GROWTH THEORY: THE ECONOMY IN THE VERY LONG RUN

Part III

MACROECONOMICS

LECTURE 8

ECONOMIC GROWTH I:

CAPITAL ACCUMULATION & POPULATION GROWTH

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Outline

8-1 The Accumulation of Capital
8-2 The Golden Rule Level of Capital
8-3 Population Growth

Outline

The Solow growth model shows how

1. **saving,**
2. **population growth,**
3. **technological progress**

Affect

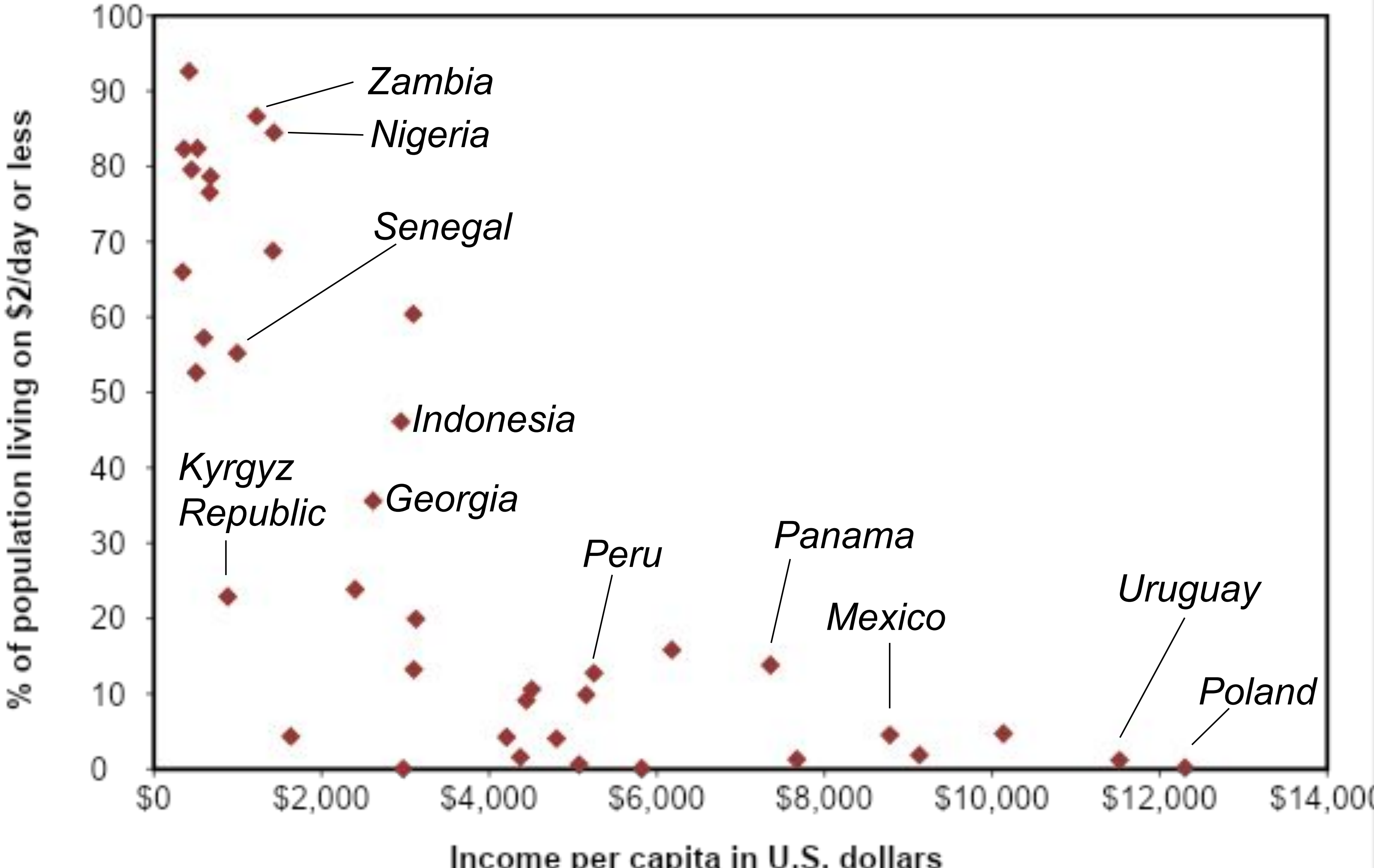
- **Level & Growth** of output

TABLE 8-1

International Differences in the Standard of Living

Country	Income per person (2010)	Country	Income per person (2010)
United States	$47,140	Indonesia	2,580
Germany	43,330	Philippines	2,050
Japan	42,150	India	1,340
Russia	9,910	Nigeria	1,180
Brazil	9,390	Vietnam	1,100
Mexico	9,330	Pakistan	1,050
China	4,260	Bangladesh	640

Income and poverty in the world
selected countries, 2010



- The Supply in the **Solow** model is based on the PF:

$$Y = F(K, L).$$

Assumption:

- the PF has constant returns to scale:

$zY = F(zK, zL)$, for any positive number z.

- If $z = 1/L$ →
- $Y/L = F(K/L, 1)$.

8-1 The Accumulation of Capital

- ***y* = *Y*/*L*** *is output per worker*
- ***k* = *K*/*L*** *is capital per* worker
- ***f*(*k*) = *F*(*k*, 1)**
- ***y* = *f*(*k*)**

$$Y/L = F(K/L, 1)$$

$$MPK = f(k + 1) - f(k)$$

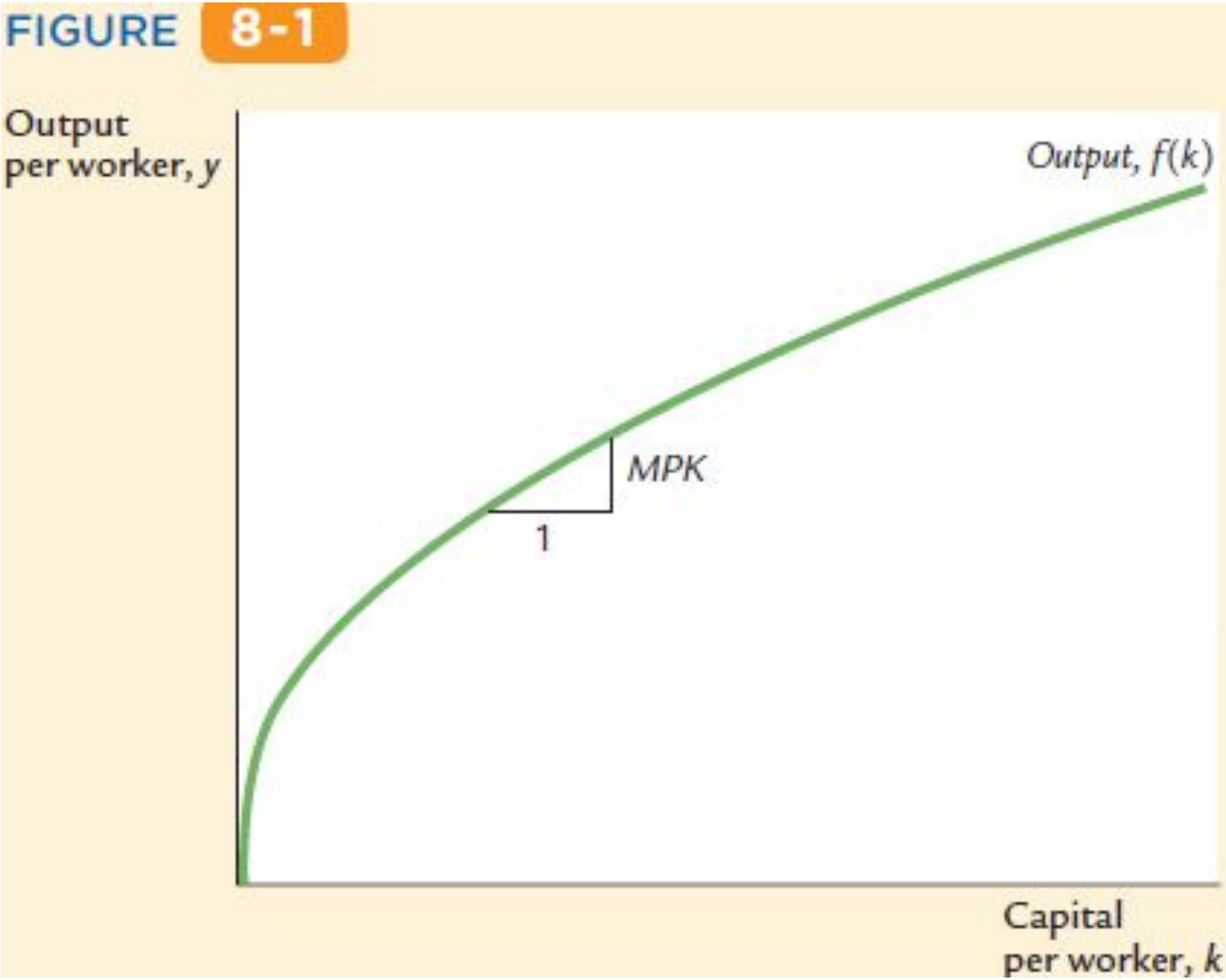
k is low →

- the average worker has only a little capital →
- an extra unit of capital is very useful and →
- He produces a lot of additional output.

k is high →

- the average worker has a lot of capital already, →
- so an extra unit increases production only slightly.

The Production Function



The PF shows how the amount of capital per worker *k* determines the amount of output per worker $y = f(k)$.

8-1 The Accumulation of Capital

- Output per worker ***y*** is divided between consumption per worker ***c*** and investment per worker ***i:***

$$y = c + i.$$

G - we can ignore here and *NX* – we **assumed** a closed economy.

- The Solow model **assumes** that people
  1. **save** a fraction ***s*** of their income
  2. **consume** a fraction ***(1 − s).***
  - We can express this idea with the following ***CF***:

$$c = (1 - s)y,$$

$$0 < s \text{ (the saving rate) } < 1$$

Gnt. policies can influence a nation's *s*

What ***s*** is **desirable ?**

Assamption:

We take the saving rate ***s*** **as given.**

To see what this ***CF*** implies for ***I***,

we substitute (1 − *s*)*y* for ***c***

in the national income accounts identity:

$$y = (1 - s)y + i \Rightarrow$$

$$i = sy$$

s is the fraction of ***y*** devoted to ***i***.

8-1 The Accumulation of Capital

- The Supply and Demand for Goods
- Growth in the Capital Stock and the Steady State
- Approaching the Steady State: A Numerical Example
- How Saving Affects Growth

The 2 main ingredients of the **Solow model**—

the PF and the CF.

For any given capital stock ***k***,

- ***$y = f(k)$***

  determines how much ***Y*** the economy produces, and

- ***s*** ***($i = sy$)***

  determines the allocation of that ***Y*** between ***C & I.***

8-1 The Accumulation of Capital

- The **capital stock (CS)** is a key determinant of **output**,
- its **changes** can lead to economic growth.

2 forces influence the CS.

- **Investment** is expenditure on new plant and equipment, and it causes the CS to rise.
- **Depreciation** is the wearing out of old capital, and it causes the **CS** to fall.

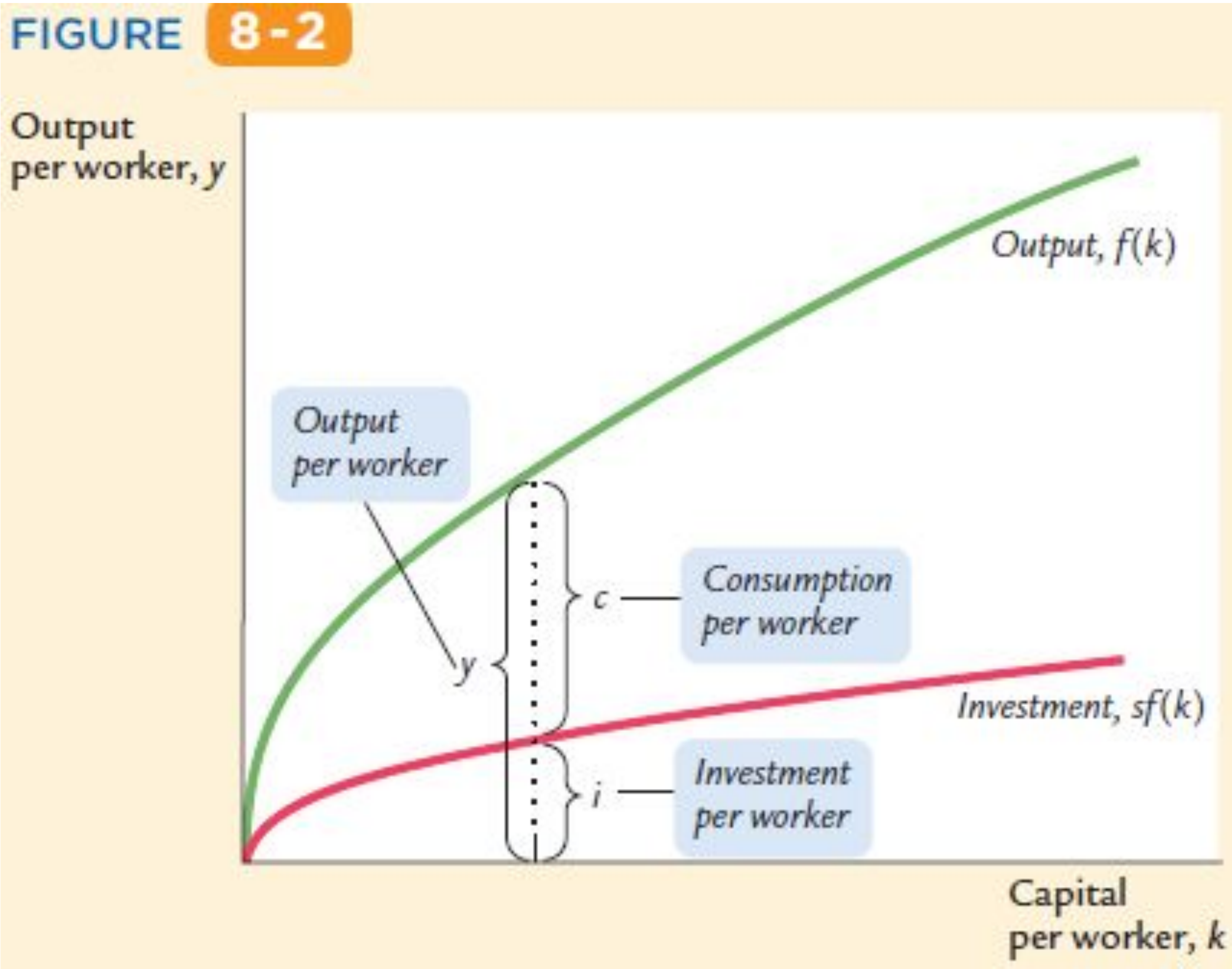
Investment per worker ***i = sy***

We can express ***i*** as a function of the ***CS*** per worker:

$$i = sf(k).$$

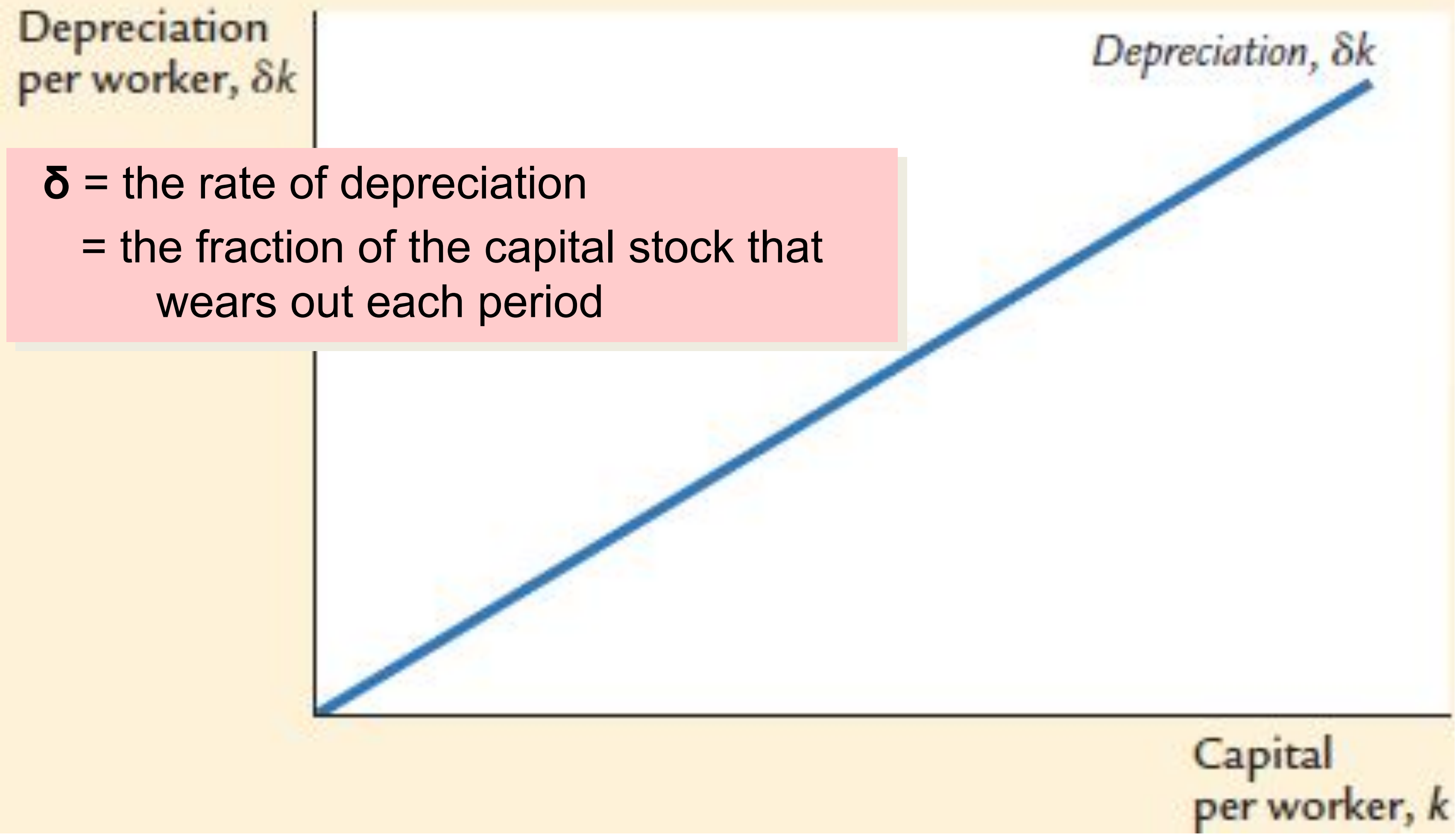
This equation relates the existing CS ***k*** *to the accumulation of* **new capital *i*.**

Output, Consumption, and Investment



- The saving rate ***s*** determines the allocation of output between C & I.
- For any level of capital *k,*
  - **output** is *f (k),* **investment** is *sf(k),* and **consumption** is *f (k) -sf(k).*

FIGURE 8-3



δ = the rate of depreciation
= the fraction of the capital stock that wears out each period

- **Depreciation is a** constant fraction of the CS wears out every year. Depreciation is therefore proportional to the capital stock.

Capital accumulation

The basic idea: Investment increases the capital stock, depreciation reduces it.

Change in capital stock = investment – depreciation

$\Delta k \quad = \quad i \quad - \quad \delta k$

Since $i = sf(k)$, this becomes:

$$\Delta k = sf(k) - \delta k$$

The equation of motion for *k*

$$\Delta k = s f(k) - \delta k$$

- **The Solow model's central equation**
- **Determines behavior of capital over time…**
- **…which, in turn, determines behavior of all of the other endogenous variables because they all depend on *k*.**

E.g.,

income per person: $y = f(k)$

consumption per person: $c = (1-s) f(k)$

The steady state

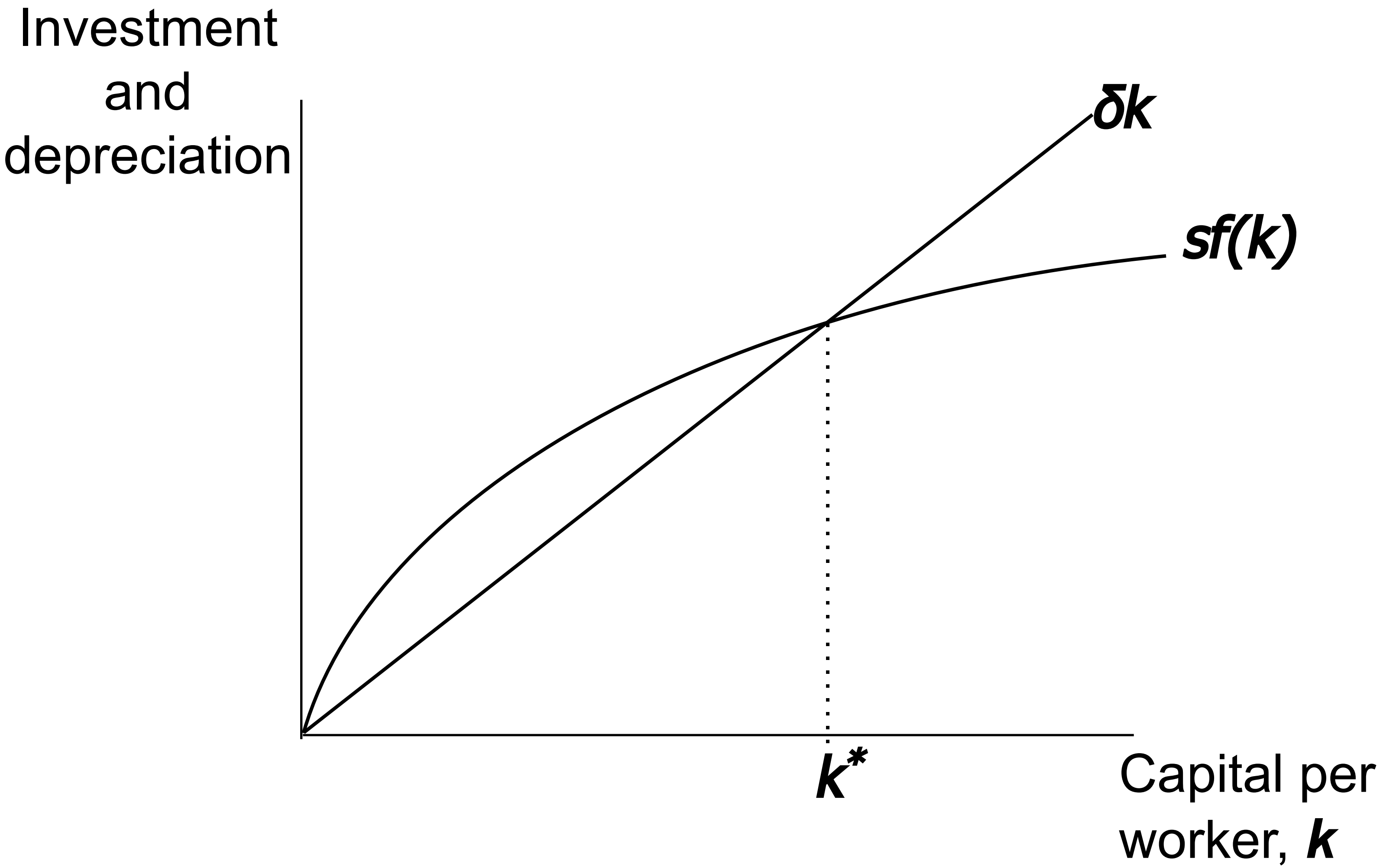
$$\Delta k = s f(k) - \delta k$$

If investment is just enough to cover depreciation
$[sf(k) = \delta k]$,
then capital per worker will remain constant:

$$\Delta k = 0.$$

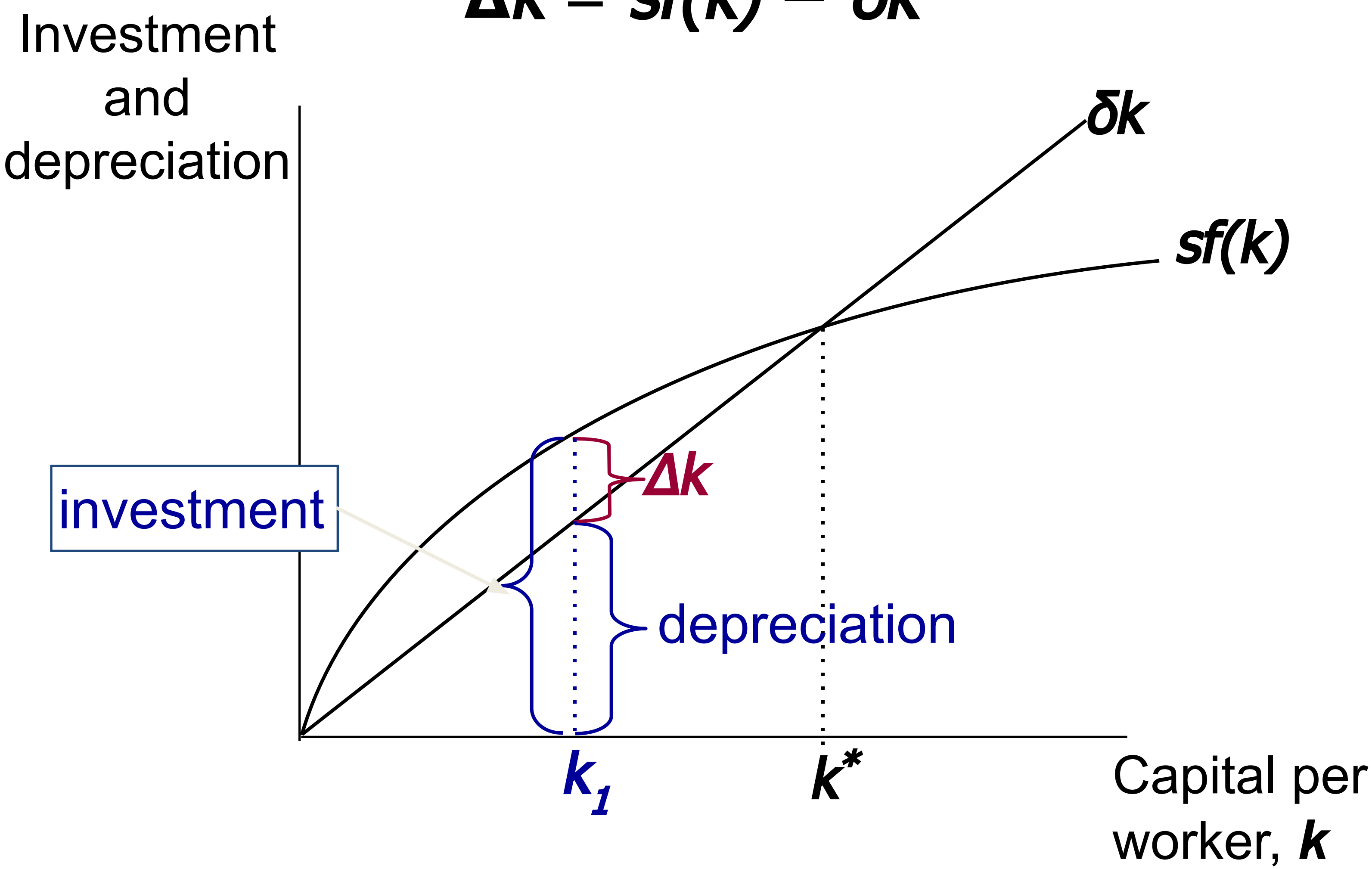
This occurs at one value of *k*, denoted k^*, called the *steady state capital stock*.

The steady state



Moving toward the steady state

$$\Delta k = sf(k) - \delta k$$



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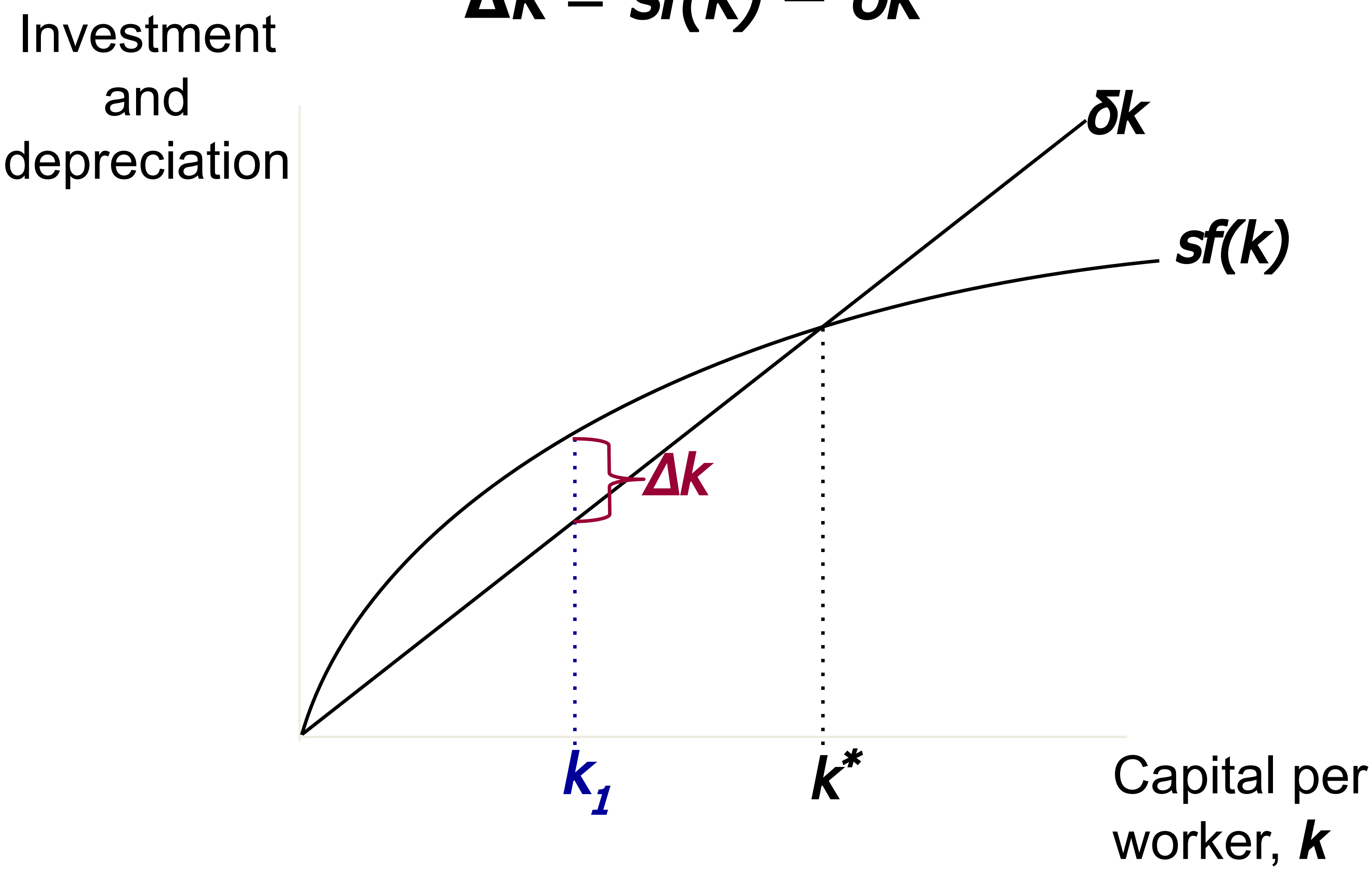

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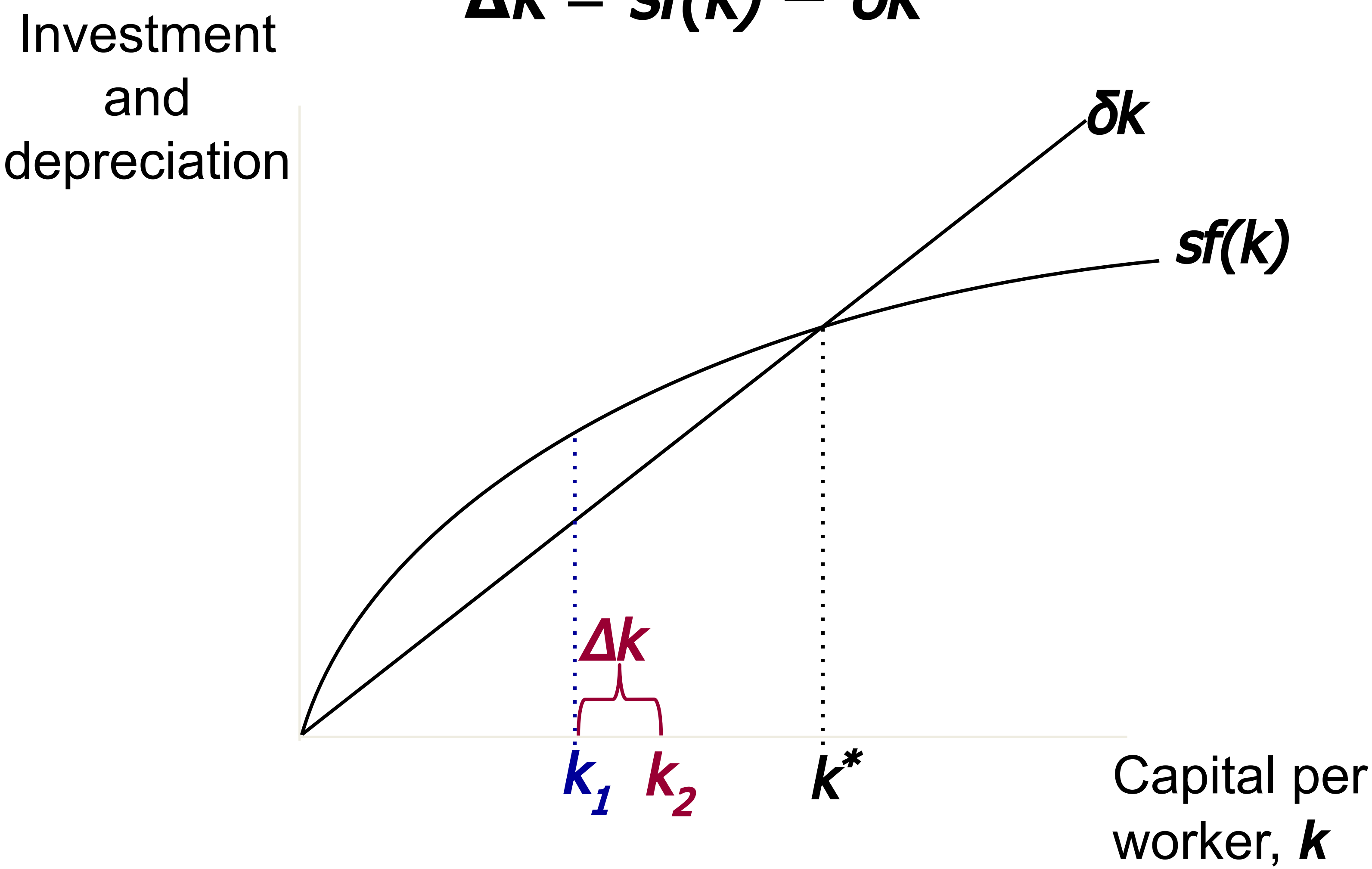
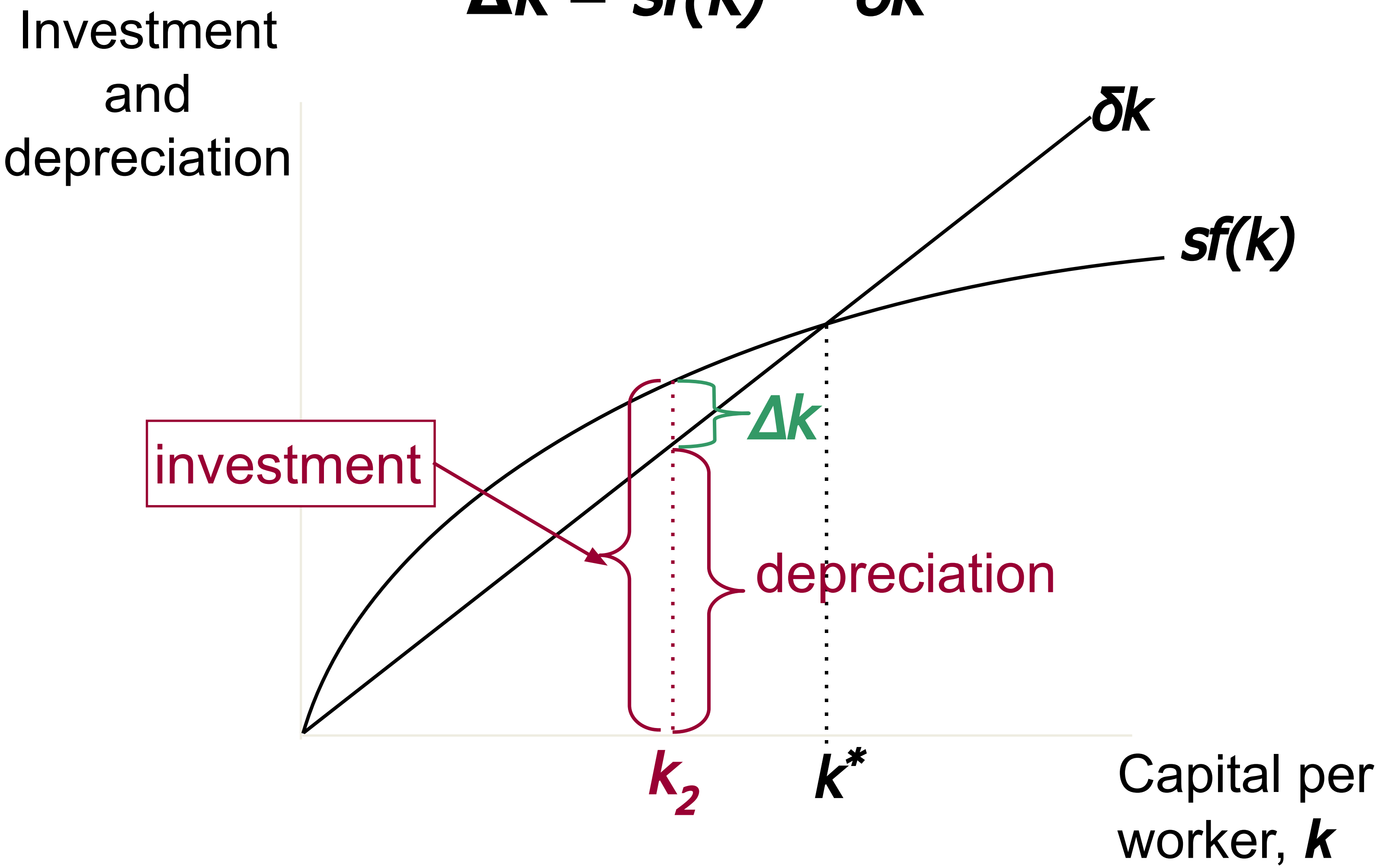


Moving toward the steady state

$$\Delta k = sf(k) - \delta k$$



Moving toward the steady state

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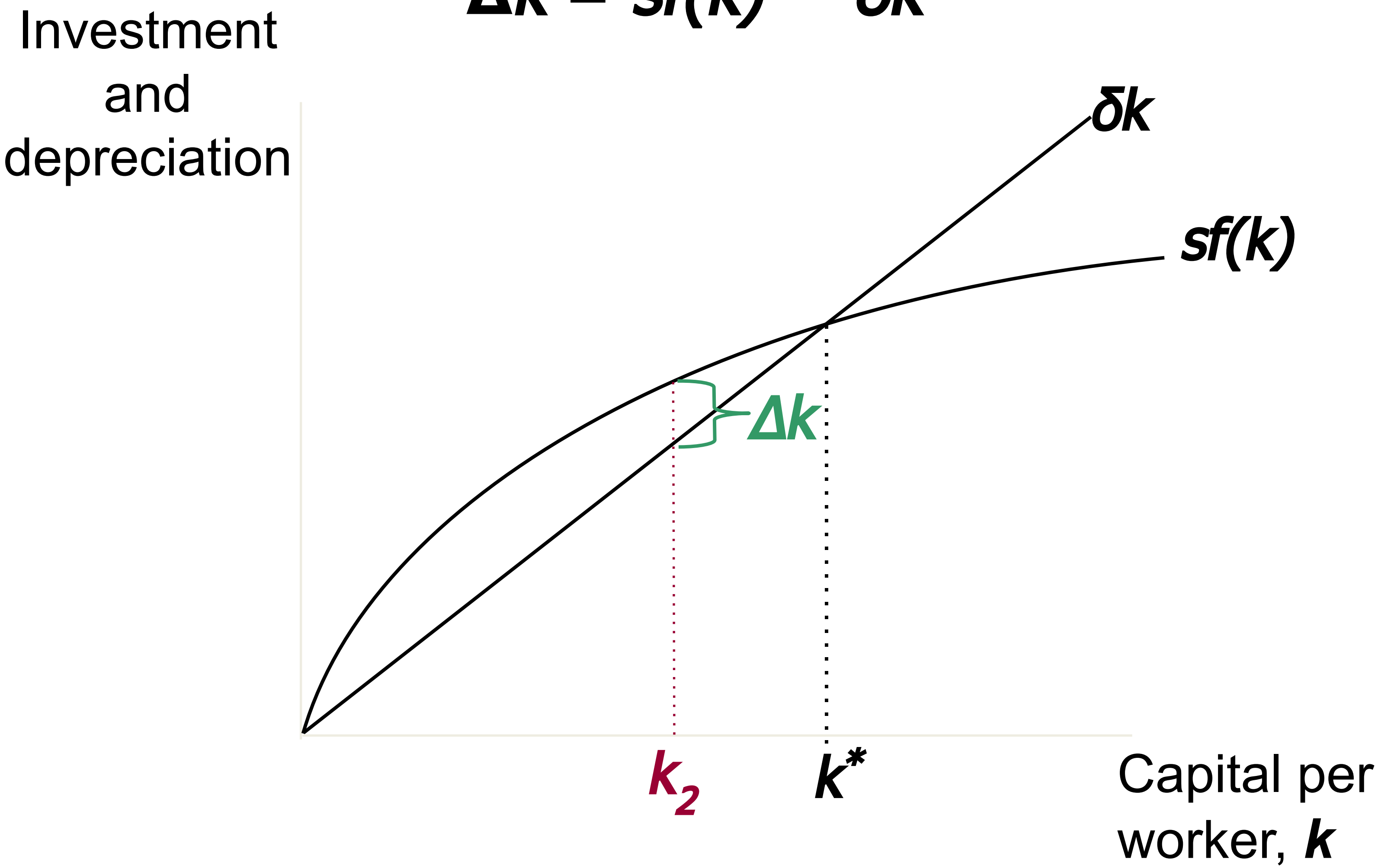

Moving toward the steady state

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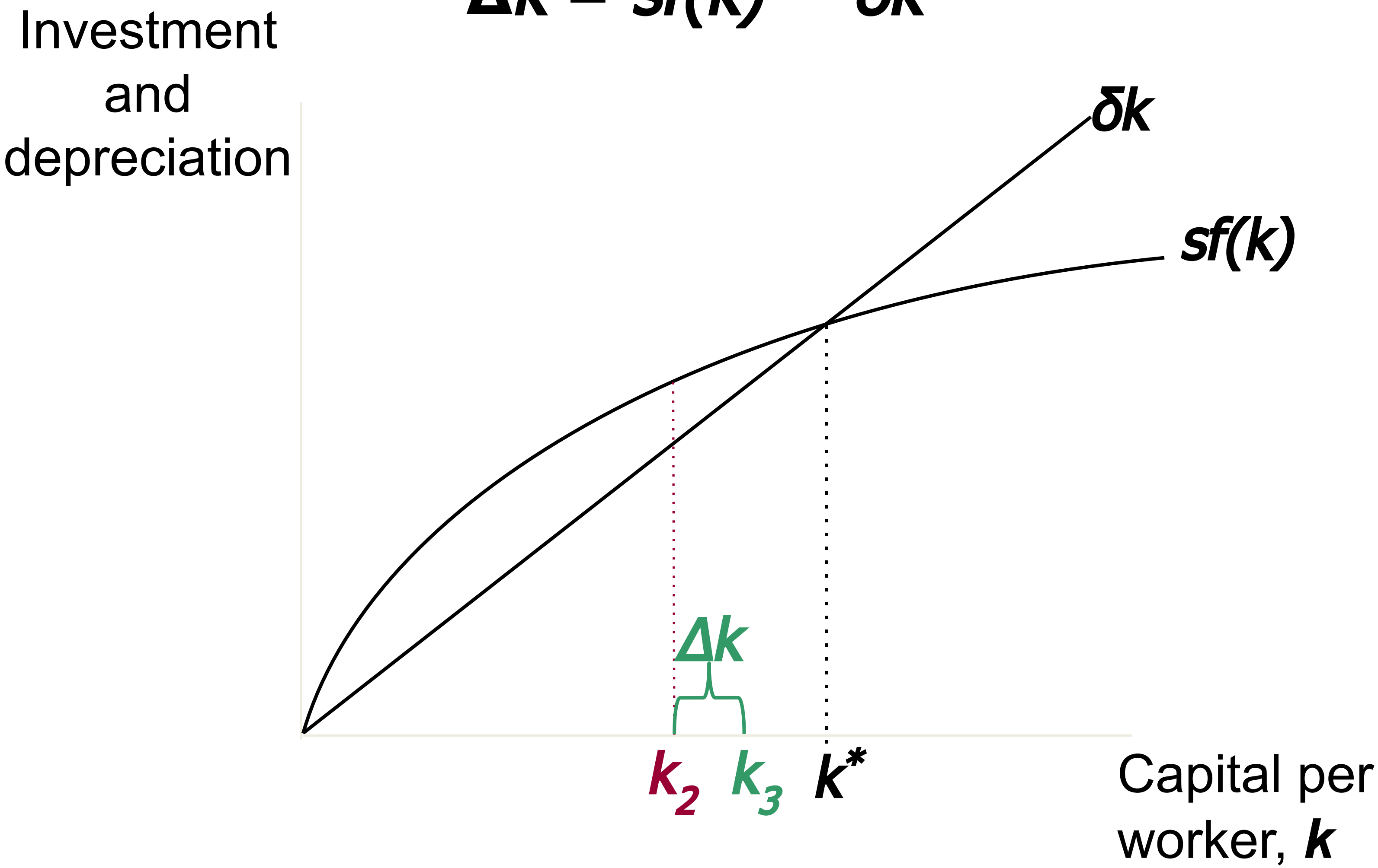
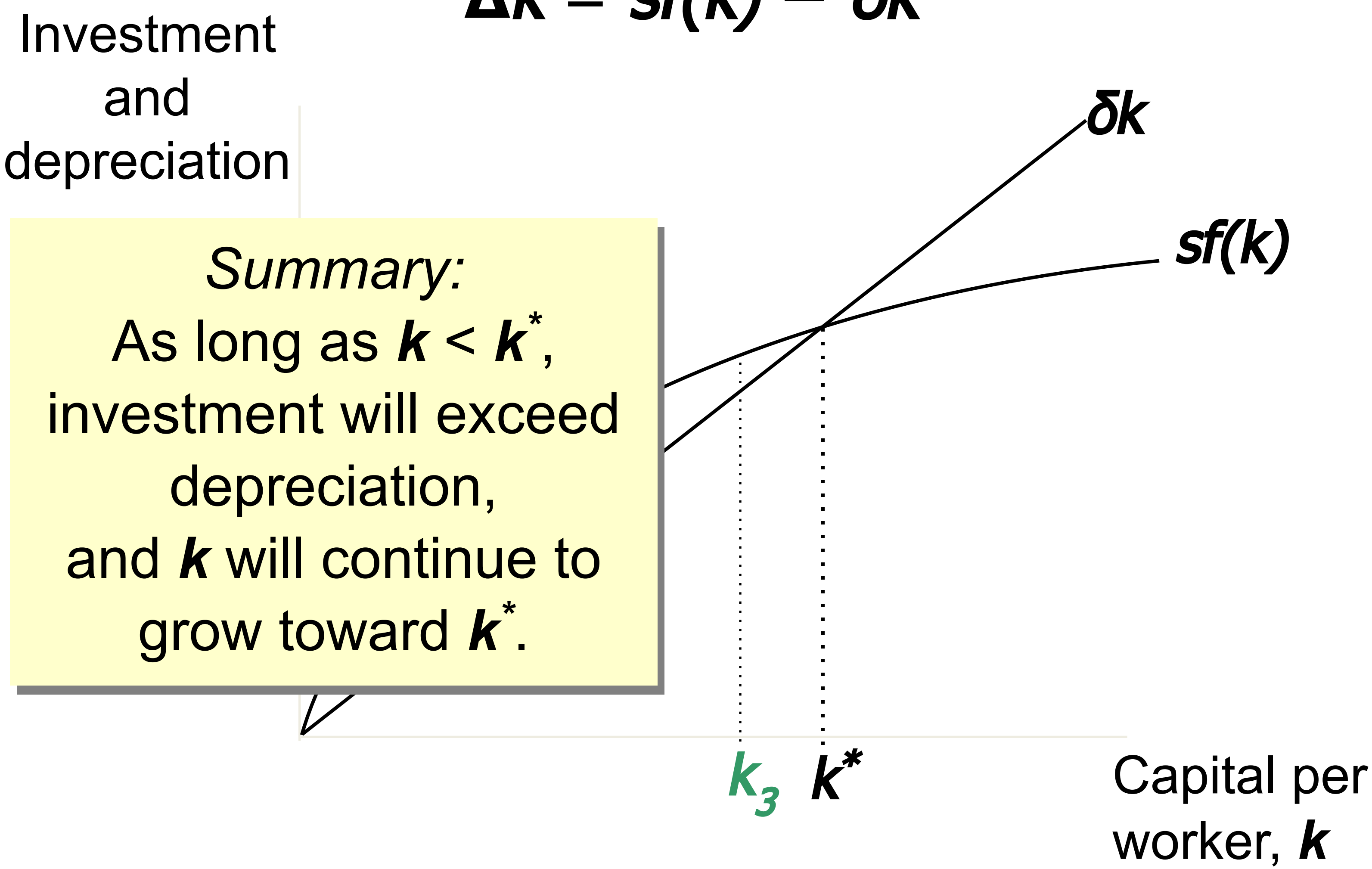


Moving toward the steady state

$$\Delta k = sf(k) - \delta k$$



Summary:
As long as $\boldsymbol{k} < \boldsymbol{k}^*$,
investment will exceed depreciation,
and $\boldsymbol{k}$ will continue to grow toward $\boldsymbol{k}^*$.

Now you try:

Draw the Solow model diagram, labeling the steady state k^*.

On the horizontal axis, pick a value greater than k^* for the economy's initial capital stock. Label it k_1.

Show what happens to k over time. Does k move toward the steady state or away from it?

A numerical example

Production function (aggregate):

$$Y = F(K, L) = \sqrt{K \times L} = K^{1/2} L^{1/2}$$

To derive the per-worker production function, divide through by ***L***:

$$\frac{Y}{L} = \frac{K^{1/2} L^{1/2}}{L} = \left(\frac{K}{L}\right)^{1/2}$$

Then substitute ***y*** = ***Y/L*** and ***k*** = ***K/L*** to get

$$y = f(k) = k^{1/2}$$

A numerical example, *cont.*

Assume:

- $s = 0.3$
- $\delta = 0.1$
- initial value of $k = 4.0$

Approaching the steady state: A numerical example

Assumptions: $y = \sqrt{k}$; $s = 0.3$; $\delta = 0.1$; initial $k = 4.0$

Year	***k***	***y***	***c***	***i***	**□*k***	**□*k***
1	**4.000**	**2.000**	**1.400**	**0.600**	**0.400**	**0.200**
2	**4.200**	**2.049**	**1.435**	**0.615**	**0.420**	**0.195**
3	**4.395**	**2.096**	**1.467**	**0.629**	**0.440**	**0.189**
4	4.584	2.141	1.499	0.642	0.458	0.184
…						
10	5.602	2.367	1.657	0.710	0.560	0.150
…						
25	7.351	2.706	1.894	0.812	0.732	0.080
…						
100	8.962	2.994	2.096	0.898	0.896	0.002
…						
□	9.000	3.000	2.100	0.900	0.900	0.000

Exercise: Solve for the steady state

Continue to assume

$s = 0.3$, $\delta = 0.1$, and $y = k^{1/2}$

Use the equation of motion

$$\Delta k = s\,f(k) - \delta k$$

to solve for the steady-state values of ***k***, ***y***, and ***c***.

Solution to exercise:

$$\Delta k = 0$$ def. of steady state

$$s\,f(k^*) = \delta k^*$$ eq'n of motion with $\Delta k = 0$

$$0.3\sqrt{k^*} = 0.1k^*$$ using assumed values

$$3 = \frac{k^*}{\sqrt{k^*}} = \sqrt{k^*}$$

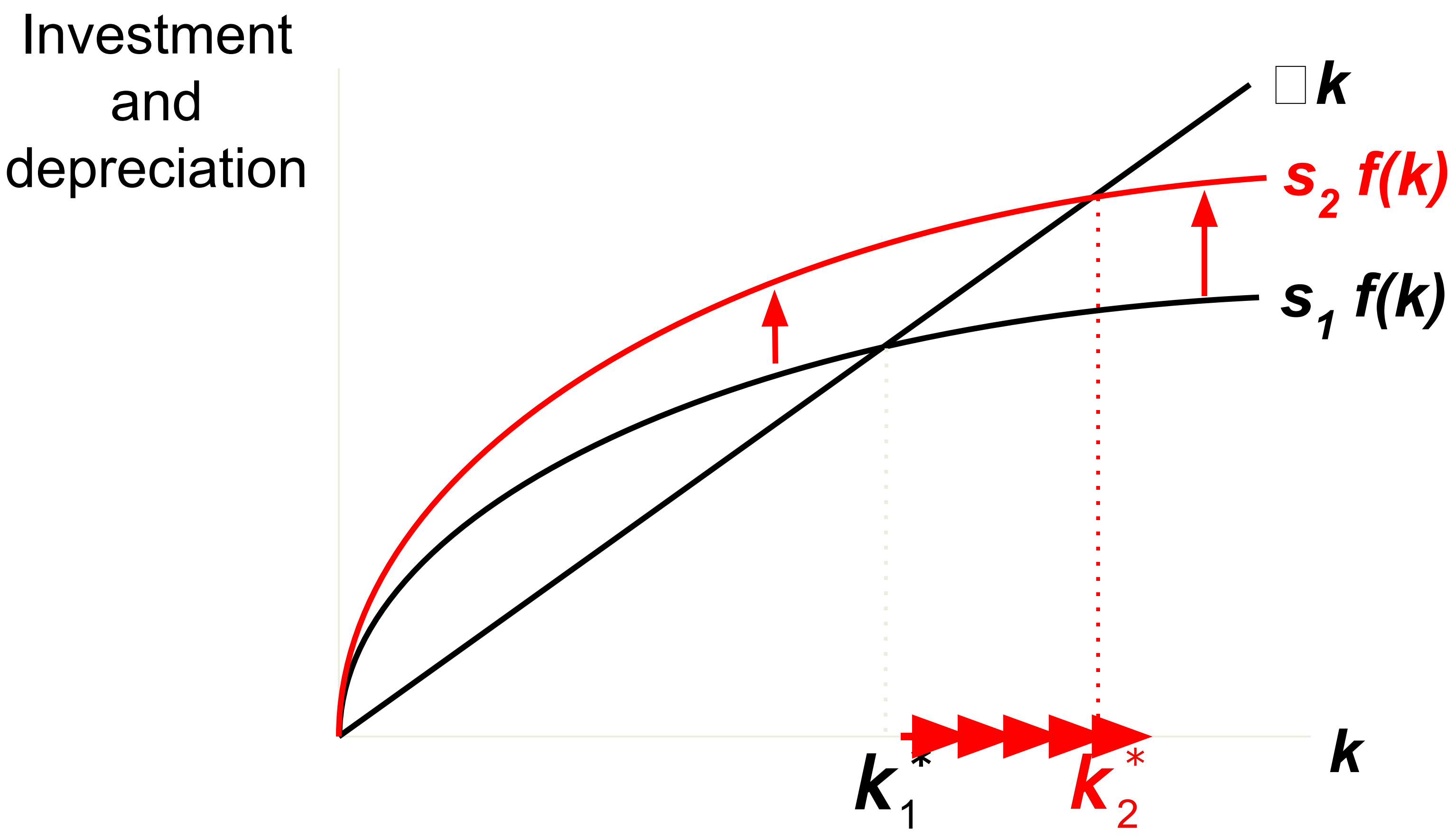
Solve to get: $k^* = 9$ and $y^* = \sqrt{k^*} = 3$

Finally, $c^* = (1 - s)y^* = 0.7 \times 3 = 2.1$

An increase in the saving rate

An increase in the saving rate raises investment…

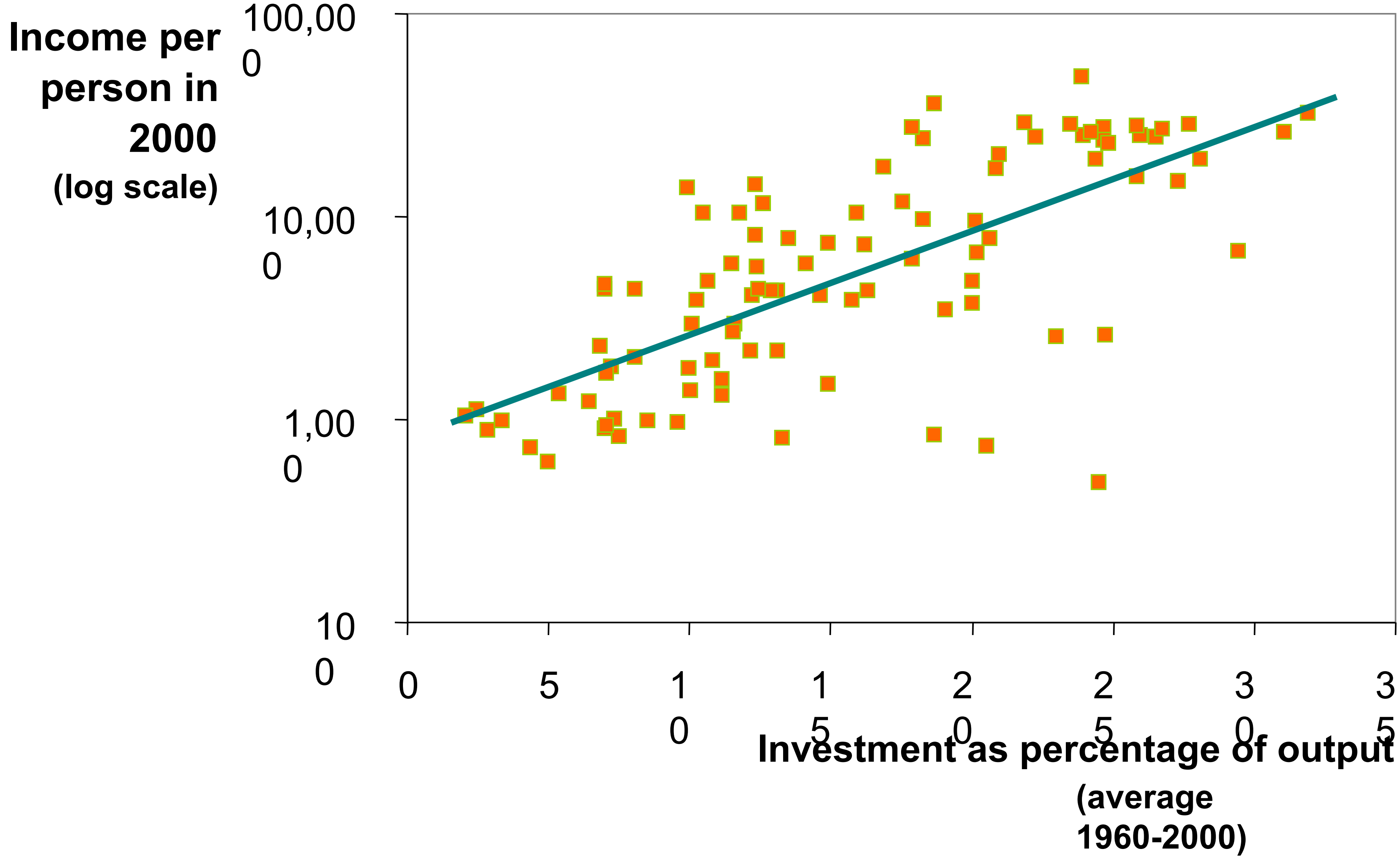
…causing ***k*** to grow toward a new steady state:



Prediction:

- Higher $s \Rightarrow$ higher k^*.
- And since $y = f(k)$,
  higher $k^* \Rightarrow$ higher y^*.
- Thus, the Solow model predicts that countries with higher rates of saving and investment
  will have higher levels of capital and income per worker in the long run.

International evidence on investment rates and income per person



The Golden Rule: Introduction

- **Different values of s lead to different steady states. How do we know which is the "best" steady state?**
- **The "best" steady state has the highest possible consumption per person: $c^* = (1-s)\,f(k^*)$.**
- **An increase in s**
  1. leads to higher k^* and y^*, which raises c^*
  2. reduces consumption's share of income $(1-s)$, which lowers c^*.
- **So, how do we find the s and k^* that maximize c^*?**

The Golden Rule capital stock

k^*_{gold} = **the Golden Rule level of capital, the steady state value of k that maximizes consumption.**

To find it, first express c^* in terms of k^*:

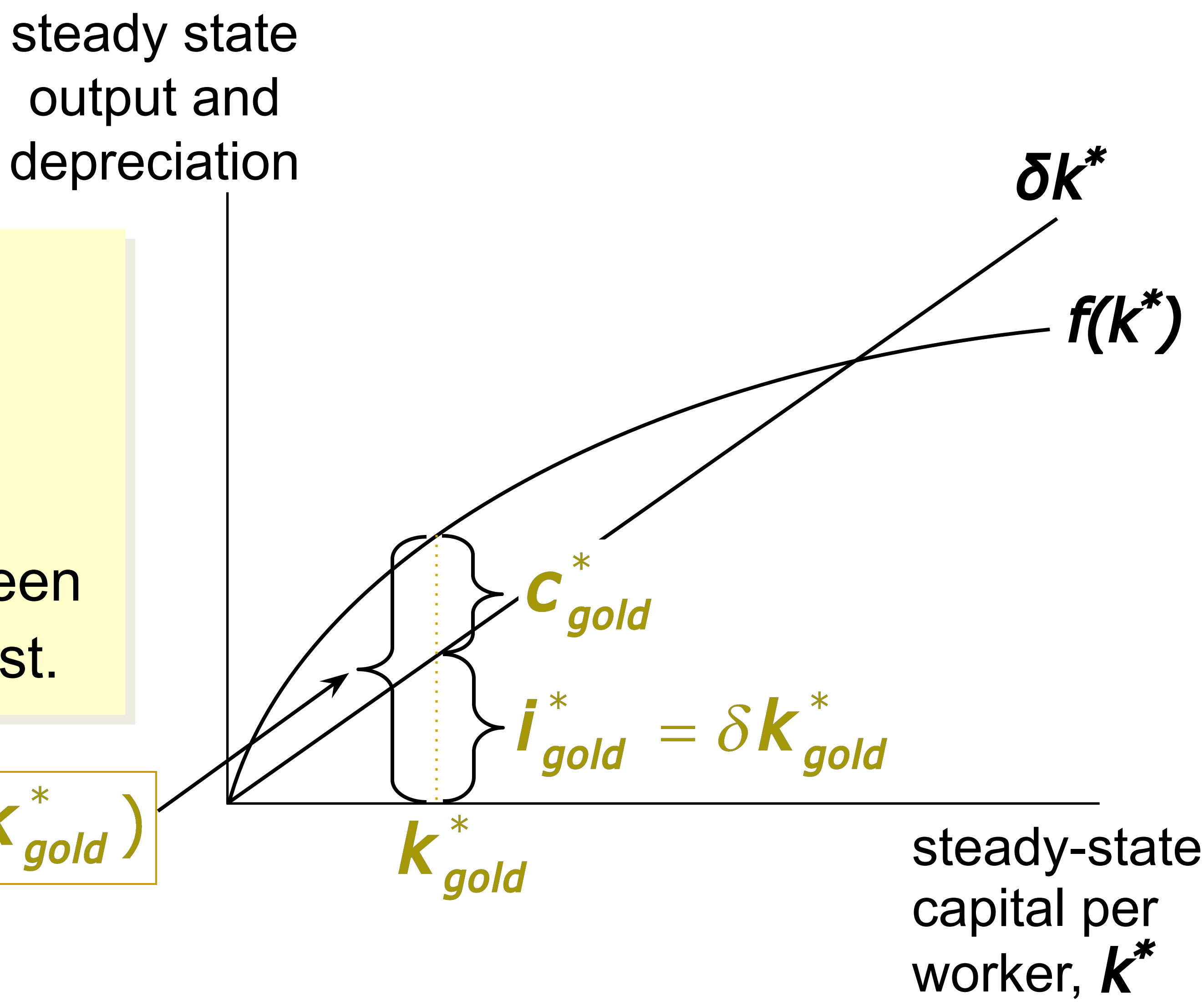
$$c^* = y^* - i^*$$
$$= f(k^*) - i^*$$
$$= f(k^*) - \delta k^*$$

In the steady state:
$i^* = \delta k^*$
because $\Delta k = 0$.

The Golden Rule capital stock

Then, graph $f(k^*)$ and δk^*, look for the point where the gap between them is biggest.

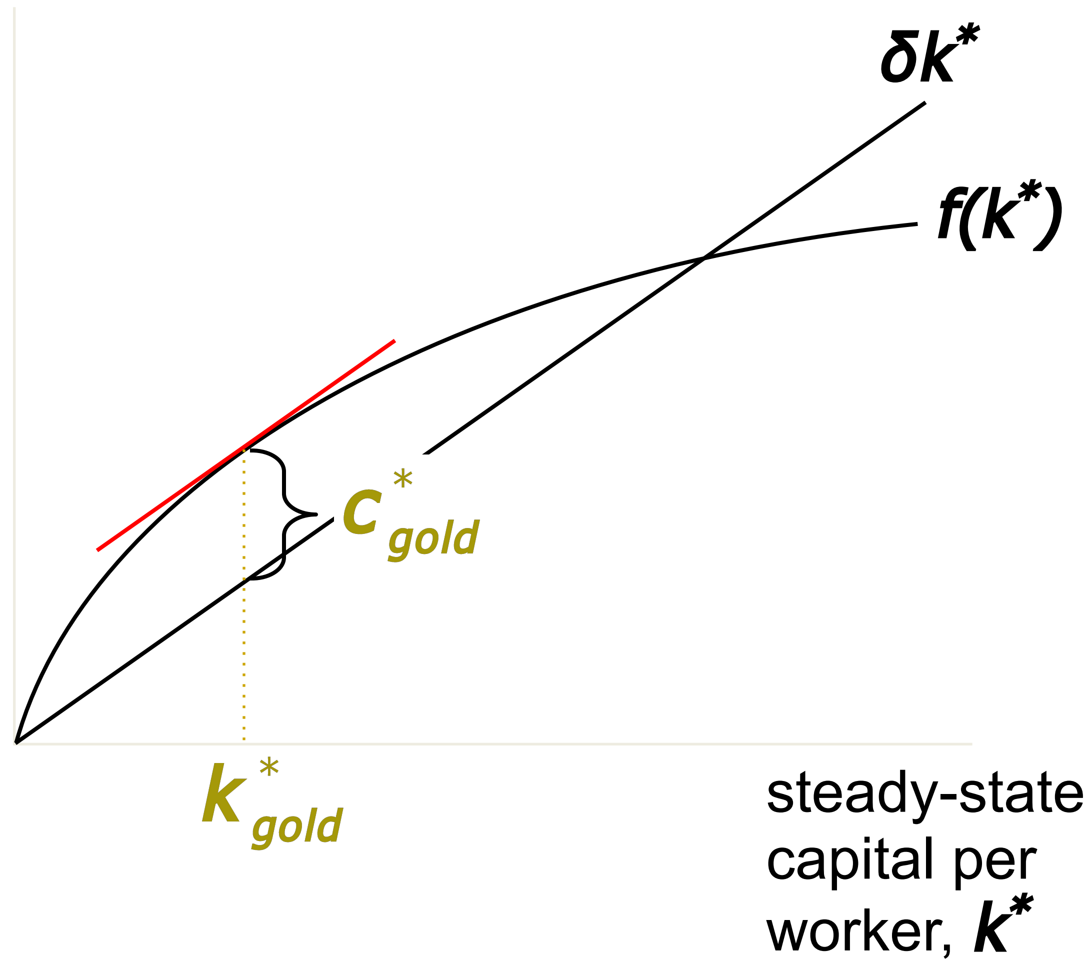
$$y^*_{gold} = f(k^*_{gold})$$



The Golden Rule capital stock

$c^* = f(k^*) - \delta k^*$
is biggest where the slope of the production function equals the slope of the depreciation line:

$\text{MPK} = \delta$



The transition to the Golden Rule steady state

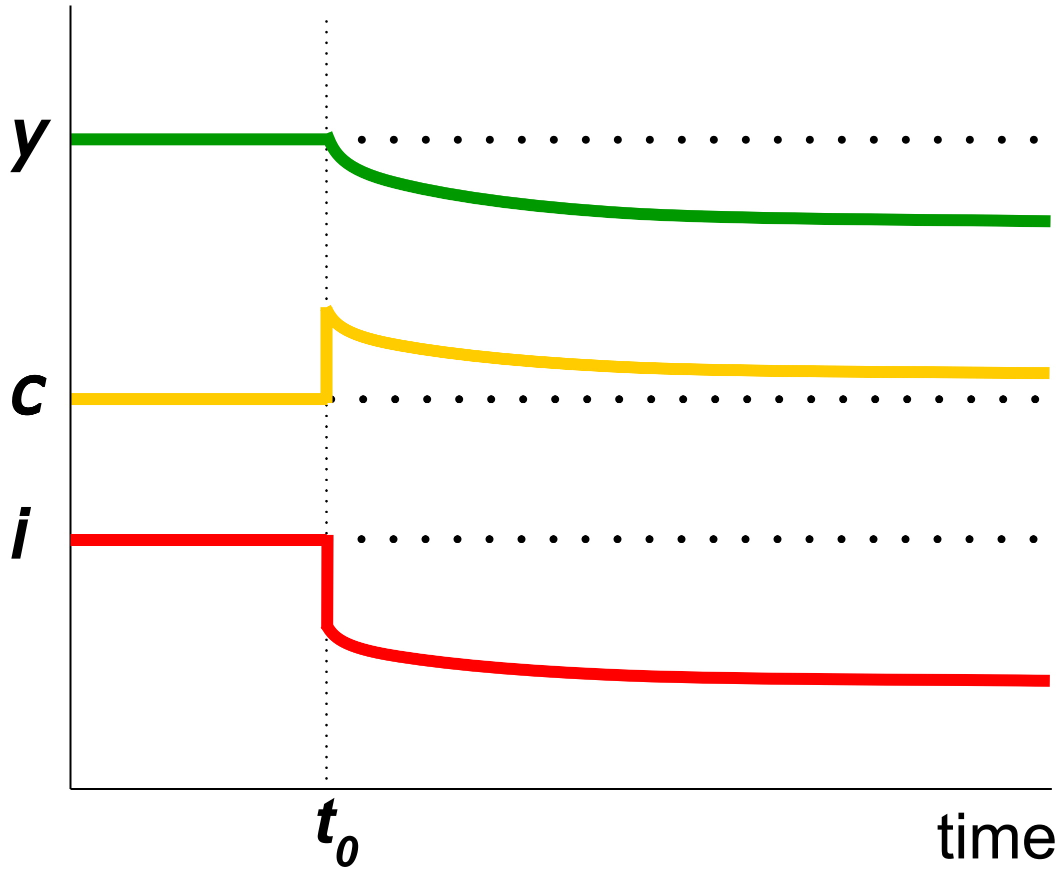
- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust *s*.
- This adjustment leads to a new steady state with higher consumption.

- But what happens to consumption during the transition to the Golden Rule?

Starting with too much capital

If $k^* > k^*_{gold}$ then increasing c^* requires a fall in s.

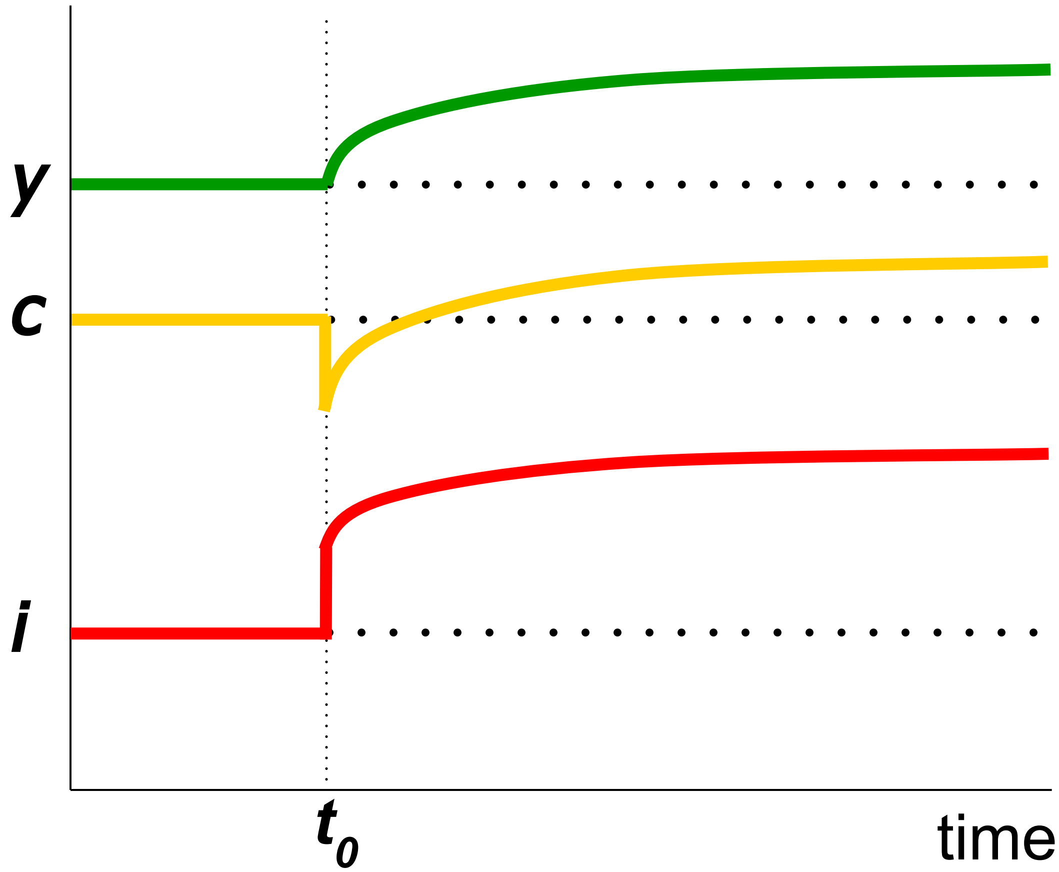
In the transition to the Golden Rule, consumption is higher at all points in time.



Starting with too little capital

If $k^* < k^*_{gold}$ then increasing c^* requires an increase in s.

Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.



Population growth

- Assume that the population (and labor force) grow at rate *n*. (*n* is exogenous.)

- EX: Suppose L = 1,000 in year 1 and the population is growing at 2% per year (n = 0.02).
- Then $\Delta L = n L = 0.02 \times 1{,}000 = 20$, so L = 1,020 in year 2.

$$\frac{\Delta L}{L} = n$$

Break-even investment

- $(\delta + n)k$ = **break-even investment**, **the amount of investment necessary to keep *k* constant.**
- **Break-even investment includes:**
  - δk to replace capital as it wears out
  - nk to equip new workers with capital

    (Otherwise, ***k*** would fall as the existing capital stock would be spread more thinly over a larger population of workers.)

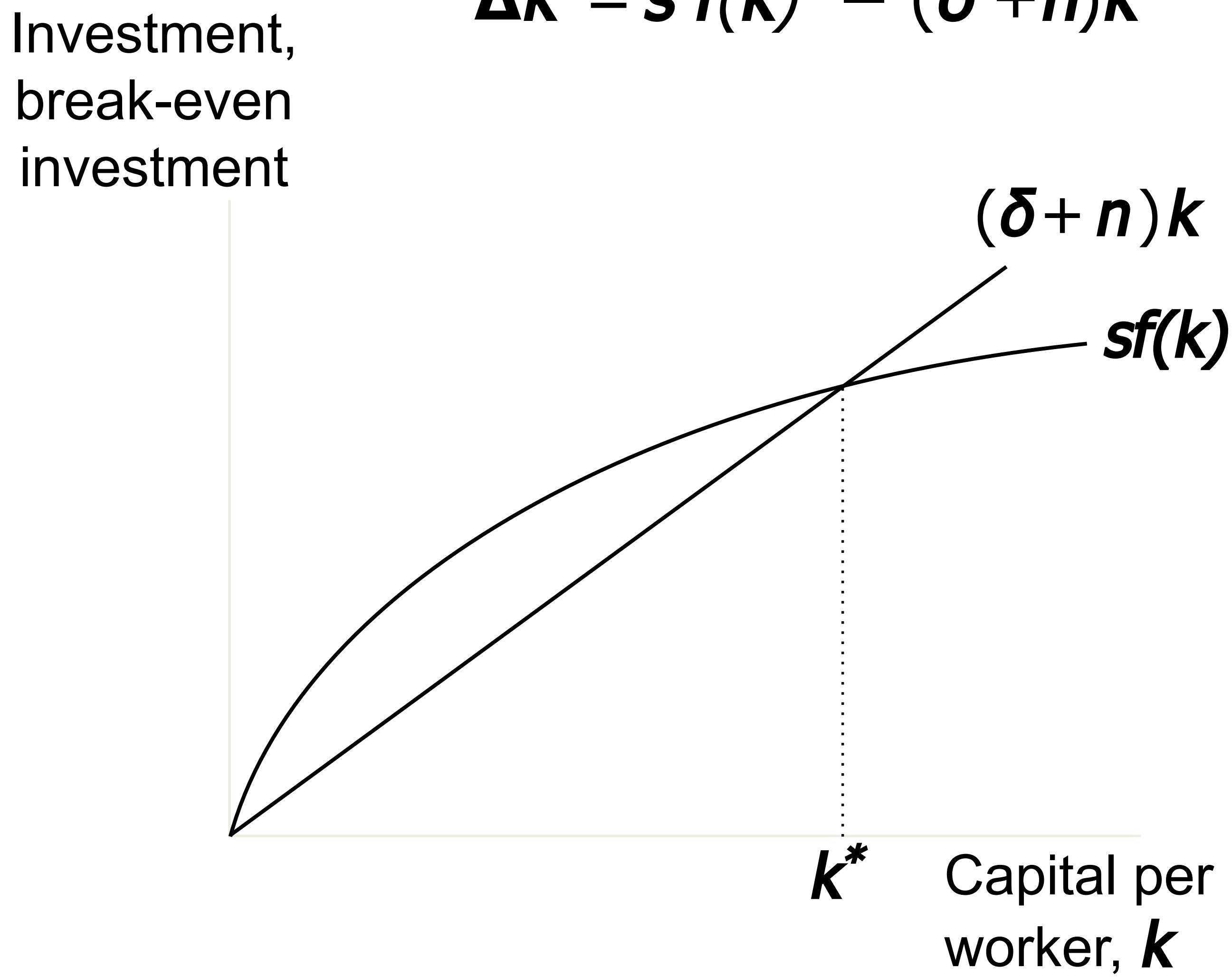
The equation of motion for *k*

- **With population growth, the equation of motion for *k* is**

$$\Delta k = \underbrace{s f(k)}_{\text{actual investment}} - \underbrace{(\delta + n)\, k}_{\text{break-even investment}}$$

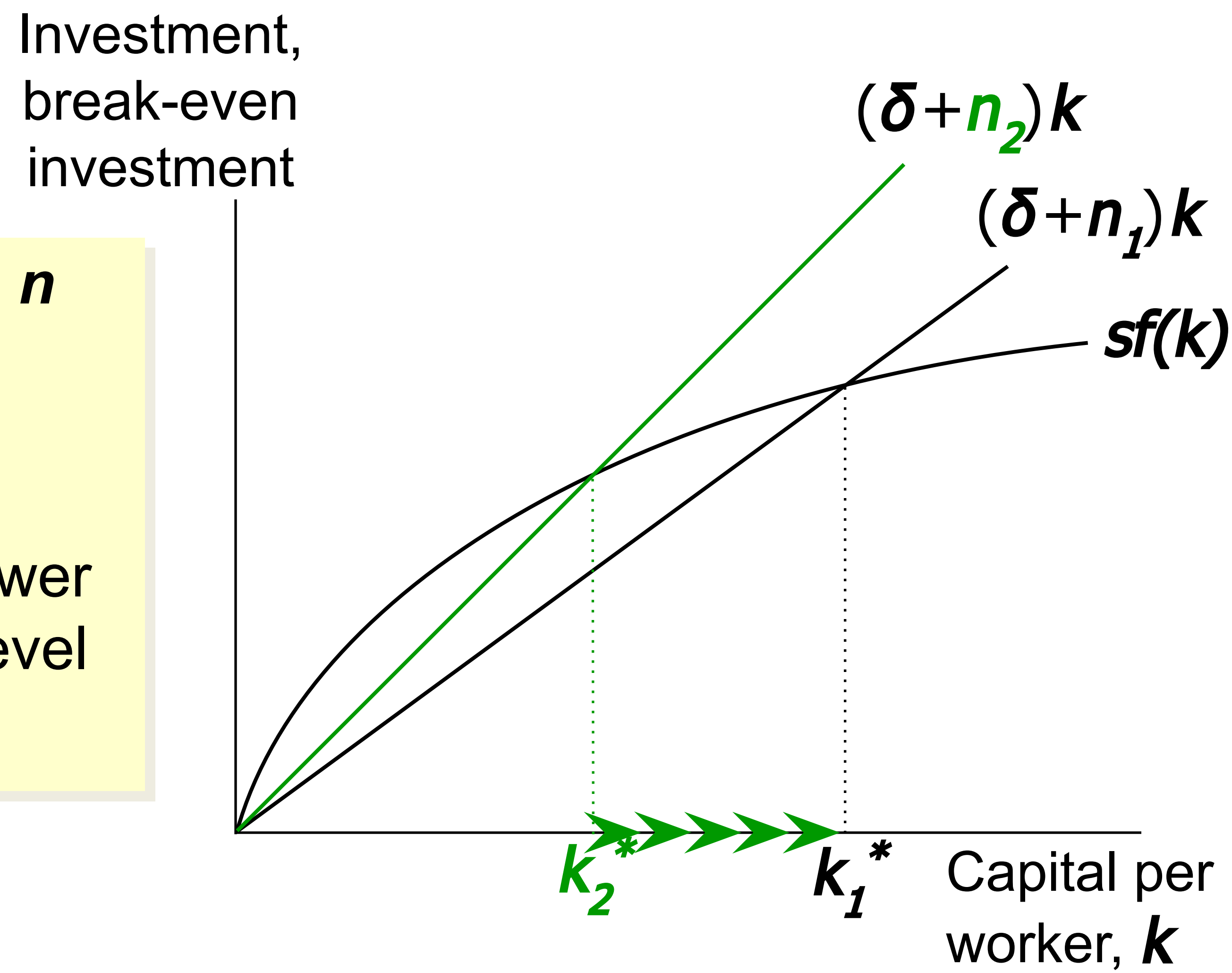
The Solow model diagram

$$\Delta k = s\,f(k) - (\delta + n)k$$



The impact of population growth

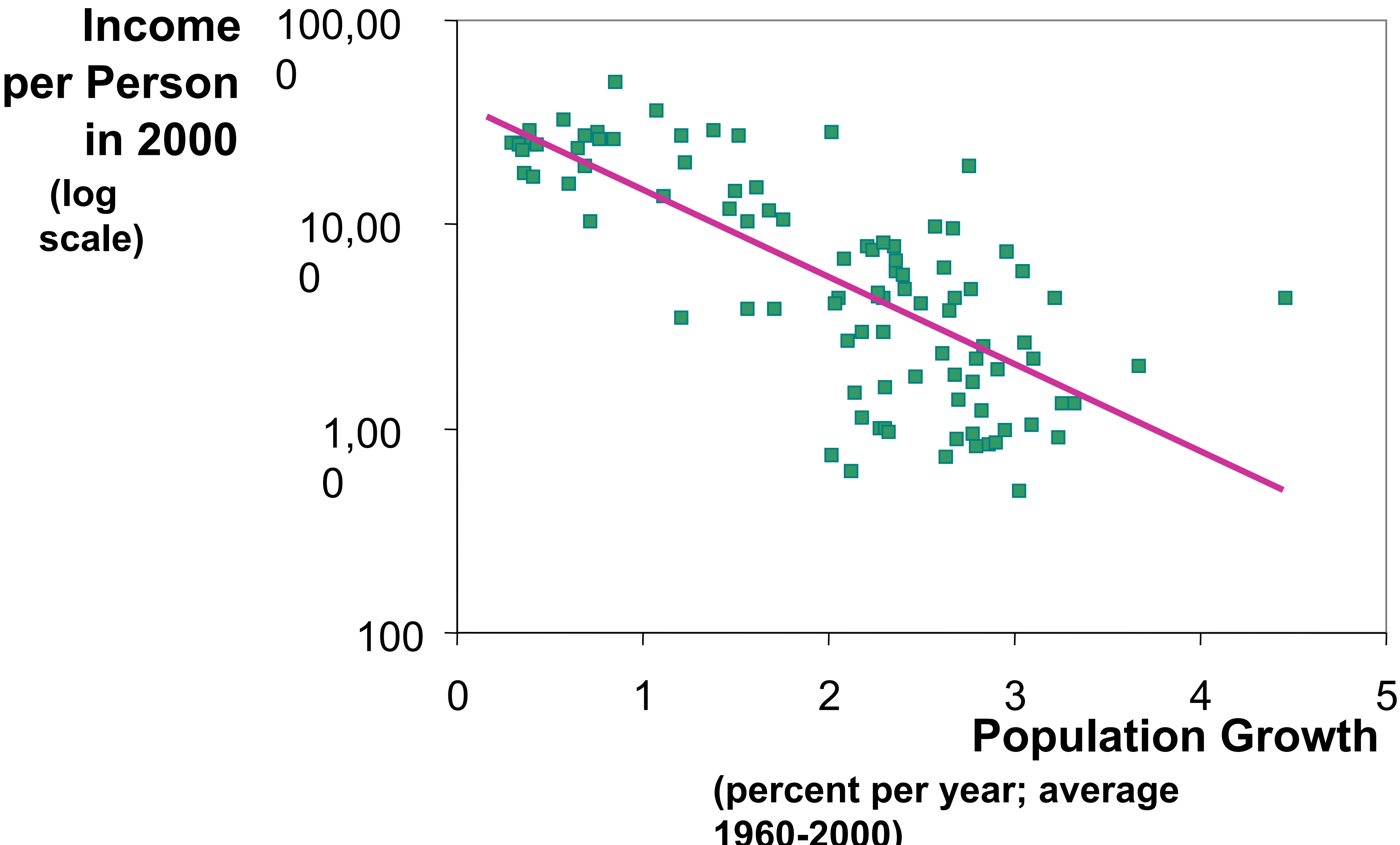
An increase in ***n*** causes an increase in break-even investment, leading to a lower steady-state level of ***k***.



Prediction:

- Higher $n \Rightarrow$ lower k^*.
- And since $y = f(k)$,
  lower $k^* \Rightarrow$ lower y^*.
- Thus, the Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run.

International evidence on population growth and income per person



The Golden Rule with population growth

To find the Golden Rule capital stock, express $\boldsymbol{c}^*$ in terms of $\boldsymbol{k}^*$:

$$c^* = y^* - i^*$$

$$= f(k^*) - (\delta + n)\, k^*$$

$\boldsymbol{c}^*$ is maximized when

$$\text{MPK} = \delta + n$$

or equivalently,

$$\text{MPK} - \delta = n$$

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate.

Alternative perspectives on population growth

The Malthusian Model (1798)

- Predicts population growth will outstrip the Earth's ability to produce food, leading to the impoverishment of humanity.
- Since Malthus, world population has increased sixfold, yet living standards are higher than ever.
- Malthus omitted the effects of technological progress.

Alternative perspectives on population growth

The Kremerian Model (1993)

- Posits that population growth contributes to economic growth.
- More people = more geniuses, scientists & engineers, so faster technological progress.
- Evidence, from very long historical periods:
  - As world pop. growth rate increased, so did rate of growth in living standards
  - Historically, regions with larger populations have enjoyed faster growth.

Chapter Summary

1. **The Solow growth model shows that, in the long run, a country's standard of living depends**
   - positively on its saving rate
   - negatively on its population growth rate
2. **An increase in the saving rate leads to**
   - higher output in the long run
   - faster growth temporarily
   - but not faster steady state growth.

Chapter Summary

3. **If the economy has more capital than the Golden Rule level, then reducing saving will increase consumption at all points in time, making all generations better off.**
   If the economy has less capital than the Golden Rule level, then increasing saving will increase consumption for future generations, but reduce consumption for the present generation.