$\forall \Box, P \rightarrow \neg \Box P$

Kurt

Gödel

28|4 1906

14|1 1978


"~~Materialism~~ is false"
- K. Gödel

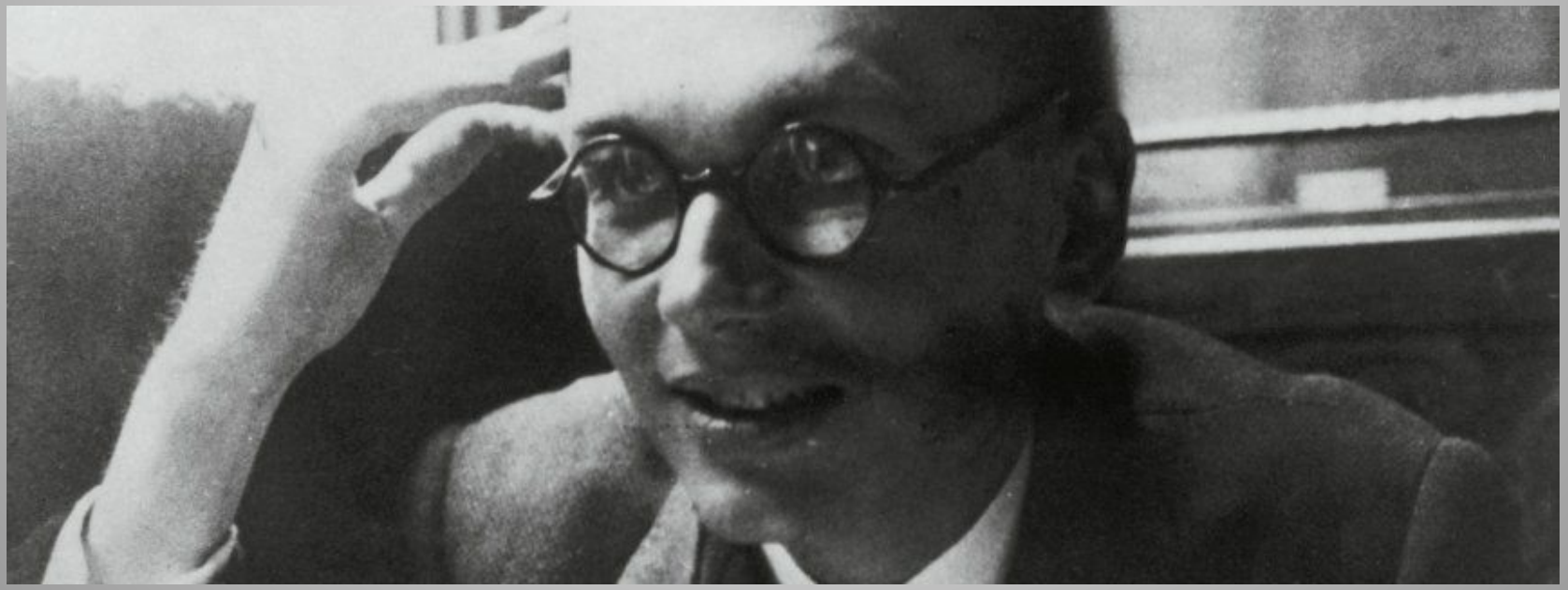
Brünn, Austria – Hungary (now: Brno, Czech Republic)

In his family young Kurt was known as *Herr Warum* ("Mr. Why") because of his insatiable curiosity.

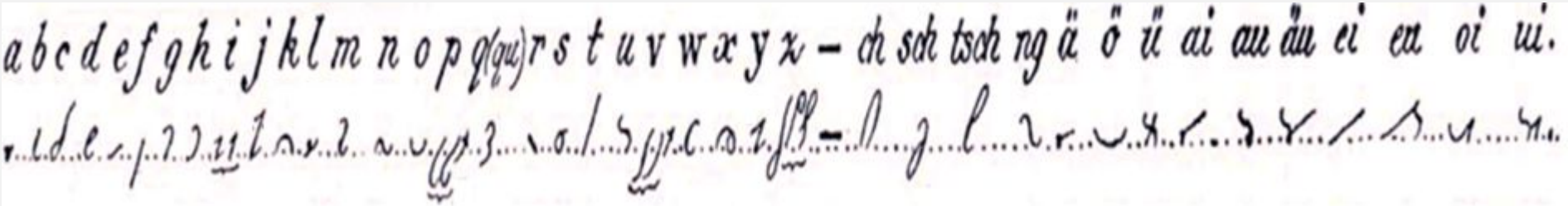
University of Vienna

- **Studied theoretical physics, Gödel also attended courses on mathematics and philosophy.**

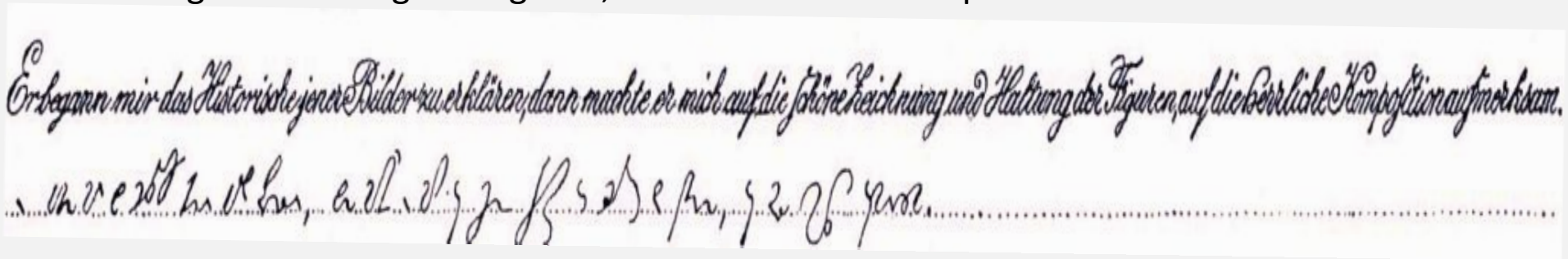
- **He was fond of Gabelsberger shorthand, criticismus of Isaac Newton, the writings of Immanuel Kant and participated in the Vienna philosophical circle.**



Gabelsberger shorthand



Er begann mir das Historische jener Bilder zu erklären, dann machte er mich auf die schöne Zeichnung und Haltung der Figuren, auf die herrliche Komposition aufmerksam.



I don't believe in empirical science. I only believe in a priori truth.

— Kurt Gödel —

Completeness Theorem

- **At the age of 23 Gödel completed his doctoral dissertation.**
- **In it he established the completeness of the first-order predicate calculus.**

Incompleteness Theorem

- **In 25 years Gödel published one of the greatest scientific achievements of the XX century.**
- **He proved for any computable axiomatic system, that is powerful enough to describe the arithmetic of the natural numbers (e.g., the Peano axioms or ZFC theory), that:**

  1. If a formal axiomatic system is consistent, it can't be complete. [I.e., there are true sentence in this system's language that can't be proved within it.]

  2. The consistency of axioms can't be proved within their own system. [I.e., the sentence that confirms the consistency of this system is unprovable in it.]

- **These theorems are directly related to the solution of Hilbert's 2nd problem:**

  they pessimistically ended a half-century Hilbert's attempts to find a set of axioms sufficient for all mathematics.


RECURSION

Recursive Functions

- problems can sometimes be expressed in terms of a simpler instance of the same problem
- Example: factorial
  - 1! = 1
  - 2! = 1 * 2
  - ...
  - (N-1)! = 1 * 2 * 3 * ... * (N-1)
  - N! = 1 * 2 * 3 * ... * (N-1) * N

2! = 1! * 2

N! = (N-1)! * N

21 year old Gödel met his would-be wife, divorced Adele Nimbursky, in a Viennese night club.

She worked there as a dancer and was 27.

The more I think about language, the more it amazes me that people ever understand each other at all.

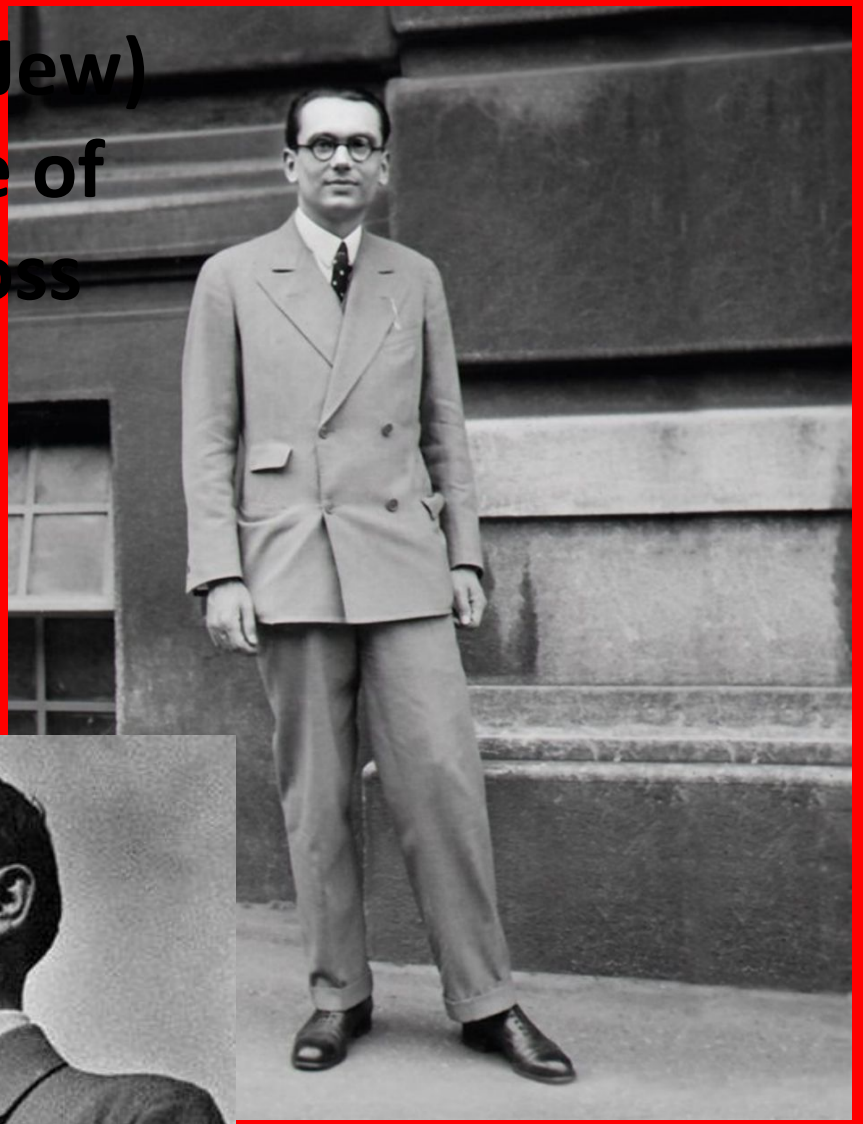
— Kurt Gödel —

Continuum hypothesis & Axiom of choice

- **Gödel proved that the negation of the continuum hypothesis is unprovable in the standard axiomatics of set theory with the axiom of choice (ZFC system), assuming these axioms are consistent.**

- **This result greatly influenced on solution of Hilbert's 1st problem.**

In 1940, Gödel (not being a Jew) left for the U.S. and because of the danger of a journey across the Atlantic traveled there through the USSR (on the Trans-Siberian Railway) and Japan.



- On his U.S. Citizenship exam Gödel had discovered **incompleteness in the U.S. Constitution**

that could allow **the U.S.________________**

to become **a DICTATORSHIP__________**

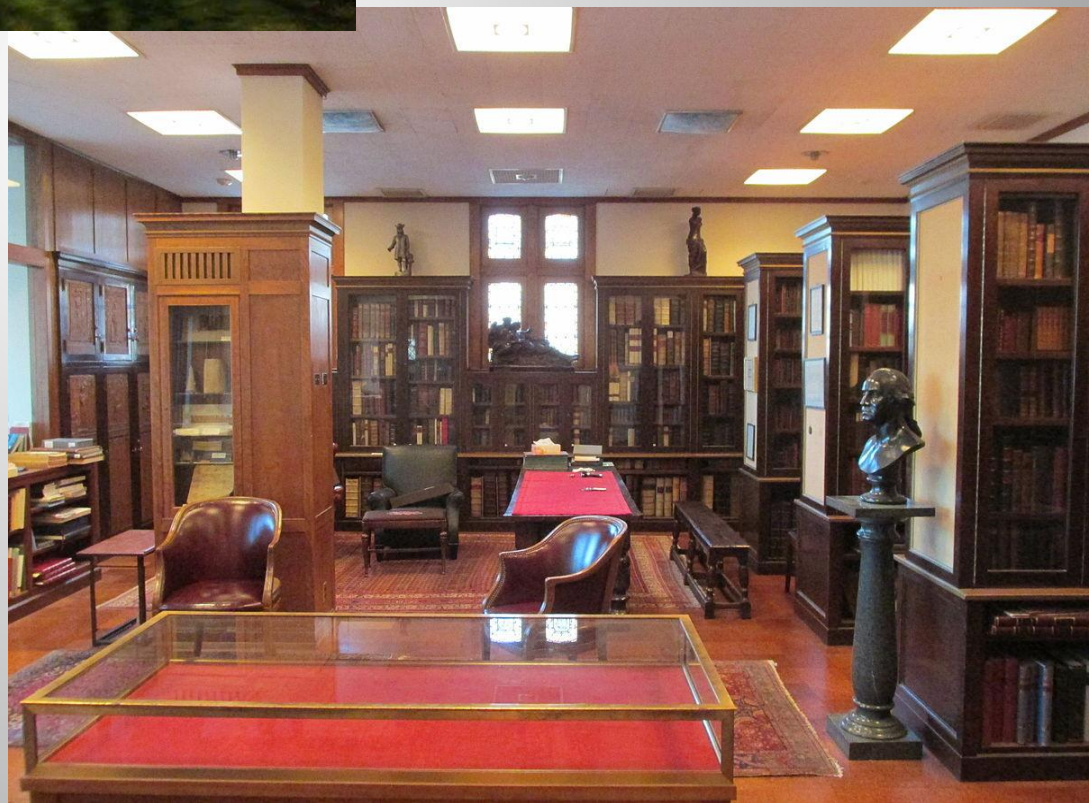
!!

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

Princeton, New Jersey

Institute for Advanced Study

(IAS)

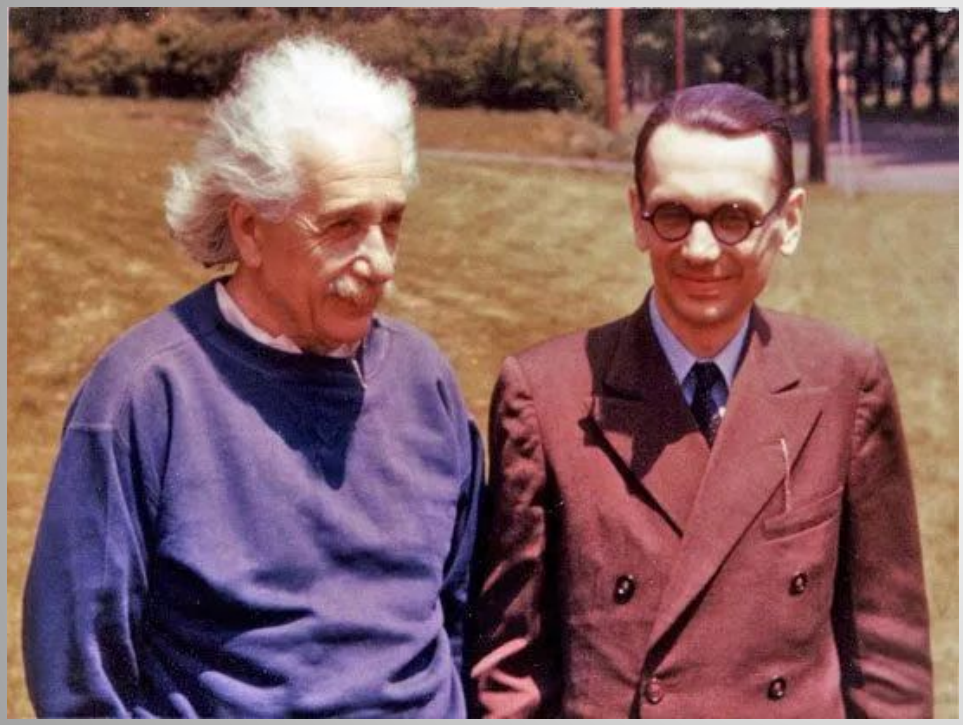


Einstein's field equations & Time travel

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **Gödel demonstrated the existence of solutions, involving closed timelike curves, to Einstein's field equations in general relativity.**
- **His "rotating universes" would allow time travel to the past and caused Einstein to have doubts about his own theory.**
- **His solutions are known as the *Gödel metric* (*an exact solution of the Einstein field equations*).**

Toward the end of his life A. Einstein confided that



his "own work no longer meant much, that he came to the Institute merely ... to have the privilege of walking home with Gödel“.

Gödel's ontological proof of God's existence

Ax. 1. $P(\varphi) \wedge \Box\, \forall x[\varphi(x) \rightarrow \psi(x)] \rightarrow P(\psi)$

Ax. 2. $P(\neg\varphi) \leftrightarrow \neg P(\varphi)$

Th. 1. $P(\varphi) \rightarrow \Diamond\, \exists x\, [\varphi(x)]$

Df. 1. $G(x) \iff \forall\varphi[P(\varphi) \rightarrow \varphi(x)]$

Ax. 3. $P(G)$

Th. 2. $\Diamond\, \exists x\, G(x)$

Df. 2. $\varphi \operatorname{ess} x \iff \varphi(x) \wedge \forall\psi\{\psi(x) \rightarrow \Box\, \forall x[\varphi(x) \rightarrow \psi(x)]\}$

Ax. 4. $P(\varphi) \rightarrow \Box\, P(\varphi)$

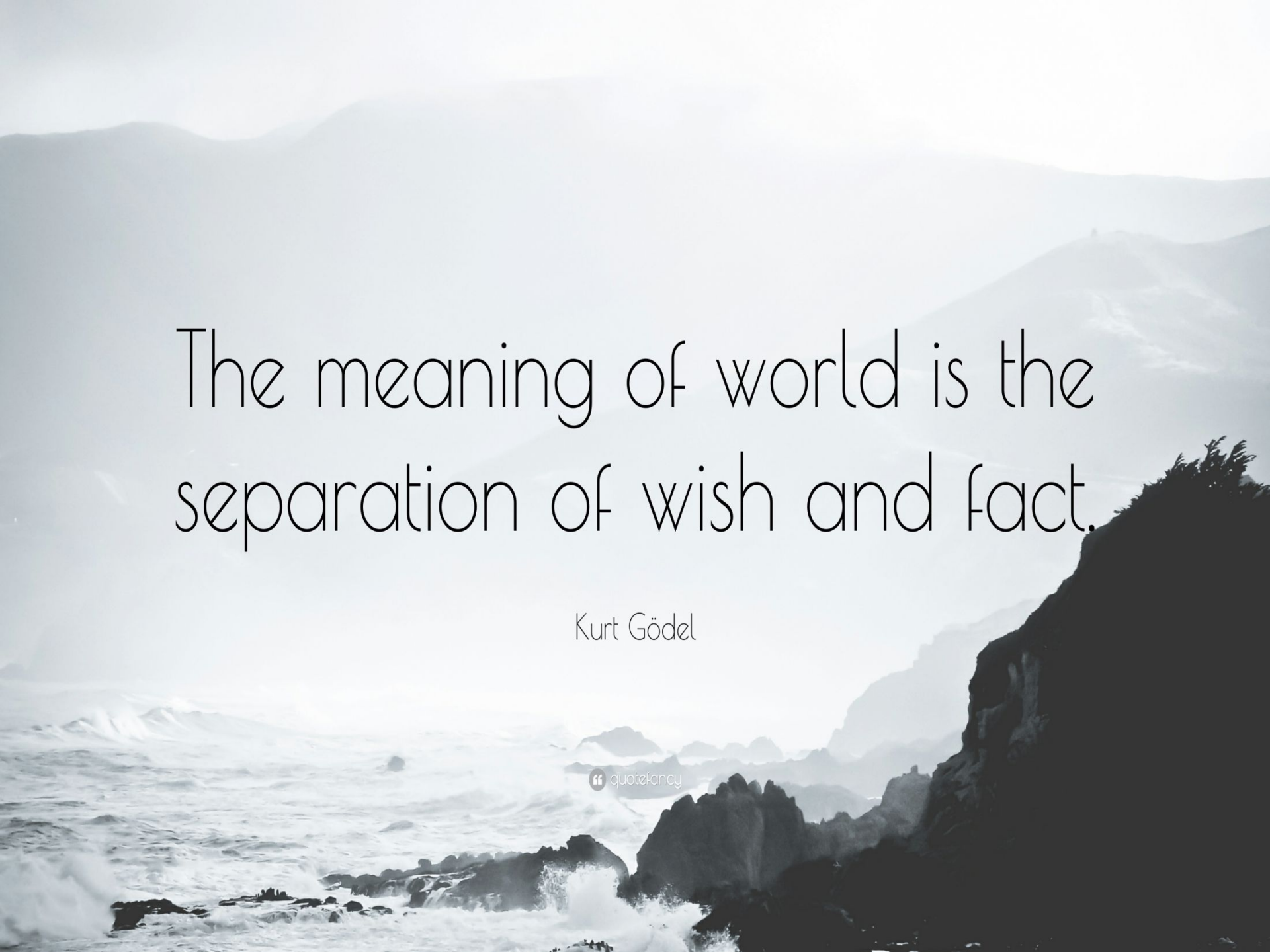
Th. 3. $G(x) \rightarrow G \operatorname{css} x$

Df. 3. $E(x) \iff \forall\varphi[\varphi \operatorname{css} x \rightarrow \Box\, \exists x\, \varphi(x)]$

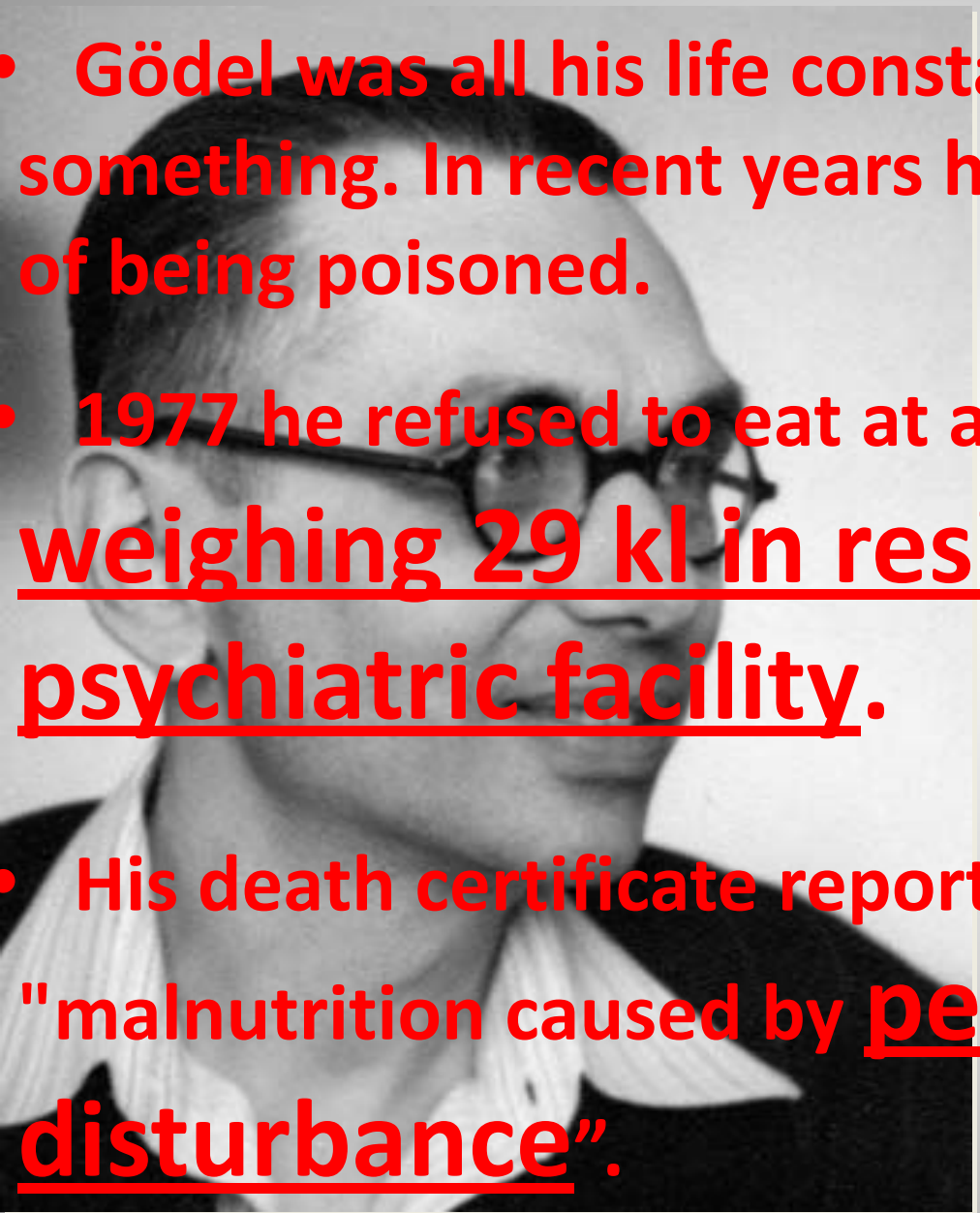
Ax. 5. $P(E)$

Th. 4. $\Box\, \exists x\, G(x)$

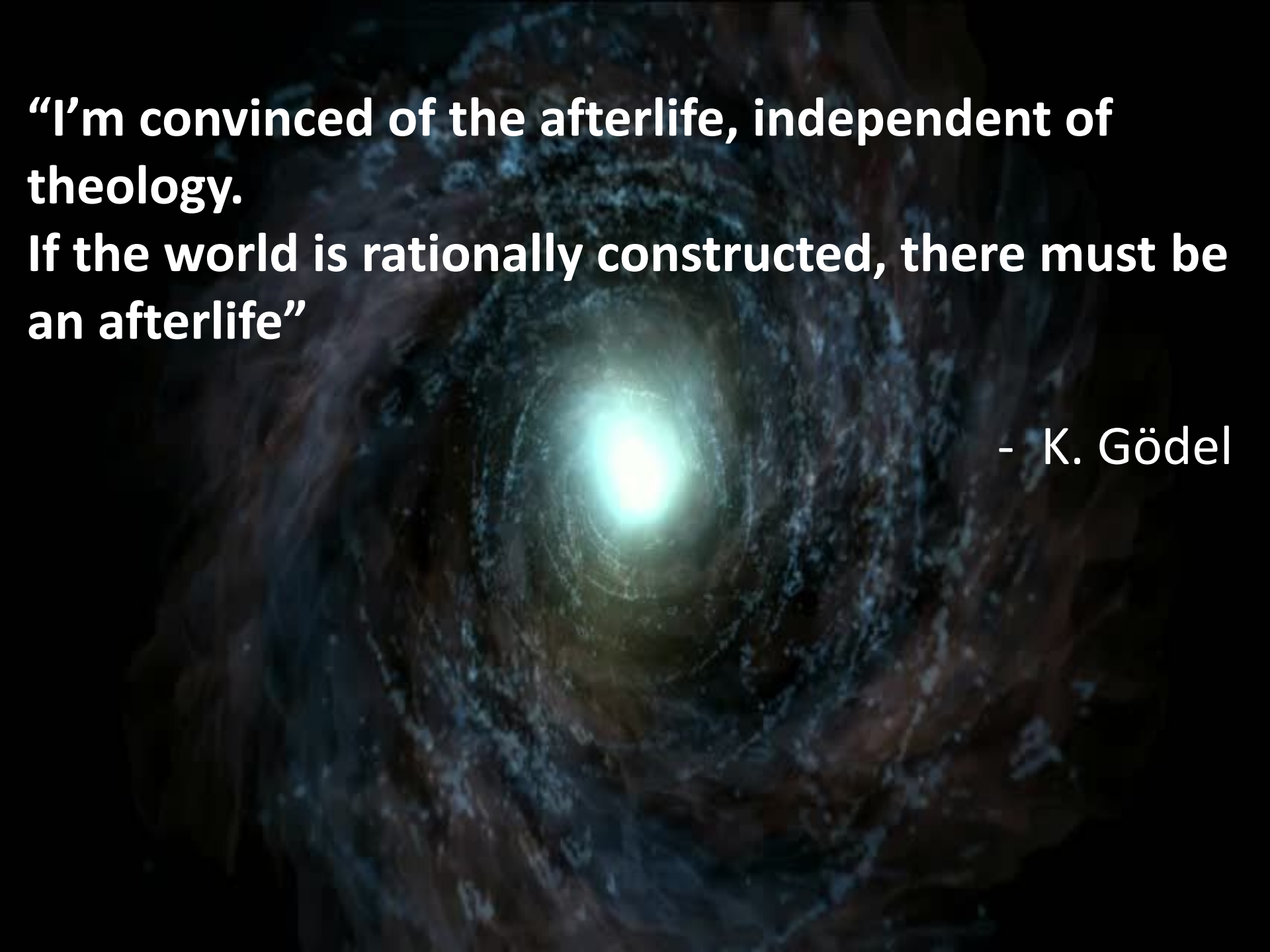
I.e.: ***<u>God exists</u>*.**


The meaning of world is the separation of wish and fact.
Kurt Gödel
quotefancy

~~D E A T H~~ . . . . . .



- Gödel was all his life constantly afraid of something. In recent years he had an obsessive fear of being poisoned.
- 1977 he refused to eat at all and **died weighing 29 kl in residential psychiatric facility**.
- His death certificate reported that he died of "malnutrition caused by **personality disturbance**".



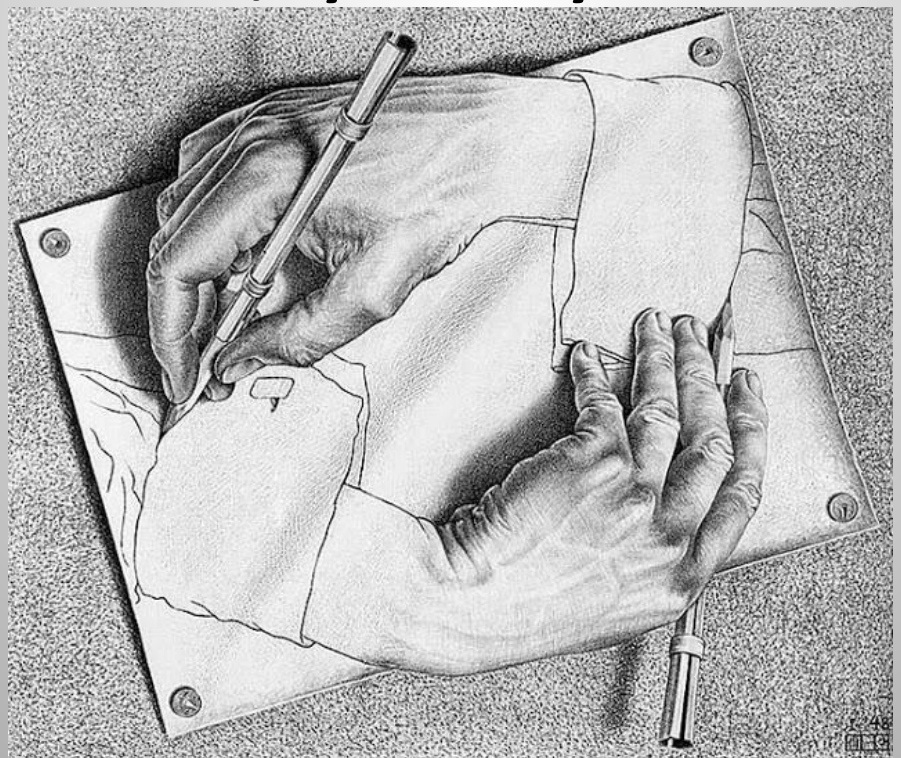
“I’m convinced of the afterlife, independent of theology.
If the world is rationally constructed, there must be an afterlife”

- K. Gödel

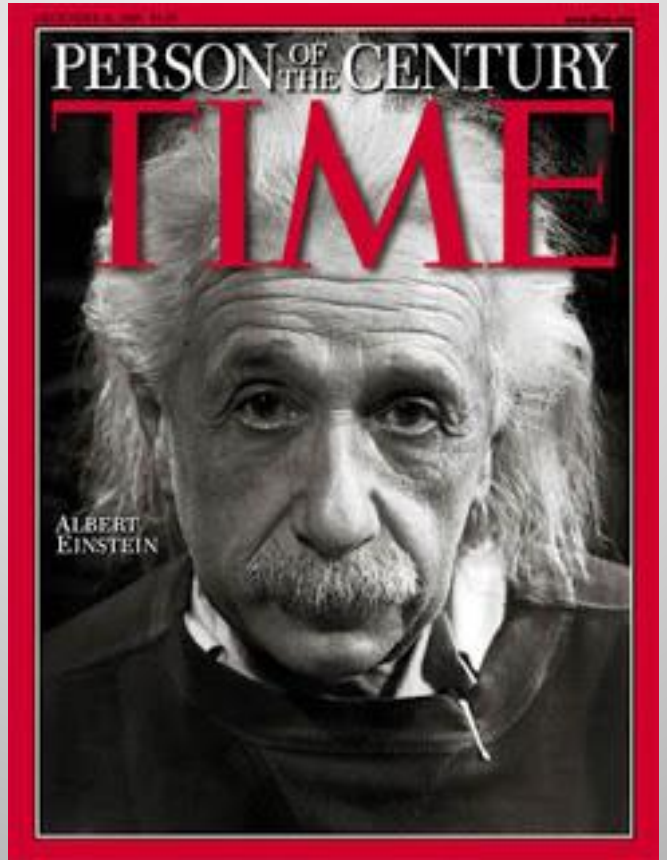
“K. Gödel - the greatest logician since Aristotle”

- H. Weyl & J. von Newman

- **D. Hofstadter published the philosophical book "Gödel, Esher, Bach: This Endless Garland".**
- **It draws parallels in the works and biographies of K. Gödel, the artist M.K. Esher and the composer I.S. Bach.**
- **It also highlights the fundamental concepts on which mathematics, symmetry and our mind are based.**



- **Magazine Time included K. Gödel in the list of 100 persons who formed the XX century.**
- **A man of the century became A. Einstein.**



P.S.:

The open problem of the modern theory of proofs

- ***Find the shortest unprovable Peano arithmetic proposition.***

  - **The proofs of Gödel's incompleteness theorems demonstrate how one can construct such sentences, but the resulting sentences of the formal language of arithmetic are too large.**