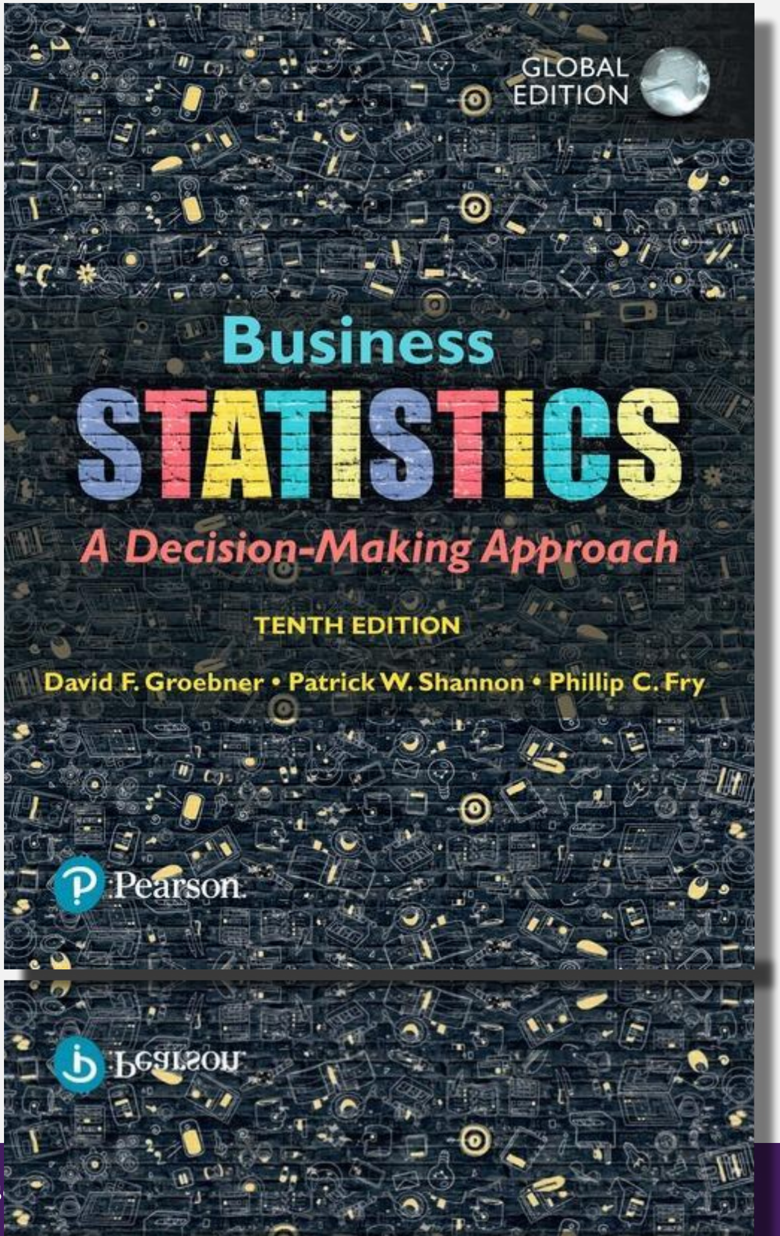
Chapter 14

Introduction to Linear Regression and Correlation Analysis



Learning Outcomes

Outcome 1. Calculate and interpret the correlation between two variables.

Outcome 2. Determine whether the correlation is significant.

Outcome 3. Calculate the simple linear regression equation for a set of data and know the basic assumptions behind regression analysis

Outcome 4. Determine whether a regression model is significant.

Outcome 5. Recognize regression analysis applications for purposes of description and prediction.

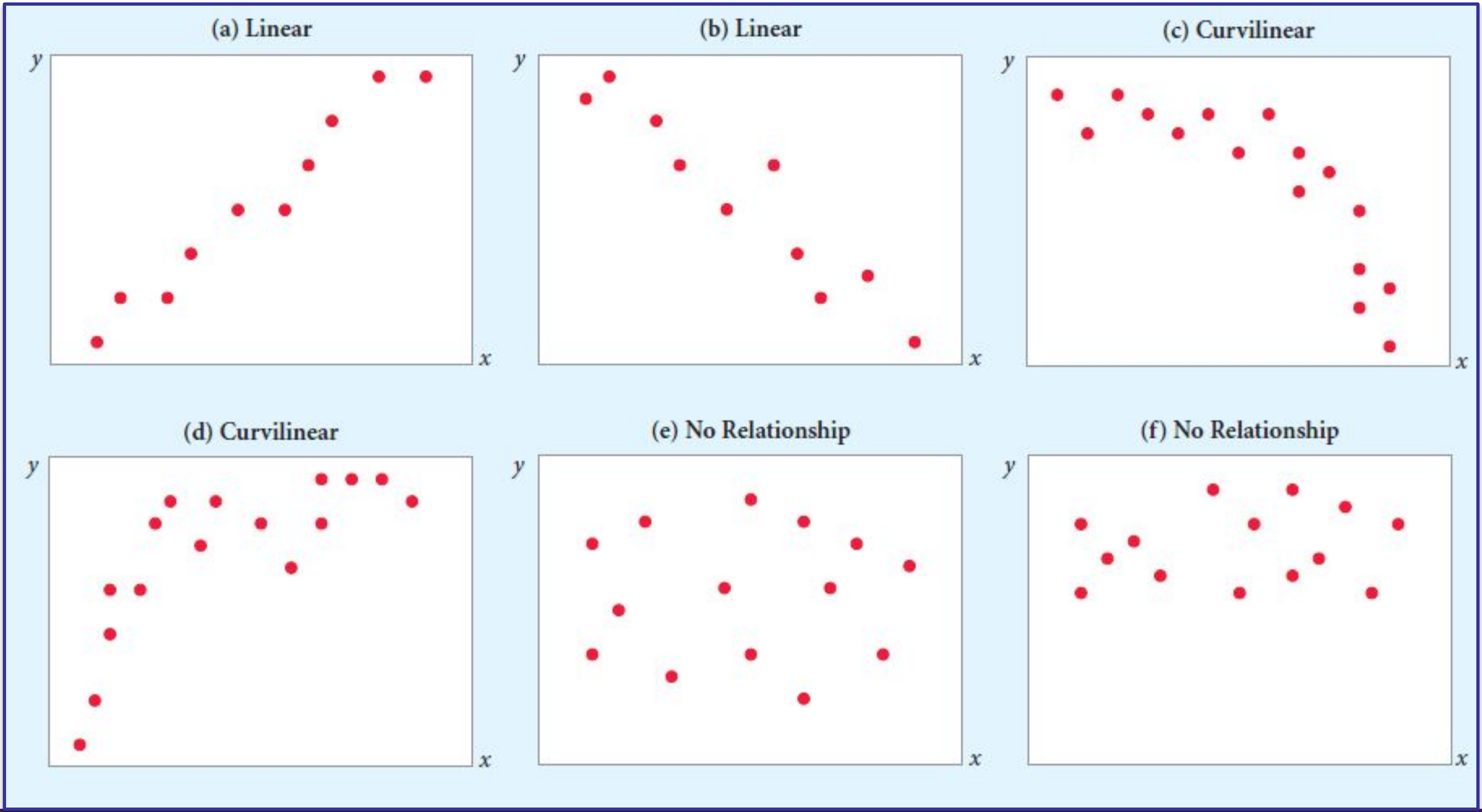
Outcome 6. Calculate and interpret confidence intervals for the regression analysis.

Outcome 7. Recognize some potential problems if regression analysis is used incorrectly.

14.1 Scatter Plots and Correlation

- Scatter Plot
  - A two-dimensional plot showing the values for the joint occurrence of two quantitative variables. The scatter plot may be used to graphically represent the relationship between two variables. It is also known as a scatter diagram.
- Correlation Coefficient
  - A quantitative measure of the strength of the linear relationship between two variables. The correlation ranges from -1.0 to + 1.0. A correlation of ±1.0 indicates a perfect linear relationship, whereas a correlation of 0 indicates no linear relationship.

Two-Variable Relationships

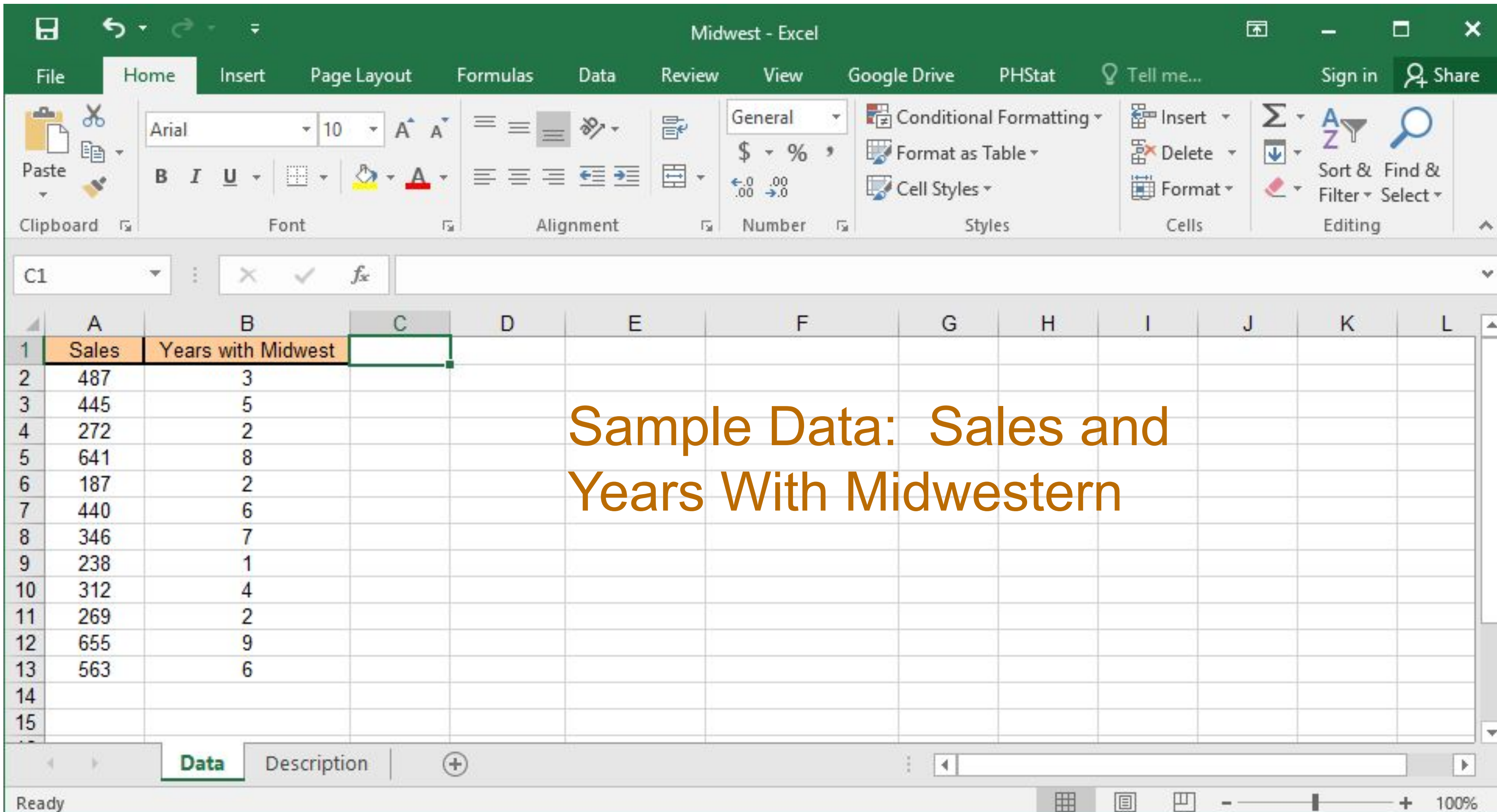


Scatter Plot – Example Using Excel 2016

The director of marketing for Midwest Distribution Company is concerned about the rapid turnover in her sales force. In the course of exit interviews, she discovered a major concern with the compensation structure. At issue is the relationship between sales and number of years with the company. The data for a random sample of 12 sales representatives was used for analysis.

Objective: Use Excel 2016 to first create a scatter plot using the data file **Midwest.xlsx**.

Scatter Plot – Example Using Excel 2016



Sales	Years with Midwest
487	3
445	5
272	2
641	8
187	2
440	6
346	7
238	1
312	4
269	2
655	9
563	6

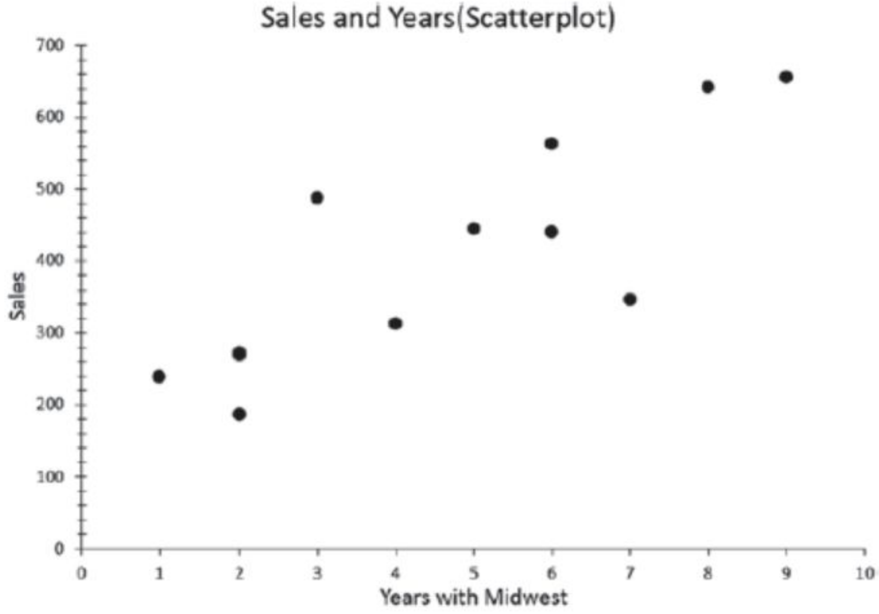
Sample Data: Sales and Years With Midwestern

Scatter Plot – Example Using Excel 2016

Excel 2016 Instructions

1. Open file: **Midwest.xlsx.**
2. Move the *Sales* column to the right of the *Years* column.
3. Select data to be used in the chart.
4. On the **Insert** tab, click **Scatter (X, Y)** or **Bubble Chart**, and then click the **Scatter** option.
5. Use the **Design** tab of **Chart Tools** to add titles and remove the grid lines.
6. Use the **Design** tab of **Chart Tools** to move the chart to a new worksheet.

FIGURE 14.3 Excel 2016 Scatter Plot of Sales vs. Years with Midwest Distribution



The relationship between Sales and Years With Midwestern appears to be positive and linear.

The Correlation Coefficient

- Sample Correlation Coefficient:

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{[\sum(x-\bar{x})^2][\sum(y-\bar{y})^2]}}$$

- Algebraic Equivalent:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

r - Sample correlation coefficient
n - Sample size
x - Value of the independent variable
y - Value of the dependent variable

The Correlation Coefficient

The Correlation Coefficient measures the strength of the linear relationship between two variables.

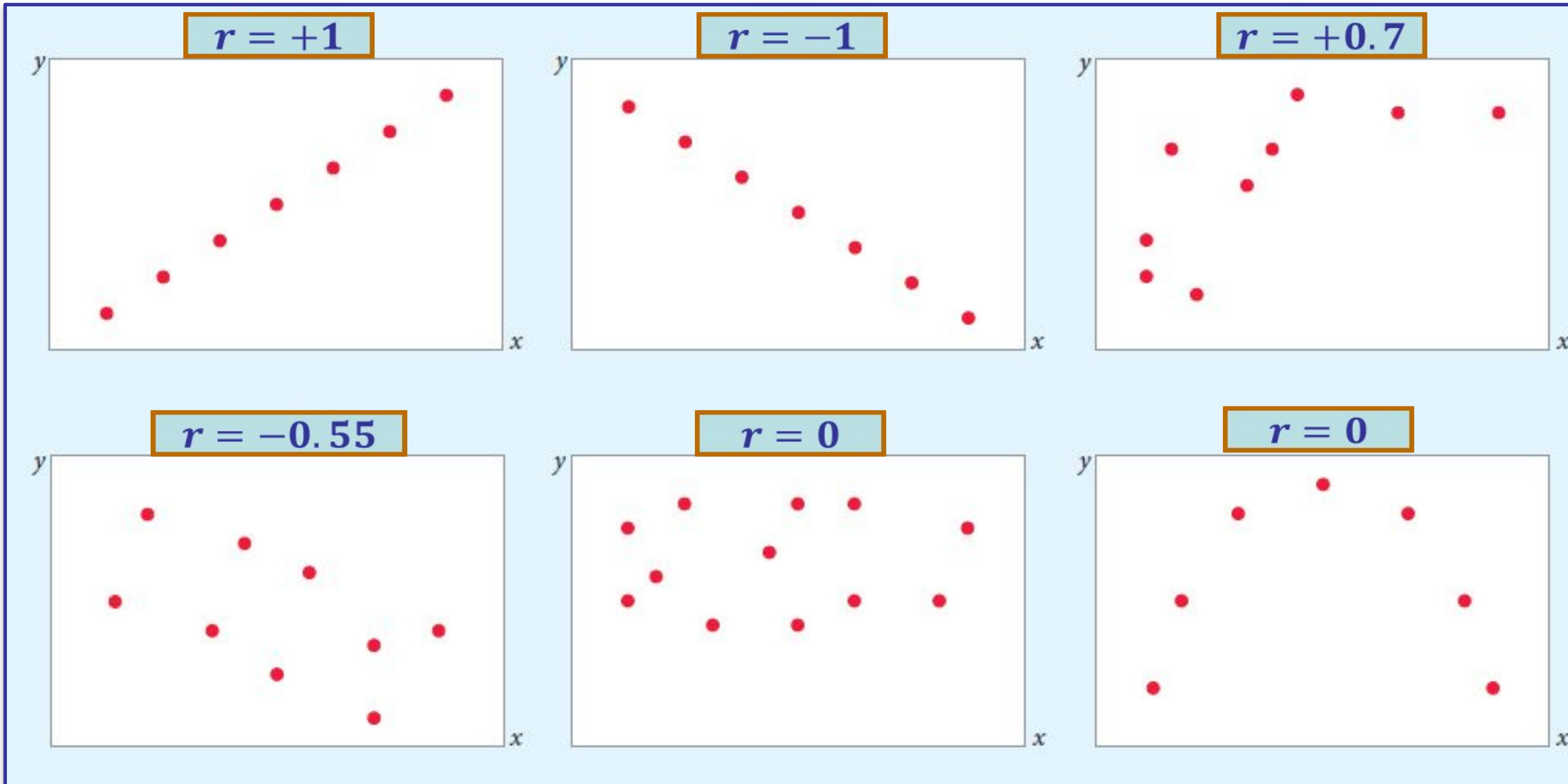
$-1.0 \leq r \leq +1.0$

r close to 1.0 implies a strong <u>positive linear</u> relationship

r close to -1.0 implies a strong <u>negative linear</u> relationship

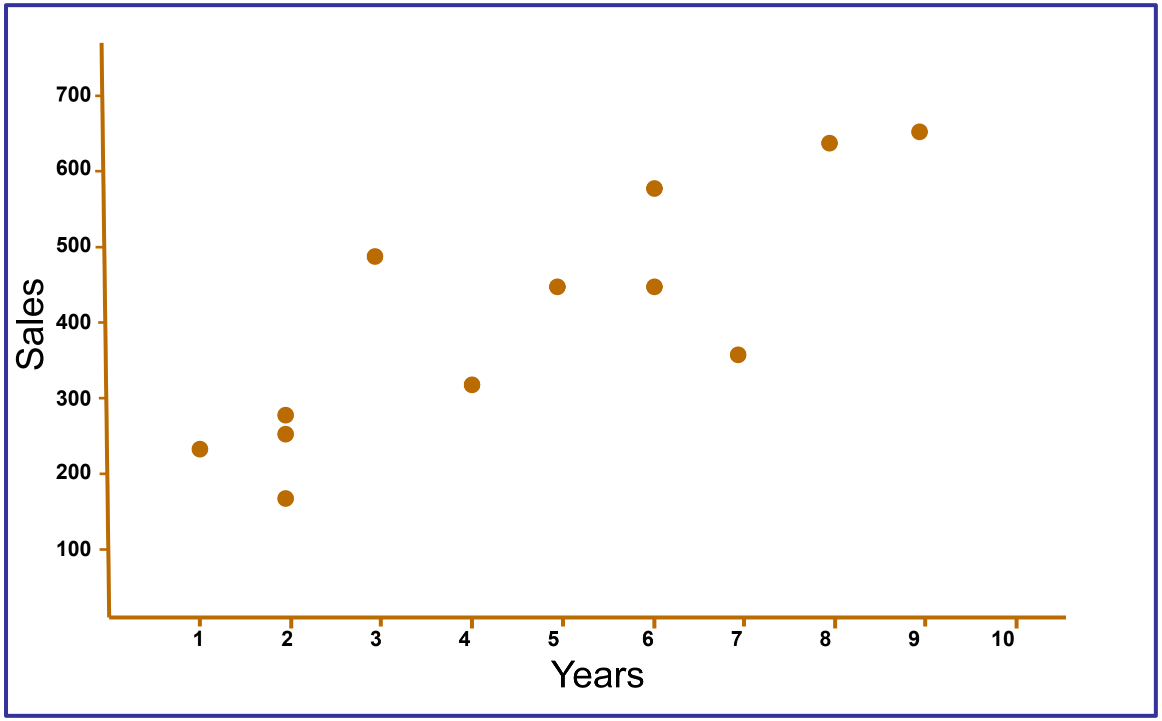
r close to 0.0 implies a weak linear relationship

Correlation between Two Variables



The Correlation Coefficient - Example

The company is studying the relationship between sales (on which commissions are paid) and number of years a sales person is with the company. A random sample of 12 sales representatives is collected. Compute the correlation coefficient.



The Correlation Coefficient – Manual Calculation Example

Sales	Years					
y	x	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
487	3	−1.58	82.42	−130.22	2.50	6,793.06
445	5	0.42	40.42	16.98	0.18	1,633.78
272	2	−2.58	−132.58	342.06	6.66	17,577.46
641	8	3.42	236.42	808.56	11.70	55,894.42
187	2	−2.58	−217.58	561.36	6.66	47,341.06
440	6	1.42	35.42	50.30	2.02	1,254.58
346	7	2.42	−58.58	−141.76	5.86	3,431.62
238	1	−3.58	−166.58	596.36	12.82	27,748.90
312	4	−0.58	−92.58	53.70	0.34	8,571.06
269	2	−2.58	−135.58	349.80	6.66	18,381.94
655	9	4.42	250.42	1,106.86	19.54	62,710.18
563	6	1.42	158.42	224.96	2.02	25,096.90
$\Sigma = 4{,}855$	$\Sigma = 55$			$\Sigma = 3{,}838.92$	$\Sigma = 76.92$	$\Sigma = 276{,}434.92$

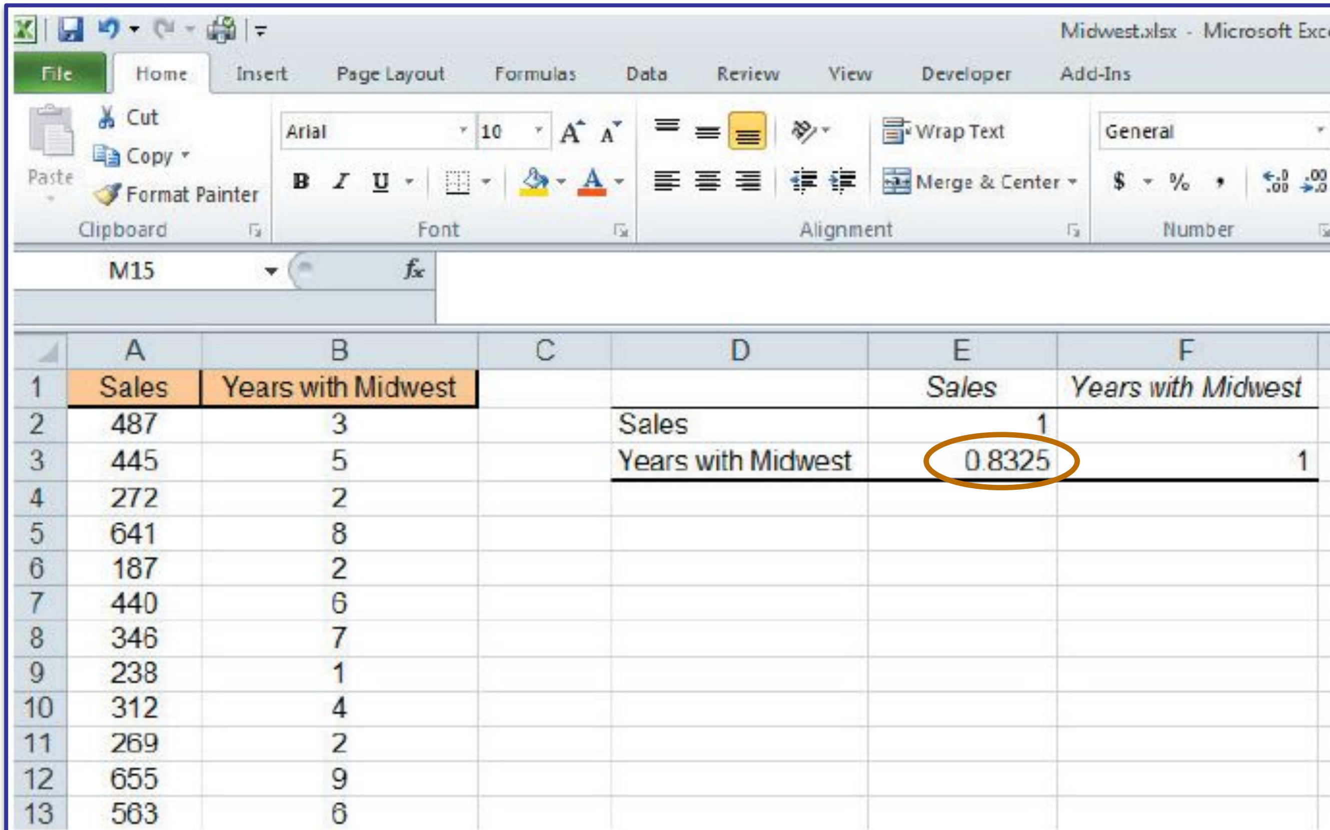
$$\bar{y} = \frac{\Sigma y}{n} = \frac{4{,}855}{12} = 404.58 \qquad \bar{x} = \frac{\Sigma x}{n} = \frac{55}{12} = 4.58$$

Using Equation 14.1,

$$r = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma(x - \bar{x})^2 \Sigma(y - \bar{y})^2}} = \frac{3{,}838.92}{\sqrt{(76.92)(276{,}434.92)}} = 0.8325$$

The Correlation Coefficient – Example Using Excel 2016

1. Open file: **Midwest.xlsx**.
2. Select **Data > Data Analysis**.
3. Select **Correlation.**
4. Define the data range.
5. Click on **Labels in First Row**.
6. Specify output choice.
7. Click **OK**.



Note: Data are taken from previous example.

Significance Test for the Correlation

- The Null and Alternative Hypotheses:

$\boldsymbol{H_0: \rho = 0}$ (no correlation)
$\boldsymbol{H_A: \rho \neq 0}$ (correlation exists)

- Test Statistic for Correlation:

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

ρ - Population correlation coefficient
t - Number of standard errors r is from 0
r - Sample correlation coefficient
n - Sample size
$df = n - 2$ - Degrees of freedom

- Assumptions:
  - The data are interval or ratio-level.
  - The two variables (*y* and *x*) are distributed as a *bivariate normal* distribution.

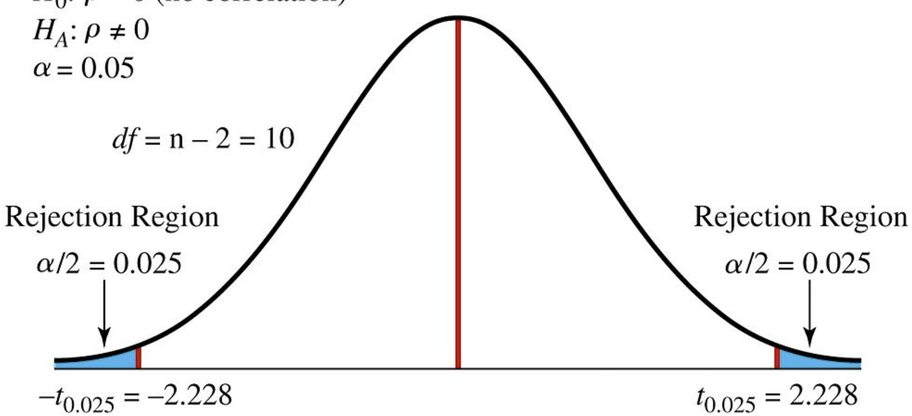
Significance Test for the Correlation - Example

Midwestern Example

	A	B	C
1		*Sales*	*Years with Midwest*
2	Sales	1	
3	Years with Midwest	0.8325	1

Hypotheses:

H_0: $\rho = 0$ (no correlation)
H_A: $\rho \neq 0$
$\alpha = 0.05$



The calculated t-value is

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{0.8325}{\sqrt{\frac{1-0.6931}{10}}} = 4.752$$

Decision Rule:

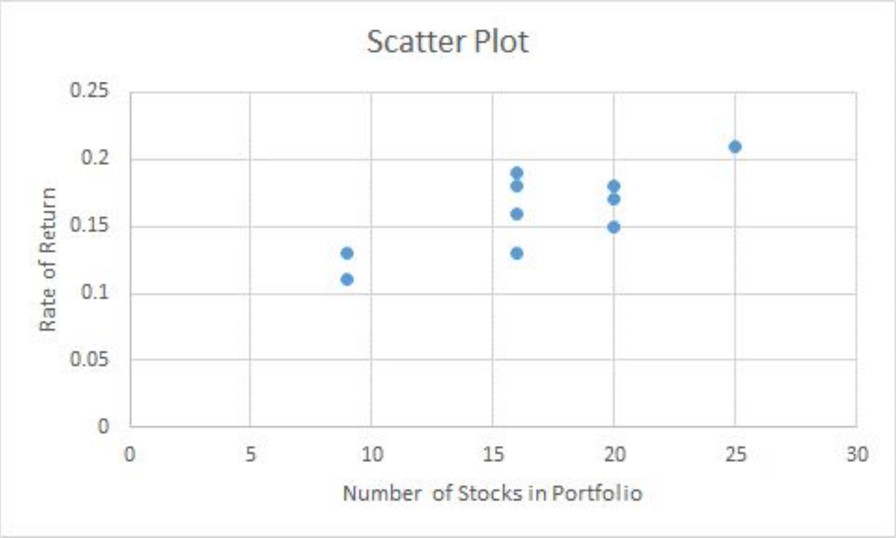
If $t > t_{0.025} = 2.228$, reject H_0.
If $t < -t_{0.025} = -2.228$, reject H_0.
Otherwise, do not reject H_0.
Because $4.752 > 2.228$, reject H_0.

Based on the sample evidence, we conclude there is a significant positive linear relationship between years with the company and sales volume.

The Correlation Coefficient – Example

A money management company is interested in determining whether there is a positive linear relationship between the number of stocks in a client's portfolio and the portfolio annual rate of return. A sample of n=10 clients has been selected. The sample data are:

Number of Stocks	Rate of Return
9	0.13
16	0.16
25	0.21
16	0.18
20	0.18
16	0.19
20	0.15
20	0.17
16	0.13
9	0.11



The Correlation Coefficient – Example

	Number of Stocks	Rate of Return
Number of Stocks	1	
Rate of Return	0.780	1

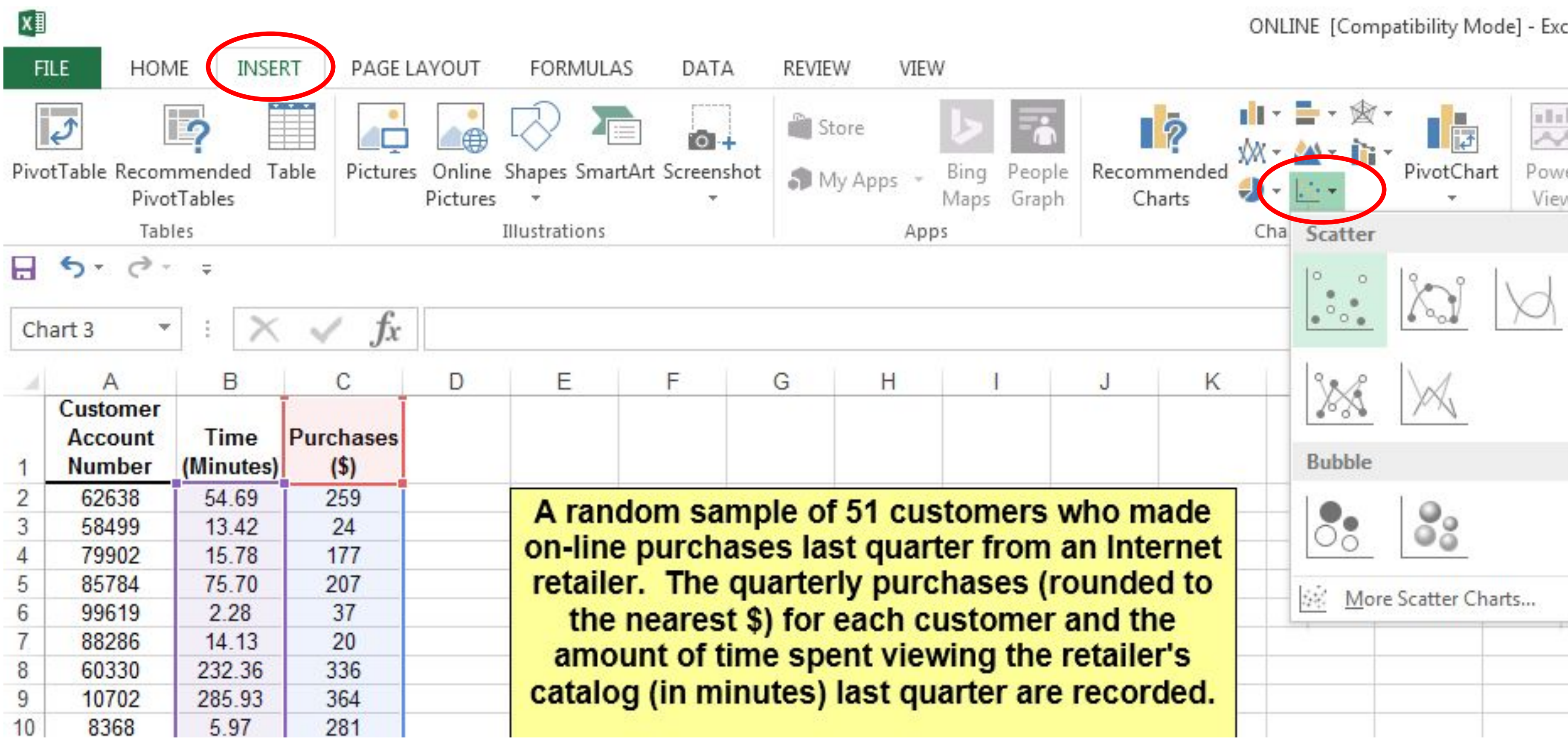
$$r = 0.780$$

$$H_o : \rho = 0.0$$
$$H_A : \rho \geq 0.0$$
$$\alpha = 0.05$$

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{0.780}{\sqrt{\frac{1-0.780^2}{10-2}}} = 3.53$$

Since t = 3.53 > $t_{0.05,df=8} = 1.8595$ reject the null hypothesis.

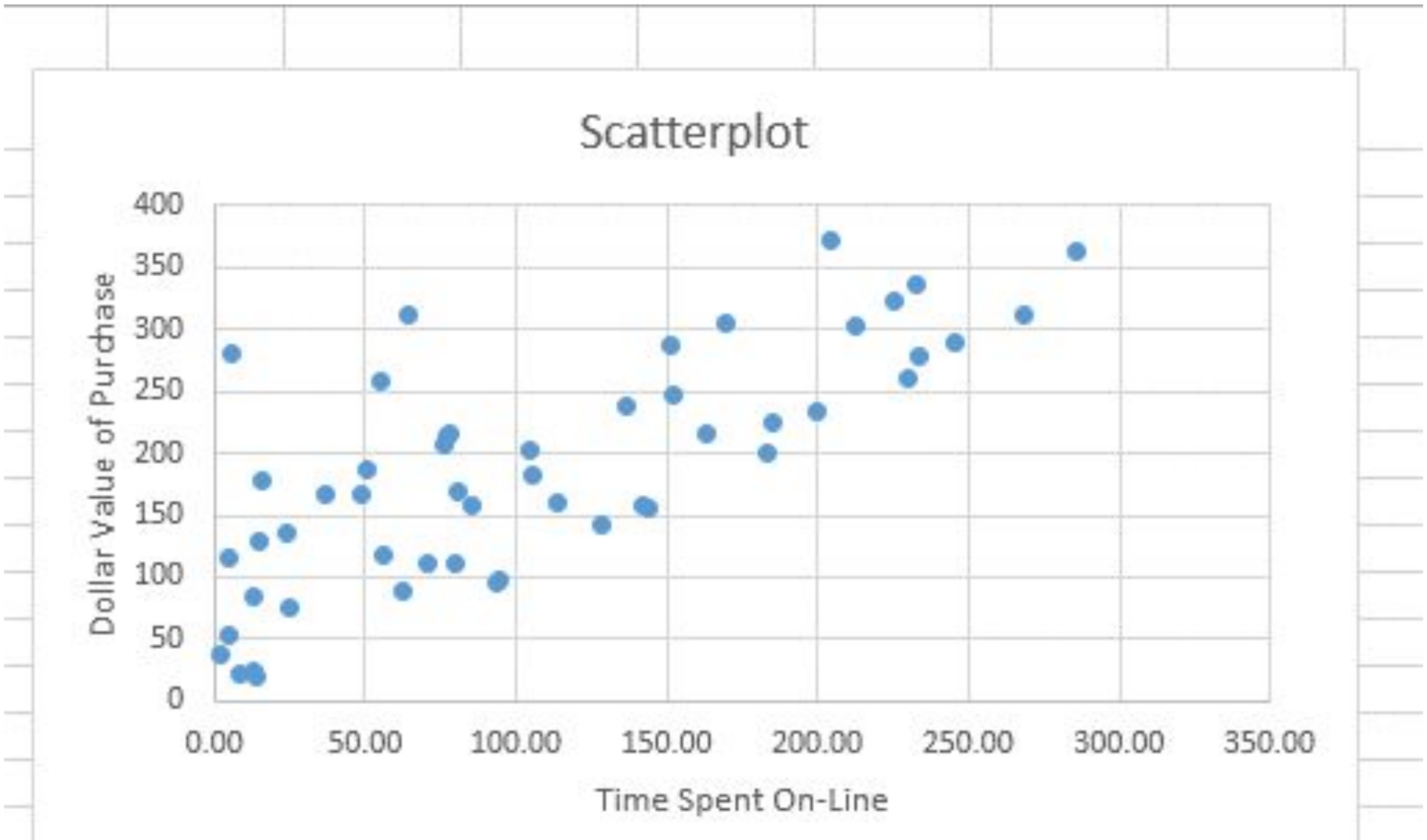
Scatter Plot and Correlation Coefficient – Example Using Excel



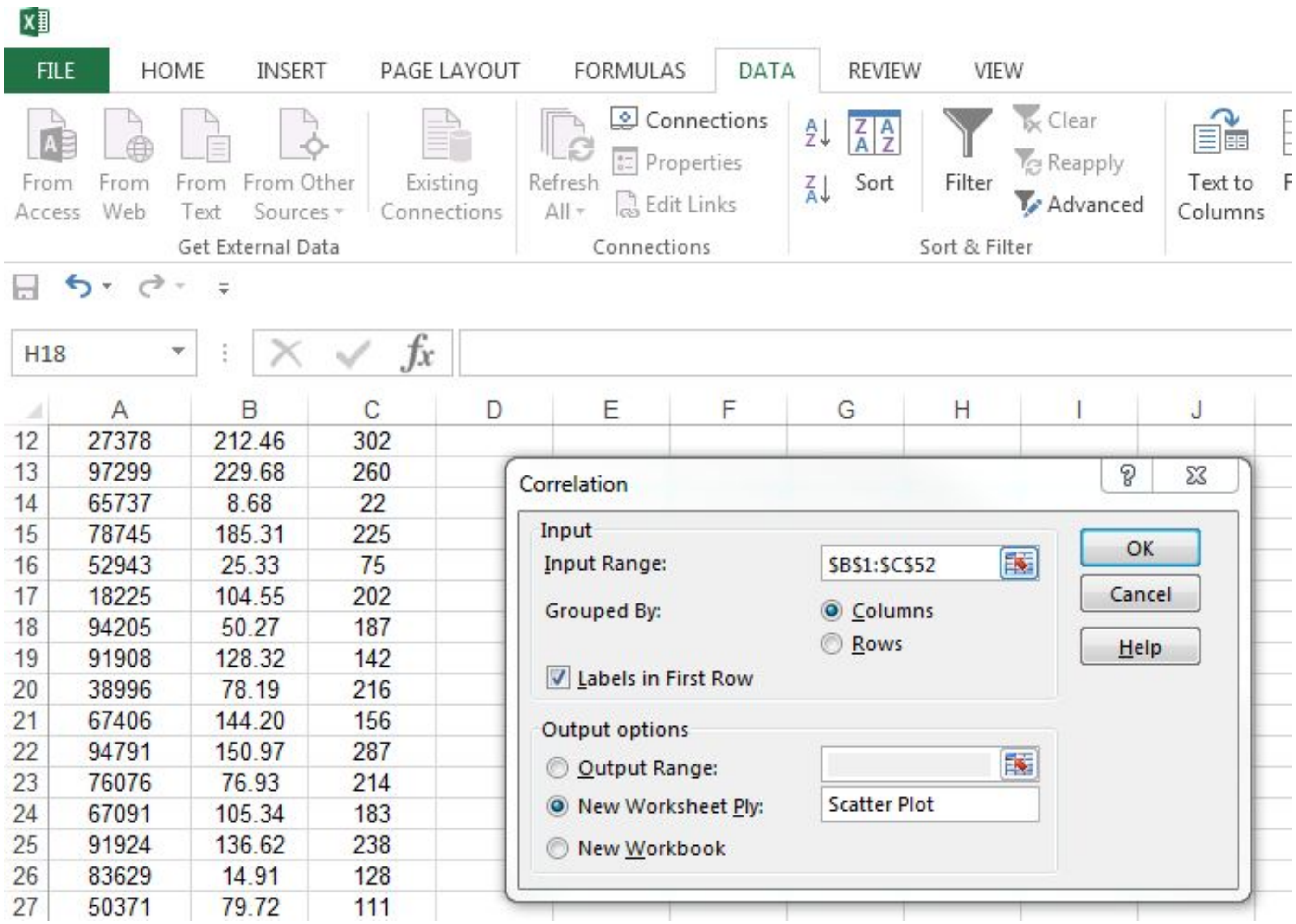
	Customer Account Number	Time (Minutes)	Purchases ($)
2	62638	54.69	259
3	58499	13.42	24
4	79902	15.78	177
5	85784	75.70	207
6	99619	2.28	37
7	88286	14.13	20
8	60330	232.36	336
9	10702	285.93	364
10	8368	5.97	281

A random sample of 51 customers who made on-line purchases last quarter from an Internet retailer. The quarterly purchases (rounded to the nearest $) for each customer and the amount of time spent viewing the retailer's catalog (in minutes) last quarter are recorded.

Scatter Plot and Correlation Coefficient – Example Using Excel



Scatter Plot and Correlation Coefficient – Example Using Excel



Using the Data Analysis Tool for calculating the correlation coefficient.

Scatter Plot and Correlation Coefficient – Example Using Excel

	A	B	C	D
1		*Time (Minutes)*	*Purchases ($)*	
2	Time (Minutes)	1		
3	Purchases ($)	0.756	1	
4				

$$H_o : \rho = 0.0$$

$$H_A : \rho \neq 0.0$$

$$\alpha = 0.05$$

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{0.756}{\sqrt{\frac{1-0.756^2}{51-2}}} = 8.08$$

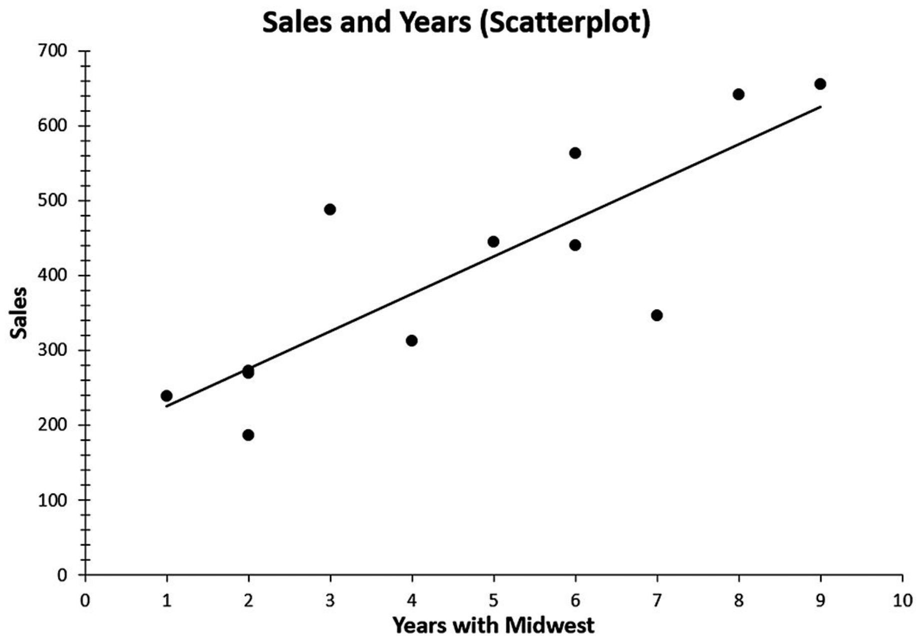
Since t = 8.08 > $t_{0.05, df=49} = 2.0096$ we reject the null hypothesis

Correlation Analysis - Summary

- Step 1: Specify the population parameter of interest
- Step 2: Formulate the appropriate null and alternative hypotheses
- Step 3: Specify the level of significance
- Step 4: Compute the correlation coefficient and the test statistic
- Step 5: Construct the rejection region and decision rule.
- Step 6: Reach a decision
- Step 7: Draw a conclusion

14.2 Simple Linear Regression Analysis

A statistical method that is used to describe the linear relationship between two variables in the form of a straight that passes through the points on a scatterplot



Simple Linear Regression Analysis

- When there are only two variables - a dependent variable, and an independent variable, the technique is referred to as <u>simple regression analysis</u>
- When the relationship between the dependent variable and the independent variable is linear, the technique is <u>simple linear regression</u>

Dependent and Independent Variables

Dependent Variable – A variable whose values are thought to be a function of, or dependent on, the values of more or more other variables. This dependent variable is referred to as the y variable and is placed on the vertical axis of a scatterplot.

The Independent Variable – A variable whose values are thought to influence the values of the dependent variable. Independent variables are also called explanatory variables. The dependent variable is referred to as the x variable and is placed on the horizontal axis of a scatterplot.

The Regression Model

- Population Model:

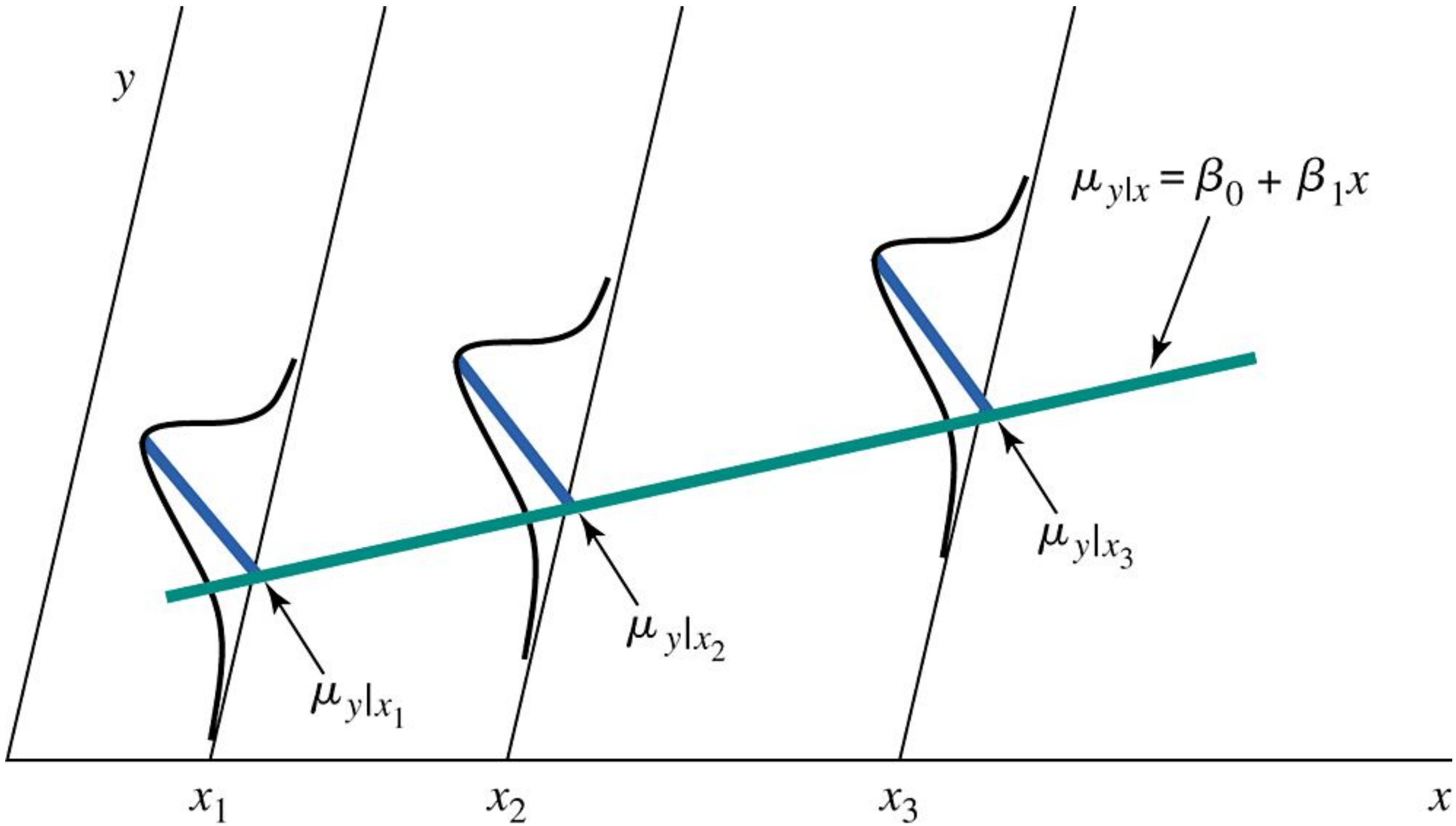
$$\boxed{y = \beta_0 + \beta_1 x + \varepsilon}$$

y - Value of the dependent variable
x - Value of the independent variable
β_0- Population's y intercept
β_1- Slope of the population regression line
ε - Random error term

Linear Regression Assumptions

1. The random errors, ε , are statistically independent
2. For each value of *x* there can exist many possible values of *y* and the distribution of *y* values is normally distributed.
3. The distributions of errors have equal variances for all possible levels of *x*
4. A straight line, called the population regression model (equation) will pass through the mean of the possible *y* values for all levels of *x*

Linear Regression Assumptions – Visual Representation



Meaning of the Regression Coefficients

- Regression Slope Coefficient, β_1
  - Measures the average change in the value of the dependent variable, y, for each unit change in x
  - Can be either positive, zero, or negative
- The Population's y Intercept, β_0
  - Indicates the mean value of y when x is 0

Estimates of the Regression Coefficients

$$\hat{y} = b_o + b_1 x$$

$where:$

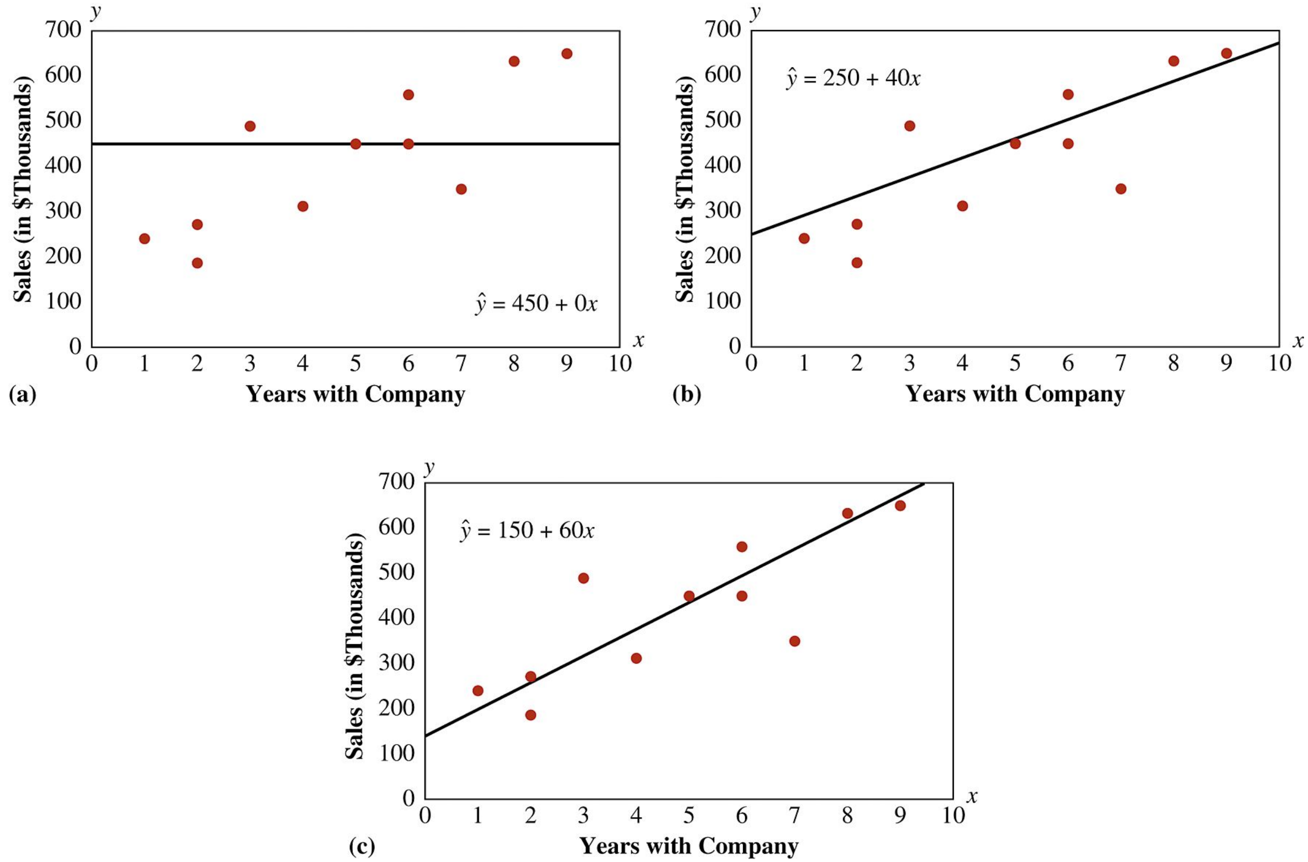
$\hat{y} =$ estimated value of the dependent variable for a given value of x

$b_1 =$ estimate of the true population regression slope coefficient

$b_o =$ estimate of the true population y intercept

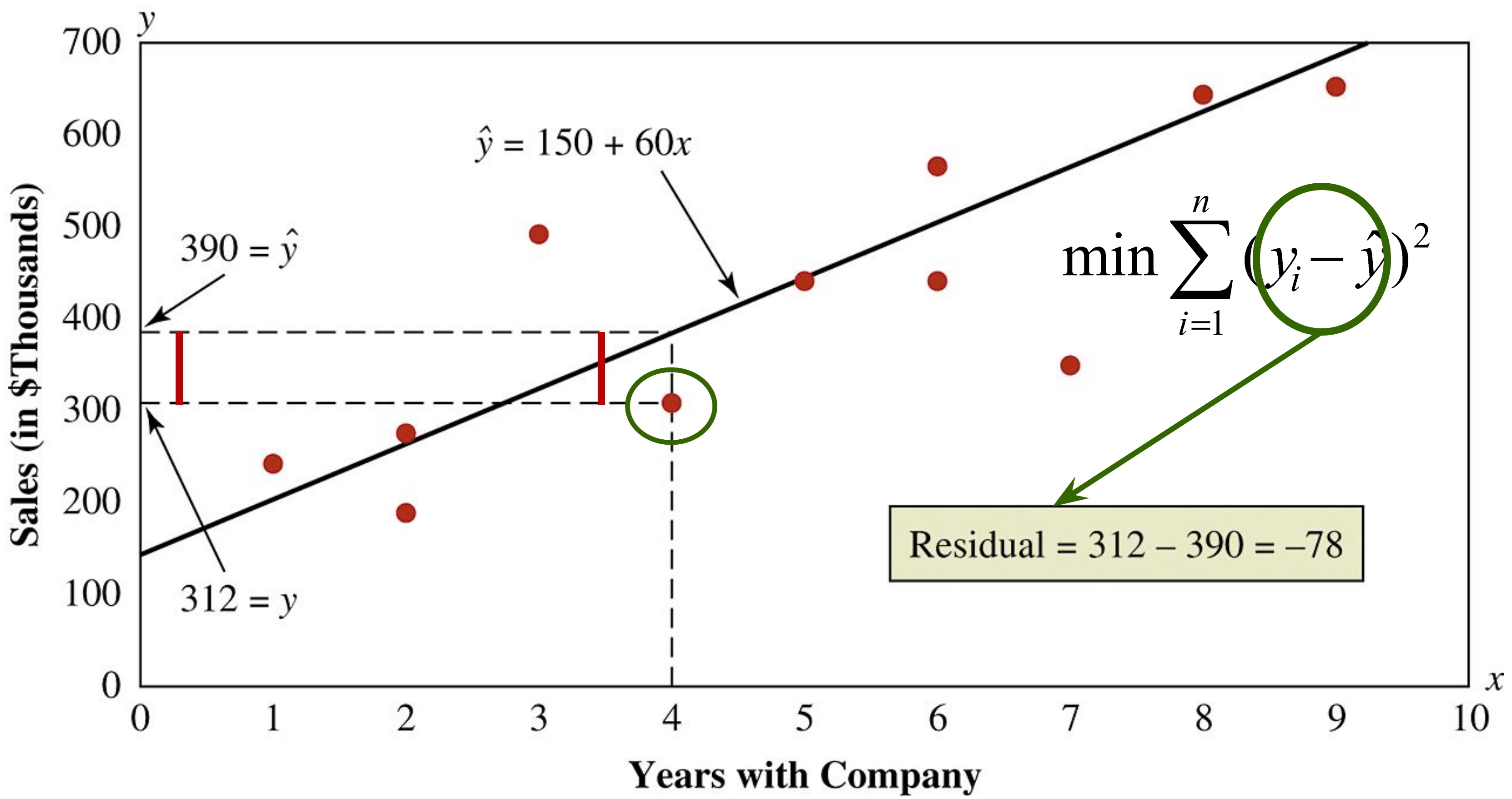
How do we determine the values for b_o and b_1?

Regression Line Examples



Which Regression Line is Best? Examine Regression Errors

Computation of Regression Error - Example



Least Squares Criterion

- The criterion for determining a regression line that <u>minimizes the sum of squared prediction errors (residuals)</u>

$$\min \boxed{\sum_{i=1}^{n} \underbrace{(y_i - \hat{y})}_{\text{Residual}}^2}$$

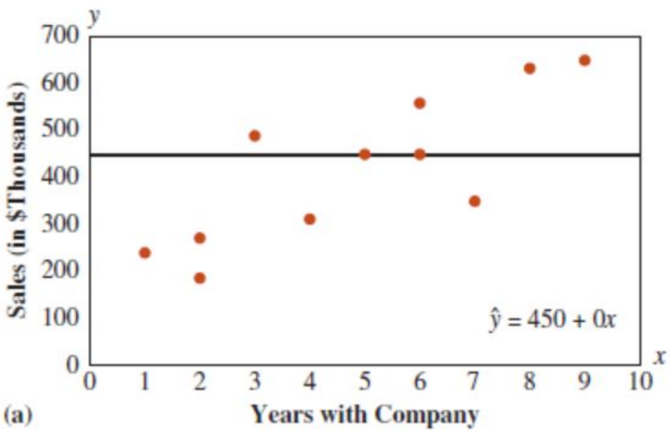
Sum of Squared Residual (Errors) = SSE

- Residual: The difference between the actual value of the dependent variable and the value predicted by the regression model.

Computation of Regression Residuals – Trial-and-Error Example

$$\hat{y} = 450 + 0x$$

Squared Residuals

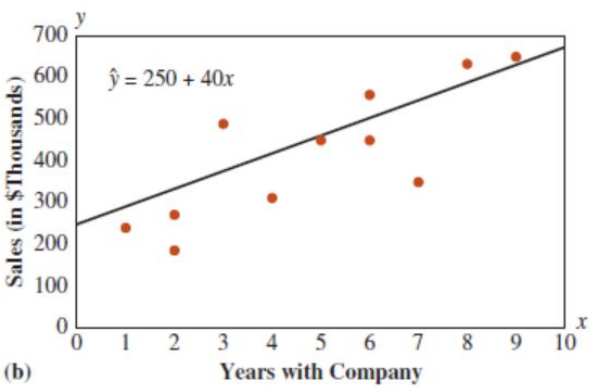


			Residual	
x	$\hat{y}$	y	$y - \hat{y}$	$(y - \hat{y})^2$
3	450	487	37	1,369
5	450	445	−5	25
2	450	272	−178	31,684
8	450	641	191	36,481
2	450	187	−263	69,169
6	450	440	−10	100
7	450	346	−104	10,816
1	450	238	−212	44,944
4	450	312	−138	19,044
2	450	269	−181	32,761
9	450	655	205	42,025
6	450	563	113	12,769
Sum of Squared Residuals (Errors) =				$\Sigma = 301{,}187$

Computation of Regression Residuals – Trial-and-Error Example

$\hat{y} = 250 + 40x$

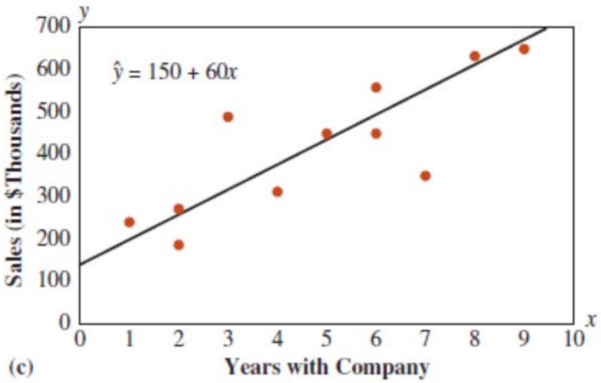
Squared Residuals



			Residual	
x	$\hat{y}$	y	$y - \hat{y}$	$(y - \hat{y})^2$
3	370	487	117	13,689
5	450	445	−5	25
2	330	272	−58	3,364
8	570	641	71	5,041
2	330	187	−143	20,449
6	490	440	−50	2,500
7	530	346	−184	33,856
1	290	238	−52	2,704
4	410	312	−98	9,604
2	330	269	−61	3,721
9	610	655	45	2,025
6	490	563	73	5,329
				$\Sigma = 102{,}307$

Computation of Regression Residuals – Trial-and-Error Example

$\hat{y} = 150 + 60x$



Squared Residuals

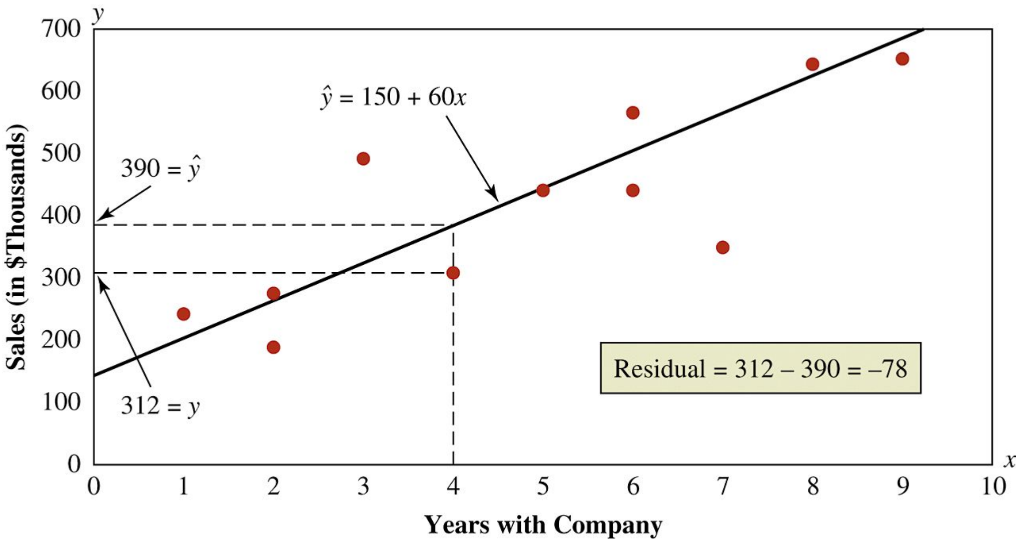
			Residual	
x	$\hat{y}$	y	$y - \hat{y}$	$(y - \hat{y})^2$
3	330	487	157	24,649
5	450	445	−5	25
2	270	272	2	4
8	630	641	11	121
2	270	187	−83	6,889
6	510	440	−70	4,900
7	570	346	−224	50,176
1	210	238	28	784
4	390	312	−78	6,084
2	270	269	−1	1
9	690	655	−35	1,225
6	510	563	53	2,809
				$\sum = 97{,}667$

Least Squares Criterion

We need a more direct approach than trial-and-error! The answer lies in finding the slope and intercept such that the sum of squared residuals is minimized for the sample data.

Sum of Squared Residuals (Errors) = SSE

$$\min \sum_{i=1}^{n} (y_i - \hat{y})^2$$



The least squares equations:

$$b_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \quad \text{and} \quad b_o = \bar{y} - b_1\bar{x}$$

Least Squares Equations – Manual Calculations Example

y	x	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$
487	3	-1.6	82.4	-131.84	2.56
445	5	0.4	40.4	16.16	0.16
272	2	-2.6	-132.6	344.76	6.76
641	8	3.4	236.4	803.76	11.56
187	2	-2.6	-217.6	565.76	6.76
440	6	1.4	35.4	49.56	1.96
346	7	2.4	-58.6	-140.64	5.76
238	1	-3.6	-166.6	599.76	12.96
312	4	-0.6	-92.6	55.56	0.36
269	2	-2.6	-135.6	352.56	6.76
655	9	4.4	250.4	1101.76	19.36
563	6	1.4	158.4	221.76	1.96
$\bar{y} = 404.6$	$\bar{x} = 4.6$			$\sum = 3838.92$	$\sum = 76.92$

$$b_1 = \frac{\sum_{i=1}^{n}(x-\bar{x})(y-\bar{y})}{\sum_{i=1}^{n}(x-\bar{x})^2} = \frac{3838.92}{76.92} = 49.91$$

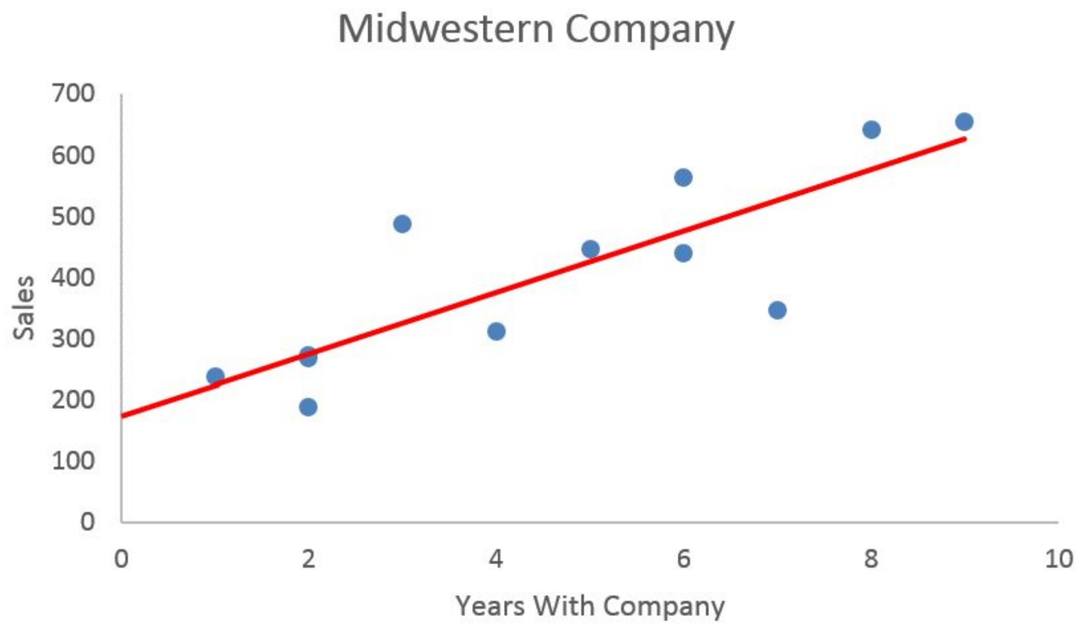
$$b_o = \bar{y} - b_1\bar{x} = 404.6 - (49.91)(4.6) = 175.01$$

Estimated Regression Equation -Example

$$b_1 = \frac{\sum_{i=1}^{n}(x-\overline{x})(y-\overline{y})}{\sum_{i=1}^{n}(x-\overline{x})^2} = \frac{3838.92}{76.92} = 49.91$$

$$b_o = \overline{y} - b_1\overline{x} = 404.6 - (49.91)(4.6) = 175.01$$

$$\hat{y} = 175.01 + 49.91(x)$$



Minimum Sum of Squares Residuals-Example

y	x	$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$
487	3	324.74	162.26	26328.31
445	5	424.56	20.44	417.79
272	2	274.83	-2.83	8.01
641	8	574.29	66.71	4450.22
187	2	274.83	-87.83	7714.11
440	6	474.47	-34.47	1188.18
346	7	524.38	-178.38	31819.42
238	1	224.92	13.08	171.09
312	4	374.65	-62.65	3925.02
269	2	274.83	-5.83	33.99
655	9	624.2	30.8	948.64
563	6	474.47	88.53	7837.56
				$\sum = 84{,}842.35$

The Least Squares equations minimize SSE

Excel 2016 Regression Results

1. Open file.
2. Select **Data > Data Analysis**.
3. Select **Regression**.
4. Define y and x variable data range.
5. Select **Labels**.
6. Select **Residuals**.
7. Select output location.
8. Click **OK**.

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
3	*Regression Statistics*						
4	Multiple R	0.8325					
5	R Square	0.6931					
6	Adjusted R Square	0.6624					
7	Standard Error	92.1055					
8	Observations	12					
10	ANOVA						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
12	Regression	1	191,600.62	191,600.62	22.59	0.0008	
13	Residual	10	84,834.29	8,483.43			
14	Total	11	276,434.92				
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17	Intercept	175.8288	54.9899	3.1975	0.0095	53.3037	298.3539
18	Years with Midwest	49.9101	10.5021	4.7524	0.0008	26.5100	73.3102
19							
20	RESIDUAL OUTPUT						
21	*Observation*	*Predicted*	*Residuals*				
22	1	325.56	161.44				
23	2	425.38	19.62				
24	3	275.65	-3.65				

$$\min \sum_{i=1}^{n} (y - \hat{y})^2 = 84,834.29$$

$$\hat{y} = 175.83 + 49.91(x)$$

(Regression results differ slightly from manual calculations due to rounding.)

Test for Significance of the Regression Slope Coefficient

- Hypotheses:

$$\begin{aligned} H_0&: \beta_1 = 0 \\ H_A&: \beta_1 \neq 0 \end{aligned}$$

- A slope of 0 would imply that there is no linear relationship between x and y variables and that the x variable, in its linear form, is of no use in explaining the variation in y.

Test Statistic for Test of the Significance of the Slope Coefficient

- Hypotheses:

$$H_0: \beta_1 = 0$$
$$H_A: \beta_1 \neq 0$$

- Test Statistic:

$$t = \frac{b_1 - \beta_1}{s_{b_1}} \qquad df = n - 2$$

b_1 - Sample regression slope coefficient
β_1 - Hypothesized slope (usually β_1= 0)
s_{b_1} - Estimator of the standard error of the slope

$$H_o: \mu \leq 25$$
$$H_A: \mu > 25$$

$$H_0: \mu = 16$$
$$H_A: \mu \neq 16$$
$$\alpha = 0.10$$

Point Estimate $= \bar{x}$

Standard Error $= \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Test Statistic

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{26 - 25}{\frac{3}{\sqrt{64}}} = 2.67$$

Test Statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{15.93 - 16}{\frac{0.50}{\sqrt{16}}} = -0.56$$

Standard Error of the Slope

- Simple Regression Estimator for the Standard Error of the Slope:

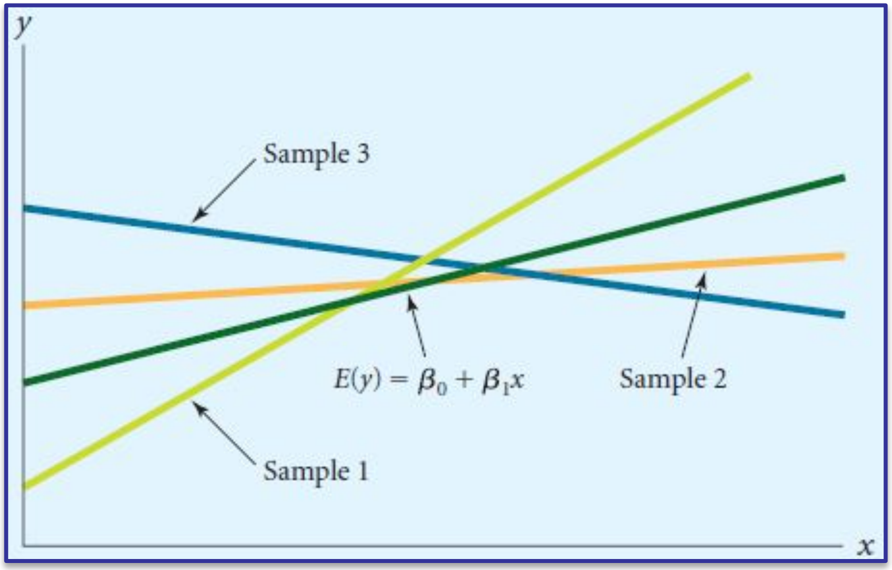
$$s_{b_1} = \frac{s_\varepsilon}{\sqrt{\sum (x_i - \bar{x})^2}}$$

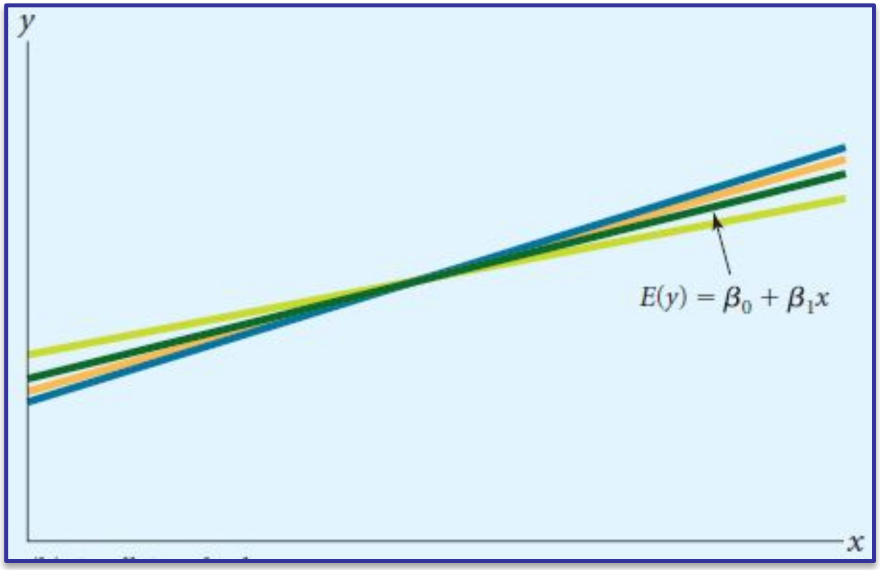
s_{b_1} - Standard deviation of the regression slope (*standard error of the slope*)

s_ε - Sample standard error of the estimate (the measure of deviation of the actual y-values around the regression line)

$$\sqrt{\frac{SSE}{n-2}}$$

Standard Error of the Slope





Large Standard Error

Small Standard Error

Standard Error of the Slope-Example

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
3	*Regression Statistics*						
4	Multiple R	0.8325					
5	R Square	0.6931					
6	Adjusted R Square	0.6624					
7	Standard Error	92.1055					
8	Observations	12					
10	ANOVA						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
12	Regression	1	191,600.62	191,600.62	22.59	0.0008	
13	Residual	10	84,834.29	8,483.43			
14	Total	11	276,434.92				
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17	Intercept	175.8288	54.9899	3.1975	0.0095	53.3037	298.3539
18	Years with Midwest	49.9101	10.5021	4.7524	0.0008	26.5100	73.3102
19							
20	RESIDUAL OUTPUT						
21	*Observation*	*Predicted*	*Residuals*				
22	1	325.56	161.44				
23	2	425.38	19.62				
24	3	275.65	-3.65				

$$S_\varepsilon = \sqrt{\frac{SSE}{n-2}} = \text{Standard Error of the Estimate}$$

MSE

$$S_{b_1} = \sqrt{\frac{S_\varepsilon}{\sum(x-\bar{x})^2}} = \text{Standard Error of Slope Coefficient}$$

Test Statistic for Test of the Significance of the Slope Coefficient

$$H_o : B_1 = 0.0$$

$$H_1 : B_1 \neq 0.0$$

$$\alpha = 0.05$$

$$H_0 : \beta_1 = 0$$
$$H_A : \beta_1 \neq 0$$

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
3	*Regression Statistics*						
4	Multiple R	0.8325					
5	R Square	0.6931					
6	Adjusted R Square	0.6624					
7	Standard Error	92.1055					
8	Observations	12					
10	ANOVA						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
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22	1	325.56	161.44				
23	2	425.38	19.62				
24	3	275.65	-3.65				

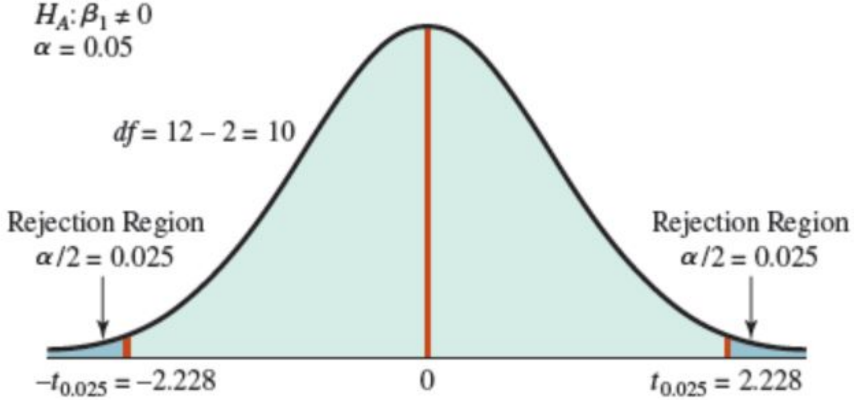
Test Statistic

$$t = \frac{b_1 - B_1}{S_{b_1}} = \frac{49.91 - 0.0}{10.5021} = 4.752$$

Test Statistic for Test of the Significance of the Slope Coefficient

Hypotheses:

$H_0: \beta_1 = 0$
$H_A: \beta_1 \neq 0$
$\alpha = 0.05$



The calculated t is

$$t = \frac{b_1 - \beta_1}{s_{b_1}} = \frac{49.91 - 0}{10.50} = 4.752$$

Decision Rule:

If $t > t_{0.025} = 2.228$, reject $H_{0.}$
If $t < -t_{0.025} = -2.228$, reject $H_{0.}$
Otherwise, do not reject $H_{0.}$

Because $4.752 > 2.228$, we reject the null hypothesis and conclude that the true slope is not 0. Thus, the simple linear relationship that utilizes the independent variable, years with the company, is useful in explaining the variation in the dependent variable, sales volume.

p-value for Test of the Significance of the Slope Coefficient

$$H_o : B_1 = 0.0$$

$$H_1 : B_1 \neq 0.0$$

$$\alpha = 0.05$$

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
3	*Regression Statistics*						
4	Multiple R	0.8325					
5	R Square	0.6931					
6	Adjusted R Square	0.6624					
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21	*Observation*	*Predicted*	*Residuals*				
22	1	325.56	161.44				
23	2	425.38	19.62				
24	3	275.65	-3.65				

p-value

Because p-value = 0.0008 < alpha/2 = 0.025, reject H_o

Review: The Correlation Coefficient – Manual Calculation Example

Sales y	Years x	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
487	3	−1.58	82.42	−130.22	2.50	6,793.06
445	5	0.42	40.42	16.98	0.18	1,633.78
272	2	−2.58	−132.58	342.06	6.66	17,577.46
641	8	3.42	236.42	808.56	11.70	55,894.42
187	2	−2.58	−217.58	561.36	6.66	47,341.06
440	6	1.42	35.42	50.30	2.02	1,254.58
346	7	2.42	−58.58	−141.76	5.86	3,431.62
238	1	−3.58	−166.58	596.36	12.82	27,748.90
312	4	−0.58	−92.58	53.70	0.34	8,571.06
269	2	−2.58	−135.58	349.80	6.66	18,381.94
655	9	4.42	250.42	1,106.86	19.54	62,710.18
563	6	1.42	158.42	224.96	2.02	25,096.90
Σ = 4,855	Σ = 55			Σ = 3,838.92	Σ = 76.92	Σ = 276,434.92

$$\bar{y} = \frac{\Sigma y}{n} = \frac{4{,}855}{12} = \boxed{404.58} \qquad \bar{x} = \frac{\Sigma x}{n} = \frac{55}{12} = 4.58$$

Using Equation 14.1,

$$r = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma(x - \bar{x})^2 \Sigma(y - \bar{y})^2}} = \frac{3{,}838.92}{\sqrt{(76.92)(276{,}434.92)}} = 0.8325$$

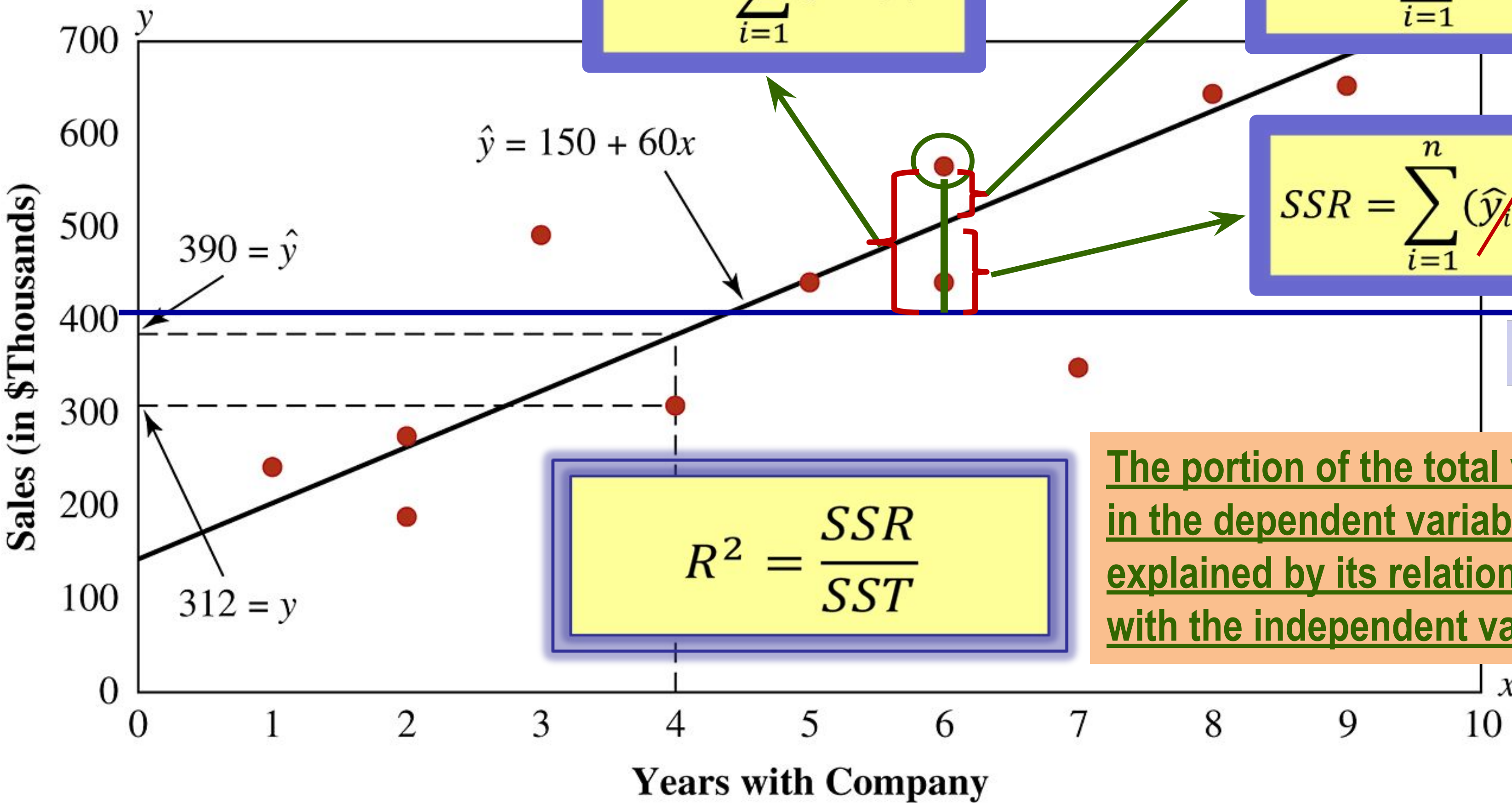
Sums of Squares

$$SST = SSR + SSE$$

$$SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$$

$$SSE = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$$



$$R^2 = \frac{SSR}{SST}$$

<u>The portion of the total variation in the dependent variable that is explained by its relationship with the independent variable</u>

Sums of Squares

- Total Sum of Squares:

$$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

Sum of Squared Residual (Errors) = SSE

$$SSE = \sum_{i=1}^{n} (y_i - \cancel{\hat{y}_i})^2$$

- Sum of Squares Regression:

$$SSR = \sum_{i=1}^{n} (\cancel{\hat{y}_i} - \bar{y})^2$$

n - Sample size
y_i - i^{th} value of the dependent variable
$\bar{y}$ - Average value of the dependent variable
$\hat{y}_i$ - i^{th} predicted value of y given the i^{th} value of x

$$SST = SSR + SSE$$

Sums of Squares - Example

$S_e = \sqrt{SSE}$ = Standard Error of the Estimate

SSR

SSE

SST

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
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17	Intercept	175.8288	54.9899	3.1975	0.0095	53.3037	298.3539
18	Years with Midwest	49.9101	10.5021	4.7524	0.0008	26.5100	73.3102
19							
20	RESIDUAL OUTPUT						
21	*Observation*	*Predicted*	*Residuals*				
22	1	325.56	161.44				
23	2	425.38	19.62				
24	3	275.65	-3.65				

SST = SSR + SSE

The Coefficient of Determination R^2

- The portion of the total variation in the dependent variable that is explained by its relationship with the independent variable

$$\boxed{R^2 = \frac{SSR}{SST}}$$

SSR - Sum of squares regression
SST - Total sum of squares
$0 \leq R^2 \leq 1.0$

- Coefficient of Determination for the Single Independent Variable Case

$$\boxed{R^2 = r^2}$$

r - Sample correlation coefficient

The Coefficient of Determination R^2

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
3	*Regression Statistics*						
4	Multiple R	0.8325					
5	R Square	0.6931					
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20	RESIDUAL OUTPUT						
21	*Observation*	*Predicted*	*Residuals*				
22	1	325.56	161.44				
23	2	425.38	19.62				
24	3	275.65	-3.65				

$$R^2 = \frac{SSR}{SST} = \frac{191,600.62}{276,434.92} = 0.6931$$

This means 69.31% of variation in the sales data can be explained by the linear relationship b/w sales and years of experience.

Test Statistic for Significance of the Coefficient of Determination

$$H_o : \rho^2 = 0.0$$
$$H_A : \rho^2 > 0.0$$

The independent variable does not explain a significant proportion of the total variation in the dependent variable

Test Statistic

$$F = \frac{\dfrac{SSR}{1}}{\dfrac{SSE}{n-2}} = \frac{MSR}{MSE}$$

$$df, D_1 = 1 \text{ and } D_2 = n-2$$

Test Statistic for Significance of the Coefficient of Determination

$$H_o: \rho^2 = 0.0$$

$$H_A: \rho^2 > 0.0$$

$$\alpha = 0.05$$

Test Statistic

$$F = \frac{MSR}{MSE}$$

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
3	*Regression Statistics*						
4	Multiple R	0.8325					
5	R Square	0.6931					
6	Adjusted R Square	0.6624					
7	Standard Error	92.1055					
8	Observations	12					
10	ANOVA						
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19							
20	RESIDUAL OUTPUT						
21	*Observation*	*Predicted*	*Residuals*				
22	1	325.56	161.44				
23	2	425.38	19.62				
24	3	275.65	-3.65				

Because F = 22.59 > $F_{critical,0.05}$=4.965, reject the null hypothesis

p-value for Significance of the Coefficient of Determination

$$H_o : \rho^2 = 0.0$$

$$H_A : \rho^2 > 0.0$$

$$\alpha = 0.05$$

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
3	*Regression Statistics*						
4	Multiple R	0.8325					
5	R Square	0.6931					
6	Adjusted R Square	0.6624					
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17	Intercept	175.8288	54.9899	3.1975	0.0095	53.3037	298.3539
18	Years with Midwest	49.9101	10.5021	4.7524	0.0008	26.5100	73.3102
19							
20	RESIDUAL OUTPUT						
21	*Observation*	*Predicted*	*Residuals*				
22	1	325.56	161.44				
23	2	425.38	19.62				
24	3	275.65	-3.65				

p-value = 0.0008

Because p-value = 0.0008 < alpha = 0.05, reject the null hypothesis

This means the independent variable explains a significant proportion of the variation in the dependent variable.

14.3 Uses for Regression Analysis

- Description – When we are primarily interested in analyzing the relationship between the x and y variables as measured by the regression slope coefficient
- Prediction – When we are primarily interested in predicting what the value of the y variable will be when we know a value of the x variable.

Regression Analysis for Description - Example

The Environmental Protection Agency (EPA) is interested in the relationship between vehicle mileage and the CO_2 emitted by the vehicle. To analyze the relationship, staff members have collected sample data from 58 vehicles and used Excel to compute the following regression output.

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2	*Regression Statistics*						
3	Multiple R	0.9589					
4	R Square	0.9194					
5	Adjusted R Square	0.9180					
6	Standard Error	16.2705					
7	Observations	58					
8							
9	ANOVA						
10		*df*	*SS*	*MS*	*F*	*Significance F*	
11	Regression	1	169115.67	169115.67	638.83	0.0000	
12	Residual	56	14824.81	264.73			
13	Total	57	183940.48				
14							
15		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
16	Intercept	703.39	14.20	49.53	0.0000	674.94	731.84
17	Combined MPG	-13.64	0.54	-25.28	0.0000	-14.72	-12.56

$$\hat{y} = 703.39 - 13.64(mpg)$$

$$H_o : B_1 = 0.0$$

$$H_1 : B_1 \neq 0.0$$

$$\alpha = 0.05$$

Because p-value = 0.0000 < 0.05/2 we reject the null hypothesis

Regression Analysis for Description – Regression Slope Analysis

- Confidence Interval Estimate for the Regression Slope:

$$b_1 \pm t s_{b_1} \quad \text{or} \quad b_1 \pm t \frac{s_\varepsilon}{\sqrt{\sum (x_i - \bar{x})^2}}$$

s_{b_1} - Standard deviation of the regression slope coefficient
s_ε - Sample standard error of the estimate
$df = n - 2$ – Degrees of freedom

Regression Analysis for Description

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2	*Regression Statistics*						
3	Multiple R	0.9589					
4	R Square	0.9194					
5	Adjusted R Square	0.9180					
6	Standard Error	16.2705					
7	Observations	58					
8							
9	ANOVA						
10		*df*	*SS*	*MS*	*F*	*Significance F*	
11	Regression	1	169115.67	169115.67	638.83	0.0000	
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16	Intercept	703.39	14.20	49.53	0.0000	674.94	731.84
17	Combined MPG	-13.64	0.54	-25.28	0.0000	-14.72	-12.56

$$b_1 \pm t_{0.05,df=56} s_{b_1}$$

Based on the sample data, with 95% confidence, we believe that for each increase on one mpg, the mean change in CO_2 is between -14.72 and -12.56 grams with a point estimate of -13.64 grams

Regression Analysis for Prediction

Hospital administrators wish to predict the total hospital bill based on knowing the patient's length of stay in the hospital. Data were collected for 138 patients and the following regression output was produced by Excel 2016

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.77
R Square	0.60
Adjusted R Square	0.59
Standard Error	2894.78
Observations	138.00

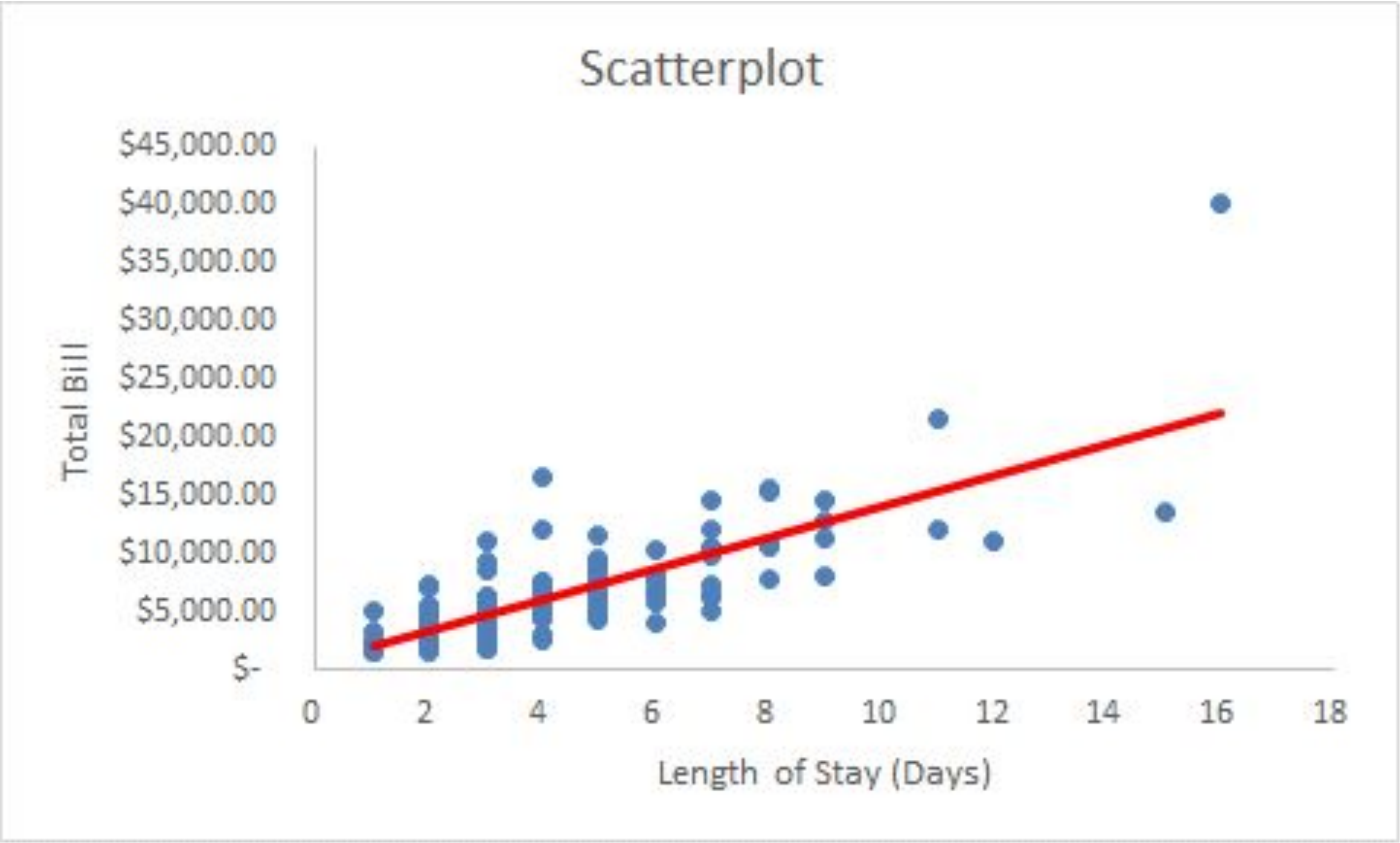
$$\hat{y} = 527.61 + 1352.80(days)$$

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1683440143.46	1683440143.46	200.89	0.00
Residual	136	1139647630.81	8379761.99		
Total	137	2823087774.28			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	527.61	483.81	1.09	0.28	-429.17	1484.38
Length of Stay	1352.80	95.44	14.17	0.00	1164.05	1541.54

Regression Analysis for Prediction – Scatterplot Example



$$\hat{y} = 527.61 + 1352.80(days)$$

Regression Analysis for Prediction – Point Estimate

Relevant Range for the x variable = 1 to 16 days

$$\hat{y} = 527.61 + 1352.80(days)$$

Point Prediction Value for x = 5 days

$$\hat{y} = 527.61 + 1352.80(5) = \$7,291.59$$

Point Prediction Value for x = 9 days

$$\hat{y} = 527.61 + 1352.80(9) = \$12,702.81$$

Confidence Interval for the Average y, Given x

Confidence Interval for $E(y)|x_p$

$$\hat{y} \pm t s_\varepsilon \sqrt{\frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$$

$\hat{y}$ - Point estimate of the dependent variable
t - Critical value with $n - 2$ degrees of freedom
n - Sample size
x_p - Specific value of the independent variable
$\bar{x}$ - Mean of the independent variable observations in the sample
s_ε - Estimate of the standard error of the estimate

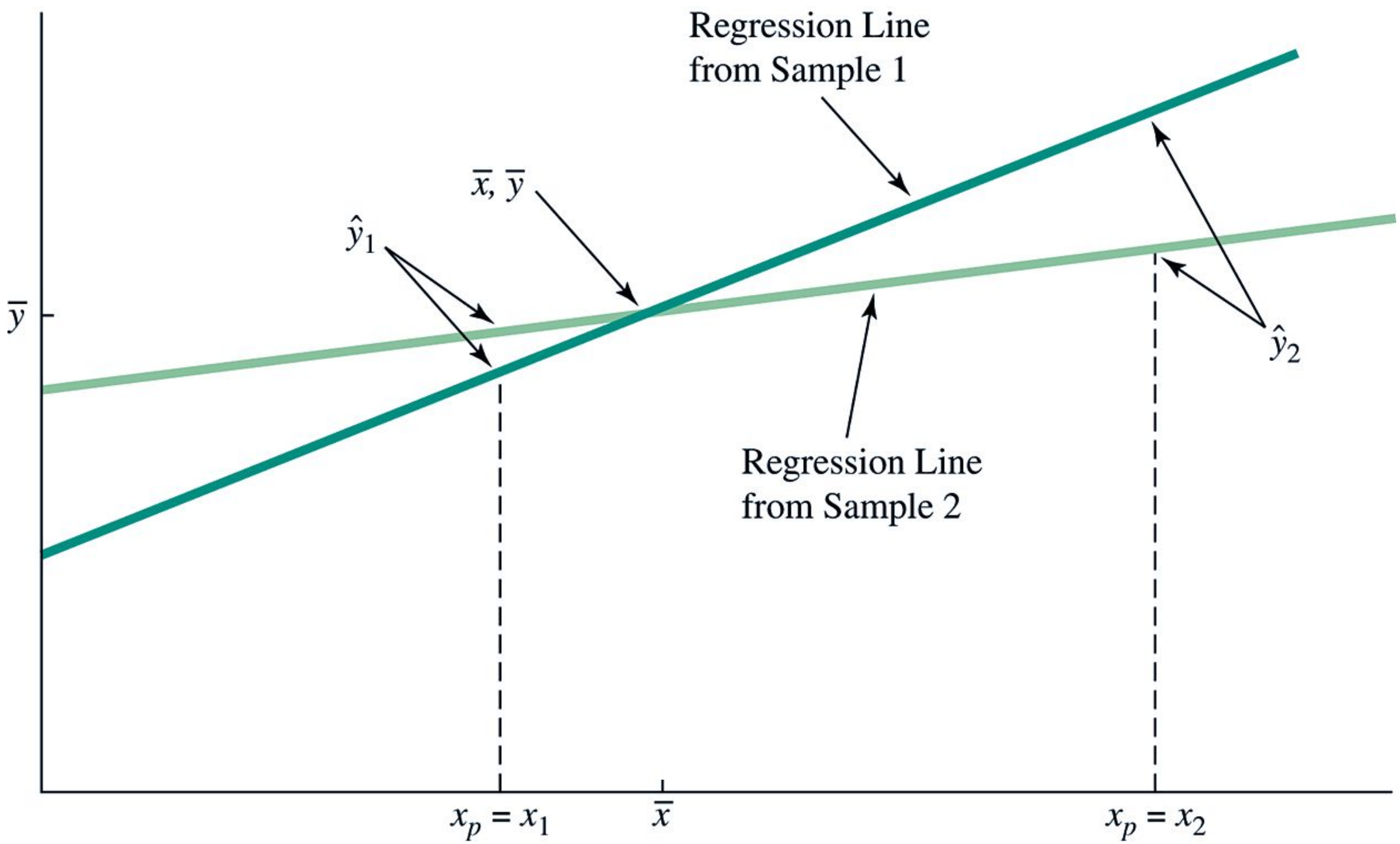
Prediction Interval for a Particular y, Given x

Prediction Interval for $y|x_p$

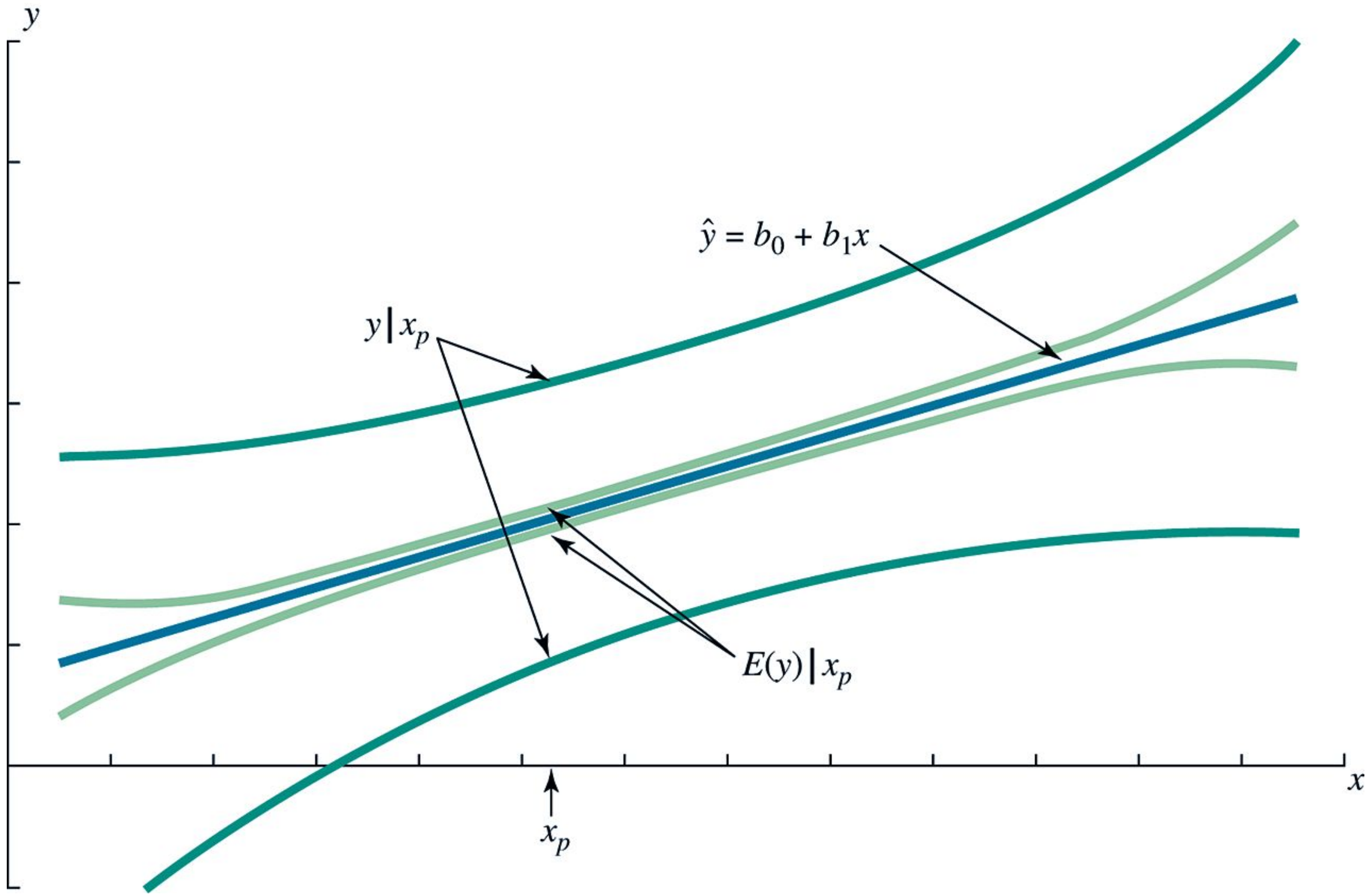
$$\hat{y} \pm t s_\varepsilon \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$$

The term $(x_p - \bar{x})^2$ has a particular effect on the confidence and prediction intervals. The farther x_p (the value of the independent variable used to predict y), is from $\bar{x}$, greater the interval becomes.

Potential Variation in y as x_p Moves Farther from $\overline{x}$



Confidence and Prediction Intervals



Confidence and Prediction Intervals Using Excel 2016 and XLSTAT – Hospital Example

$$x_p = 5 \text{ days}$$

Predictions for the new observations:

Observation	Pred(Total Chges)	Std. dev. on pred. (Mean)	Lower bound 95% (Mean)	Upper bound 95% (Mean)	Std. dev. on pred. (Observation)	Lower bound 95% (Observation)	Upper bound 95% (Observation)
PredObs1	7,291.59	253.83	6,789.63	7,793.54	2,905.89	1,545.01	13,038.16

Point Estimate

Condence Interval
6,789.63 ——— 7,793.54

Prediction Interval
1,545.01-———13,038.16