ТЕМА:

Пределы

ВИДЫ НЕОПРЕДЕЛЕННОСТИ:

$$\frac{0}{0}, \qquad \frac{\infty}{\infty}, \qquad \infty - \infty, \infty \cdot \mathbf{0}$$

При пределах:

$\alpha(0)$-бесконечно малое значение

∞- бесконечно большое значение

a- постоянная величина

$$\frac{0}{a} = 0, \qquad \frac{a}{0} = \infty, \qquad \frac{a}{\infty} = 0, \qquad \frac{\infty}{a} = \infty$$

- $$"\frac{\mathbf{0}}{\mathbf{0}}"$$

Примеры.

1)
$$\lim_{x\to-2}\frac{x^3+4x^2+4x}{(x+2)(x-3)}=\lim_{x\to-2}\frac{x(x+2)^2}{(x+2)(x-3)}=\lim_{x\to-2}\frac{x(x+2)}{x-3}=\frac{-2(-2+2)}{-2-3}=\frac{0}{-5}=0\ ;$$

2) $$\lim_{x\to0}\frac{\sqrt[3]{1+mx}-1}{x}=\left|\begin{matrix}1+mx=t^3\\ x=\frac{t^3-1}{m}\\ t\to1\end{matrix}\right|=$$

$$\lim_{t\to1}\frac{m(t-1)}{t^3-1}=\lim_{t\to1}\frac{m(t-1)}{(t-1)(t^2+t+1)}=\lim_{t\to1}\frac{m}{t^2+t+1}=\frac{m}{3};$$

3)

$$\lim_{x\to\pi}\frac{\sqrt{1-tgx}-\sqrt{1+tgx}}{sin2x}=\lim_{x\to\pi}\frac{(\sqrt{1-tgx}-\sqrt{1+tgx})(\sqrt{1-tgx}+\sqrt{1+tgx})}{sin2x(\sqrt{1-tgx}+\sqrt{1+tgx})}=$$

$$\lim_{x\to\pi}\frac{(1-tgx)-(1+tgx)}{sin2x(\sqrt{1-0}+\sqrt{1+0})}=\lim_{x\to\pi}\frac{-2tgx}{2\cdot 2sinxcosx}=-\lim_{x\to\pi}\frac{1}{2cos^2x}=-\frac{1}{2}.$$

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$$"\frac{\infty}{\infty}"$$

$$4)\ \lim_{x\to\infty}\frac{x^4-5x}{x^2-3x+1}=\lim_{x\to\infty}\frac{\frac{x^4}{x^4}-\frac{5x}{x^4}}{\frac{x^2}{x^4}-\frac{3x}{x^4}+\frac{1}{x^4}}=$$

$$\lim_{x\to\infty}\frac{1-\frac{5}{x^3}}{\frac{1}{x^2}-\frac{3}{x^3}+\frac{1}{x^4}}=\frac{1-\frac{5}{\infty}}{\frac{1}{\infty}-\frac{3}{\infty}+\frac{1}{\infty}}=\frac{1}{0}=\infty;$$

$$5)\ \lim_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}=\lim_{x\to\infty}\frac{\frac{\sqrt{x}}{x}-\frac{6x}{x}}{\frac{3x}{x}+\frac{1}{x}}=$$

$$\lim_{x\to\infty}\frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}}=\frac{\frac{1}{\infty}-6}{3+\frac{1}{\infty}}=\frac{0-6}{3+0}=-2;$$

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"∞−∞"

6) $\lim_{x\to+\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2-x}\right)=\lim_{x\to+\infty}\frac{x^2+x+1-x^2+x}{\sqrt{x^2+x+1}+\sqrt{x^2-x}}=$

$\lim_{x\to+\infty}\frac{2x+1}{\sqrt{x^2+x+1}+\sqrt{x^2-x}}=\lim_{x\to+\infty}\frac{2+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}}}=1$

$$"\mathbf{0} \cdot \infty"$$

$$7)\ \lim_{x\to 1}(1-x)tg\frac{\pi}{2}x = \begin{vmatrix} 1-x=t \\ x=1-t \\ t\to 0 \end{vmatrix} = \lim_{t\to 0} t\cdot tg\left(\frac{\pi}{2}-\frac{\pi t}{2}\right) =$$

$$\lim_{t\to 0} t\cdot ctg\frac{\pi t}{2} = \lim_{t\to 0}\frac{t}{sin\frac{\pi t}{2}}\cdot cos\frac{\pi t}{2} = \frac{2}{\pi};$$