Electrical Communication ELC318

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Logistics

- **Email**: ahesham.inquiries@gmail.com

- **Include [**Power3rd20**]** in the email subject

- **Grades**:
    - 30% mid-term exam
    - 70% Final exam

Intended Learning Outcomes (ILO's)

- To **review/calculate** Fourier Transform of signals

- To **recognize** the different types of digital **modulation** and **demodulation** techniques

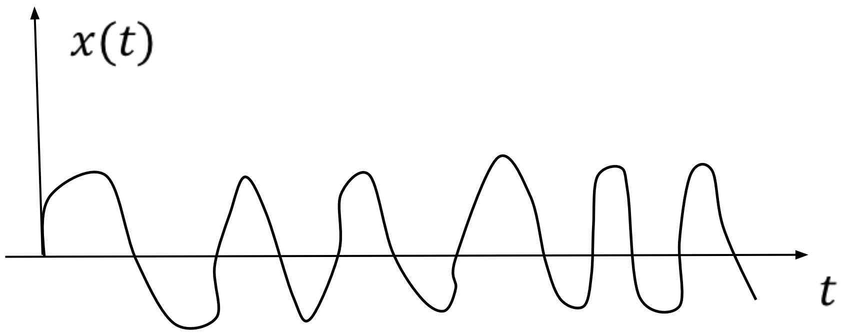
- To **Calculate** main communication system parameters: bandwidth.

- To **Choose** the best modulation/ demodulation technique for a practical engineering system and analyze the system

Fourier Transform

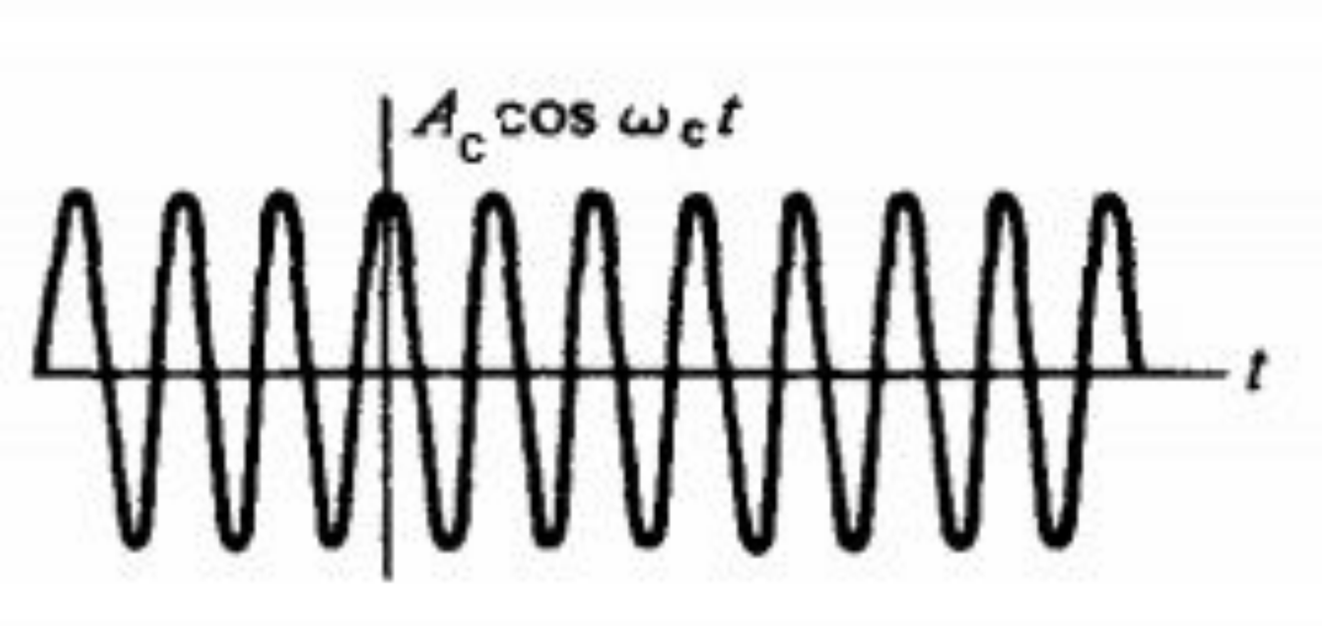
Electrical Signal

- A signal $x(t)$is function of time t and could represent an electrical waveform



Electrical Signal

- Assume $x(t) = A_c \cos(\omega_c t)$
- What does $x(t)$ represent?



Electrical Signal

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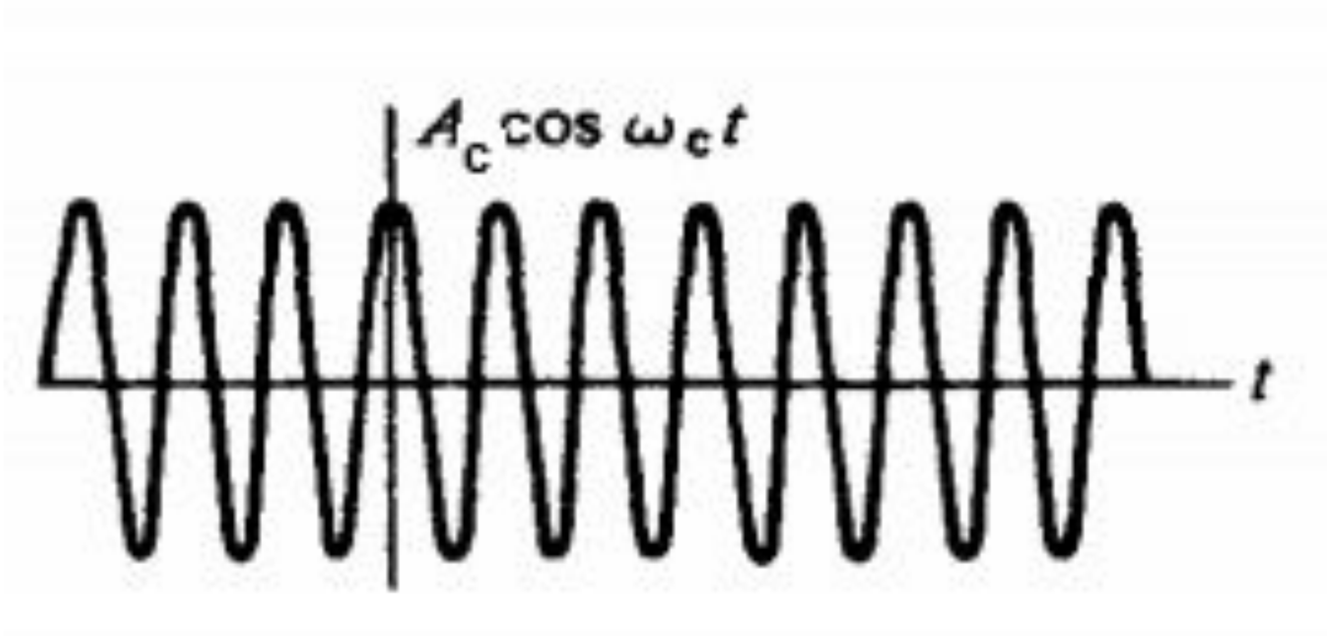
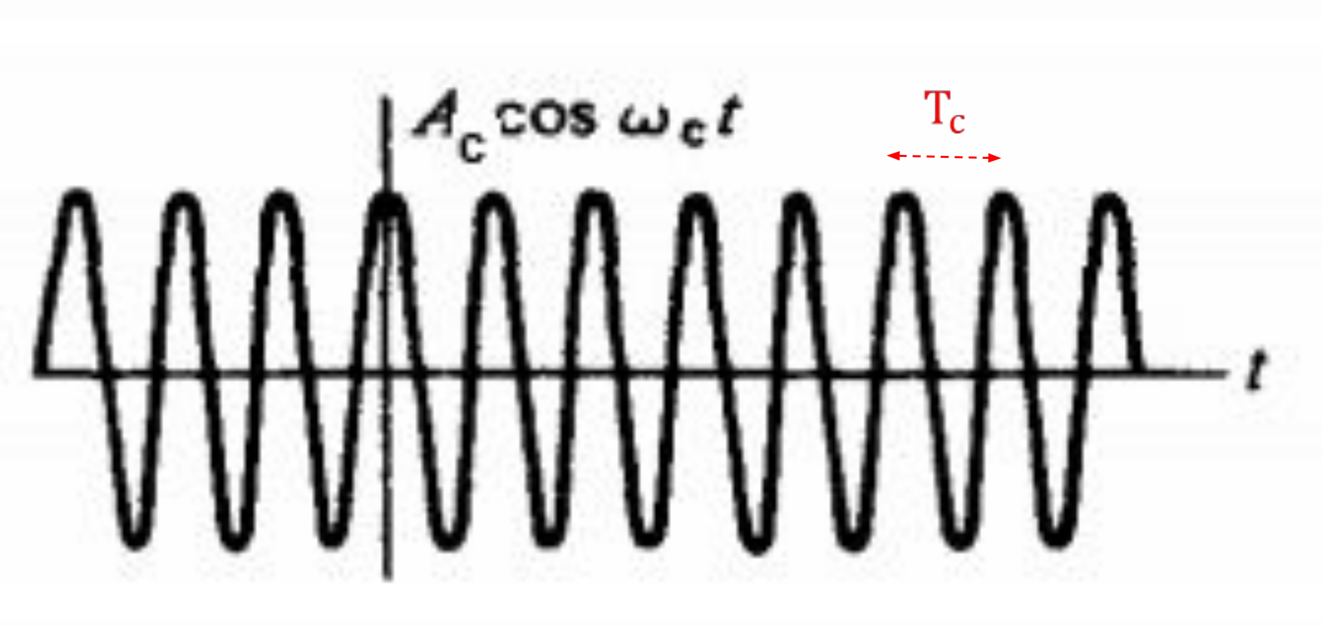
- What does $x(t)$ represent?
  - Voltage? Current?



Electrical Signal

- Assume $x(t) = A_c \cos(\omega_c t)$
- What does $x(t)$ represent?
  - Voltage? Current? → Electrical Signal in general in Time Domain



$$\omega_c = 2\,\pi\,F_c$$

$$F_c = \frac{1}{T_c}$$

If $T_c = 0.02$, what is the value of F_c?

Electrical Signal

- If I tell you: “A cosine signal has amplitude 1 Volt, and Frequency 50Hz” then you can draw the signal in time domain
- Is there any other representation of this Cosine signal?

Electrical Signal

- If I tell you: “A cosine signal has amplitude 1 Volt, and Frequency 50Hz” then you can draw the signal in time domain
- Is there any other representation of this Cosine signal?
  - Yes: Frequency domain

Fourier Transform

- Frequency domain is another way to represent a signal
- Frequency domain of a signal is obtained using **Fourier Transform** equation:

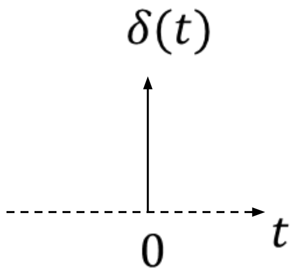
$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi\, a_k\, \delta(\omega - k\, \omega_c)$$

- The parameter "a_k" is called Fourier series coefficient

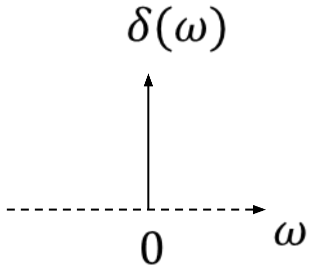
$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi\, a_k\, \delta(\omega - k\, \omega_c)$$
$$= \cdots + 2\pi\, a_{-2}\, \delta(\omega + 2\omega_c) + 2\pi\, a_{-1}\, \delta(\omega + \omega_c) + 2\pi\, a_0\, \delta(\omega) + 2\pi\, a_1\, \delta(\omega - \omega_c) + 2\pi\, a_2\, \delta(\omega - 2\omega_c) + \cdots$$

Fourier Transform

- $\delta(t)$ is called "delta" function in time domain

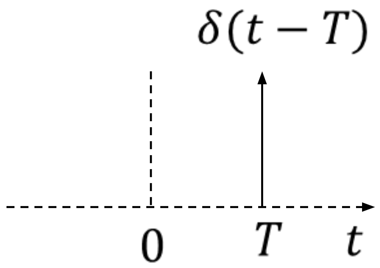


- $\delta(\omega)$ is also called "delta" function, but in ω domain

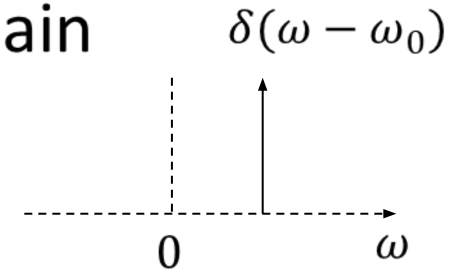


Fourier Transform

- $\delta(t - T)$ is "delta" function at $t = T$



- $\delta(\omega - \omega_o)$ is also called "delta" function, but in ω domain



Fourier Transform

- Before discussing how to calculate Fourier series coefficients, let's take an example
- Assume $x(t) = \cos(\omega_c t)$ and that $a_1 = \frac{1}{2}$ and $a_{-1} = \frac{1}{2}$
- Fourier Transform:

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi\, a_k\, \delta(\omega - k\, \omega_c) = \pi\delta(\omega + \ \omega_c) + \pi\delta(\omega - k\, \omega_c)$$

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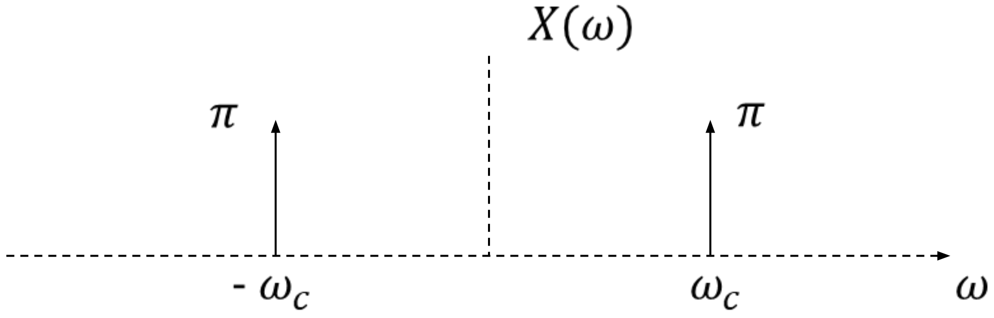

Fourier Transform

- How to compute Fourier series coefficients?
- Fourier series equation:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j\,k\,\omega_c\,t}$$

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Fourier Transform

- How to compute Fourier series coefficients?
- Fourier series equation:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j\,k\,\omega_c\,t}$$

- Example: $x(t) = \cos(\omega_c t)$

Since $\cos(\omega_c t)$ can be written as $\cos(\omega_c t) = \frac{1}{2} e^{-j\,\omega_c\,t} + \frac{1}{2} e^{j\,\omega_c\,t}$

Then $a_{-1} = \frac{1}{2}$ and $a_1 = \frac{1}{2}$

Fourier Transform

- How to compute Fourier series coefficients?
- Fourier series equation:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j\,k\,\omega_c\,t}$$

- Example (2): $x(t) = \sin(\omega_c t)$

Since $\sin(\omega_c t)$ can be written as $\sin(\omega_c t) = \frac{-1}{2j} e^{-j\,\omega_c\,t} + \frac{1}{2j} e^{j\,\omega_c\,t}$

Then $a_{-1} = \frac{-1}{2j}$ and $a_1 = \frac{1}{2j}$

Fourier Transform

- How to compute Fourier series coefficients?
- Fourier series equation:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j\,k\,\omega_c\,t}$$

- Example (3): $x(t) = \sin(2\omega_c t)$

Since $\sin(2\omega_c t)$ can be written as $\sin(\omega_c t) = \frac{-1}{2j} e^{-j\,2\omega_c\,t} + \frac{1}{2j} e^{j\,2\omega_c\,t}$

Then $a_{-2} = \frac{-1}{2j}$ and $a_2 = \frac{1}{2j}$

Fourier Transform

- What we have seen so far is sine and cosine which are “periodic signals”
- What if the signal non-periodic? How to obtain Fourier Transform

Fourier Transform of Non-Periodic Signals

- Fourier Transform of non-periodic signal:

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)\, e^{-j\,\omega t}\, dt$$

Fourier Transform of Non-Periodic Signals

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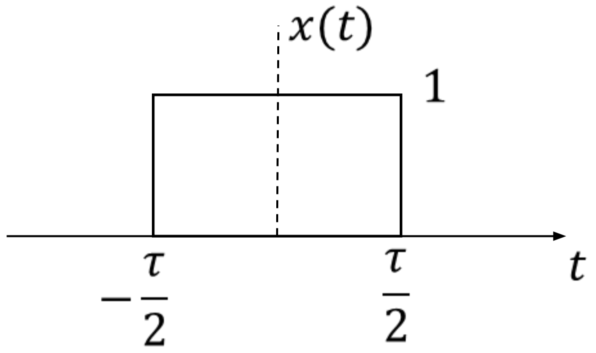
- Example: Rectangle pulse



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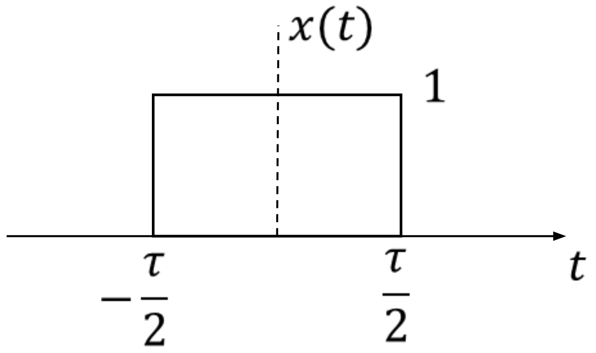


$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)\, e^{-j\,\omega t}\, dt$$

$$= \int_{t=-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\,\omega t}\, dt$$

$$= \frac{1}{-j\omega}\left(e^{-j\frac{\omega\tau}{2}} - e^{j\frac{\omega\tau}{2}} \right)$$

Fourier Transform of Non-Periodic Signals

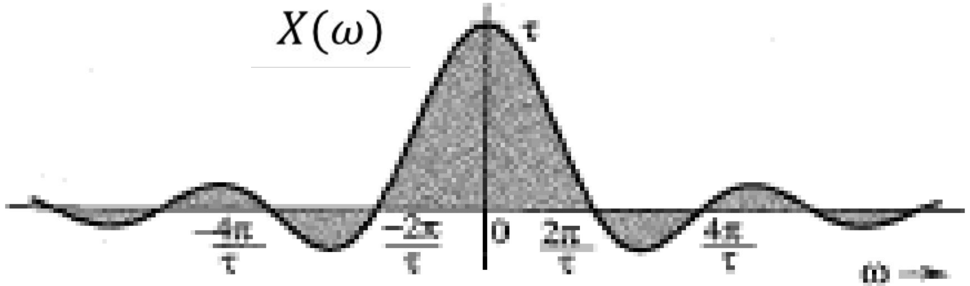
- Note that

$$\frac{1}{-j\omega}\left(e^{-j\frac{\omega\tau}{2}}-e^{j\frac{\omega\tau}{2}}\right)=\frac{2}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$ (show that)

- This expression can be written in another form

$$\frac{2}{\omega}\sin\left(\frac{\omega\tau}{2}\right)=\frac{\tau}{\frac{\omega\tau}{2}}\sin\left(\frac{\omega\tau}{2}\right)\equiv\tau\ sinc\ \left(\frac{\omega\tau}{2}\right)$$

$$sinc\ \left(\frac{\omega\tau}{2}\right)=\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\frac{\omega\tau}{2}}$$



Conclusion

- Fourier Transform obtains the signal in the frequency domain

- Fourier Transform for periodic signals:

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi\, a_k\, \delta(\omega - k\, \omega_c)$$

- Fourier Transform for non-periodic signals:

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)\, e^{-j\,\omega t}\, dt$$