6038$$

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Step-3 Calculating the Consistency

Criteria weights	0.6038	0.1365	0.1957	0.0646
	Price or Cost	Storage Space	Camera	Looks
Price or Cost	1	5	4	7
Storage Space	0.2	1	0.5	3
Camera	0.25	2	1	3
Looks	0.14	0.33	0.33	1

AHP

Criteria weights	0.6038	0.1365	0.1957	0.0646
	Price or Cost	Storage Space	Camera	Looks
Price or Cost	1×0.6038	5×0.1365	4×0.1957	7×0.0646
Storage Space	0.2×0.6038	1×0.1365	0.5×0.1957	3×0.0646
Camera	0.25×0.6038	2×0.1365	1×0.1957	3×0.0646
Looks	0.14×0.6038	0.33×0.1365	0.33×0.1957	1×0.0646

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	Price or Cost	Storage Space	Camera	Looks	Weighted Sum value
Price or Cost	0.6038	0.6825	0.7832	0.4522	2.5217
Storage Space	0.1208	0.1365	0.0979	0.1938	0.5490
Camera	0.1510	0.2730	0.1958	0.1938	0.8136
Looks	0.0863	0.0455	0.0653	0.0646	0.2616

$$0.6038+0.6825+0.7832+0.4522= 2.5217$$

AHP

	Price or Cost	Storage Space	Camera	Looks	Weighted Sum value	Criteria weights
Price or Cost	0.6038	0.6825	0.7832	0.4522	2.5217	0.6038
Storage Space	0.1208	0.1365	0.0979	0.1938	0.5490	0.1365
Camera	0.1510	0.2730	0.1958	0.1938	0.8136	0.1958
Looks	0.0863	0.0455	0.0653	0.0646	0.2616	0.0646

AHP

	Price or Cost	Storage Space	Camera	Looks	Weighted Sum value	Criteria weights	
Price or Cost	0.6038	0.6825	0.7832	0.4522	2.5217	0.6038	$\frac{2.5217}{0.6038}$
Storage Space	0.1208	0.1365	0.0979	0.1938	0.5490	0.1365	$\frac{0.5490}{0.1365}$
Camera	0.1510	0.2730	0.1958	0.1938	0.8136	0.1958	$\frac{0.8136}{0.1958}$
Looks	0.0863	0.0455	0.0653	0.0646	0.2616	0.0646	$\frac{0.2616}{0.0646}$

AHP

	Price or Cost	Storage Space	Camera	Looks	Weighted Sum value	Criteria weights	
Price or Cost	0.6038	0.6825	0.7832	0.4522	2.5217	0.6038	4.1762
Storage Space	0.1208	0.1365	0.0979	0.1938	0.5490	0.1365	4.0225
Camera	0.1510	0.2730	0.1958	0.1938	0.8136	0.1958	4.1553
Looks	0.0863	0.0455	0.0653	0.0646	0.2616	0.0646	4.0488

$$\lambda_{max} = \frac{4.1762 + 4.0225 + 4.1553 + 4.0488}{4} = 4.1007$$

AHP

	Price or Cost	Storage Space	Camera	Looks
Price or Cost	0.6038	0.6825	0.7832	0.4522
Storage Space	0.1208	0.1365	0.0979	0.1938
Camera	0.1510	0.2730	0.1958	0.1938
Looks	0.0863	0.0455	0.0653	0.0646

$$\lambda_{max} = 4.1007$$

$$\text{Consistency Index (C.I.)} = \frac{\lambda_{max} - n}{n - 1} = \frac{4.1007 - 4}{4 - 1} = 0.03358$$

where n is the number of compared elements

AHP

	Price or Cost	Storage Space	Camera	Looks
Price or Cost	0.6038	0.6825	0.7832	0.4522
Storage Space	0.1208	0.1365	0.0979	0.1938
Camera	0.1510	0.2730	0.1958	0.1938
Looks	0.0863	0.0455	0.0653	0.0646

Consistency Index (C.I.)= 0.03358

Consistency Ratio= Consistency Index (C.I.)/RI

AHP

	Price or Cost	Storage Space	Camera	Looks						
Price or Cost	0.6038	0.6825	0.7832	0.4522						
St S n	1	2	3	4	5	6	7	8	9	10
Ca RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49
L										

Consistency Index (C.I.)= 0.03358

Consistency Ratio= Consistency Index (C.I.)/Random Index(R.I.)

$$\text{Consistency Ratio} = \frac{0.03358}{0.90} = 0.037311$$

AHP

	Price or Cost	Storage Space	Camera	Looks
Price or Cost	0.6038	0.6825	0.7832	0.4522
Storage Space	0.1208	0.1365	0.0979	0.1938
Camera	0.1510	0.2730	0.1958	0.1938
Looks	0.0863	0.0455	0.0653	0.0646

Consistency Ratio (CR) = 0.037311 < 0.10

AHP

	Criteria weights
Price or Cost	0.6038
Storage Space	0.1365
Camera	0.1958
Looks	0.0646

Consistency Ratio = $0.037311 < 0.10$