

МЕТОДЫ РЕШЕНИЯ ТРИГОНОМЕТРИЧЕСКИХ УРАВНЕНИЙ

Учитель математики МБОУ СОШ №9 г. Уфы В.М.Хабибуллина

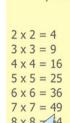
 $\frac{a}{\sin B} = \frac{b}{\sin B} = \frac{c}{\sin B}$



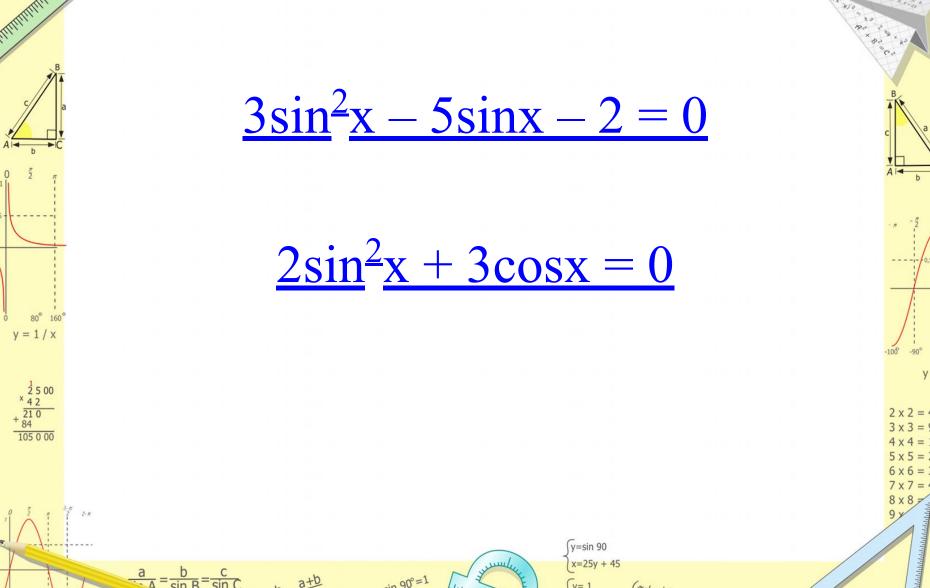


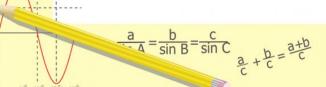


 $(x+y)(x-y) = x^2 - y^2$

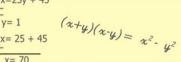


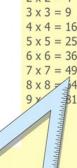
•сведения уравнения к квадратному **МЕТОДЫ** •разложения на множители y = 1/x2 5 00 × 4 2 + 21 0 + 84 •решение однородных 105 0 00 уравнений $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $(x+y)(x-y) = x^2 - y^2$











$$3\sin^2 x - 5\sin x - 2 = 0$$

$$3\sin^2 x - 3\sin x - 2 = 0$$

$$\sin x = t \qquad \qquad \sin x = -\frac{1}{3}$$

$$3t^2 - 5t - 2 = 0$$

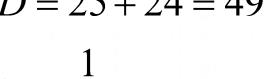
$$5t - 3t - 2 - 0$$

$$D = 25 + 24 = 49$$

t = 2

$$D = 25 + 24 = 49$$

$$D = 25 + 24 = 49$$



$$\sin x = 2 \rightarrow$$
 He ume

$$x = (-1)^{n+1} \arcsin \frac{1}{3} + \pi n, n \in \mathbb{Z}$$

$$y = \sin 90$$

$$\frac{a}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



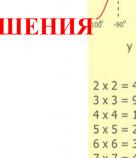
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

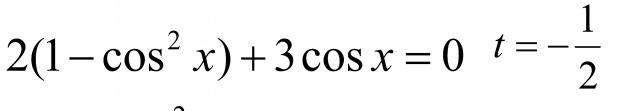
$$y = \sin 90$$

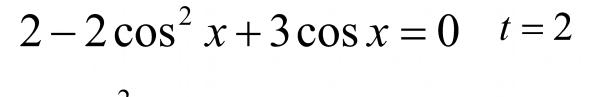
 $x = 25y + 45$
 $y = 1$ ($x \neq 4$)
 $x = 25 + 45$

 $x = \left(-1\right)^{n+1} \arcsin \frac{1}{2} + \pi n, n \in \mathbb{Z}$



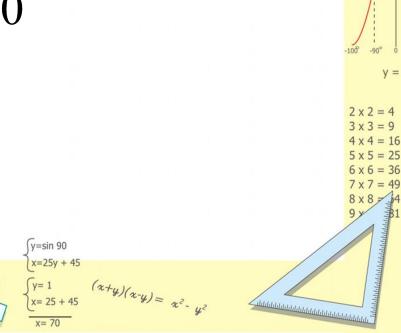
$$2\sin^2 x + 3\cos x = 0 \qquad D = 9 + 16 = 25$$

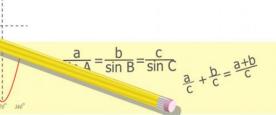




$$-2\cos^2 x + 3\cos x + 2 = 0$$
$$\cos x = t$$

$$2t^2 - 3t - 2 = 0$$







$$\cos x = 2 \rightarrow$$
 НЕ ИМЕЕТ РЕШЕНИЯ

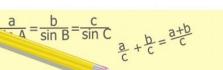
$$\cos x = -\frac{1}{2}$$

$$x = \pm \left(\pi - \frac{\pi}{3}\right) + 2\pi n, n \in \mathbb{Z}$$

$$x = \pm \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z}$$

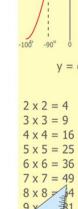
105 0 00

OTBET: $x = \pm \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z}$









МЕТОД РАЗЛОЖЕНИЯ НА МНОЖИТЕЛИ

$$2\cos^2 x + \sqrt{3}\cos x = 0$$
$$\cos x \left(2\cos x + \sqrt{3}\right) = 0$$

$$\cos x = 0$$

$$2\cos x$$

$$\iota - \iota$$

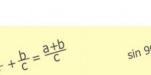
$$2\cos x + \sqrt{3} = 0$$
$$2\cos x + \sqrt{3} = 0$$

$$x_1 = \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$$

$$x_2 = \pm \left(\pi - \frac{\pi}{6}\right) + 2\pi k$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$





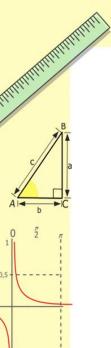
$$= 1 \qquad (x+y)(x-y) = x^2 - y^2$$

$$= 70$$

 $x_2 = \pm \frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$

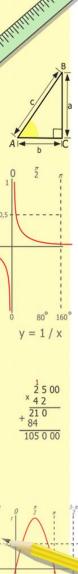


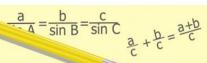
МЕТОД РАЗЛОЖЕНИЯ НА МНОЖИТЕЛИ



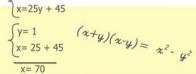
OTBET:
$$x_1 = \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$$

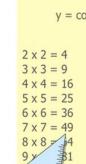
$$x_2 = \pm \frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$$











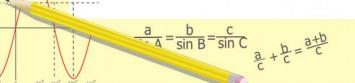
ОПРЕДЕЛЕНИЕ:

Уравнение вида $a\sin x + b\cos x = 0$

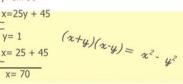
называют *однородным* тригонометрическим уравнением *первой* степени.

Алгоритм решения однородного тригонометрического уравнения первой степени:

Деление обеих частей уравнения на cosx, $cosx \neq 0$.

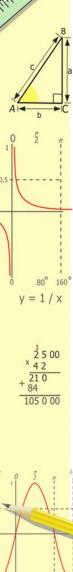






ОПРЕДЕЛЕНИЕ:

Уравнение вида $a \sin^2 x + b \sin x \cos x + c \cos^2 x = 0$ называют однородным тригонометрическим уравнением второй степени.





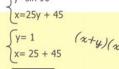
 $(x+y)(x-y) = x^2 - y^2$

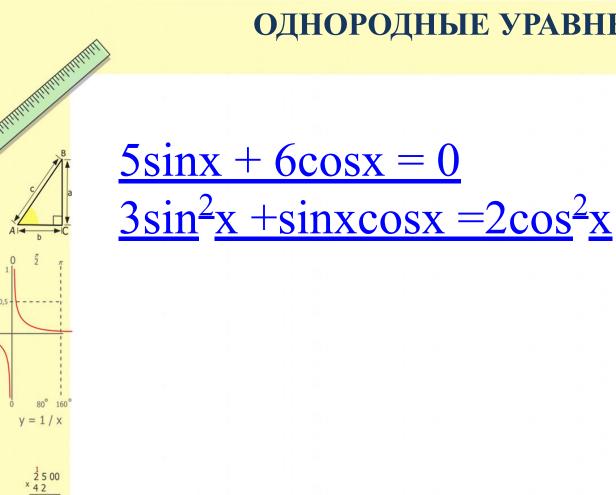
Алгоритм решения однородного тригонометрического уравнения второй степени:

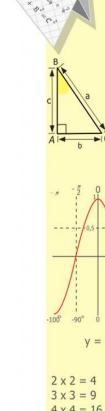
- 1.Посмотреть, есть ли в уравнении член $asin^2 x$.
- 2. Если член $asin^2 x$ в уравнении содержится (т.е. $a \neq 0$), то уравнение решается делением обеих частей уравнения на $cos^2 x$ и последующим введение новой переменной.
- 3. Если член $asin^2 x$ в уравнении не содержится (т.е. a = 0), то уравнение решается методом разложения на множители: за скобки выносят cosx.

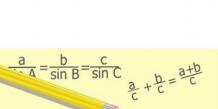






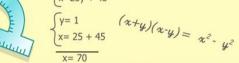






105 0 00





$$5\sin x + 6\cos x = 0$$

$$5\frac{\sin x}{\cos x} + 6\frac{\cos x}{\cos x} = \frac{0}{\cos x}$$

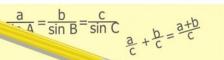
$$5tgx + 6 = 0$$

2 5 00

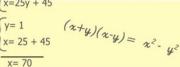
105 0 00

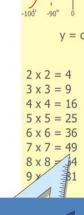
$$tgx = -\frac{6}{5} \qquad x = -arctg \frac{6}{5} + \pi n, n \in \mathbb{Z}$$

OTBET: $x = -arctg \frac{6}{5} + \pi n, n \in \mathbb{Z}$



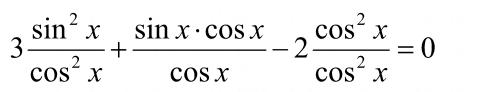






$$\sin x \cos x - 2\cos^2 x$$

$$3\frac{\sin^2 x}{\sin^2 x} + \frac{\sin x \cdot \cos x}{\cos^2 x} - 2\frac{\cos^2 x}{\cos^2 x} = 0$$



$$3\frac{}{\cos^2 x} + \frac{}{\cos x} - 2\frac{}{\cos^2 x} = 0$$
$$3tg^2 x + tgx - 2 = 0$$

$$\cos^2 x \qquad \cos x \qquad \cos^2 x$$
$$3tg^2x + tgx - 2 = 0$$

$$tgx = t tgx = -1$$

t = -1

 $t = \frac{2}{3}$

$$t = t$$

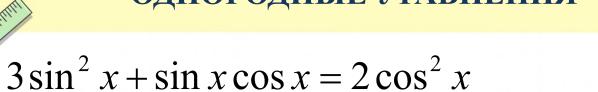
$$+ t - 2 = 0$$

$$x_1 = -arctg1 + \pi n$$

$$x - 2 = 0$$
 $x_1 = -arctg1 + \pi n$
 $+ 24 = 25$ $x_1 = -\frac{\pi}{4} + \pi n, n \in \mathbb{Z}$

$$3t^{2} + t - 2 = 0 x_{1} = -arctg1 + \pi n$$

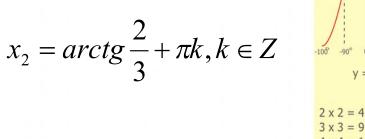
$$D = 1 + 24 = 25 \pi$$











$$\frac{a}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^{\circ} = 1$$

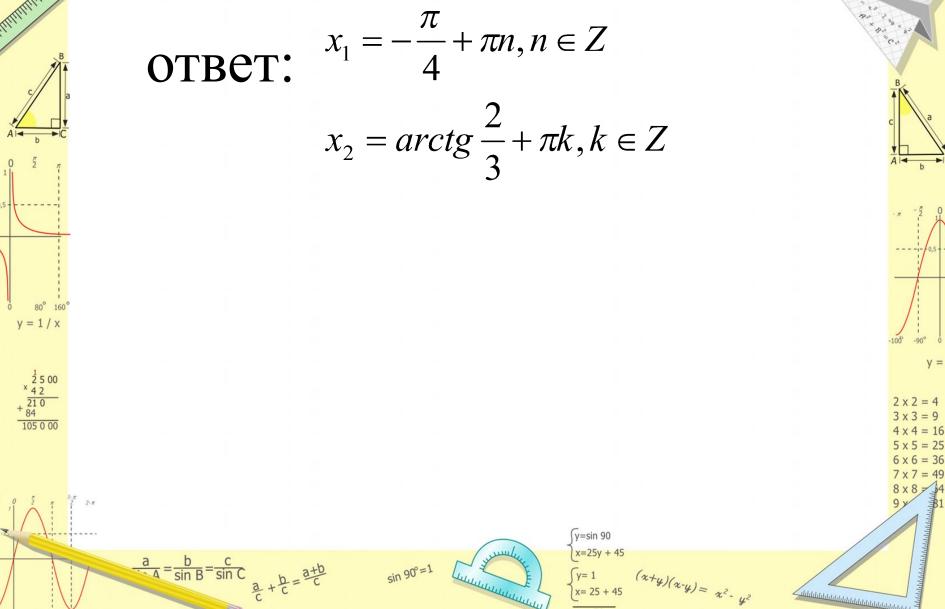
$$\sin 90^{\circ} = 1$$

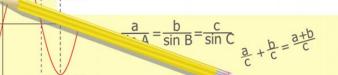
$$\sin 90^{\circ} = 1$$

$$\sin 90^{\circ} = 1$$

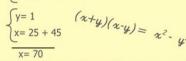
$$\cos x = 25 + 45$$

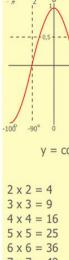
$$(x+y)(x-y) = x^{\circ}.$$











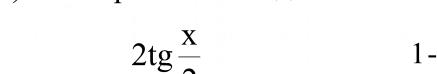
УРАВНЕНИЯ ВИДА: $A \cos x + B \sin x = C$, $A, B, C \neq 0$

методы:

1) Универсальная подстановка

$$2tg\frac{X}{2}$$

становка
$$\cos x = \frac{1 - tg^2 \frac{x}{2}}{1 + tg^2 \frac{x}{2}}; \quad \Pi$$
роверка обязательна!



$$a \cos x + b \sin x$$
 заменим на $C \sin(x+\phi)$, где $C = \sqrt{a^2 + b^2}$;

$$y = 2 \times 2 = 4$$
 $3 \times 3 = 9$

$$sin\phi = \frac{a}{C};$$
 $cos\phi = \frac{b}{C};$ ϕ - вспомогательный аргумент.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^{\circ} = 1$$

$$x = 25 + 45$$

$$x = 70$$

$$x = 30$$

$$x = 30$$

$$x = 30$$

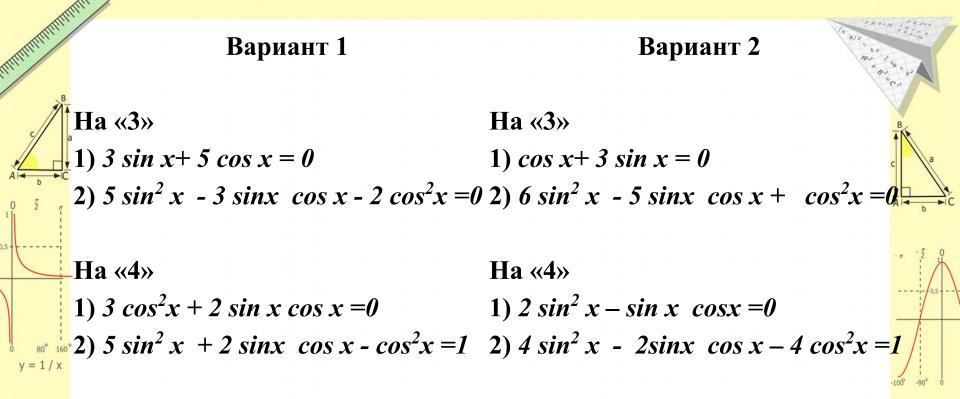
$$x = 30$$

$$x = 70$$

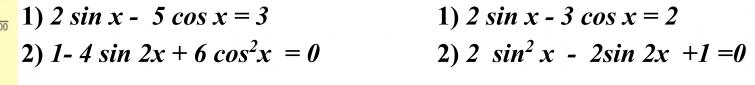
$$x = 30$$

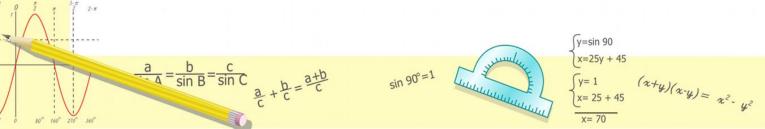
$$x = 70$$

САМОСТОЯТЕЛЬНАЯ РАБОТА



Ha **«5»**

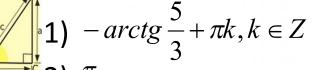




Ha **«**5»

Вариант 1

Вариант 2



$$K \in \mathbb{Z}$$

$$) - arctg - + \pi k, k \in Z$$

$$\frac{\pi}{4} + \pi k$$
, $-arctg 0$, $4 + \pi k$

1)
$$-arctg \frac{1}{3} + \pi k, k \in \mathbb{Z}$$

2) $arctg \frac{1}{3} + \pi k, arctg \frac{1}{2} + \pi n, k, n \in \mathbb{Z}$

2)
$$\frac{\pi}{4} + \pi k, -arctg \ 0, 4 + \pi n, k, n \in \mathbb{Z}$$

1)
$$\pi k, arctg \frac{1}{2} + \pi n, k, n \in \mathbb{Z}$$

1)
$$\frac{\pi}{2} + \pi k, -arctg1, 5 + \pi n, k, n \in \mathbb{Z}$$

2)
$$-\frac{\pi}{4} + \pi k, arctg \frac{5}{3} + \pi n, k, n \in \mathbb{Z}$$

2)
$$-\frac{\pi}{4} + \pi k, arctg = 0.5 + \pi n, k, n \in \mathbb{Z}$$

$$\in Z$$

$$\in Z$$

1)
$$-2arctg5 + 2\pi k, \frac{\pi}{2} + 2\pi n, k, n \in \mathbb{Z}$$

2) $\frac{\pi}{2} + \pi k, arctg \frac{1}{2} + \pi n, k, n \in \mathbb{Z}$

1)
$$2arctg(-1 \pm \sqrt{5}) + 2\pi k, k \in \mathbb{Z}$$
 1) $-2arctg5 + 2\pi k, \frac{\pi}{2} + 2\pi n, k$
2) $\frac{\pi}{4} + \pi k, arctg7 + \pi n, k, n \in \mathbb{Z}$ 2) $\frac{\pi}{4} + \pi k, arctg\frac{1}{3} + \pi n, k, n \in \mathbb{Z}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$