Data Mining:
Concepts and Techniques

— Chapter 2 —

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Chapter 2: Getting to Know Your Data

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary

Types of Data Sets

- Record
  - Relational records
  - Data matrix, e.g., numerical matrix, crosstabs
  - Document data: text documents: term-frequency vector
  - Transaction data
- Graph and network
  - World Wide Web
  - Social or information networks
  - Molecular Structures
- Ordered
  - Video data: sequence of images
  - Temporal data: time-series
  - Sequential Data: transaction sequences
  - Genetic sequence data
- Spatial, image and multimedia:
  - Spatial data: maps
  - Image data:
  - Video data:

	team	coach	play	ball	score	game	win	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Important Characteristics of Structured Data

- Dimensionality
  - Curse of dimensionality
- Sparsity
  - Only presence counts
- Resolution
  - Patterns depend on the scale
- Distribution
  - Centrality and dispersion

Data Objects

- Data sets are made up of data objects.
- A **data object** represents an entity.
- Examples:
  - sales database: customers, store items, sales
  - medical database: patients, treatments
  - university database: students, professors, courses
- Also called *samples* , *examples, instances, data points, objects, tuples*.
- Data objects are described by **attributes**.
- Database rows -> data objects; columns ->attributes.

Attributes

- **Attribute (**or **dimensions, features, variables**): a data field, representing a characteristic or feature of a data object.
  - *E.g., customer _ID, name, address*
- Types:
  - Nominal
  - Binary
  - Numeric: quantitative
    - Interval-scaled
    - Ratio-scaled

Attribute Types

- **Nominal:** categories, states, or "names of things"
  - *Hair_color = {auburn, black, blond, brown, grey, red, white}*
  - marital status, occupation, ID numbers, zip codes
- **Binary**
  - Nominal attribute with only 2 states (0 and 1)
  - <u>Symmetric binary</u>: both outcomes equally important
    - e.g., gender
  - <u>Asymmetric binary</u>: outcomes not equally important.
    - e.g., medical test (positive vs. negative)
    - Convention: assign 1 to most important outcome (e.g., HIV positive)
- **Ordinal**
  - Values have a meaningful order (ranking) but magnitude between successive values is not known.
  - *Size = {small, medium, large},* grades, army rankings

Numeric Attribute Types

- Quantity (integer or real-valued)
- **Interval**
    - Measured on a scale of **equal-sized units**
    - Values have order
        - E.g., *temperature in C˚or F˚, calendar dates*
    - No true zero-point
- **Ratio**
    - Inherent **zero-point**
    - We can speak of values as being an order of magnitude larger than the unit of measurement (10 K˚ is twice as high as 5 K˚).
        - e.g., *temperature in Kelvin, length, counts, monetary quantities*

Discrete vs. Continuous Attributes

- **Discrete Attribute**
  - Has only a finite or countably infinite set of values
    - E.g., zip codes, profession, or the set of words in a collection of documents
  - Sometimes, represented as integer variables
  - Note: Binary attributes are a special case of discrete attributes
- **Continuous Attribute**
  - Has real numbers as attribute values
    - E.g., temperature, height, or weight
  - Practically, real values can only be measured and represented using a finite number of digits
  - Continuous attributes are typically represented as floating-point variables

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Basic Statistical Descriptions of Data

- Motivation
  - To better understand the data: central tendency, variation and spread
- Data dispersion characteristics
  - median, max, min, quantiles, outliers, variance, etc.
- Numerical dimensions correspond to sorted intervals
  - Data dispersion: analyzed with multiple granularities of precision
  - Boxplot or quantile analysis on sorted intervals
- Dispersion analysis on computed measures
  - Folding measures into numerical dimensions
  - Boxplot or quantile analysis on the transformed cube

Measuring the Central Tendency

- Mean (algebraic measure) (sample vs. population):

  Note: *n* is sample size and *N* is population size.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \frac{\sum x}{N}$$

  - Weighted arithmetic mean:
  - Trimmed mean: chopping extreme values

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

- Median:
  - Middle value if odd number of values, or average of the middle two values otherwise
  - Estimated by interpolation (for *grouped data*):

$$median = L_1 + \left(\frac{n/2 - (\sum freq)_l}{freq_{median}}\right) width$$

age	$frequency$
1–5	200
6–15	450
16–20	300
21–50	1500
51–80	700
81–110	44

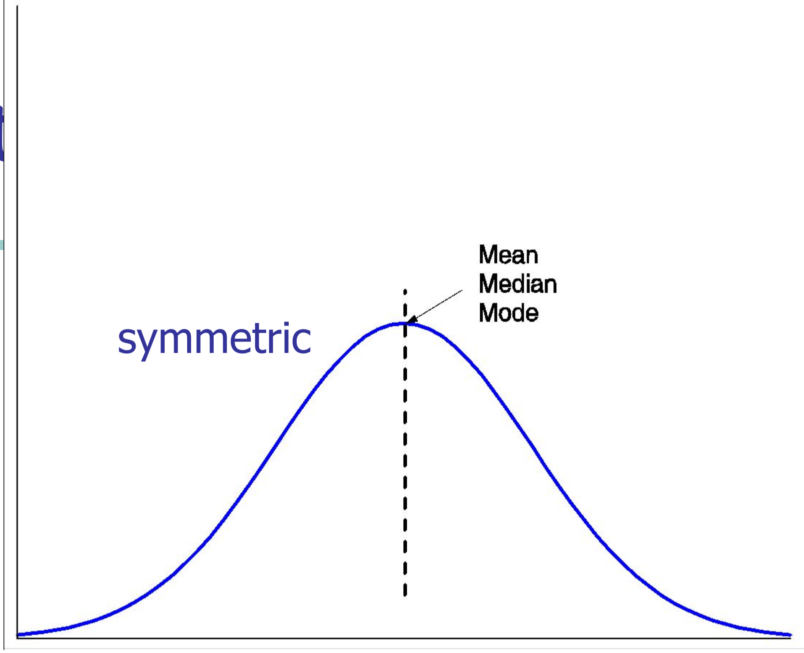
Median interval → 21–50

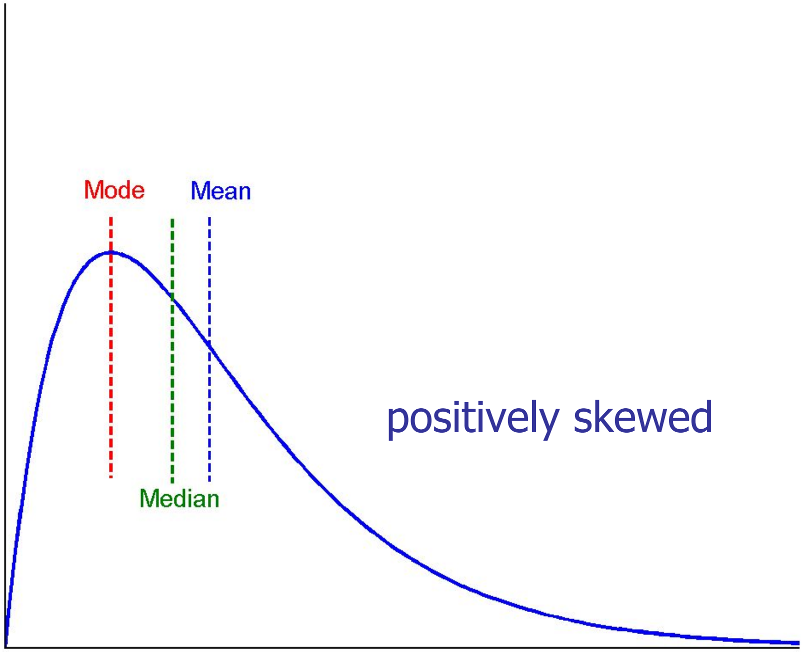
- Mode
  - Value that occurs most frequently in the data
  - Unimodal, bimodal, trimodal
  - Empirical formula:

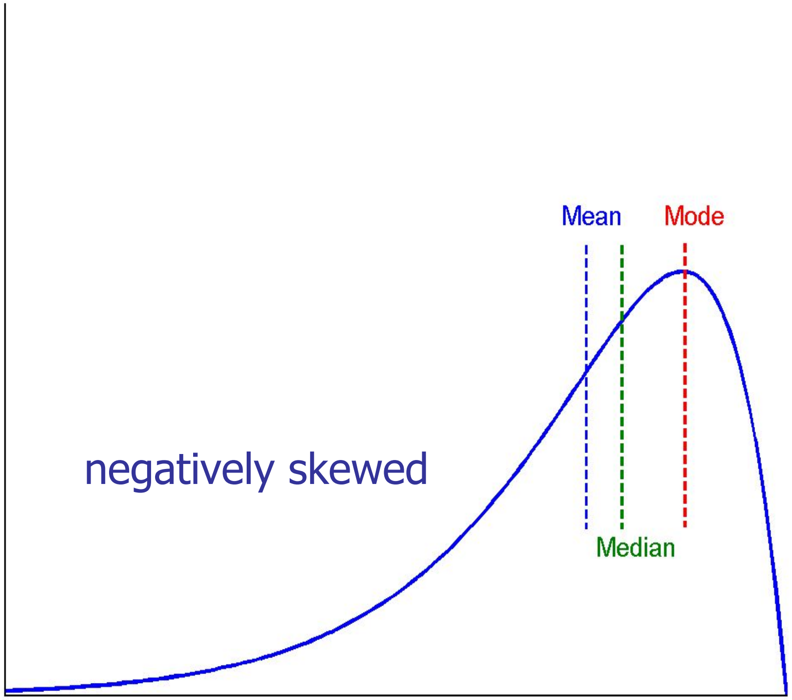
$$mean - mode = 3 \times (mean - median)$$

Symmetric vs. Skewed Dat

- Median, mean and mode of symmetric, positively and negatively skewed data







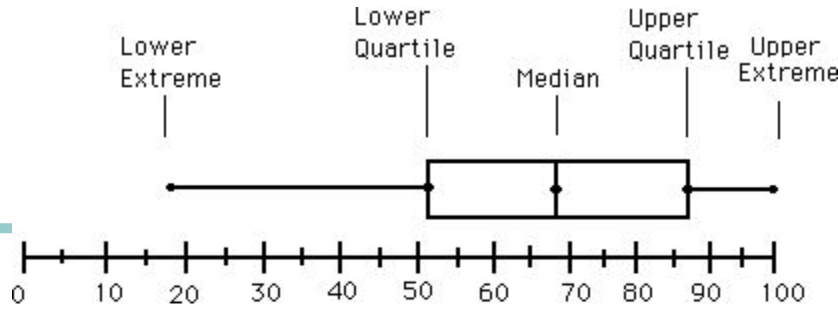
Measuring the Dispersion of Data

- Quartiles, outliers and boxplots
  - **Quartiles**: Q_1 (25th percentile), Q_3 (75th percentile)
  - **Inter-quartile range**: IQR = $Q_3 - Q_1$
  - **Five number summary**: min, Q_1, median, Q_3, max
  - **Boxplot**: ends of the box are the quartiles; median is marked; add whiskers, and plot outliers individually
  - **Outlier**: usually, a value higher/lower than 1.5 x IQR
- Variance and standard deviation (*sample: s, population: σ*)
  - **Variance**: (algebraic, scalable computation)

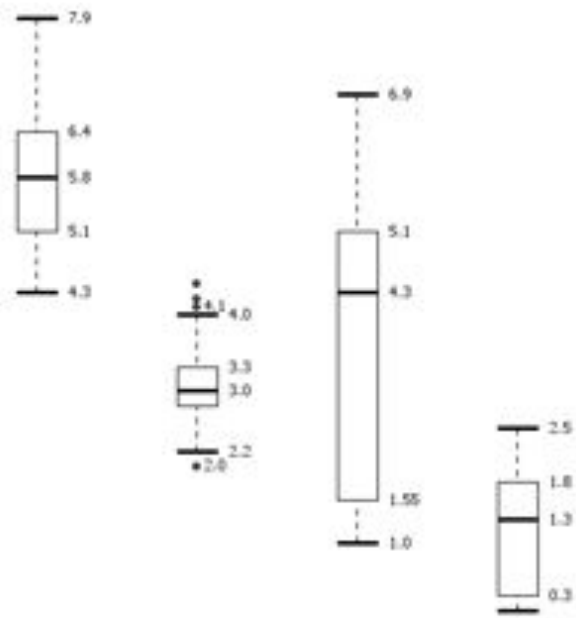
$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n}x_i\right)^2\right] \qquad \sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \mu)^2 = \frac{1}{N}\sum_{i=1}^{n}x_i^2 - \mu^2$$

  - **Standard deviation** *s (or σ)* is the square root of variance s^2 *(or σ^2)*

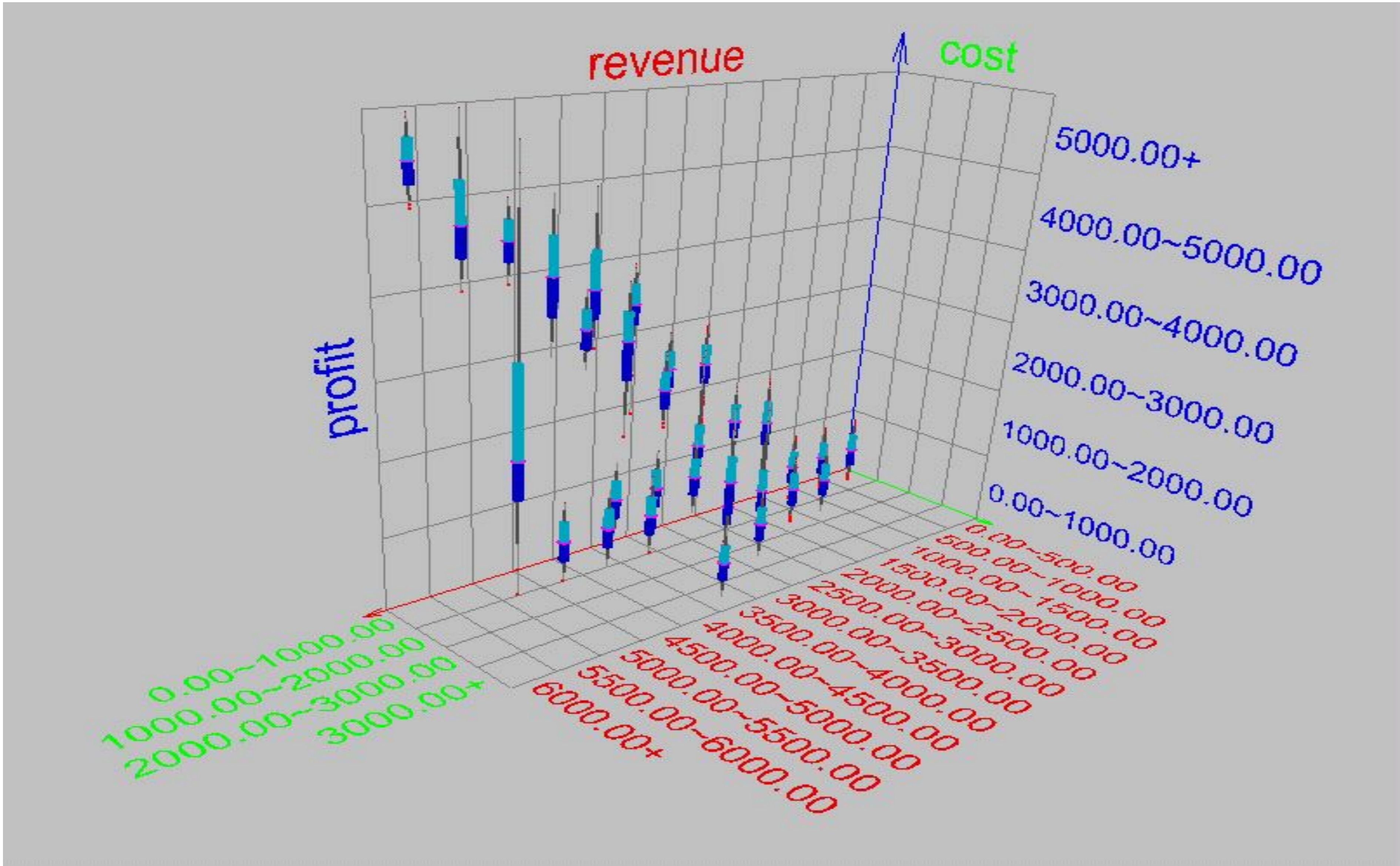
Boxplot Analysis



- **Five-number summary** of a distribution
  - Minimum, Q1, Median, Q3, Maximum
- **Boxplot**
  - Data is represented with a box
  - The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
  - The median is marked by a line within the box
  - Whiskers: two lines outside the box extended to Minimum and Maximum
  - Outliers: points beyond a specified outlier threshold, plotted individually

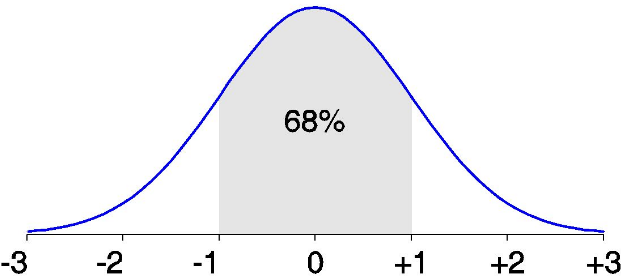


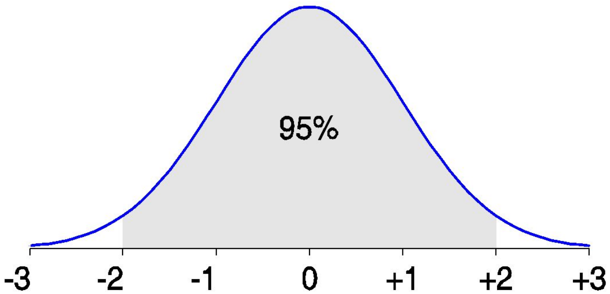
Visualization of Data Dispersion: 3-D Boxplots

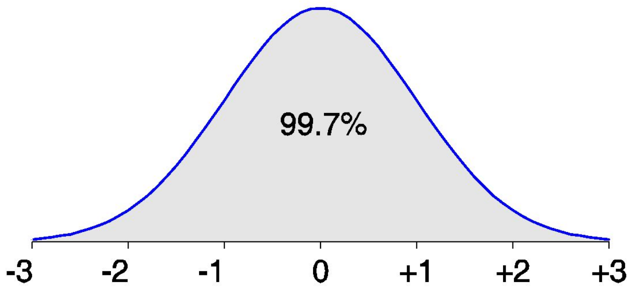


Properties of Normal Distribution Curve

- The normal (distribution) curve
  - From $\mu-\sigma$ to $\mu+\sigma$: contains about 68% of the measurements (μ: mean, σ: standard deviation)
  - From $\mu-2\sigma$ to $\mu+2\sigma$: contains about 95% of it
  - From $\mu-3\sigma$ to $\mu+3\sigma$: contains about 99.7% of it





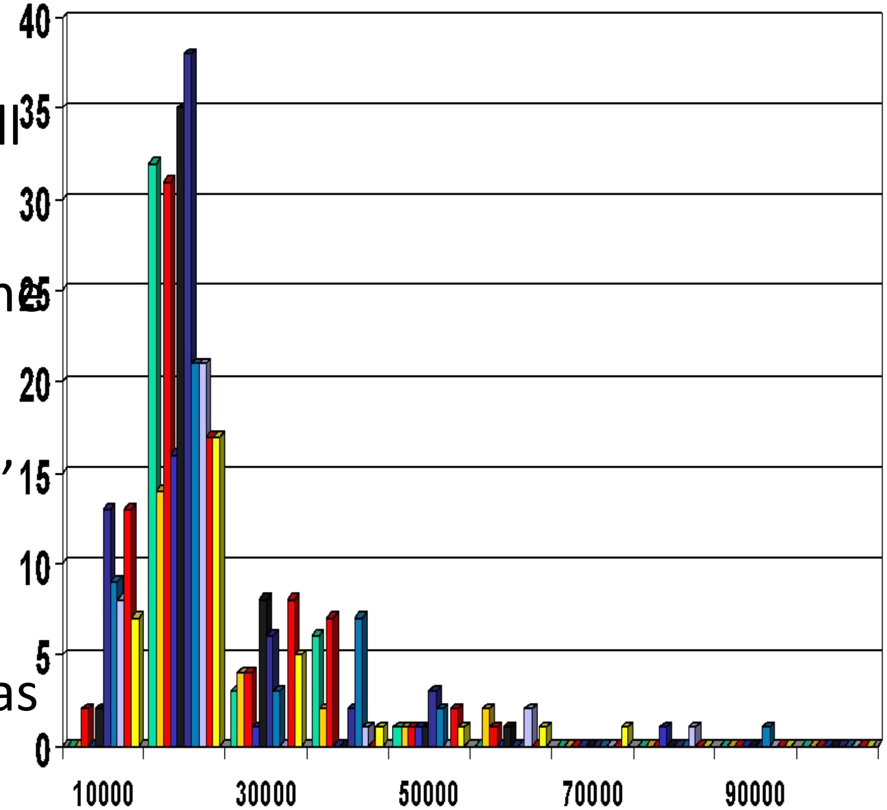


Graphic Displays of Basic Statistical Descriptions

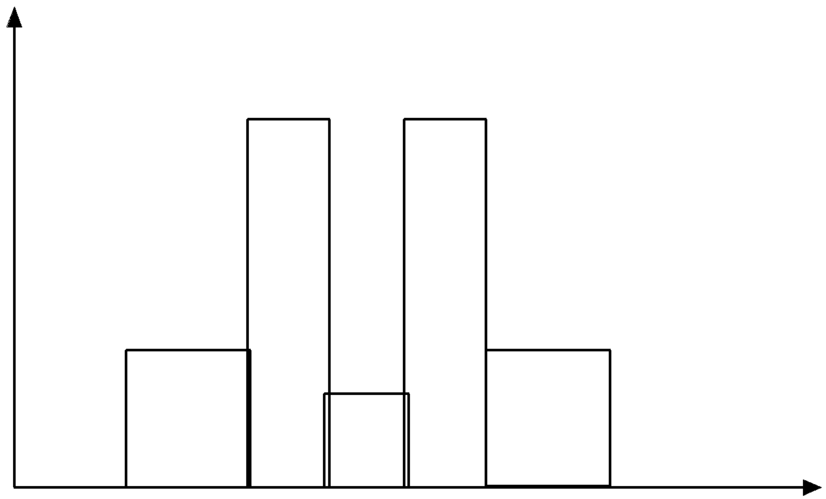
- **Boxplot**: graphic display of five-number summary
- **Histogram**: x-axis are values, y-axis repres. frequencies
- **Quantile plot**: each value x_i is paired with f_i indicating that approximately 100 f_i % of data are $\leq x_i$
- **Quantile-quantile (q-q) plot**: graphs the quantiles of one univariant distribution against the corresponding quantiles of another
- **Scatter plot**: each pair of values is a pair of coordinates and plotted as points in the plane

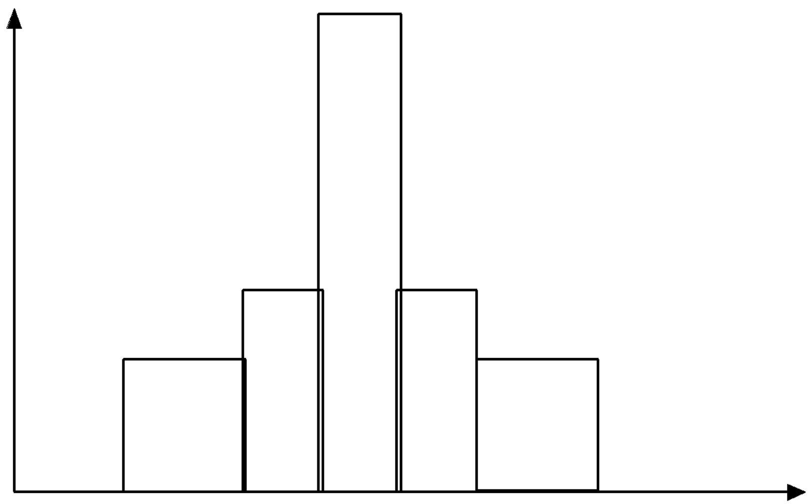
Histogram Analysis

- Histogram: Graph display of tabulated frequencies, shown as bars
- It shows what proportion of cases fall into each of several categories
- Differs from a bar chart in that it is the *area* of the bar that denotes the value, not the height as in bar charts, a crucial distinction when the categories are not of uniform width
- The categories are usually specified as non-overlapping intervals of some variable. The categories (bars) must be adjacent



Histograms Often Tell More than Boxplots

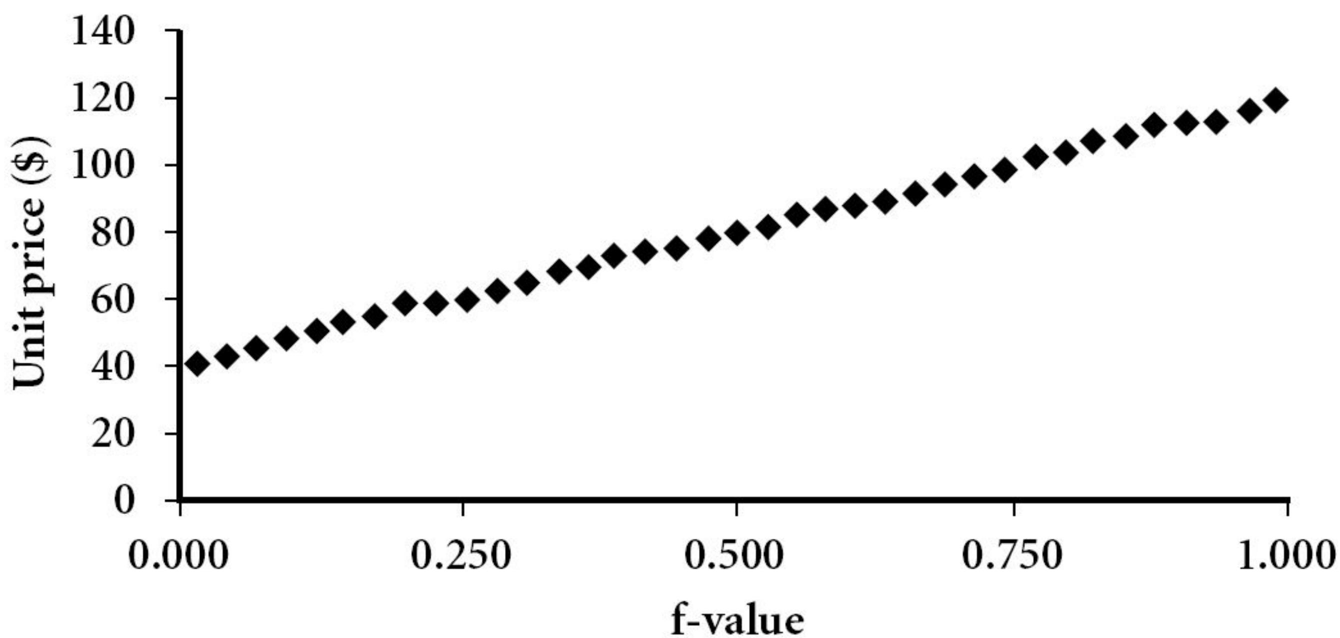




- The two histograms shown in the left may have the same boxplot representation
  - The same values for: min, Q1, median, Q3, max
- But they have rather different data distributions

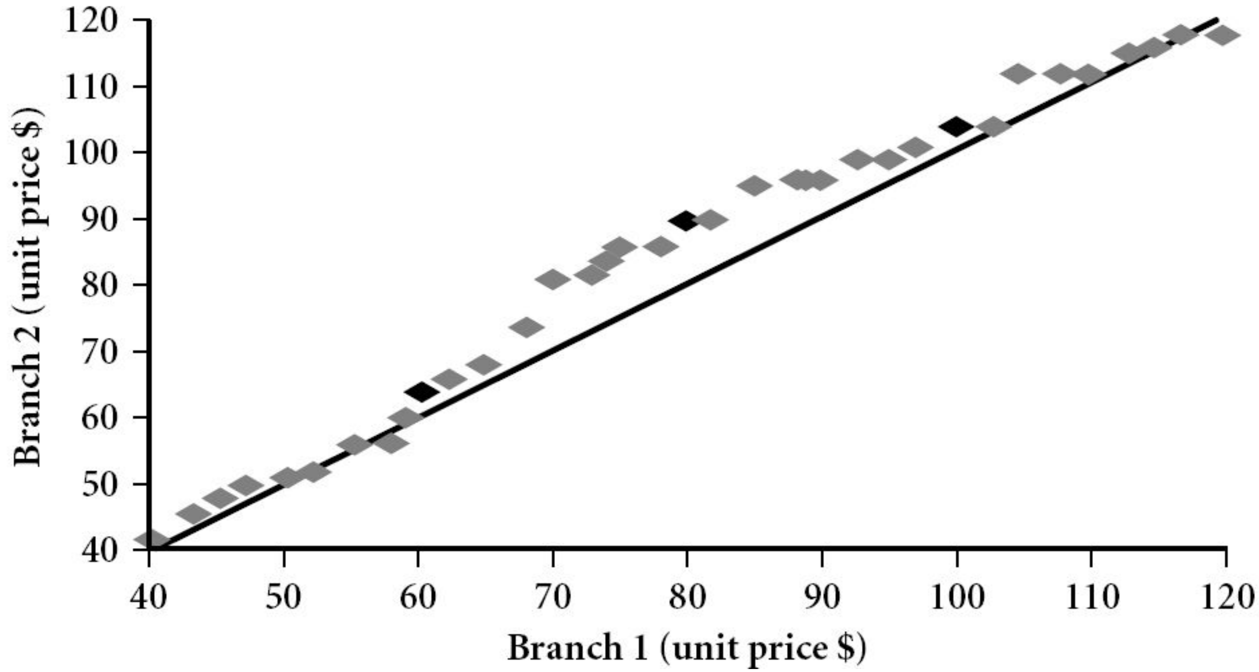
Quantile Plot

- Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)
- Plots **quantile** information
  - For a data x_i data sorted in increasing order, f_i indicates that approximately 100 f_i% of the data are below or equal to the value x_i



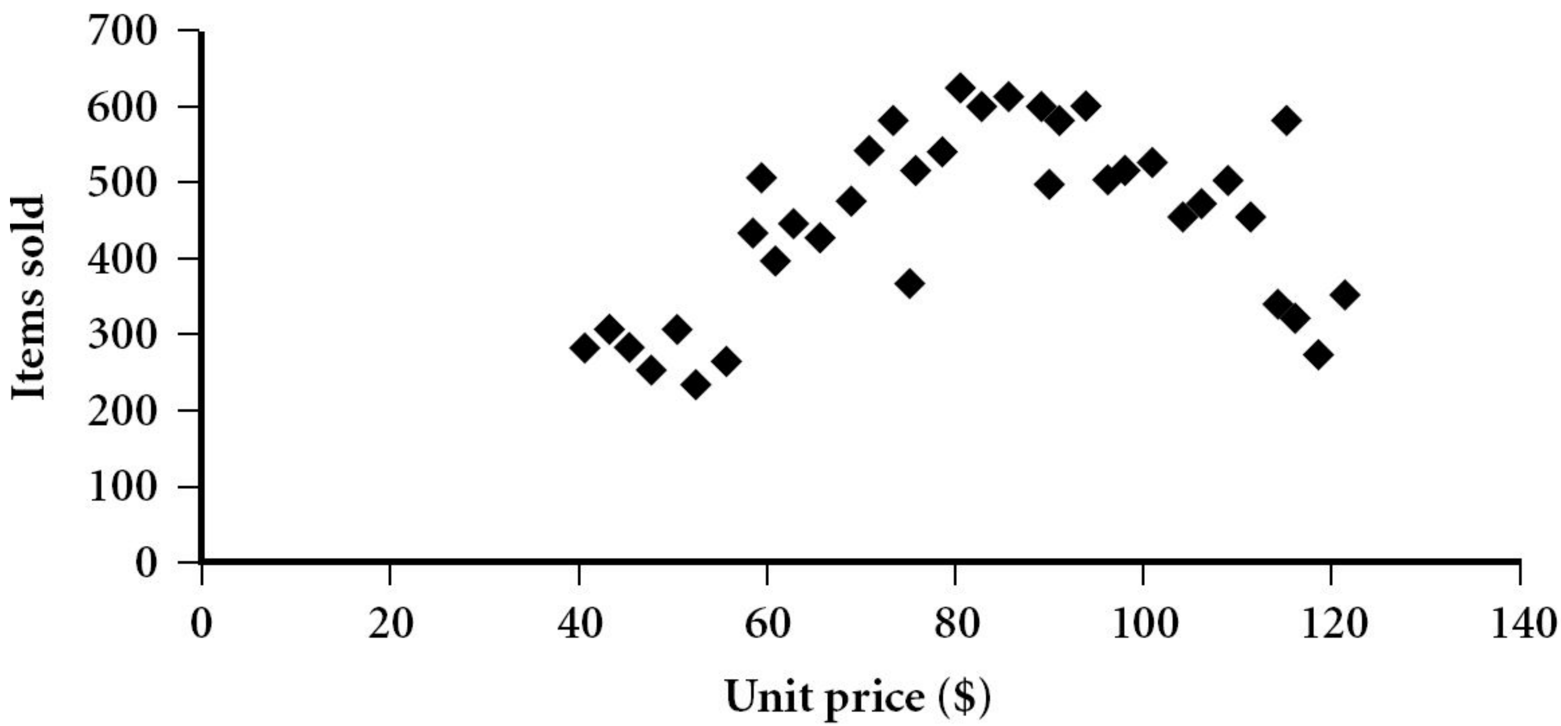
Quantile-Quantile (Q-Q) Plot

- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- View: Is there is a shift in going from one distribution to another?
- Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.

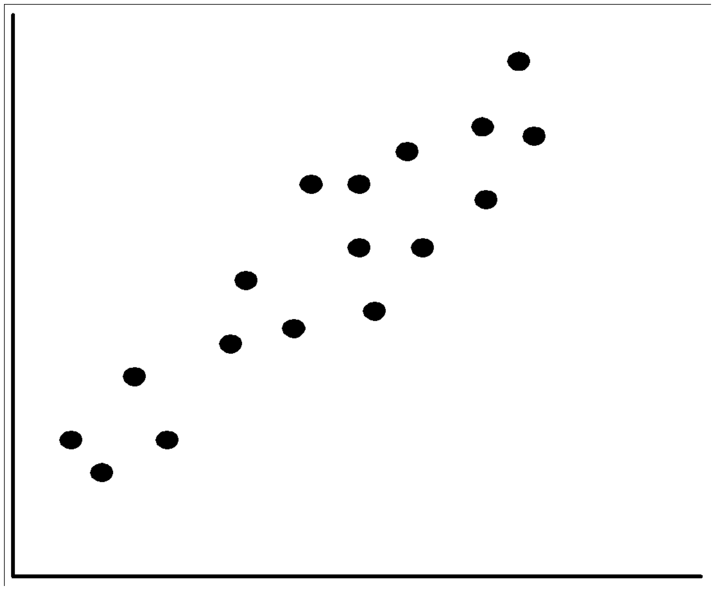


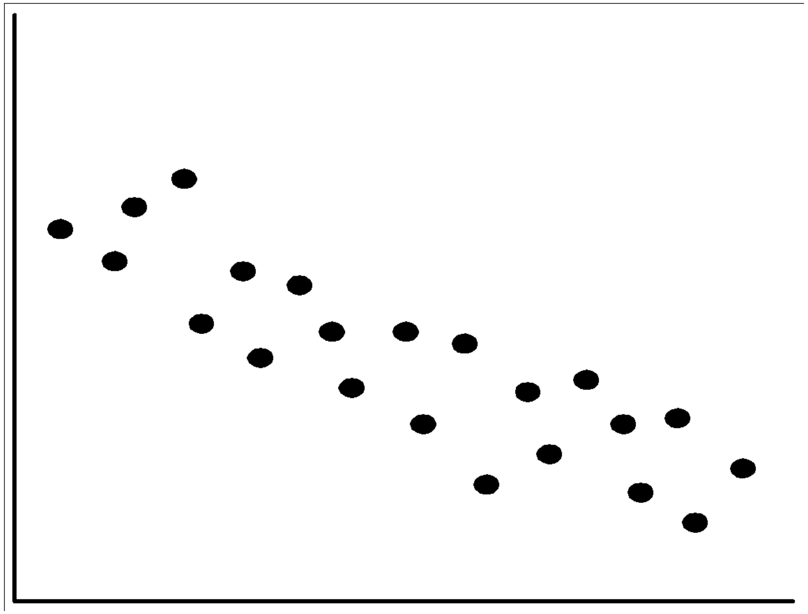
Scatter plot

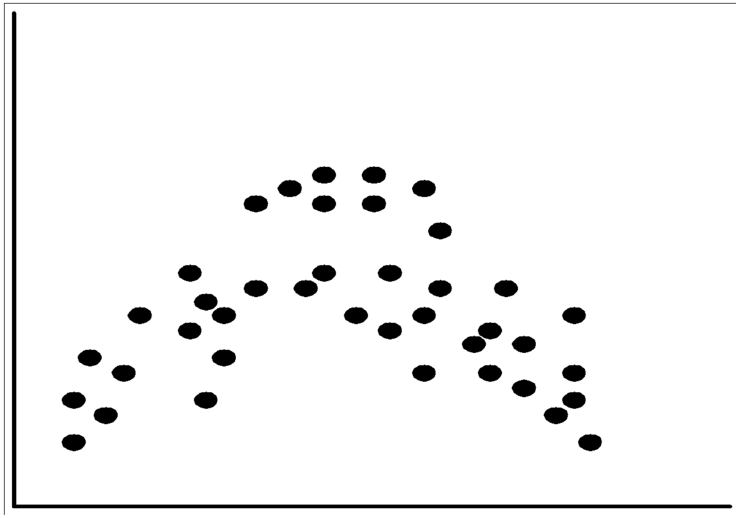
- Provides a first look at bivariate data to see clusters of points, outliers, etc
- Each pair of values is treated as a pair of coordinates and plotted as points in the plane



Positively and Negatively Correlated Data

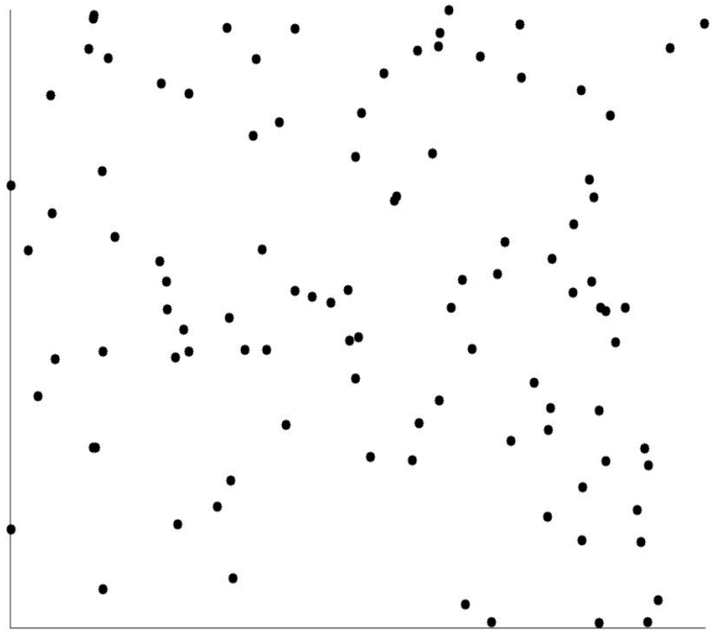


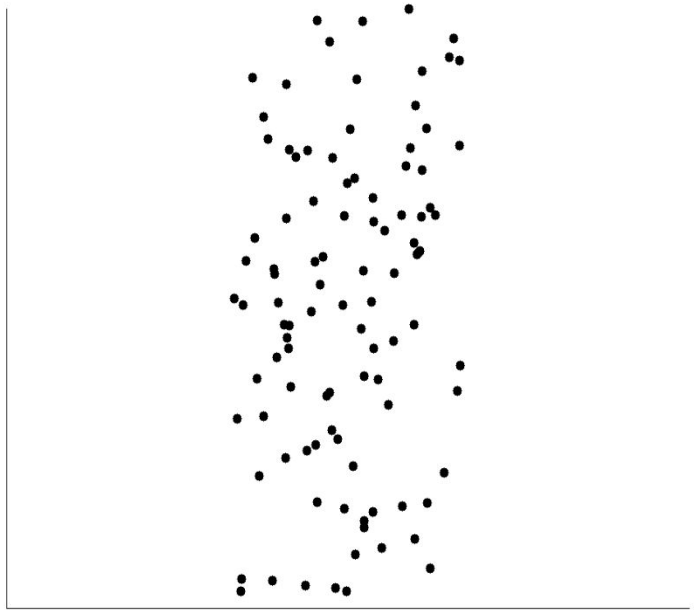




- The left half fragment is positively correlated
- The right half is negative correlated

Uncorrelated Data





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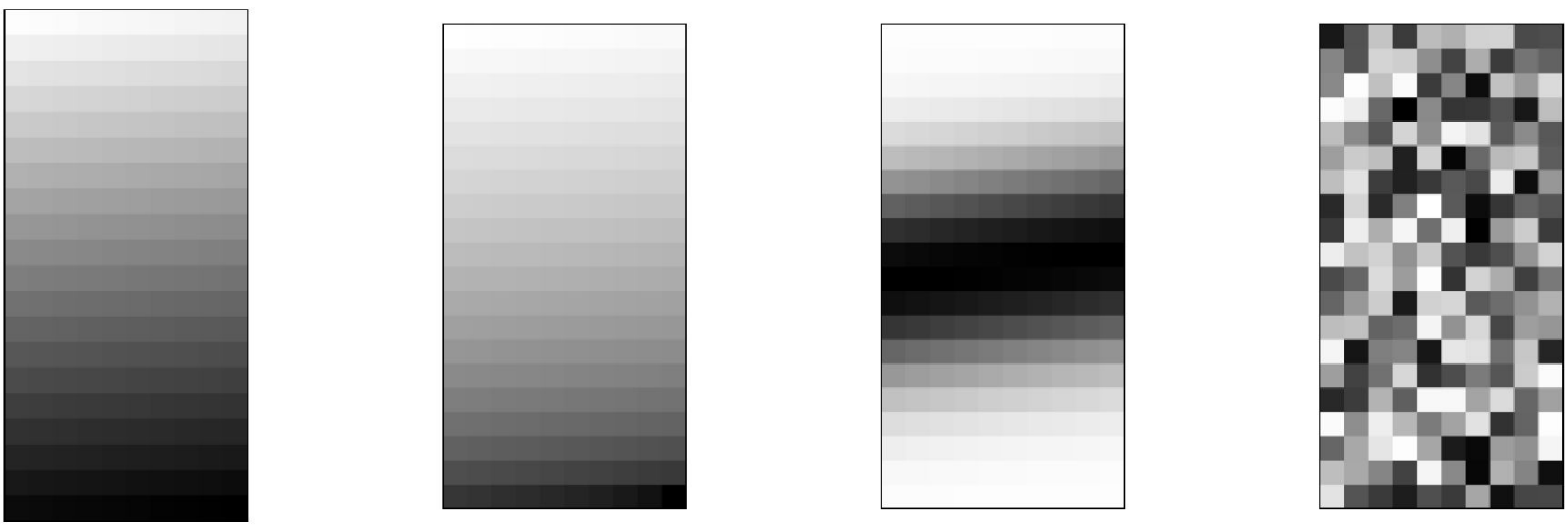
Data Visualization

- Why data visualization?
  - Gain insight into an information space by mapping data onto graphical primitives
  - Provide qualitative overview of large data sets
  - Search for patterns, trends, structure, irregularities, relationships among data
  - Help find interesting regions and suitable parameters for further quantitative analysis
  - Provide a visual proof of computer representations derived
- Categorization of visualization methods:
  - Pixel-oriented visualization techniques
  - Geometric projection visualization techniques
  - Icon-based visualization techniques
  - Hierarchical visualization techniques
  - Visualizing complex data and relations

Pixel-Oriented Visualization Techniques

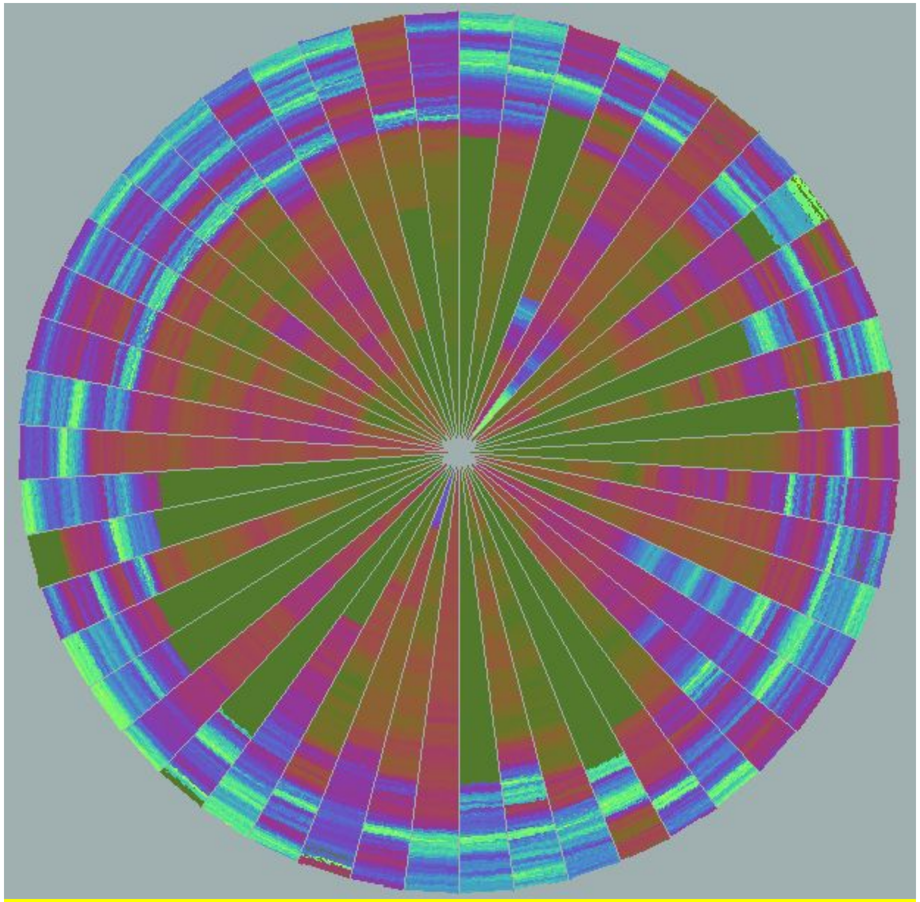
- For a data set of m dimensions, create m windows on the screen, one for each dimension
- The m dimension values of a record are mapped to m pixels at the corresponding positions in the windows
- The colors of the pixels reflect the corresponding values



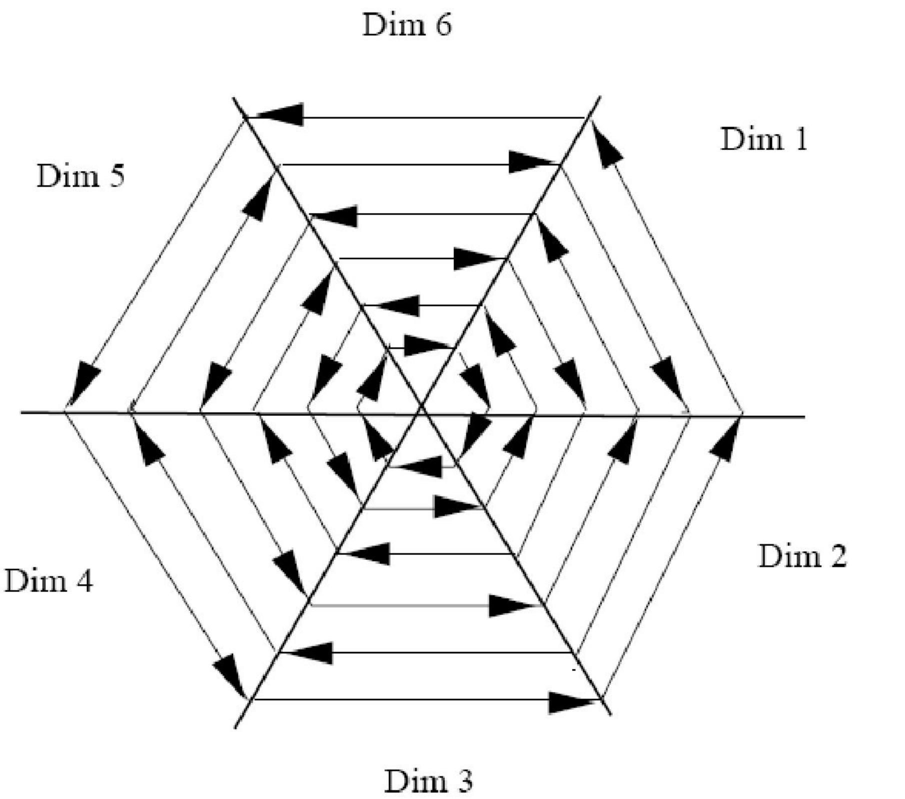
(a) Income (b) Credit Limit (c) transaction volume (d) age

Laying Out Pixels in Circle Segments

- To save space and show the connections among multiple dimensions, space filling is often done in a circle segment



Representing about 265,000 50-dimensional Data Items with the 'Circle Segments' Technique



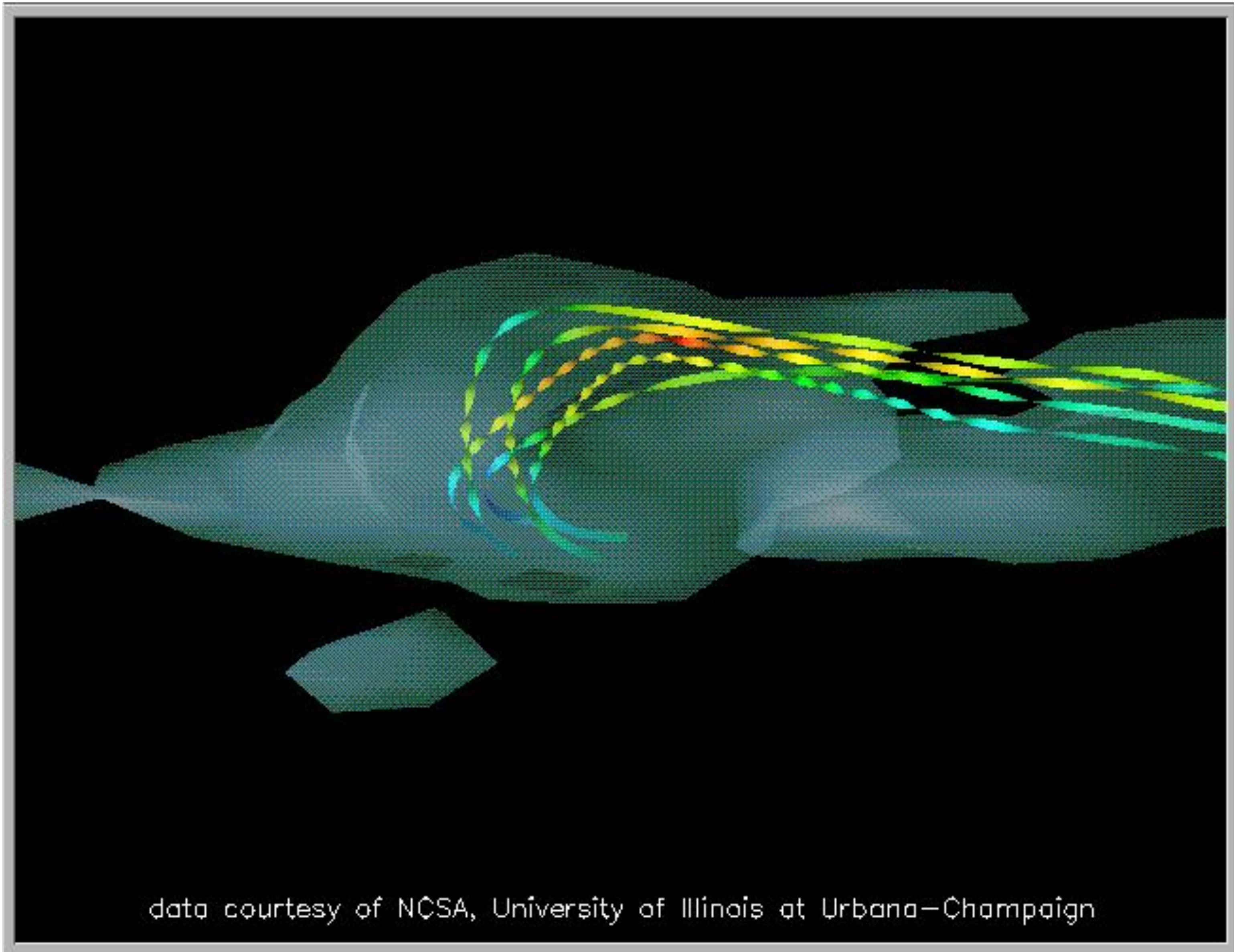
(b) Laying out pixels in circle segment

Geometric Projection Visualization Techniques

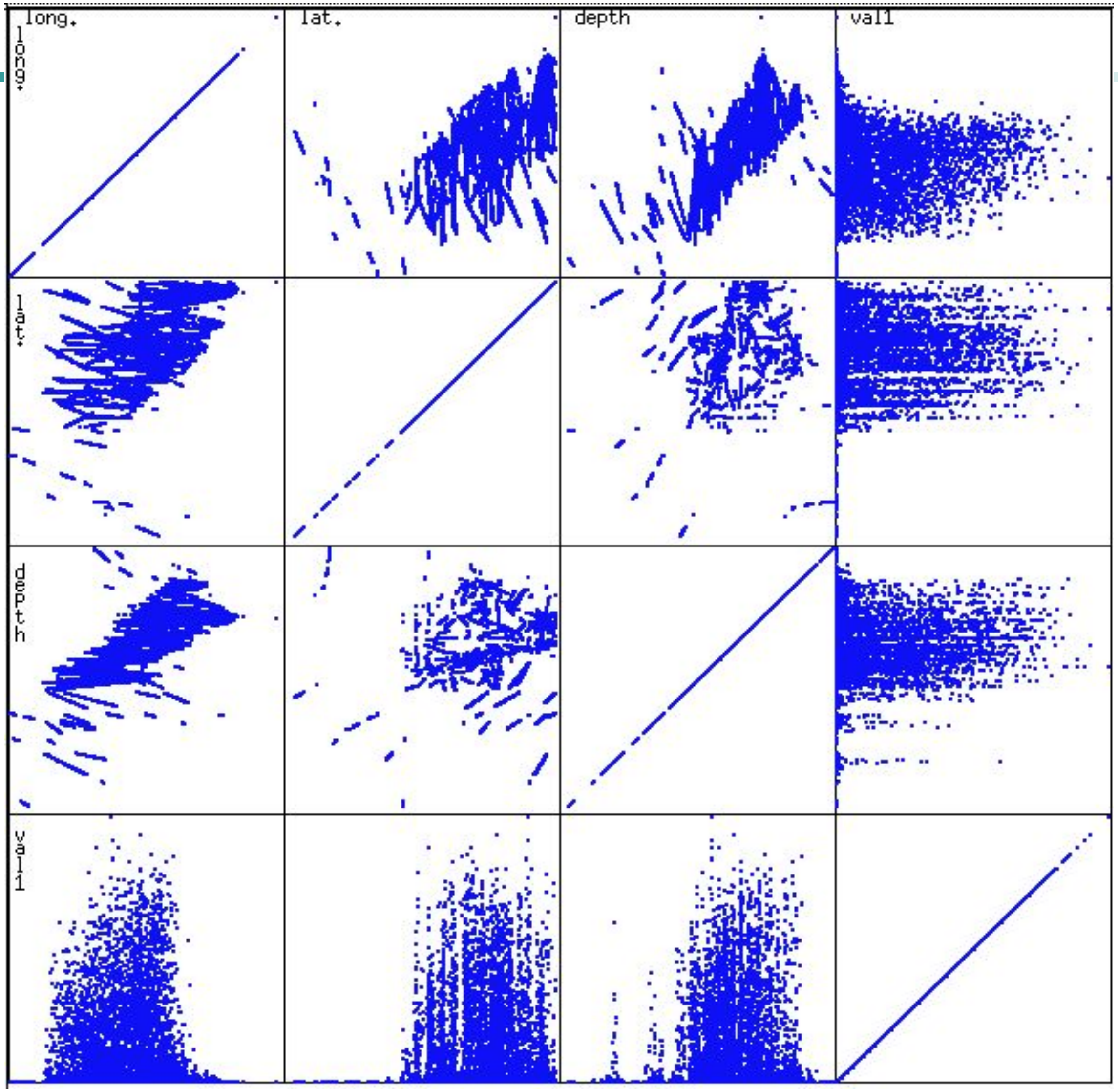
- Visualization of geometric transformations and projections of the data
- Methods
  - Direct visualization
  - Scatterplot and scatterplot matrices
  - Landscapes
  - Projection pursuit technique: Help users find meaningful projections of multidimensional data
  - Prosection views
  - Hyperslice
  - Parallel coordinates

Direct Data Visualization

Ribbons with Twists Based on Vorticity



Scatterplot Matrices



Matrix of scatterplots (x-y-diagrams) of the k-dim. data [total of (k2/2-k) scatterplots]

Landscapes

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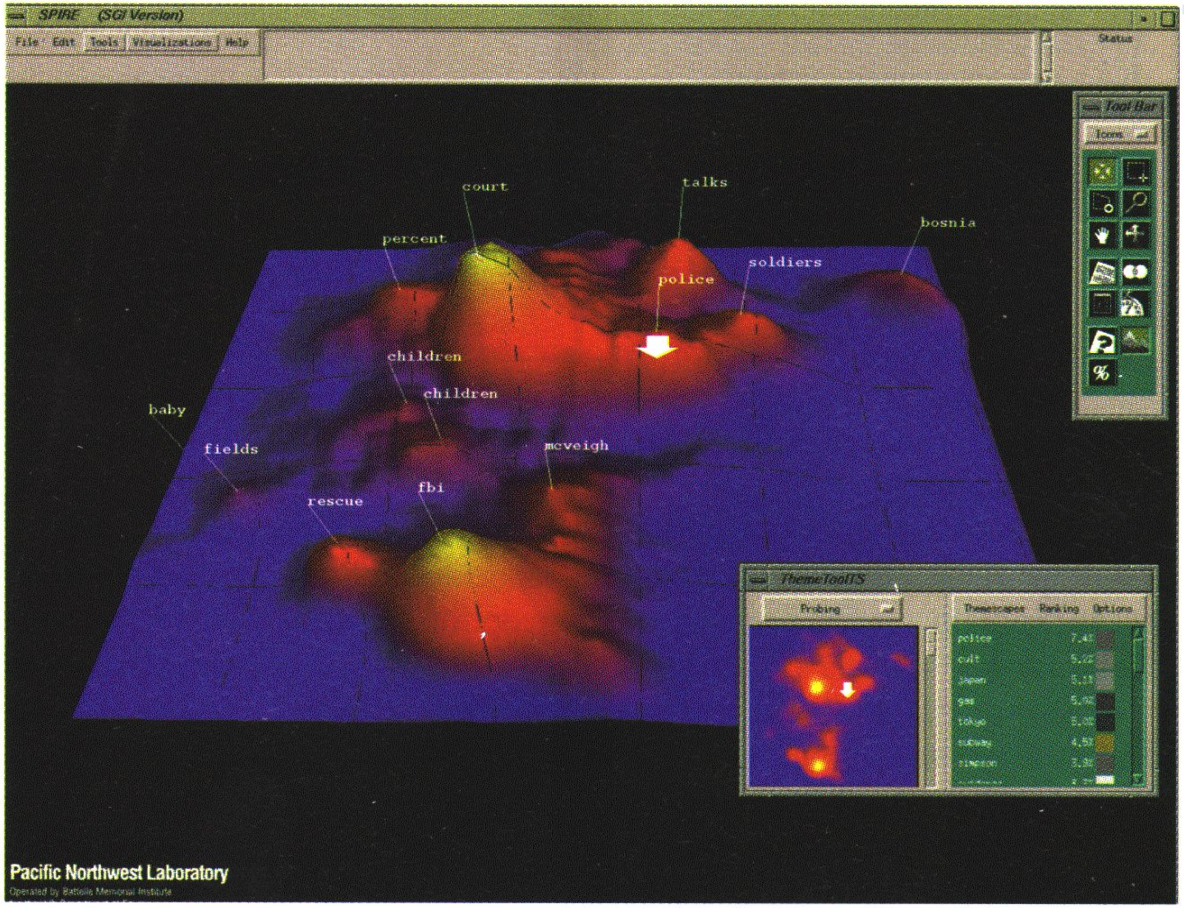


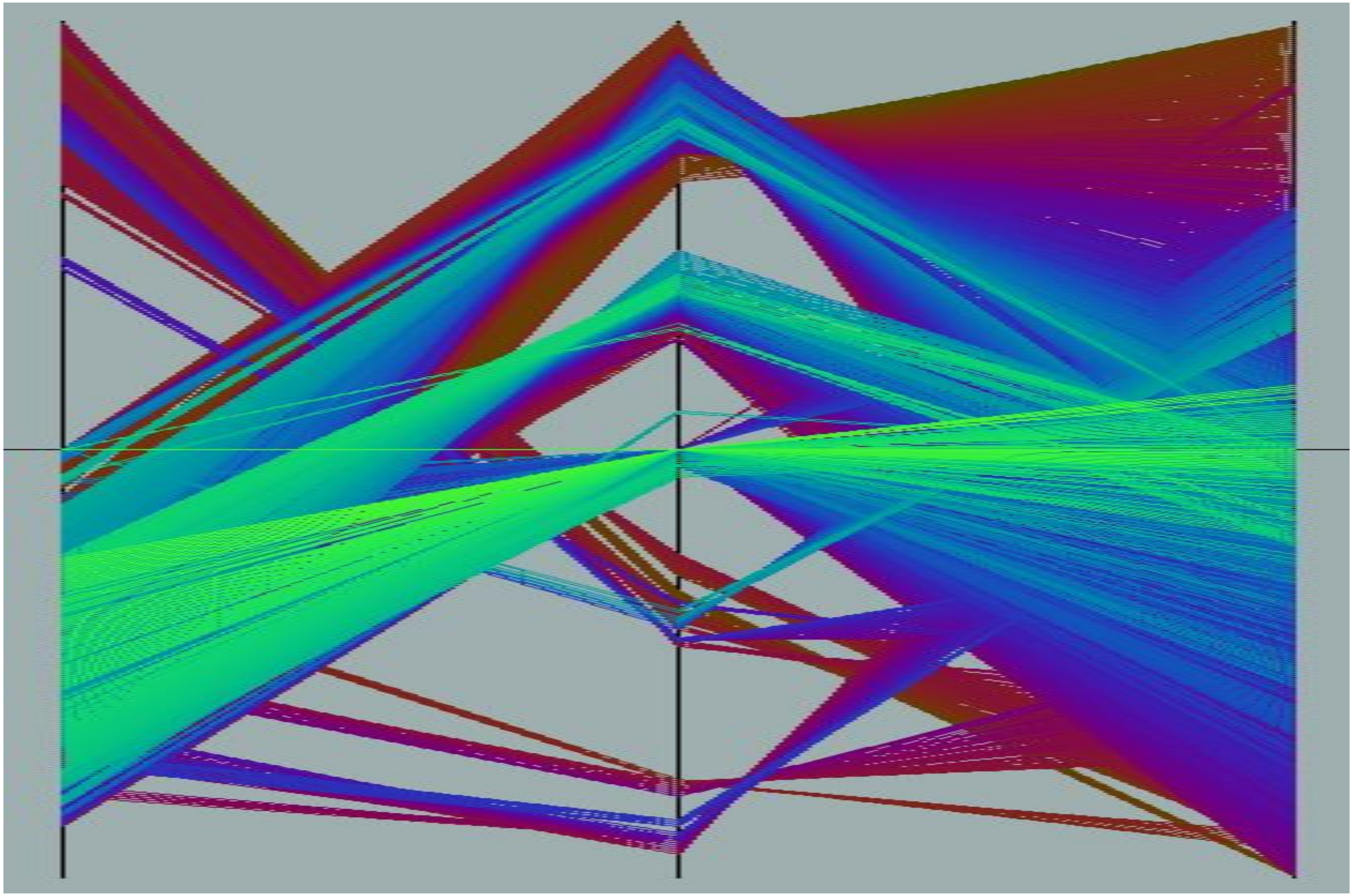
news articles visualized as a landscape

- Visualization of the data as perspective landscape
- The data needs to be transformed into a (possibly artificial) 2D spatial representation which preserves the characteristics of the data

Parallel Coordinates

- n equidistant axes which are parallel to one of the screen axes and correspond to the attributes
- The axes are scaled to the [minimum, maximum]: range of the corresponding attribute
- Every data item corresponds to a polygonal line which intersects each of the axes at the point which corresponds to the value for the attribute

Parallel Coordinates of a Data Set

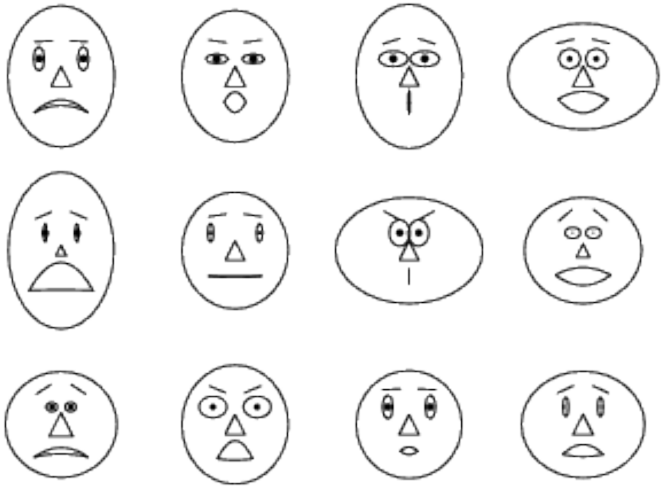


Icon-Based Visualization Techniques

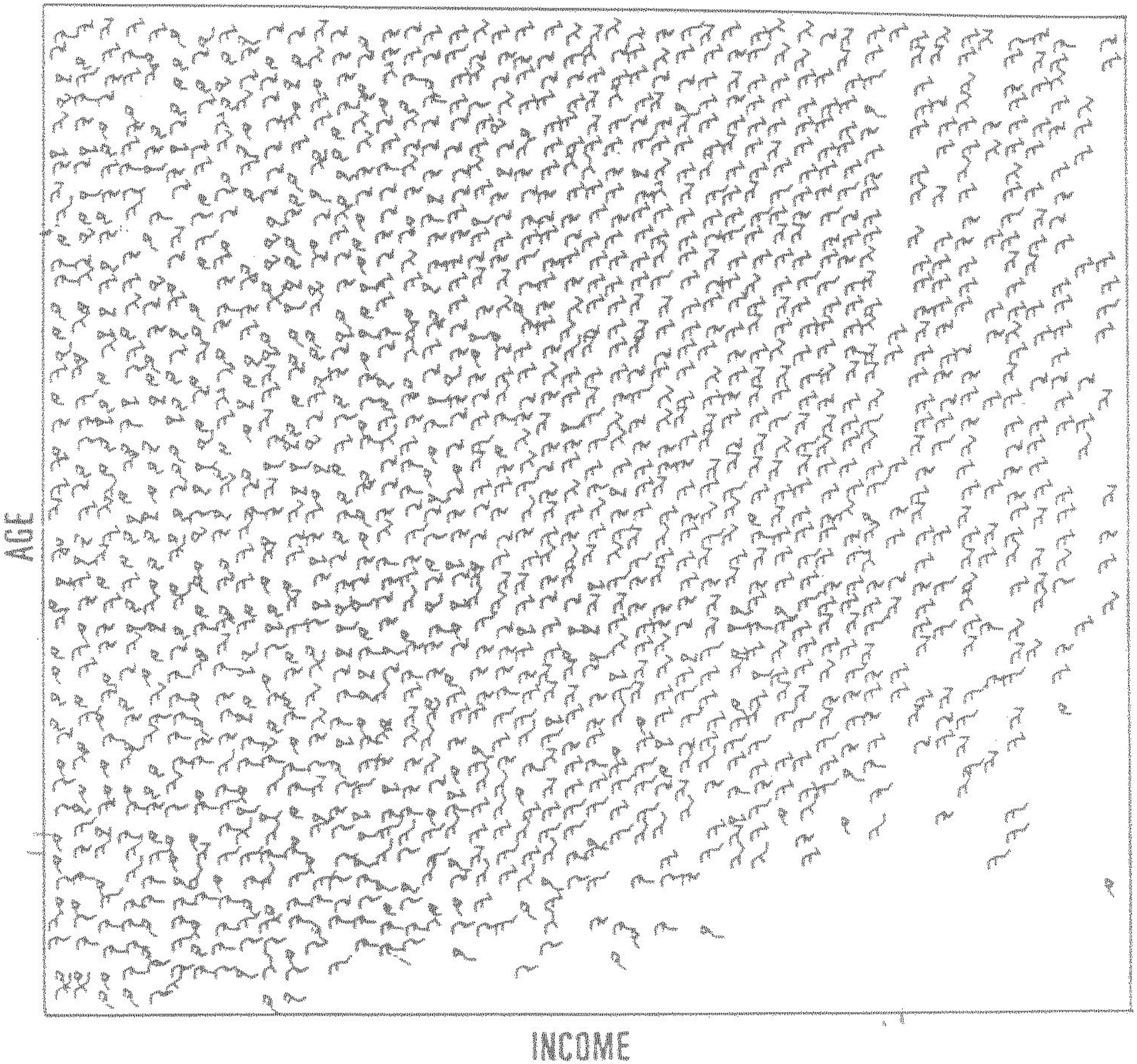
- Visualization of the data values as features of icons
- Typical visualization methods
  - Chernoff Faces
  - Stick Figures
- General techniques
  - Shape coding: Use shape to represent certain information encoding
  - Color icons: Use color icons to encode more information
  - Tile bars: Use small icons to represent the relevant feature vectors in document retrieval

Chernoff Faces

- A way to display variables on a two-dimensional surface, e.g., let x be eyebrow slant, y be eye size, z be nose length, etc.
- The figure shows faces produced using 10 characteristics--head eccentricity, eye size, eye spacing, eye eccentricity, pupil size, eyebrow slant, nose size, mouth shape, mouth size, and mouth opening): Each assigned one of 10 possible values, generated using *Mathematica* (S. Dickson)
- REFERENCE: Gonick, L. and Smith, W. *The Cartoon Guide to Statistics.* New York: Harper Perennial, p. 212, 1993
- Weisstein, Eric W. "Chernoff Face." From *MathWorld*--A Wolfram Web Resource. mathworld.wolfram.com/ChernoffFace.html



Stick Figure



A census data figure showing age, income, gender, education, etc.

A 5-piece stick figure (1 body and 4 limbs w. different angle/length)

Hierarchical Visualization Techniques

- Visualization of the data using a hierarchical partitioning into subspaces
- Methods
  - Dimensional Stacking
  - Worlds-within-Worlds
  - Tree-Map
  - Cone Trees
  - InfoCube

Dimensional Stacking

- Partitioning of the n-dimensional attribute space in 2-D subspaces, which are 'stacked' into each other
- Partitioning of the attribute value ranges into classes. The important attributes should be used on the outer levels.
- Adequate for data with ordinal attributes of low cardinality
- But, difficult to display more than nine dimensions
- Important to map dimensions appropriately

Dimensional Stacking

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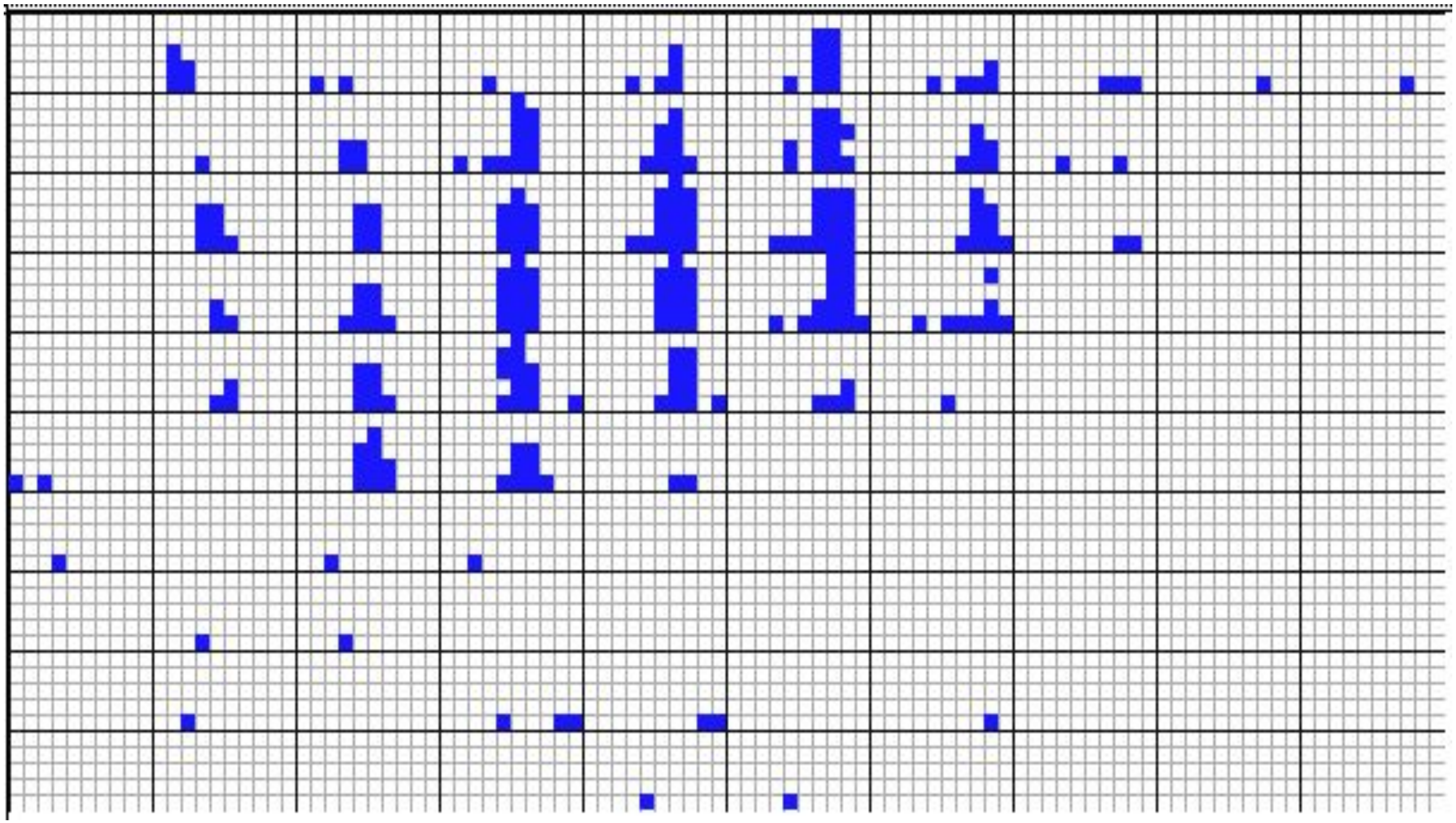
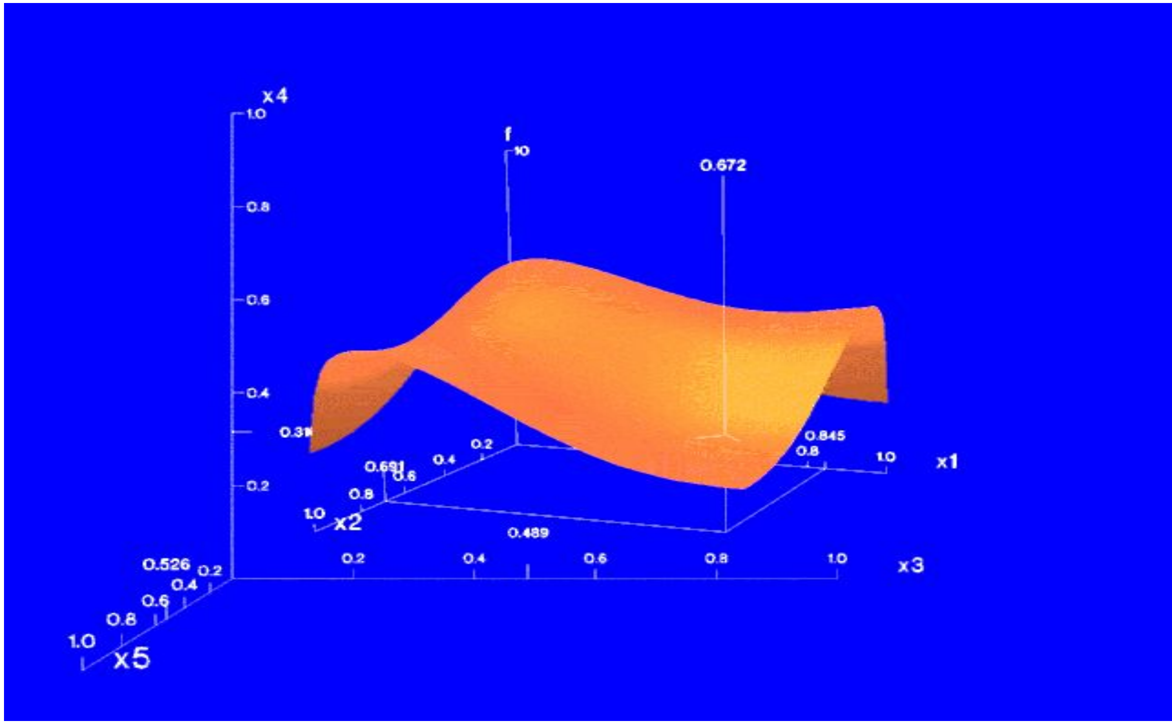


Visualization of oil mining data with longitude and latitude mapped to the outer x-, y-axes and ore grade and depth mapped to the inner x-, y-axes

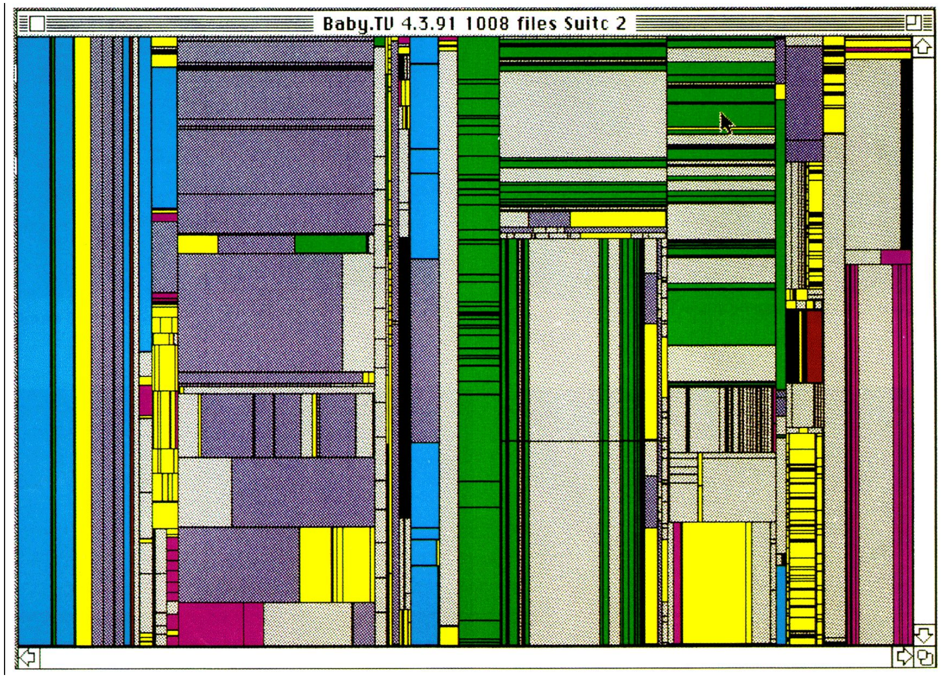
Worlds-within-Worlds

- Assign the function and two most important parameters to innermost world
- Fix all other parameters at constant values - draw other (1 or 2 or 3 dimensional worlds choosing these as the axes)
- Software that uses this paradigm
  - N–vision: Dynamic interaction through data glove and stereo displays, including rotation, scaling (inner) and translation (inner/outer)
  - Auto Visual: Static interaction by means of queries

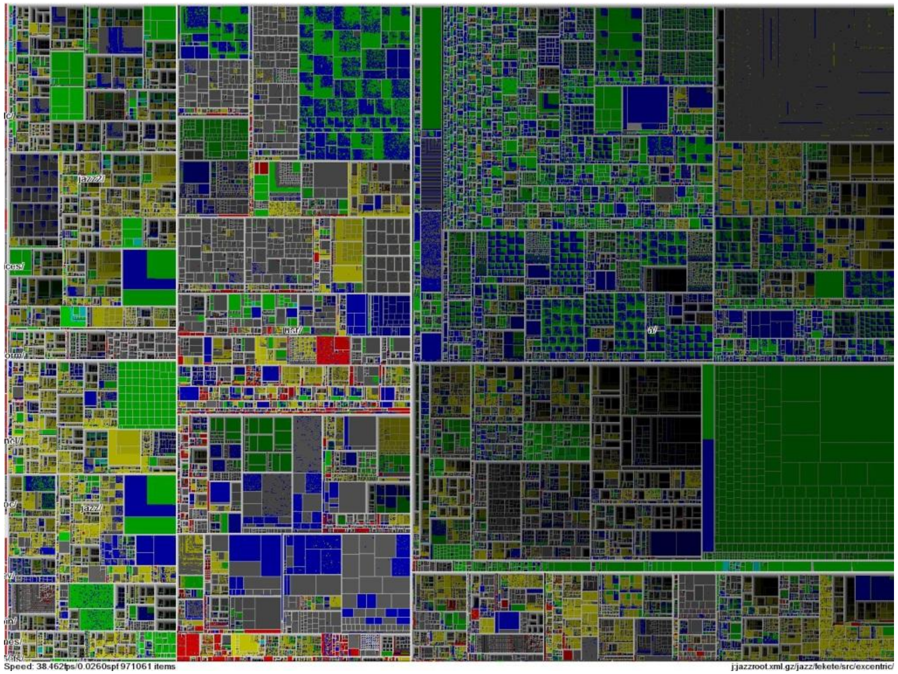


Tree-Map

- Screen-filling method which uses a hierarchical partitioning of the screen into regions depending on the attribute values
- The x- and y-dimension of the screen are partitioned alternately according to the attribute values (classes)



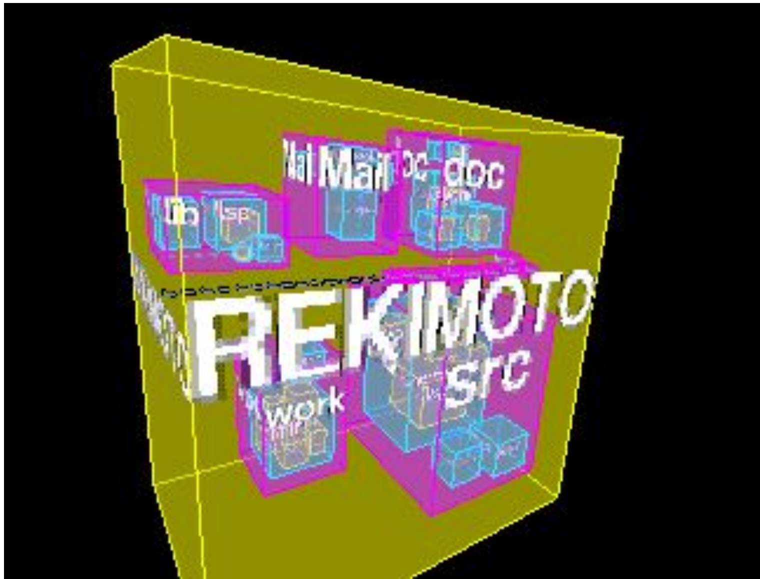
Schneiderman@UMD: Tree-Map of a File System



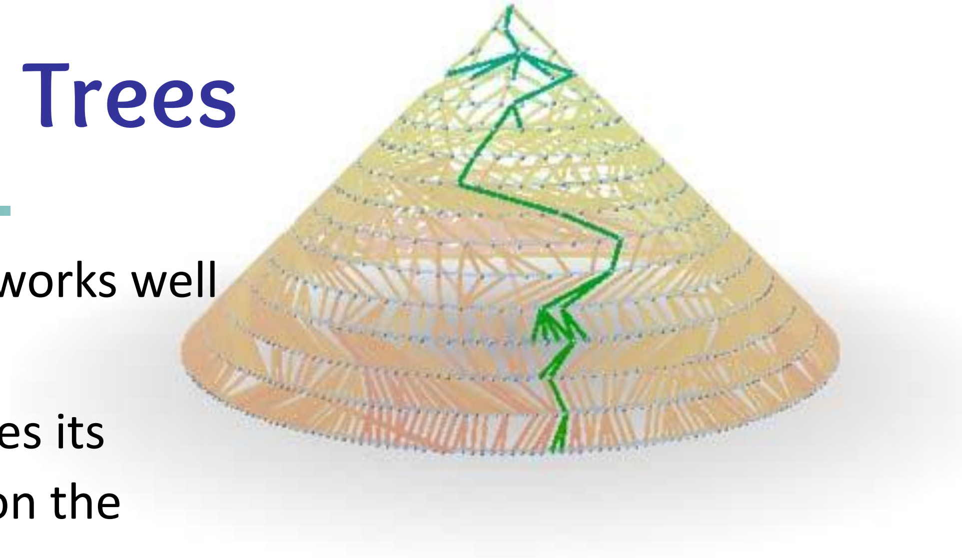
Schneiderman@UMD: Tree-Map to support large data sets of a million items

InfoCube

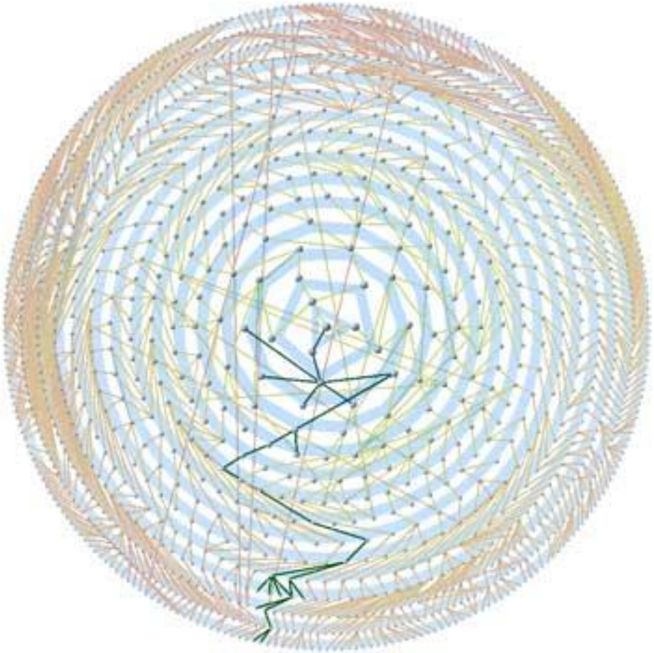
- A 3-D visualization technique where hierarchical information is displayed as nested semi-transparent cubes
- The outermost cubes correspond to the top level data, while the subnodes or the lower level data are represented as smaller cubes inside the outermost cubes, and so on



Three-D Cone Trees

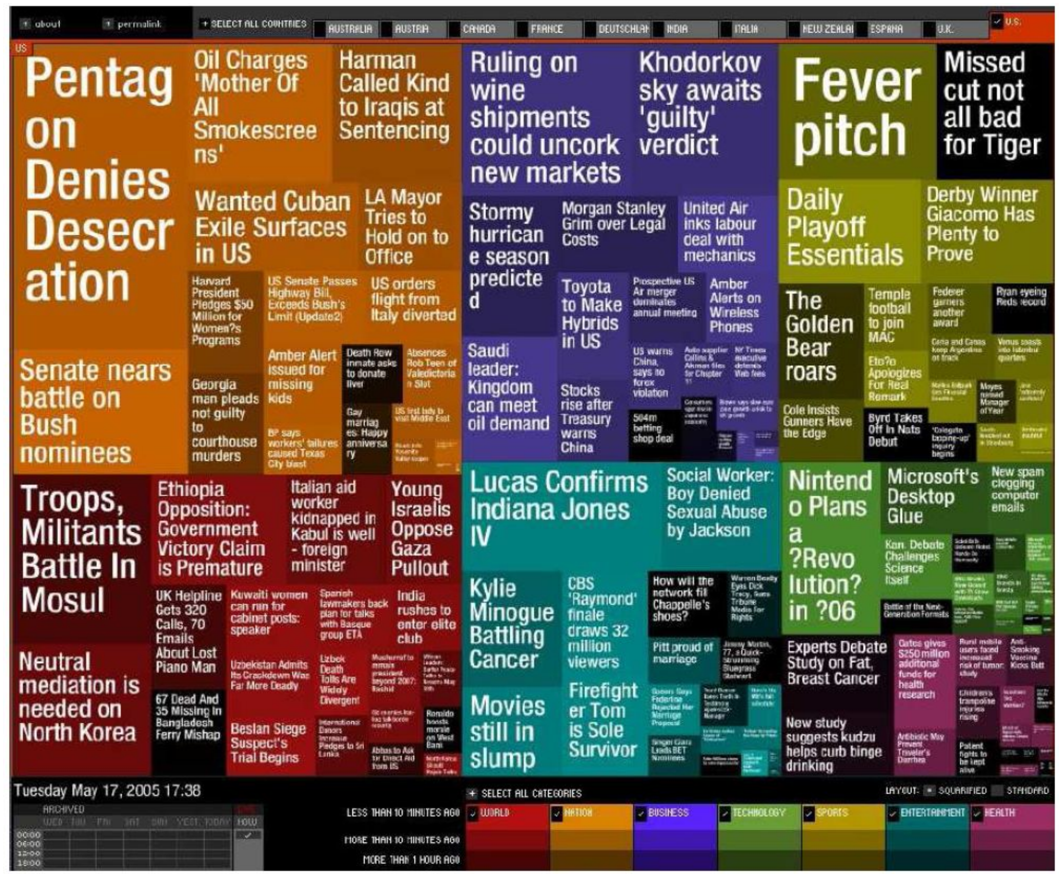


- *3D cone tree* visualization technique works well for up to a thousand nodes or so
- First build a *2D circle tree* that arranges its nodes in concentric circles centered on the root node
- Cannot avoid overlaps when projected to 2D
- G. Robertson, J. Mackinlay, S. Card. “Cone Trees: Animated 3D Visualizations of Hierarchical Information”, *ACM SIGCHI'91*
- Graph from Nadeau Software Consulting website: Visualize a social network data set that models the way an infection spreads from one person to the next



Visualizing Complex Data and Relations

- Visualizing non-numerical data: text and social networks
- Tag cloud: visualizing user-generated tags
  - The importance of tag is represented by font size/color
- Besides text data, there are also methods to visualize relationships, such as visualizing social networks



Newsmap: Google News Stories in 2005

Chapter 2: Getting to Know Your Data

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary

Similarity and Dissimilarity

- **Similarity**
  - Numerical measure of how alike two data objects are
  - Value is higher when objects are more alike
  - Often falls in the range [0,1]
- **Dissimilarity** (e.g., distance)
  - Numerical measure of how different two data objects are
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- **Proximity** refers to a similarity or dissimilarity

Data Matrix and Dissimilarity Matrix

- Data matrix
  - n data points with p dimensions
  - Two modes

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

- Dissimilarity matrix
  - n data points, but registers only the distance
  - A triangular matrix
  - Single mode

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \cdots & \cdots & 0 \end{bmatrix}$$

Proximity Measure for Nominal Attributes

- Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)
- Method 1: Simple matching
  - *m*: # of matches, *p*: total # of variables

$$d(i,j) = \frac{p-m}{p}$$

- Method 2: Use a large number of binary attributes
  - creating a new binary attribute for each of the *M* nominal states

Proximity Measure for Binary Attributes

- A contingency table for binary data

		Object j		
		1	0	sum
Object i	1	q	r	$q+r$
	0	s	t	$s+t$
	sum	$q+s$	$r+t$	p

- Distance measure for symmetric binary variables:

$$d(i, j) = \frac{r+s}{q+r+s+t}$$

- Distance measure for asymmetric binary variables:

$$d(i, j) = \frac{r+s}{q+r+s}$$

- Jaccard coefficient (*similarity* measure for *asymmetric* binary variables):

$$sim_{Jaccard}(i, j) = \frac{q}{q+r+s}$$

- Note: Jaccard coefficient is the same as "coherence":

$$coherence(i, j) = \frac{sup(i, j)}{sup(i) + sup(j) - sup(i, j)} = \frac{q}{(q+r) + (q+s) - q}$$

Dissimilarity between Binary Variables

- Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

  - Gender is a symmetric attribute
  - The remaining attributes are asymmetric binary
  - Let the values Y and P be 1, and the value N 0

$$d(jack, mary) = \frac{0+1}{2+0+1} = 0.33$$

$$d(jack, jim) = \frac{1+1}{1+1+1} = 0.67$$

$$d(jim, mary) = \frac{1+2}{1+1+2} = 0.75$$

Standardizing Numeric Data

- Z-score: $z = \frac{x - \mu}{\sigma}$
  - X: raw score to be standardized, μ: mean of the population, σ: standard deviation
  - the distance between the raw score and the population mean in units of the standard deviation
  - negative when the raw score is below the mean, "+" when above
- An alternative way: Calculate the mean absolute deviation

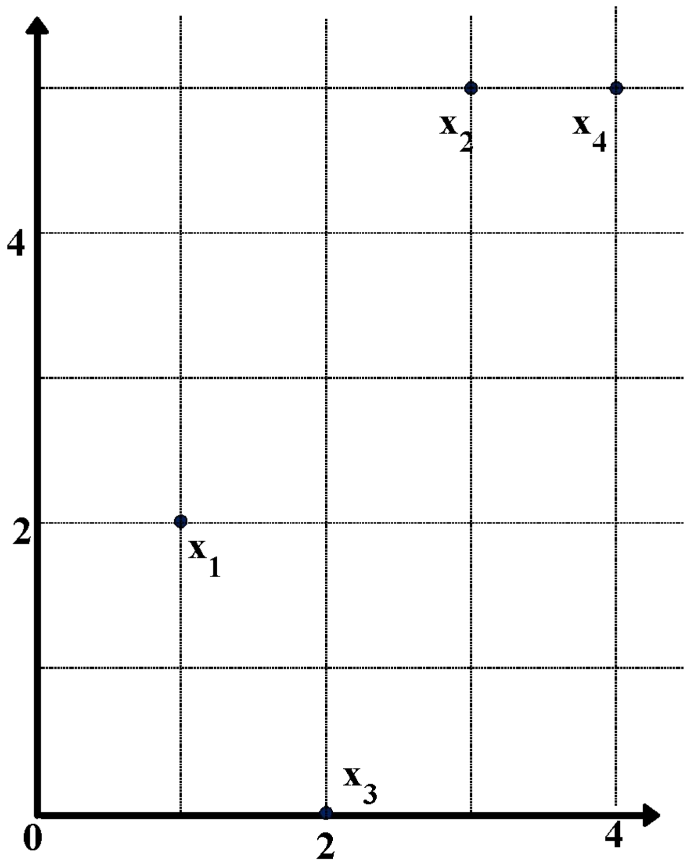
$$s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + ... + |x_{nf} - m_f|)$$

where

$$m_f = \frac{1}{n}(x_{1f} + x_{2f} + ... + x_{nf}).$$

  - standardized measure (*z-score*): $z_{if} = \frac{x_{if} - m_f}{s_f}$
- Using mean absolute deviation is more robust than using standard deviation

Example:
Data Matrix and Dissimilarity Matrix



Data Matrix

point	attribute1	attribute2
x1	1	2
x2	3	5
x3	2	0
x4	4	5

Dissimilarity Matrix
(with Euclidean Distance)

	x1	*x2*	*x3*	*x4*
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

Distance on Numeric Data: Minkowski Distance

- *Minkowski distance*: A popular distance measure

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

where $i = (x_{i1}, x_{i2}, \ldots, x_{ip})$ and $j = (x_{j1}, x_{j2}, \ldots, x_{jp})$ are two *p*-dimensional data objects, and *h* is the order (the distance so defined is also called L-*h* norm)

- Properties
  - $d(i, j) > 0$ if $i \neq j$, and $d(i, i) = 0$ (Positive definiteness)
  - $d(i, j) = d(j, i)$ (Symmetry)
  - $d(i, j) \leq d(i, k) + d(k, j)$ (Triangle Inequality)
- A distance that satisfies these properties is a metric

Special Cases of Minkowski Distance

- h = 1: Manhattan (city block, L_1 norm) distance
  - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i,j) = |x_{i_1} - x_{j_1}| + |x_{i_2} - x_{j_2}| + \dots + |x_{i_p} - x_{j_p}|$$

- h = 2: (L_2 norm) Euclidean distance

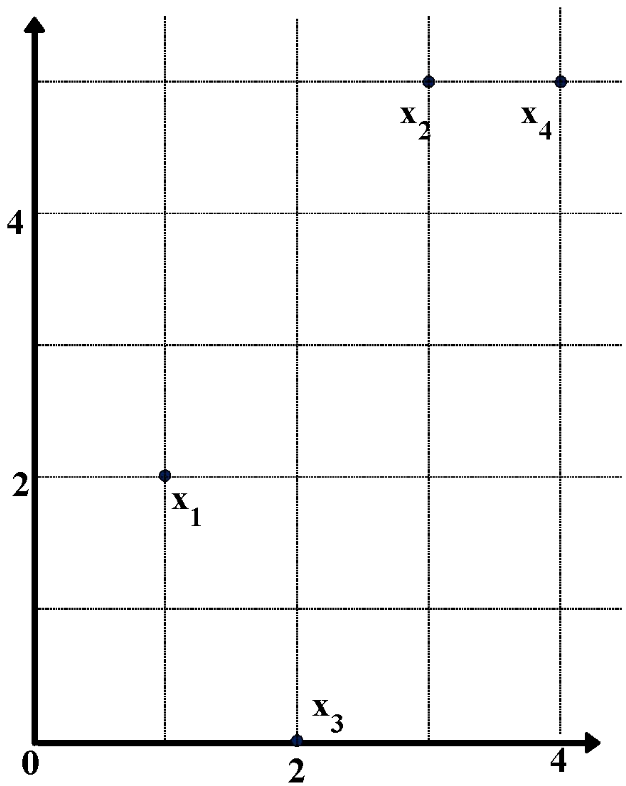
$$d(i,j) = \sqrt{(|x_{i_1} - x_{j_1}|^2 + |x_{i_2} - x_{j_2}|^2 + \dots + |x_{i_p} - x_{j_p}|^2)}$$

- $h \to \infty$. "supremum" (L_{max} norm, L_∞ norm) distance.
  - This is the maximum difference between any component (attribute) of the vectors

$$d(i,j) = \lim_{h \to \infty} \left(\sum_{f=1}^{p} |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_f^p |x_{if} - x_{jf}|$$

Example: Minkowski Distance

point	attribute 1	attribute 2
x1	1	2
x2	3	5
x3	2	0
x4	4	5



Dissimilarity Matrices

Manhattan (L_1)

L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

Euclidean (L_2)

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

Supremum

L_∞	x1	x2	x3	x4
x1	0			
x2	3	0		
x3	2	5	0	
x4	3	1	5	0

Ordinal Variables

- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank
- Can be treated like interval-scaled
  - replace x_{if} by their rank $r_{if} \in \{1, \ldots, M_f\}$
  - map the range of each variable onto [0, 1] by replacing *i*-th object in the *f*-th variable by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

  - compute the dissimilarity using methods for interval-scaled variables

Attributes of Mixed Type

- A database may contain all attribute types
  - Nominal, symmetric binary, asymmetric binary, numeric, ordinal
- One may use a weighted formula to combine their effects

$$d(i,j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}$$

  - f is binary or nominal:

    $d_{ij}^{(f)} = 0$ if $x_{if} = x_{jf}$, or $d_{ij}^{(f)} = 1$ otherwise
  - f is numeric: use the normalized distance
  - f is ordinal
    - Compute ranks r_{if} and
    - Treat z_{if} as interval-scaled

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

Cosine Similarity

- A **document** can be represented by thousands of attributes, each recording the *frequency* of a particular word (such as keywords) or phrase in the document.

Document	*team*	*coach*	*hockey*	*baseball*	*soccer*	*penalty*	*score*	*win*	*loss*	*season*
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

- Other vector objects: gene features in micro-arrays, …
- Applications: information retrieval, biologic taxonomy, gene feature mapping, ...
- Cosine measure: If d_1 and d_2 are two vectors (e.g., term-frequency vectors), then

  $$\cos(d_1, d_2) = (d_1 \cdot d_2) / ||d_1||\ ||d_2||,$$

  where $\cdot$ indicates vector dot product, $||d||$: the length of vector d

Example: Cosine Similarity

- $\cos(d_1, d_2) = (d_1 \cdot d_2) / ||d_1|| \, ||d_2||$,
  where · indicates vector dot product, $||d|$: the length of vector d

- Ex: Find the **similarity** between documents 1 and 2.

d_1 **=** (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)
d_2 **=** (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)

$d_1 \cdot d_2$ = 5*3+0*0+3*2+0*0+2*1+0*1+0*1+2*1+0*0+0*1 = 25
$||d_1||$= $(5*5+0*0+3*3+0*0+2*2+0*0+0*0+2*2+0*0+0*0)^{0.5}$=$(42)^{0.5}$ = 6.481
$||d_2||$= $(3*3+0*0+2*2+0*0+1*1+1*1+0*0+1*1+0*0+1*1)^{0.5}$=$(17)^{0.5}$ = 4.12
$\cos(d_1, d_2)$ = 0.94

KL Divergence: Comparing Two Probability Distributions

- *The Kullback-Leibler (KL) divergence:* Measure the difference between two probability distributions over the same variable *x*
  - From information theory, closely related to *relative entropy*, *information divergence*, and *information for discrimination*
- $D_{KL}(p(x) \,||\, q(x))$: divergence of $q(x)$ from $p(x)$, measuring the information lost when $q(x)$ is used to approximate $p(x)$
  - Discrete form:

$$D_{KL}(p(x)||q(x)) = \sum_{x \in X} p(x) \ln \frac{p(x)}{q(x)}$$

- The KL divergence measures the expected number of extra bits required to code samples from $p(x)$ ("true" distribution) when using a code based on $q(x)$, which represents a theory, model, description, or approximation of $p(x)$
- Its continuous form:

$$D_{KL}(p(x)||q(x)) = \int_{-\infty}^{\infty} p(x) \ln \frac{p(x)}{q(x)} dx$$

- The KL divergence: not a distance measure, not a metric: asymmetric, not satisfy triangular inequality

How to Compute the KL Divergence?

$$D_{KL}(p(x)||q(x)) = \sum_{x \in X} p(x) \ln \frac{p(x)}{q(x)}$$

- *Base on the formula,* $D_{KL}(P,Q) \geq 0$ and $D_{KL}(P \,||\, Q) = 0$ if and only if $P = Q$.
- How about when p = 0 or q = 0?
  - $lim_{p \to 0}\, p \log p = 0$
  - when $p \,!= 0$ but $q = 0$, $D_{KL}(p \,||\, q)$ is defined as ∞, i.e., if one event e is possible (*i.e.*, $p(e) > 0$), and the other predicts it is absolutely impossible (*i.e.*, $q(e) = 0$), then the two distributions are absolutely different
- However, in practice, P and Q are derived from frequency distributions, not counting the possibility of unseen events. Thus *smoothing* is needed
- Example: $P : (a : 3/5, b : 1/5, c : 1/5)$. $Q : (a : 5/9, b : 3/9, d : 1/9)$
  - need to introduce a small constant ϵ, e.g., $\epsilon = 10^{-3}$
  - The sample set observed in P, $SP = \{a, b, c\}$, $SQ = \{a, b, d\}$, $SU = \{a, b, c, d\}$
  - Smoothing, add missing symbols to each distribution, with probability ϵ
  - $P' : (a : 3/5 - \epsilon/3, b : 1/5 - \epsilon/3, c : 1/5 - \epsilon/3, d : \epsilon)$
  - $Q' : (a : 5/9 - \epsilon/3, b : 3/9 - \epsilon/3, c : \epsilon, d : 1/9 - \epsilon/3)$.
  - $D_{KL}(P' \,||\, Q')$ can be computed easily

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Summary

- Data attribute types: nominal, binary, ordinal, interval-scaled, ratio-scaled
- Many types of data sets, e.g., numerical, text, graph, Web, image.
- Gain insight into the data by:
  - Basic statistical data description: central tendency, dispersion, graphical displays
  - Data visualization: map data onto graphical primitives
  - Measure data similarity
- Above steps are the beginning of data preprocessing
- Many methods have been developed but still an active area of research

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