Scaling of relative permeability functions as a method of regularizing numerical solution of the water – oil displacement problem

Student: Vlasov Eugeniy

Group: 23mTF173

The problem of displacement of oil by water in a porous medium

The equation for the determination of water saturation in the one-dimensional Buckley-Leverett problem:

$$\frac{\partial S}{\partial t} + c\frac{\partial F(S)}{\partial x} = 0$$

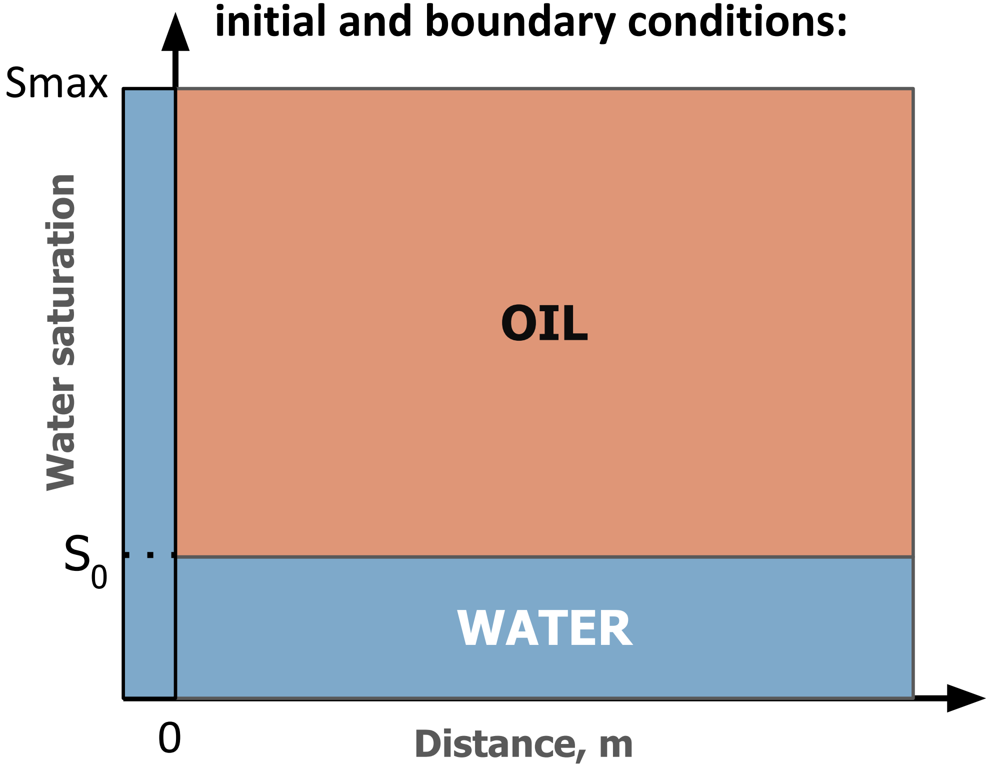
where c = v/m > 0 – the average velocity of water;

$$F(S) = \frac{f_w(S)/\mu_w}{\frac{f_w(S)}{\mu_w} + \frac{f_o(S)}{\mu_o}}$$

– Backley-Leverett function; $f_w(S), f_o(S)$ – a relative permeability function, respectively for water and oil; μ_w,μ_o – dynamic viscosity of water and oil respectively.

initial and boundary conditions:

$S(0,x) = S_0$, $0 \leqslant x \leqslant L$; $S(t,0) = Smax$, $0 < t < \infty$



Numerical diffusion in the problem of displacement of oil by water

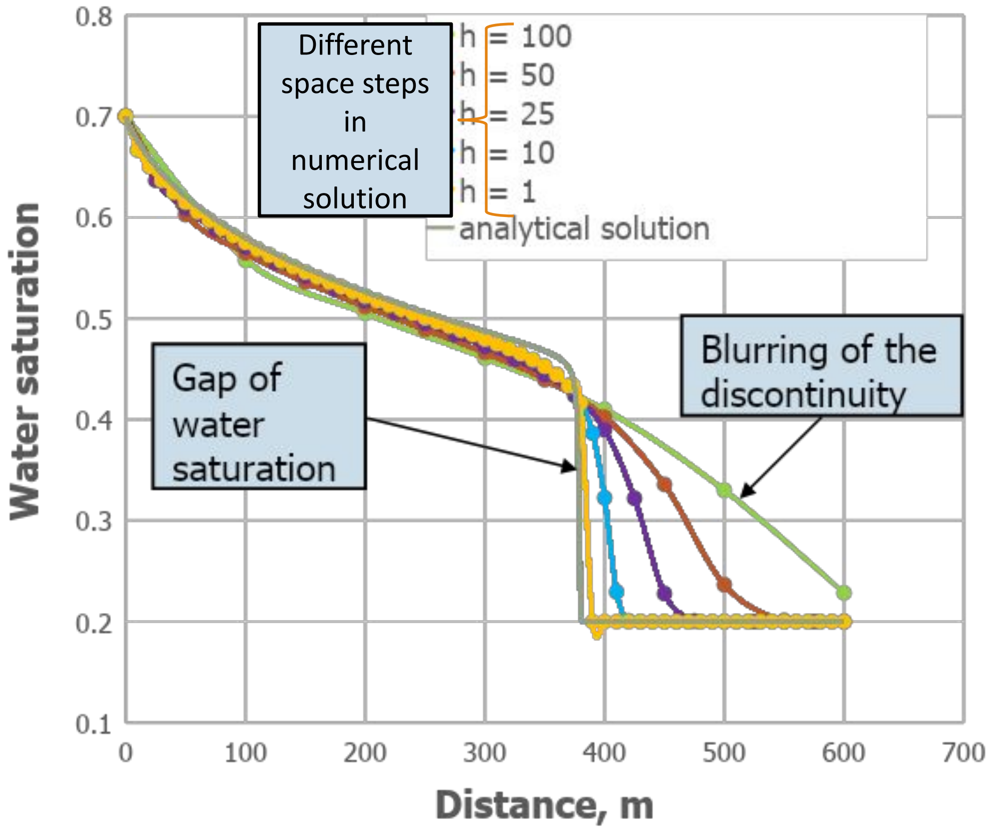
Equation has an analytic solution by applying a self-similar variable $\xi = x/ct$

$$-\xi \frac{\partial S}{\partial \xi} + \frac{\mathrm{dF(S)}}{dS}\frac{\partial S}{\partial \xi} = \frac{\partial S}{\partial \xi}\left(\frac{\mathrm{dF(S)}}{dS} - \xi\right) = 0 \rightarrow \begin{cases} S = S_0 \\ \xi = \dfrac{x}{ct} = \dfrac{\mathrm{dF(S)}}{dS} \end{cases}$$

Courant - Isacson - Rees numerical scheme:

$$S_k^{n+1} = S_k^n + \frac{\tau c}{h}(F_{k-1}^n - F_k^n),$$

Where n, k – the number of the time layer and mesh node, respectively; τ - constant time step; h –distance between adjacent nodes of a difference grid; $F_k = F(S_k^n)$.



$$S_k^{n+2} = S_k^{n+1} + \frac{F_{k-1}^{n+1} - F_k^{n+1}}{2} > S_k^{n+1} > S_0,$$

$$S_{k+1}^{n+2} = S_0 + \frac{F_k^{n+1}}{2} > S_0.$$

$$\Delta S = S_{k+1}^{n+2} - S(x=(k+1)h) = \frac{1}{2}F(S_0 + \frac{F(S_f)}{2}).$$

$$F(S) = \frac{(S - S_0)^2}{2\frac{\mu_w}{\mu_o}(S_{\max} - S)^2 + (S - S_0)^2}.$$

$$\varphi_i(S_L) = \begin{cases} f_i(S_{L\,j}), \text{если } p_j > p_{j+1} \\ f_i(S_{L\,j+1}), \text{если } p_j < p_{j+1} \end{cases},$$

$$\bar{S} = \frac{1}{L}\int_0^L S(x)\,dx = S_L + \frac{1 - F(S_L)}{F'(S_L)},$$

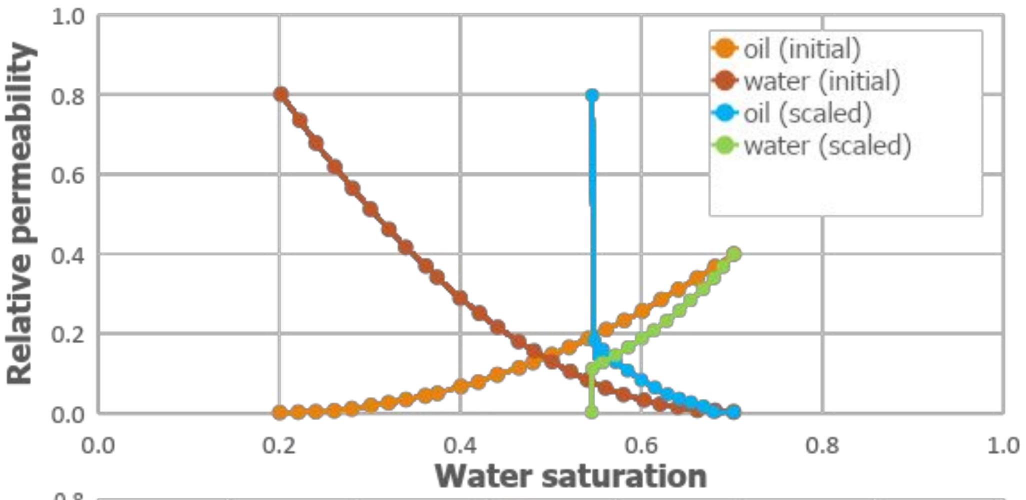
$$S_f = S_0 + \frac{F(S_f) - F(S_0)}{F'(S_f)},$$

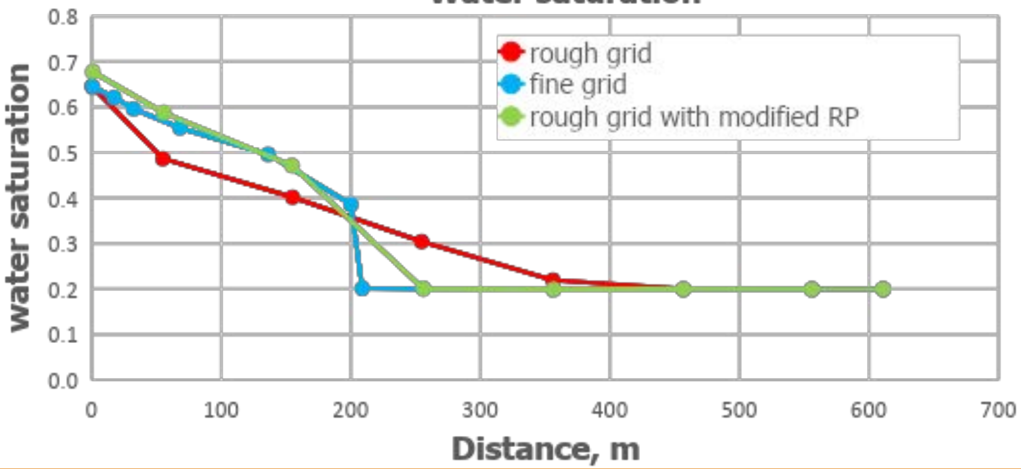
A new method for regularizing a numerical solution

To the displacement front was not continued due to the artificial diffusion, the axis of saturation in the tables of RP you want to modify on the average saturation in the cell

$$f_w(\bar{S}) = \begin{cases} f_w(S_0), if\ S_0 < S_L < S_f \\ f_w(S_L), if\ S_k > S_L > S_f \end{cases}$$

$$f_o(\bar{S}) = \begin{cases} f_{\max o}(S_L), if\ S_0 < S_L < S_f \\ f_o(S_L), if\ S_k > S_L > S_f \end{cases}$$





Thank you for your attention