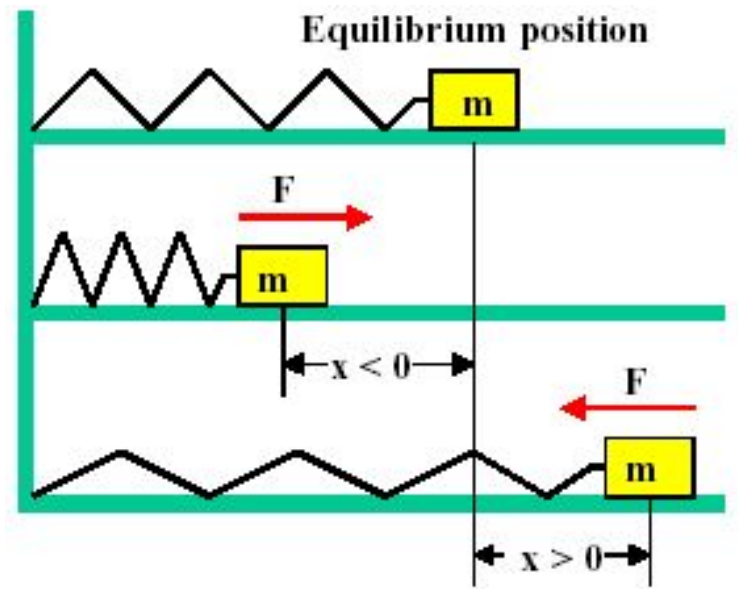
Simple Harmonic Motion

When a vibration or an oscillation repeats itself over and over, the motion is called periodic.



A mass on a spring is oscillating on a frictionless surface. As the mass moved from the equilibrium position there is a restoring force applied to it.

The restoring force is directly proportional to the displacement *x*.

$$F = -kx$$

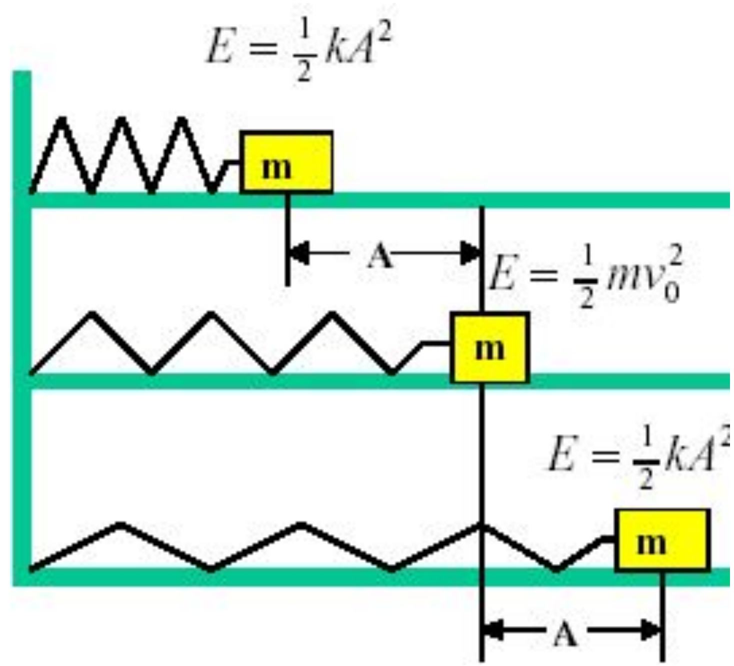
Every system that has this force exhibits a simple harmonic motion (SHM) and is called a simple harmonic oscillator.

The amplitude (A) is the maximum value of its displacement on either side of the equilibrium position.

Energy in Simple Harmonic Oscillator

A spring has Elastic Potential Energy:

$$PE = \frac{1}{2}kx^2$$



Kinetic Energy:

$$KE = \frac{1}{2}mv^2$$

Total Energy:

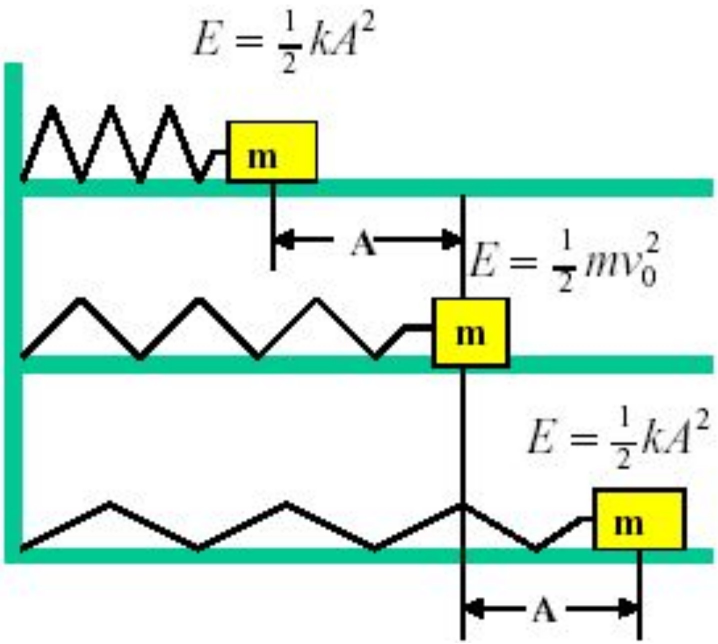
$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

When the spring is fully compressed the elastic potential energy is

$$PE = \frac{1}{2}kA^2$$

At the equilibrium position all the energy is in the form of kinetic energy

$$KE = \frac{1}{2}mv_0^2$$

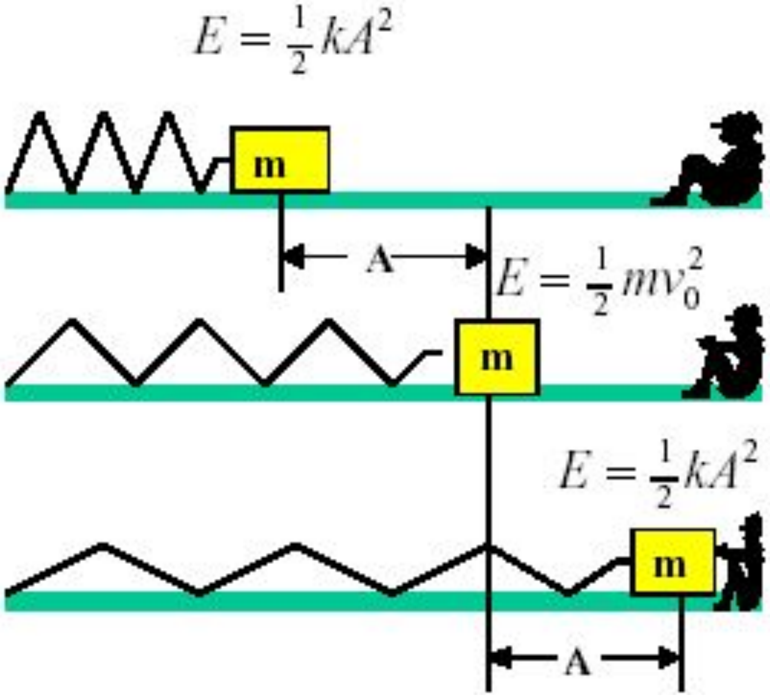


Since the total energy is conserved:

$$PE = KE \quad \Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}kA^2 \quad \Longrightarrow \quad v_0 = A\sqrt{\frac{k}{m}}$$

At any point the velocity is:

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$



PERIOD AND FREQUENCY

The period of an object in SHM is the time it takes the mass to make a complete revolution.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

UNITS:

***T* in seconds**

***f* in Hz (s^{-1})**

$$f = \frac{1}{T}$$

Simple Pendulum

For small displacements a pendulum obeys SHM.

Its period is:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

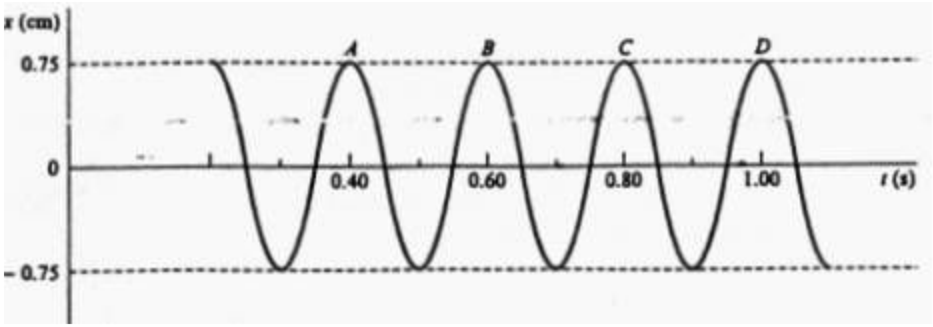
The period and frequency DO NOT depend on the mass.

11.1 For the motion shown in the figure, find:

a. Amplitude

b. Period

c. Frequency



a. Amplitude: maximum displacement from equilibrium

$$A = 0.75 \text{ cm}$$

b. T **= time for one complete cycle**

$$T = 0.2 \text{ s}$$

c. $f = 1/T = 1/0.2 = 5 \text{ Hz}$

11.2 A spring makes 12 vibrations in 40 s. Find the period and frequency of the vibration.

$$\begin{aligned} f &= vibrations/time \\ &= \mathbf{12/40} \\ &= \mathbf{0.30\ Hz} \end{aligned}$$

$$\begin{aligned} T &= 1/f \\ &= \mathbf{1/0.3} \\ &= \mathbf{3.33\ s} \end{aligned}$$

11.3 The amplitude of a *SH* oscillator is doubled. How does this affect:
a. The period,
b. The total energy, and
c. The maximum velocity of the oscillator.

a. T is independent of A so it is **unchanged**

b. $TE = 1/2\ kx^2$

$x' = 2x$ so

$TE' =$ **4TE**

c. v_{max} occurs when $x = 0$ and all energy (TE) is K

$TE' = 4TE$ then

$4 = \frac{1}{2} mv^2$ therefore v_{max} must be **doubled**

11.4 A 200-g mass vibrates horizontally without friction at the end of a horizontal spring for which k = 7.0 N/m. The mass is displaced 5.0 cm from equilibrium and released. Find:

a. Maximum speed

$m = 0.2$ kg

$k = 7$ N/m

$x_o = A = 0.05$ m

$$\frac{1}{2}kx_o^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

v_{max} **is at** $x = 0$ **then**

$$v = A\sqrt{\frac{k}{m}} = 0.05\sqrt{\frac{7}{0.2}}$$

v = **0.295 m/s**

b. Speed when it is 3.0 cm from equilibrium.

***x* = 0.03 m**

$$\frac{1}{2}kx_o^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)} \quad = \sqrt{\frac{7}{0.2}(0.05^2 - 0.03^2)}$$

***v* = 0.236 m/s**

c. What is the acceleration in each of these cases?

$$F = ma = -kx$$

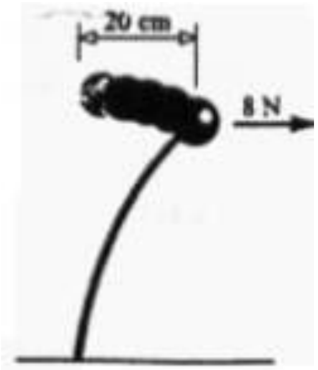
$$a = \frac{-k}{m}x$$

a. $x = 0$ **therefore** $a = 0$

b. $x = 0.03$ **m therefore**

$$a = \frac{-7}{0.2}(0.03) = -1.05 \text{ m/s}^2$$

11.5 As shown in the figure, a long, light piece of spring steel is clamped at its lower end and a 2.0-kg ball is fastened to its top end. A horizontal force of 8.0 N is required to displace the ball 20 cm to one side as shown. Assume the system to undergo *SHM* when released. Find:



a. The force constant of the spring

$F = 8$ N
$x = 0.2$ m
$m = 2$ kg

$$k = \frac{F}{x} = \frac{8}{0.2} = \mathbf{40\ N/m}$$

b. Find the period with which the ball will vibrate back and forth.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{40}} = \mathbf{1.4\ s}$$

11.6 In a laboratory experiment a student is given a stopwatch, a wooden bob, and a piece of cord. He is then asked to determine the acceleration of gravity. If he constructs a simple pendulum of length 1 m and measures the period to be 2 s, what value will he obtain for *g*?

$l = 2$ m
$T = 2$ s

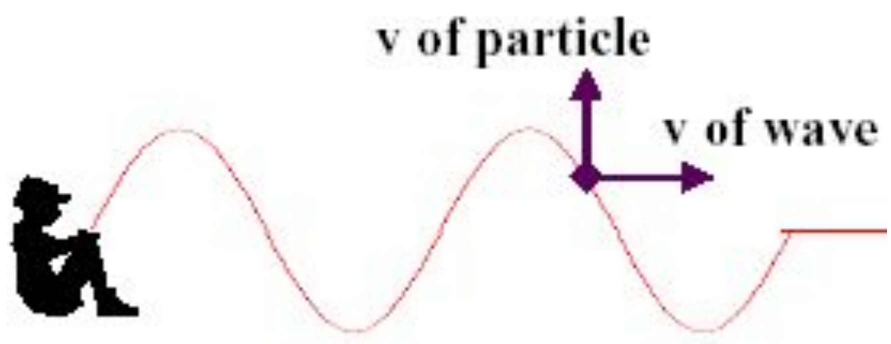
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2(1)}{2^2} = \mathbf{9.86\ m/s^2}$$

Wave Motion

A *wave* is, in general, a disturbance that moves through a medium. It carries *energy* from one location to another without transporting the material of the medium.

Examples of *mechanical waves* include water waves, waves on a string, and sound waves.

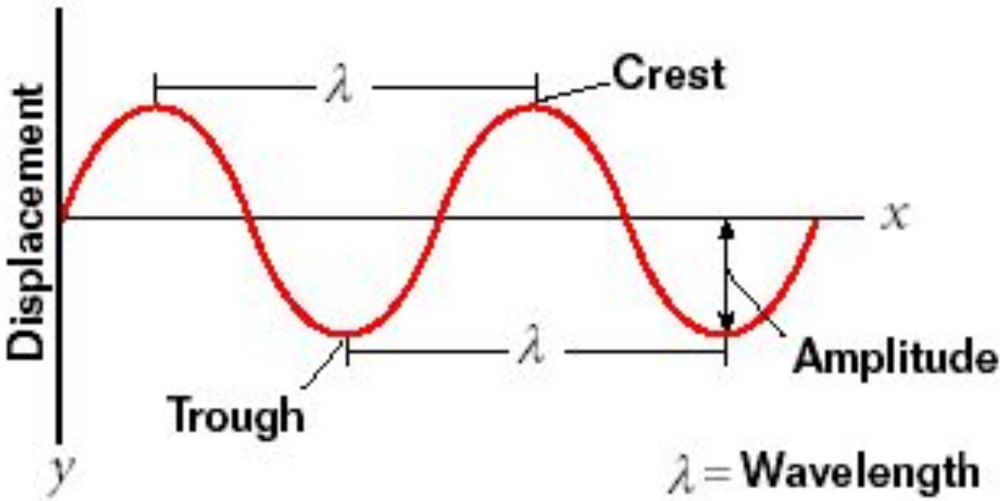


The wave caries energy from one place to the other. It does not carry the particles.

Transverse and Longitudinal Waves

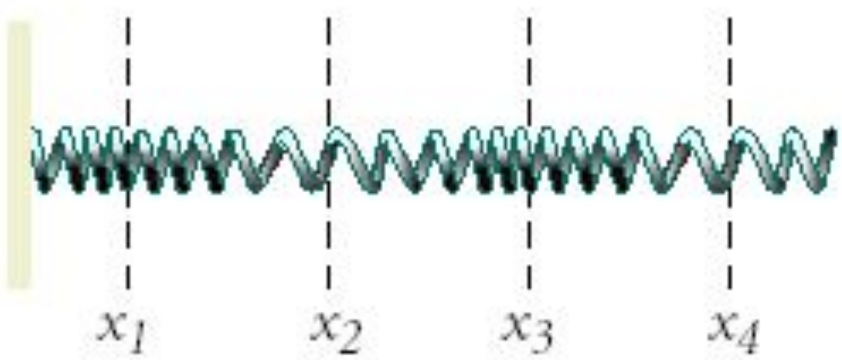
Two types of waves:

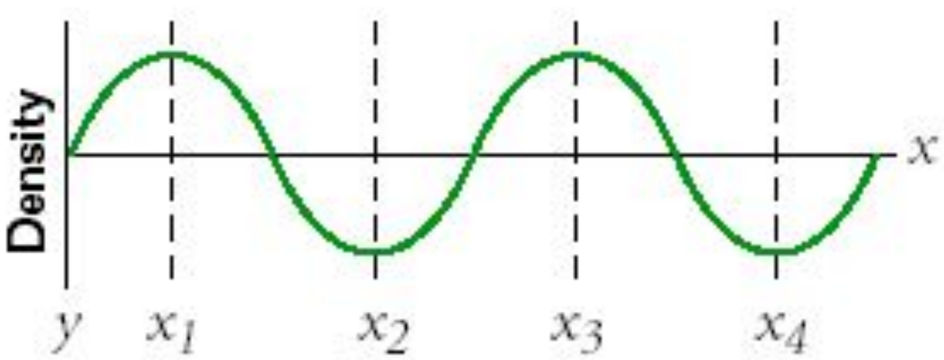
Transverse waves: The particles of the medium vibrate up and down (perpendicular to the wave).



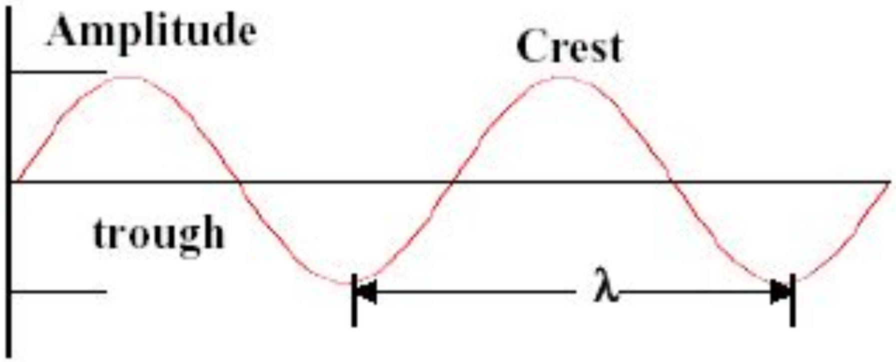
Longitudinal waves: The particles in the medium vibrate along the same direction as the wave (parallel). The medium undergoes a series of expansion and compressions. The expansions are when the coils are far apart (momentarily) and compressions are when they are when the coil is close together (momentarily).

Expansions and **compressions** are the analogs of the crests and troughs of a transverse wave.





Wave Motion



Wave velocity v is the velocity with which the wave crest is propagating. Wave velocity v depends on the medium.

A wave crest travels one wavelength in one period:

$$\boxed{v = \frac{\lambda}{T} = \lambda f}$$

On a string with tension F_T and mass per unit length of the string (linear density) m/L the velocity (m/s) of the wave is:

$$\boxed{v = \sqrt{\frac{F_T}{\left(\frac{m}{L}\right)}} = \sqrt{\frac{F_T L}{m}}}$$

11.7 Measurements show that the wavelength of a sound wave in a certain material is 18.0 cm. The frequency of the wave is 1900 Hz. What is the speed of the sound wave?

$\lambda = 0.18$ m
$f = 1900$ Hz

$$v = \lambda f$$
$$= 0.18\ (1900)$$
$$= 342 \text{ m/s}$$

11.8 A horizontal cord 5.00 m long has a mass of 1.45 g. What must be the tension in the cord if the wavelength of a 120 Hz wave is 60 cm? **a.**

L = 5 m
m = 1.45x10^{-3} kg
f = 120 Hz
λ = 0.6 m

$$v = \sqrt{\frac{F_T}{m/L}}$$

$$F_T = \frac{m}{L}v^2 = \frac{1.45x10^{-3}}{5}(72)^2 = \mathbf{1.5\ N}$$

b. How large a mass must be hung from its end to give it this tension?

$$F_T = mg$$

$$m = F_T/g$$

$$= 1.5/9.8$$

$$= \mathbf{0.153\ kg}$$

11.9 A uniform flexible cable is 20 m long and has a mass of 5.0 kg. It hangs vertically under its own weight and is vibrated from its upper end with a frequency of 7.0 Hz. **a. Find the speed of a transverse wave on the cable at its midpoint.**

$L = 20$ m
$m = 5$ kg
$f = 7$ Hz

$F_T = mg = 5\ (9.8) = 49$ N
At midpoint the cable supports half the weight so:
$F_T = 1/2\ (49) = 24.5$ N

$$v = \sqrt{\frac{F_T}{m/L}} = \sqrt{\frac{24.5}{5/20}} = 9.89 \text{ m/s}$$

b. What are the frequency and wavelength at the midpoint?

f **= 7 Hz at all points**

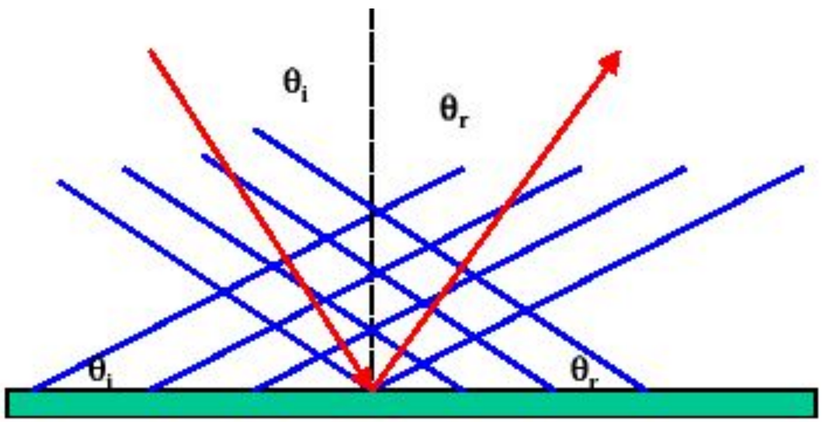
$$\lambda = \frac{v}{f} = \frac{9.89}{7} = \mathbf{1.4\ m}$$

BEHAVIOR OF WAVES

Reflection and Interference of Waves

When a wave hits a barrier or an obstacle, it is reflected.

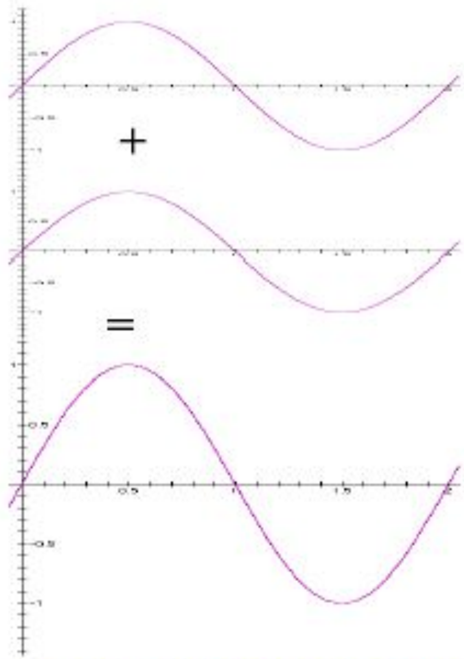
Wave in a string is inverted if the end of the string is fixed. If the end is not fixed, it will be reflected right side up.



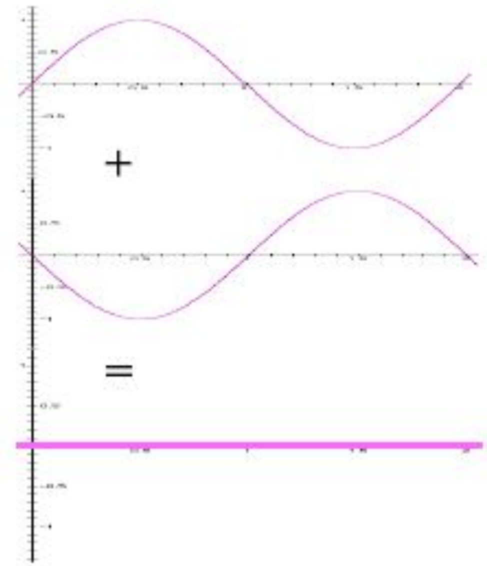
Law of Reflection:

"The angle of incident is equal the angle of reflection."

Interference: What happens when two waves pass through the same region?



Constructive interference



Destructive interference

When two crests overlap it is called constructive interference. The resultant displacement is larger then the individual ones.

When a crest and a trough interfere, it is called destructive interference. The resultant displacement is smaller.

Standing Waves

If a string is fixed on one end and oscillates on the other, the moving waves will be reflected by the fixed end. If the string vibrates at the right frequency, a *standing wave* can be produced.

The points where there is destructive interference, where the string is still are called *nodes,* the points where there are constructive interference are called *antinodes.*

The nodes and antinodes remain in a fixed position for a given frequency.

There can be *more* than one frequency for standing waves.

Frequencies at which standing waves can be produced are called the *natural (or resonant) frequencies.*

Standing Waves

A string can be fixed in both sides, like a guitar or piano string.When the string is plucked, many frequency waves will travel in both directions. Most will interfere randomly and die away. Only those with resonant frequencies will persist.

Since the ends are fixed, they will be the nodes.

The wavelengths of the standing waves have a simple relation to the length of the string.

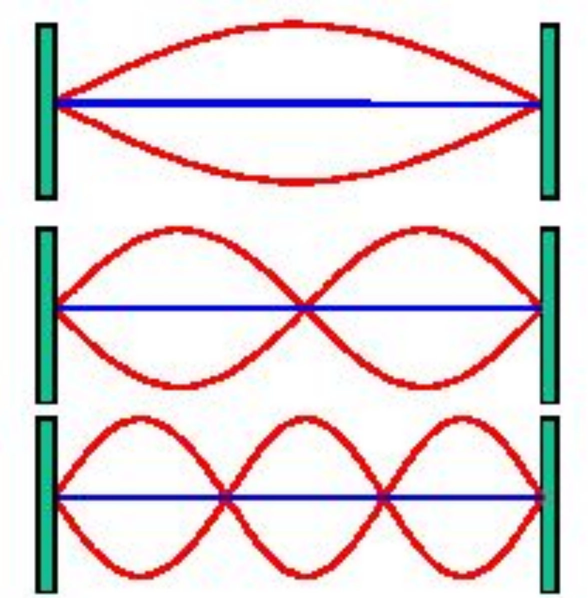
The lowest frequency called the fundamental frequency has only one antinode. That corresponds to half a wavelength:

$$\boxed{L = \frac{1}{2}\lambda_1 \Rightarrow \lambda_1 = 2L}$$

The other natural frequencies are called *overtones.* They are also called *harmonics* and they are *integer multiples* of the fundamental.

The fundamental is called the *first harmonic.*

The next frequency has two antinodes and is called the *second harmonic.*



$L = {}^{1}/_{2}\lambda_1$ Fundamental or first harmonic f_1.

$L = \lambda_2$ First overtone or second harmonic $f_2 = 2f_1$.

$L = {}^{3}/_{2}\,\lambda_3$ Second overtone or third harmonic $f_3 = 3f_1$.

$$\boxed{f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_1 \quad \mathrm{n} = 1, 2, 3 \ldots}$$

11.10 A metal string is under a tension of 88.2 N. Its length is 50 cm and its mass is 0.500 g.

a. Find the velocity of the waves on the string.

$m = 5\text{x}10^{-4}$ **kg**

$F_T = 88.2$ **N**

$L = 0.5$ **m**

$$v = \sqrt{\frac{F_T}{m/L}} = \sqrt{\frac{88.2}{5x10^{-4}/0.5}} = 297 \text{ m/s}$$

b. Determine the frequencies of its fundamental, first overtone and second overtone.

Fundamental:

$L = 1/2\,\lambda$

$\lambda = 2L$

$= 2(0.5)$

$= 1$ m

$$f = \frac{v}{\lambda} = \frac{297}{1} = 297 \text{ Hz}$$

First overtone

$$f_n = n f'$$

$$f_2 = 2(297)$$

$$= \mathbf{594\ Hz}$$

Second overtone

$$f_n = n f'$$

$$f_3 = 3(297)$$

$$= \mathbf{891\ Hz}$$

11.11 A string 2.0 m long is driven by a 240 Hz vibrator at its end. The string resonates in four segments. What is the speed of the waves on the string?

$L = 2$ m

$f = 240$ Hz

$$L = 4/2\,\lambda = 2\,\lambda$$

$$\lambda = 1/2\,L$$

$$= 1/2\,(2)$$

$$= 1 \text{ m}$$

$$v = f\lambda$$

$$= 240\,(1)$$

$$= 240 \text{ m/s}$$

11.12 A banjo string 30 cm long resonates in its fundamental to a frequency of 256 Hz. What is the tension in the string if 80 cm of the string have a mass of 0.75 g?

$L = 0.3$ m
$f' = 256$ Hz
$L = 0.8$ m
$m = 0.75\text{x}10^{-3}$ kg

$L = 1/2\ \lambda$
$\lambda = 2\ (0.3)$
$= 0.6$ m

$v = f\lambda$
$= 256(0.6)$
$= 154$ m/s

$$F_T = \frac{m}{L}v^2 = \frac{0.75x10^{-3}}{0.8}(154)^2 = \mathbf{22.2\ N}$$

11.13 A string vibrates in five segments to a frequency of 460 Hz.

a. What is its fundamental frequency?

$f_5 = 460\ \text{Hz}$

$$f_n = n f'$$
$$f' = 460/5$$
$$= 92\ \text{Hz}$$

b. What frequency will cause it to vibrate in three segments?

$$f_n = n f'$$
$$f_3 = 3(92)$$
$$= 276\ \text{Hz}$$

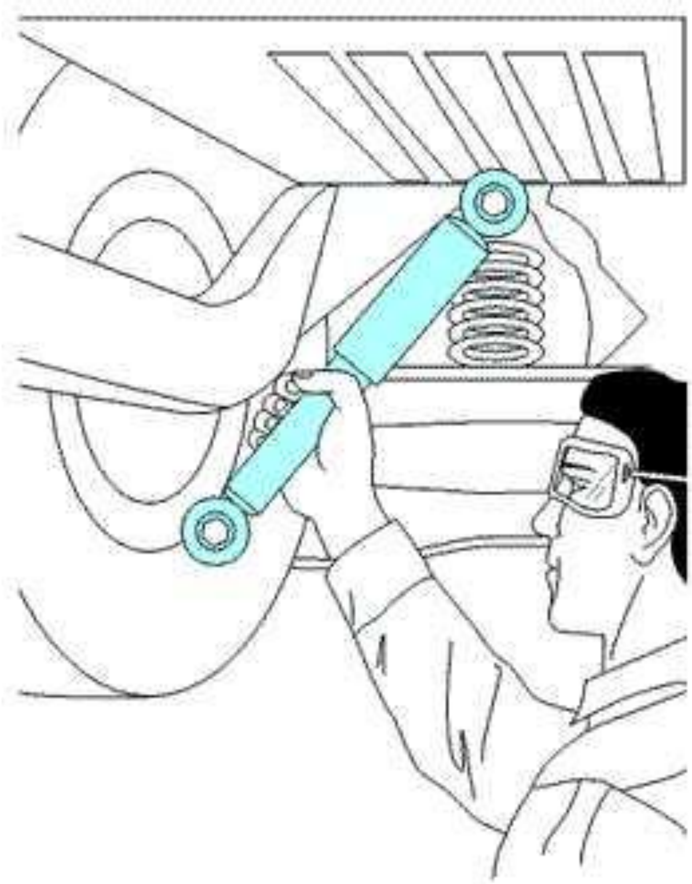
DAMPED HARMONIC MOTION

A system undergoing *SHM* will exhibit damping. *Damping* is the loss of mechanical energy as the amplitude of motion gradually decreases.

In the mechanical systems studied in the previous sections, the losses are generally due to air resistance and internal friction and the energy is transformed into heat.

For the amplitude of the motion to remain constant, it is necessary to add enough energy each second to offset the energy losses due to damping.

In many instances damping is a desired effect. For example, shock absorbers in a car remove unwanted vibration.



FORCED VIBRATIONS: RESONANCE

An object subjected to an external oscillatory force tends to vibrate. The vibrations that result are called *forced vibrations*. These vibrations have the same frequency as the external force and not the natural frequency of the object.

If the external forced vibrations have the same frequency as the natural frequency of the object, the amplitude of vibration increases and the object exhibits resonance. The natural frequency (or frequencies) at which resonance occurs is called the resonant frequency.

EXAMPLES OF RESONANCE