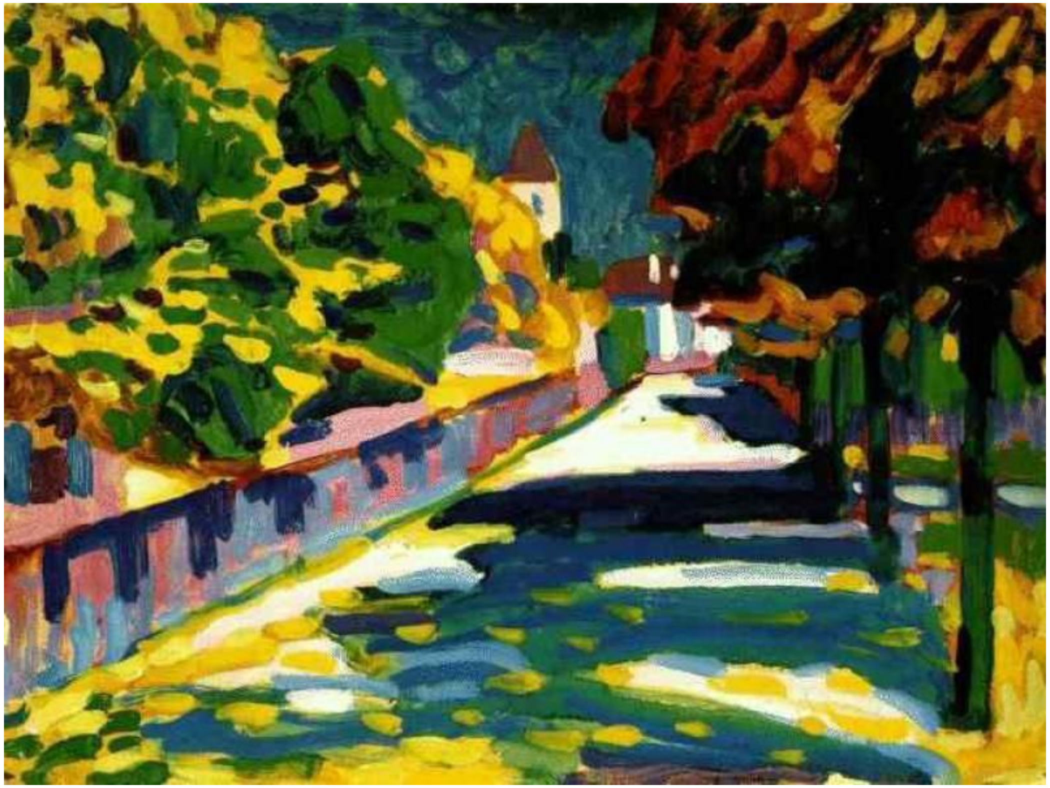

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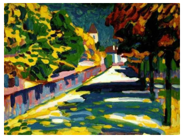
Moscow University
Risk Management

Class #8 – Linear Risks
Identification and Sensitivity Analysis

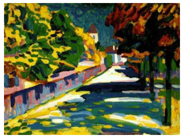
Lecturer: Luis A. B. G. Vicente

November/2015

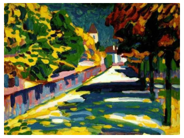


Class #8 – Linear Risks

1	Definition of risk factors
2	Linear decomposition of financial instruments into risk factors
4	Sensitivity analysis: single instruments and portfolios



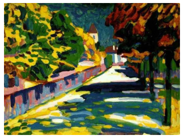
Class #8 – Linear Risks



Definition of risk factors and risk exposures

Risk factors

- Prices of financial instruments can be defined by a number of market or risk factors
- These factors, in turn, are assumed to determine the expected return on an investment
- More generically, we have $\Delta P = f(\Delta RF_1, \Delta RF_2, \dots, \Delta RF_n)$ where:
  - ΔP is the change in the price of the asset/instrument
  - $f(.)$ is price sensitivity function
  - $\Delta RF_1, \Delta RF_2, \dots, \Delta RF_n$ are the changes in the relevant risk factors



Definition of risk factors and risk exposures

Risk factors

- The function $f(.)$ is derived from some convenient pricing argument
  - Non-arbitrage pricing
  - CAPM
- Depending on the model (e.g. CAPM), ΔP needs to be interpreted as an expected change
- Important to notice that $f(.)$ corresponds to the total derivative of the price function $F(.)$

Definition of risk factors and risk exposures

Exposures

Definition: the financial amount that is exposed to a unit change given a relevant risk factor

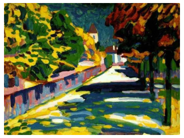
Mathematically, $E_{RF} = P \times \frac{\partial P}{\partial RF}$

Suppose the price of an hypothetical instrument is given by $P = 3 \times RF1 - 1.5 \times RF2 + 5$

Then, for a RUB 100 investment we would have:

$E_{RF1} = P \times \frac{\partial P}{\partial RF1}$=300 — This instrument entails an exposure of 300 RUB in RF1

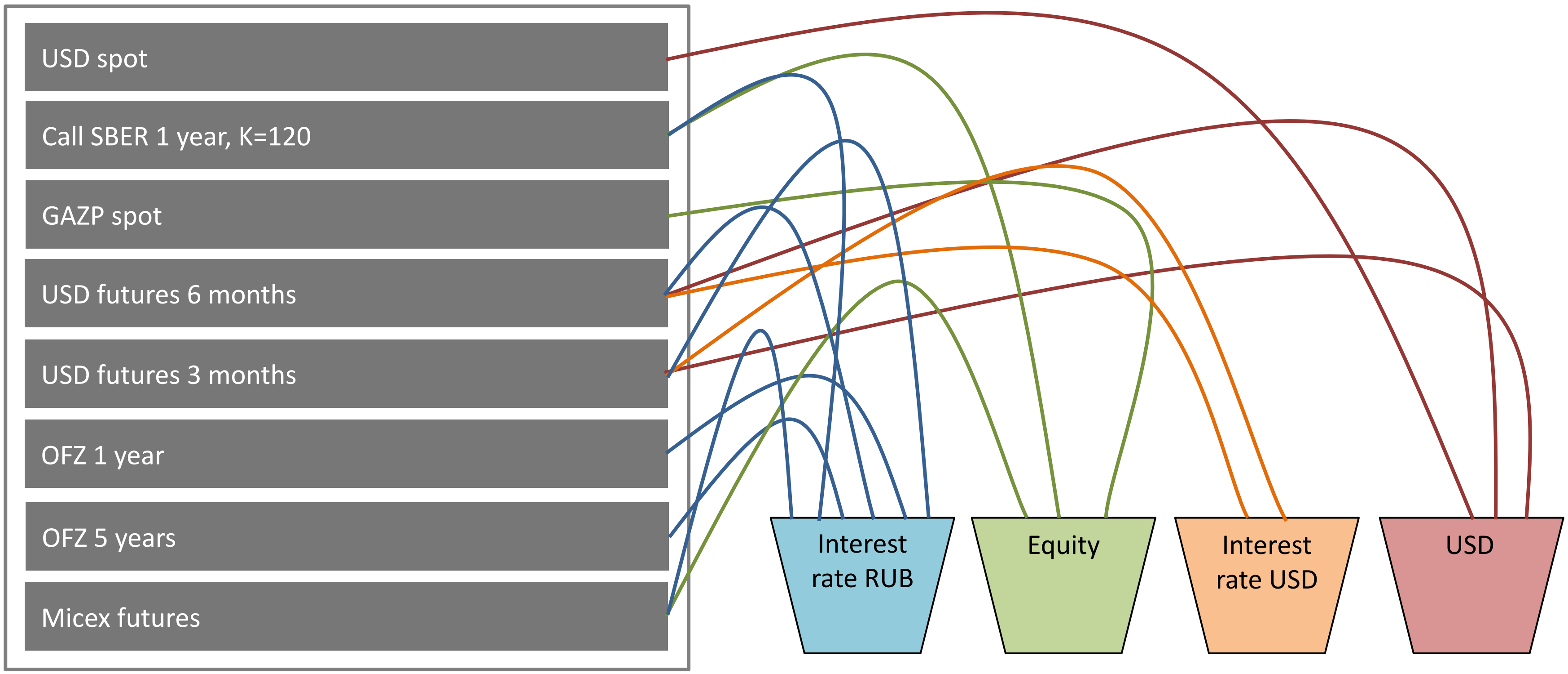
$E_{RF2} = P \times \frac{\partial P}{\partial RF2}$=-150 — This instrument entails an exposure of -150 RUB in RF2

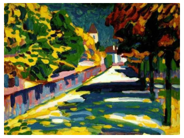


Definition of risk factors and risk exposures

Why is this concept so important for risk management?

It allows us to represent a large portfolio comprising different financial instruments by the means of a limited number of risk factors



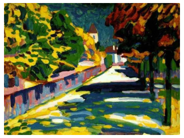


Class #8 – Linear Risks

1 Definition of risk factors and risk exposures

2 Linear decomposition of financial instruments into risk factors

3 Sensitivity analysis: single instruments and portfolios



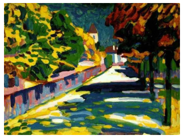
Linear decomposition of financial instruments

The general model

The rate of return of an asset is a random variable driven by a linear combination of other random variables plus noise

$$\Delta P = E_1 \times \Delta RF_1 + E_2 \times \Delta RF_2 + \cdots + E_n \times \Delta RF_n + \psi$$

ψ correspond to changes that cannot be explained by $\Delta RF_1, \Delta RF_2, \ldots, \Delta RF_n$



Linear decomposition of financial instruments

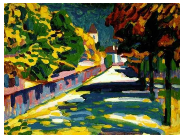
Interest rates

- We have that $P = V \times e^{-rt}$
- $\Delta P = -t \times V \times e^{-rt}, E_r = -t \times P$
- Important link: concept of duration
- Different compounding rules lead to different first order derivatives
- Suppose we have a debt with face value equal to 100 that matures in 6 months and r=10% pa
  - $E_r = -50$ — Does this make any sense?

Linear decomposition of financial instruments

Interest rates

YLD SHFT	S/A Reinv	Pricing at 11/20/15 Traded to			HORIZON SPRD*	Yield	Price	Mod Duration Bond	BMRK	%PROB
-150	5.77	AVGL	8/16/21	100	-267.5	5.770	114.13	4.37	4.14	
-100	6.27	AVGL	8/16/21	100	-267.5	6.270	111.64	4.34	4.11	
-50	6.77	AVGL	8/16/21	100	-267.5	6.770	109.23	4.31	4.08	0.0
0	7.27	AVGL	8/16/21	100	-267.5	7.270	106.89	4.27	4.05	100.0
50	7.77	AVGL	8/16/21	100	-267.5	7.770	104.61	4.24	4.02	0.0
100	8.27	AVGL	8/16/21	100	-267.5	8.270	102.41	4.21	3.99	
150	8.77	AVGL	8/10/21	99.842	-267.5	8.770	100.17	4.18	3.96	
ExVal	7.27				-267.5	7.270	106.89	4.27	4.05	



Linear decomposition of financial instruments

Equities - CAPM

We have that $\Delta P = \alpha + \beta \times \Delta rm + \varepsilon$

$E_{rm} = \beta \times P$

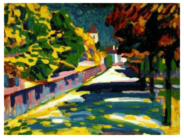
Important link: multifactor models

We can have multiple linear factors $\beta_1, \beta_2, \beta_3, \ldots$

Suppose we have a share with price equal to 50 and β equal to 1.2

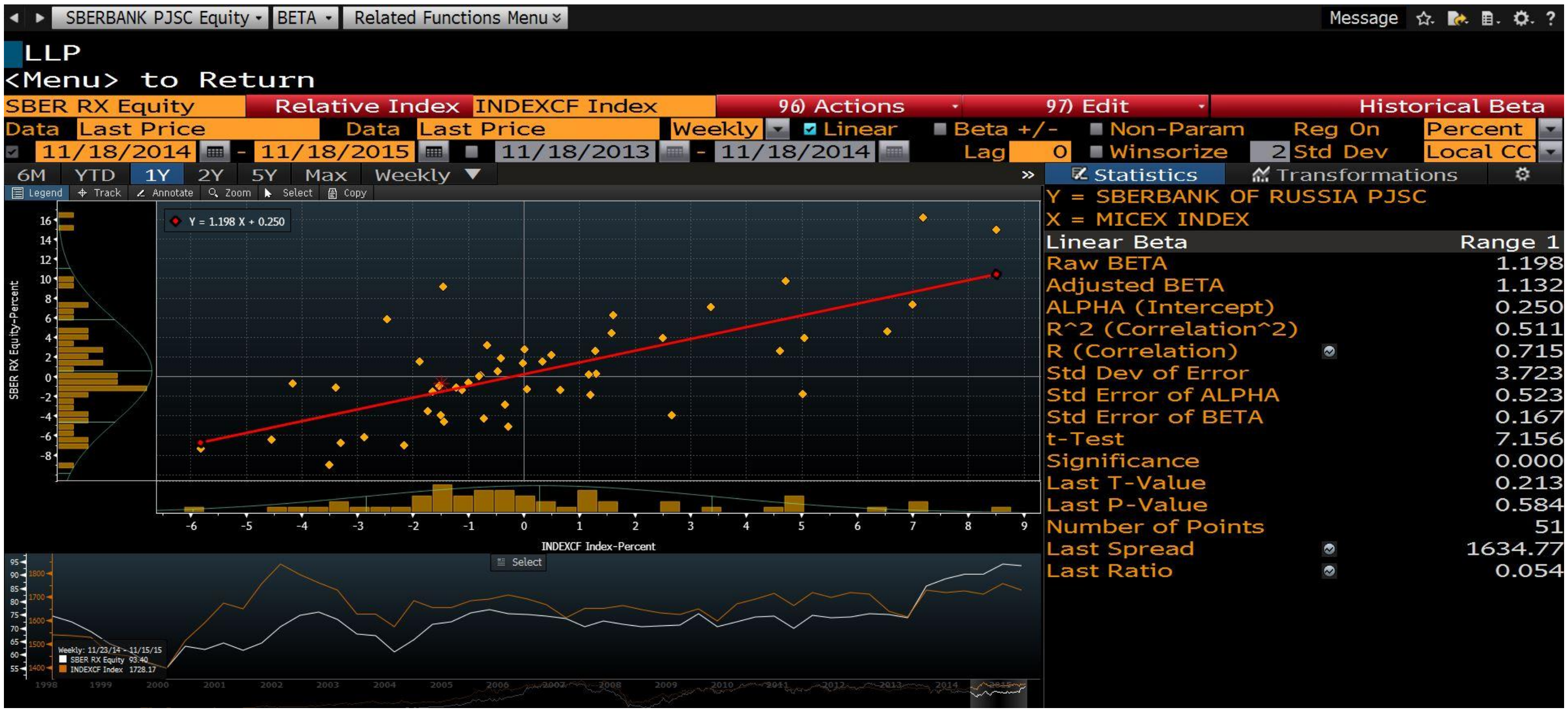
$E_r = 60$

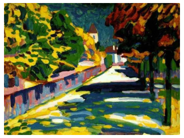
Does this make any sense?



Linear decomposition of financial instruments

Equities - CAPM





Linear decomposition of financial instruments

Derivatives – Black Scholes

From our previous class:

$$c = S \times N(d1) - K \times e^{-rt} \times N(d2)$$

$$p = K \times e^{-rt} \times N(-d2) - S \times N(-d1)$$

Delta $\Delta = \frac{\partial V}{\partial S}$ is the sensitivity to asset price.

Gamma $\Gamma = \frac{\partial^2 V}{\partial^2 S}$ is the sensitivity of delta to S

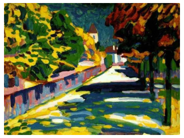
Theta $\Theta = \frac{\partial V}{\partial t}$ is time rate of change of V

Vega $v = \frac{\partial V}{\partial \sigma}$ is the sensitivity of V to sigma (volatility)

Rho $\rho = \frac{\partial V}{\partial r}$ is the sensitivity of V to the interest rate

$$d1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \sigma^2/_2\right)t}{\sigma\sqrt{t}}$$

$$d2 = \frac{ln\left(\frac{S}{K}\right) + \left(r - \sigma^2/_2\right)t}{\sigma\sqrt{t}} = d1 - \sigma\sqrt{t}$$



Linear decomposition of financial instruments

Derivatives – Black Scholes

We know that $\Delta C = N(d1)$ and $\Delta P = N(d1) - 1$

$E_s = \Delta C \times S$

Other derivatives and orders play an important role too

Suppose we call option with ΔC=0.5 and the underlying price is 80

$E_s = 40$

Does this make any sense?

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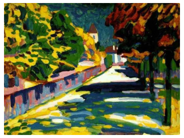


Linear decomposition of financial instruments

Derivatives – Black Scholes

Sberbank futures Dec15 Index | OMON | Related Functions Menu | Msg: +2

SBSZ5 ↓10332.00 -18.00 10332.00/10335.00 71x2 Prev 10350.0(

At 10:29d Vol 348742 Op 10345.00 Hi 10512.00 Lo 10256.00 OpenInt 943710

SBSZ5 Index 95) Actions 97) Settings Option Monitor

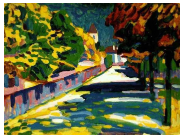
Sberbank futures Dec15 ↓10332.00 -18.00 -.1739% 10332.00 / 10335.00 Hi 10512.00 Lo 10256.00 Volm 348742 HV 36.30

Center 10309.00 Strikes 5 Exp Nov-15 on SBSZ5 Exch 92) Earnings Calendar | ACDR»

Calc Mode

81) Center Strike | 82) Calls/Puts | 83) Calls | 84) Puts | 85) Term Structure | 86) Straddle

Exp	Nov-15 on SBSZ5				Dec-15 on SBSZ5				Jan-16 on SBSH6				Mar-16 on SBSH6			
Calls/Puts	Calls		Puts		Calls		Puts		Calls		Puts		Calls		Puts	
Strike	DM	IVM	DM	IVM	DM	IVM	DM	IVM	DM	IVM	DM	IVM	DM	IVM	DM	IVM
7750							-.03	57.23								
8000							-.03	53.61								
8250							-.02	43.51								
8500							-.06	46.60							-.17	46.56
8750					.75	119.65	-.08	47.48							-.20	45.78
9000			-.01	196.49			-.11	44.25								
9250			-.13	335.97	.96	25.36	-.15	42.48			-.27	78.48				
9500	.84	294.70	-.03	156.46			-.20	40.44							-.28	46.80
9750			-.05	116.18	.74	35.17	-.27	38.94								
10000	.90	83.37	-.10	85.56	.64	38.04	-.35	37.09								
10250			-.37	75.15	.55	35.59	-.44	35.87								
10500					.45	34.73	-.50	60.43								
10750					.35	34.19										
11000					.25	33.59										
11250					.18	34.01										
11500					.13	34.69			.32	41.29						
11750					.09	34.41										
12000					.06	35.77										
12250													.27	38.81		

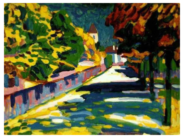


Class #8 – Linear Risks

1 Definition of risk factors and risk exposures

2 Linear decomposition of financial instruments into risk factors

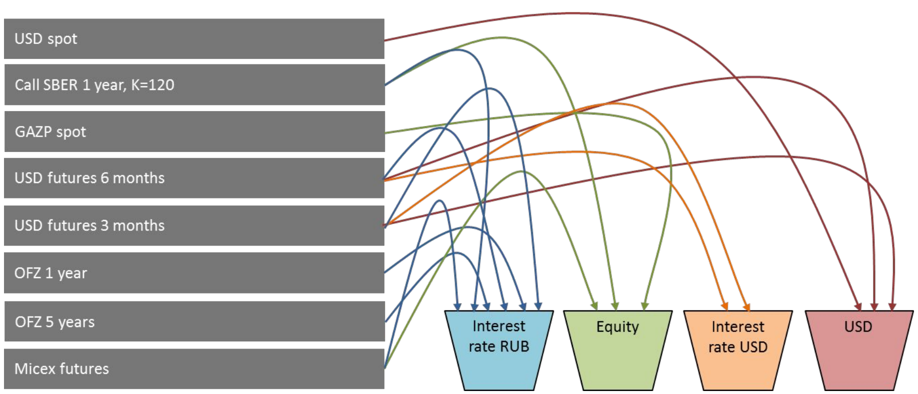
3 **Sensitivity analysis: single instruments and portfolios**

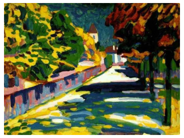


Sensitivity analysis: single instruments and portfolios

Sensitivity analysis – classical “What if” analysis

- Single instrument
  - What is the expected price change given a change in one or more risk factors?
- Portfolio
  - What is the expected change in market value given a change in one or more risk factors?
- Key tool – risk factor “bucketing”





Sensitivity analysis: single instruments and portfolios

Sensitivity analysis – classical "What if" analysis

Suppose that you have a portfolio of 5 stocks:

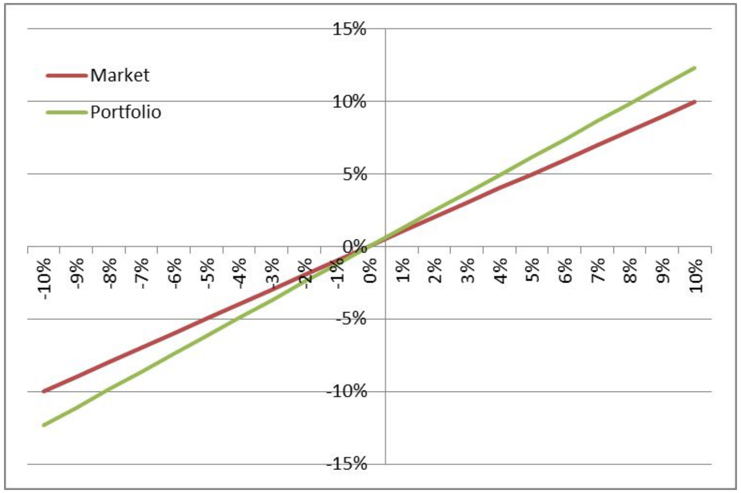
- A – Price 85.00, quantity 300, β equal to 0.75
- B – Price 5.70, quantity 2 000, β equal to 1.54
- C – Price 33.05, quantity 800, β equal to 0.88
- D – Price 12.70, quantity 800, β equal to 0.54
- E – Price 122.00, quantity 500, β equal to 1.20

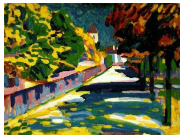
Sensitivity analysis: single instruments and portfolios

Sensitivity analysis – classical “What if” analysis

So we can represent this portfolio as an aggregate exposure to the overall equity market

	Price	Quantity	β	Financial Volume	Exposure		Weight	β
A	85.00	300	0.75	25,500.00	19,125.00		0.11	0.09
B	50.09	2000	1.54	100,180.00	154,277.20		0.45	0.69
C	33.05	800	0.88	26,440.00	23,267.20		0.12	0.10
D	12.70	800	0.54	10,160.00	5,486.40		0.05	0.02
E	122.00	500	1.20	61,000.00	73,200.00		0.27	0.33
				223,280.00	275,355.80		1.00	1.23





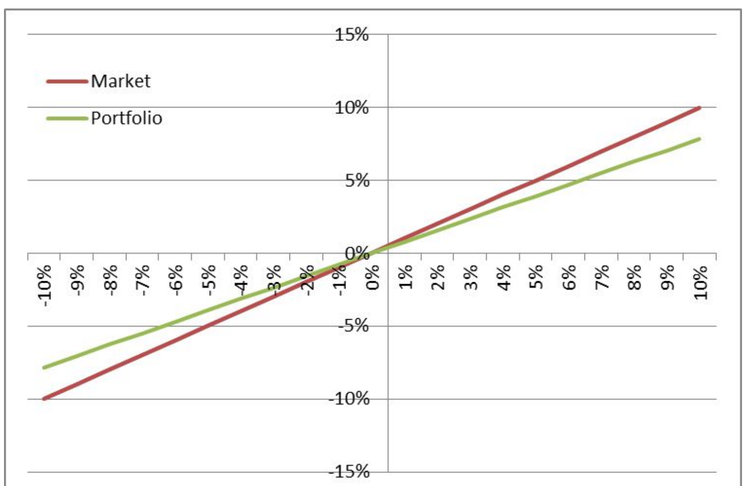
Sensitivity analysis: single instruments and portfolios

Sensitivity analysis – classical “What if” analysis

Suppose we want a more “defensive” portfolio

	Price	Quantity	β	Financial Volume	Exposure
A	85.00	300	0.75	25,500.00	19,125.00
B	50.09	0	1.54	0.00	0.00
C	33.05	800	0.88	26,440.00	23,267.20
D	12.70	8,688	0.54	110,340.12	59,583.67
E	122.00	500	1.20	61,000.00	73,200.00
				223,280.12	175,175.87

Weight	β
0.11	0.09
0.00	0.00
0.12	0.10
0.49	0.27
0.27	0.33
1.00	0.78



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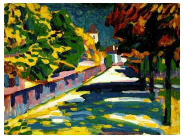


Sensitivity analysis: single instruments and portfolios

Sensitivity analysis – classical “What if” analysis

Now suppose we have a Call on B, with Δ equal to 0.50

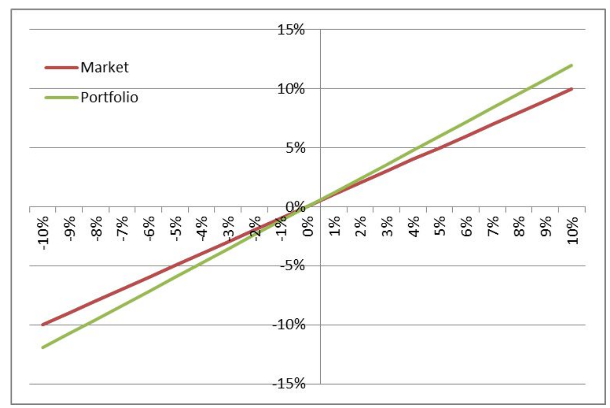
Hence, for every 1 RUB change in the price o B, the price of the call changes by 0.50 RUB

So by selling 1 000 options (going short) you could reduce your position in B by 25%

$$E_B = E_{S\_B} + E_{O\_B} = 2\,000 \times 50.09 - 1\,000 \times 50.09 \times 0.5 = 75\,139.00$$

This can be further translated into another index-equivalent exposure of 115 707.90

Weight	β
0.13	0.10
0.38	0.58
0.13	0.12
0.05	0.03
0.31	0.37
1.00	1.19



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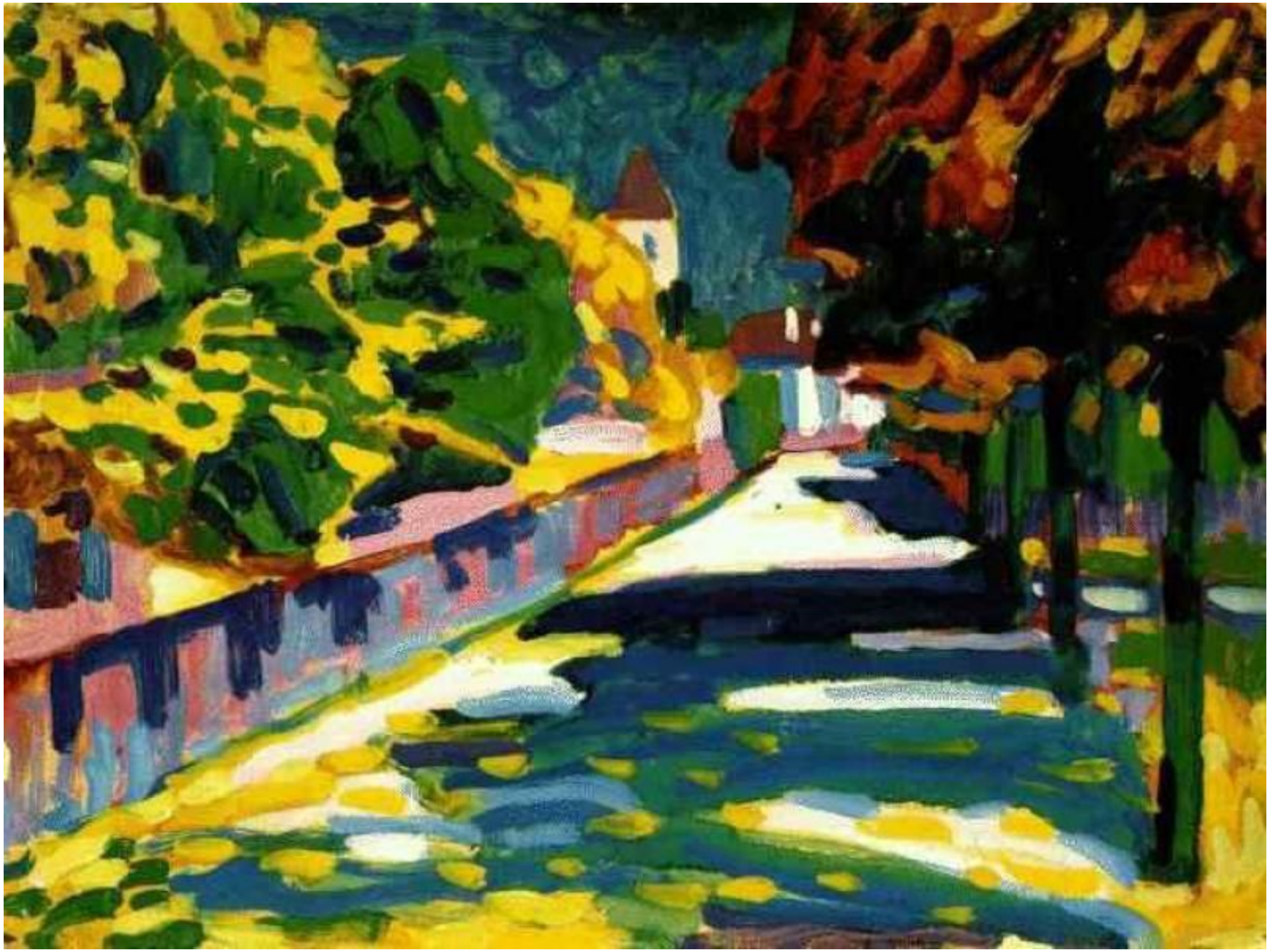

Vassily Vassilyevich Kandinsky, *Autumn in Bavaria*, 1908