Functions and Their Graphs

1.2 – Functions

Vocab

- Function = A set of ordered pairs that has each input (x) giving exactly one output (y)
- Ex: Function or not?

X	Y
-2	3
0	4
8	32
7	5

Yes

X	Y
5	3
0	4
8	32
5	-6

No;
One input gives 2 outputs

X	Y
-2	3
0	-1
8	-2
7	3

Yes

- **In a function, one input can't give 2 different outputs!**

More Vocab

- (x, y) = (input, output)
- f(x) is another way to write an output
- Domain = the set of all inputs (x)
- Range = the set of all outputs (y)
- Ex: For the function f(x) = x – 3 , evaluate the following:
  - f(-3) $f(-3) = (-3) - 3 \longrightarrow -6$
  - f(x+1)

$$f(x+1) = (x+1) - 3 \longrightarrow x - 2$$

- Ex: For the function $f(x) = 2 - x^2$, evaluate the following:
  - f(x+1)

$$f(x+1) = 2-(x+1)^2 \longrightarrow 2-(x^2+2x+1)$$

$$\longrightarrow 2-x^2-2x-1 \longrightarrow -x^2-2x+1$$

- Ex: For the function $f(x) = x^2 + x$, evaluate the following:
  - f(2x)

$$f(2x) = (2x)^2 + (2x) \longrightarrow 4x^2 + 2x$$

- Ex: For the function $f(x) = x^2 - 2x + 3$, evaluate the following:
  - $f(x+h)$

$$f(x+h) = (x+h)^2 - 2(x+h) + 3$$

$$= x^2 + 2xh + h^2 - 2x - 2h + 3$$

- Ex: For the function $f(x) = 2x^2 - 3$, evaluate the following:
  - The difference quotient $\dfrac{f(x+h)-f(x)}{h}$

$$\rightarrow \frac{(2(x+h)^2-3)-(2x^2-3)}{h}$$

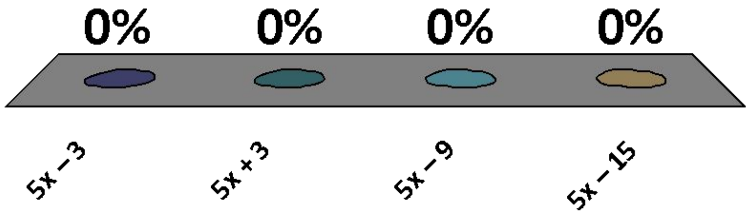
$$\rightarrow \frac{(2(x^2+2hx+h^2)-3)-2x^2+3}{h}$$

$$\rightarrow \frac{\cancel{2x^2}+4hx+h^2\cancel{-3}\cancel{-2x^2}\cancel{+3}}{h}$$

$$\rightarrow \frac{4hx+h^2}{h} \rightarrow 4x+h$$

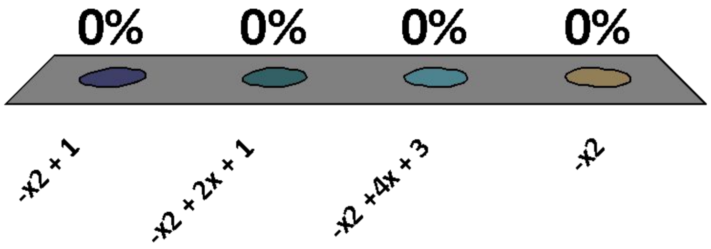
f(x) = 5x + 6. Find f(x - 3).

1. 5x – 3
2. 5x + 3
3. 5x – 9
4. 5x – 15



$f(x) = 2x - x^2$. Find $f(x + 1)$.

1. $-x^2 + 1$
2. $-x^2 + 2x + 1$
3. $-x^2 + 4x + 3$
4. $-x^2$



- Ex: The function below is a piecewise function. Find f(0) and f(1).

$$f(x) = \begin{cases} x-3, & x<1 \\ 2x-4, & x \geq 1 \end{cases}$$

  - Since 0<1, use the top function for f(0).
  - **f(0) = -3!**

  - Since 1≥1, use the bottom function for f(1).
  - **f(1) = -2!**

More Vocab

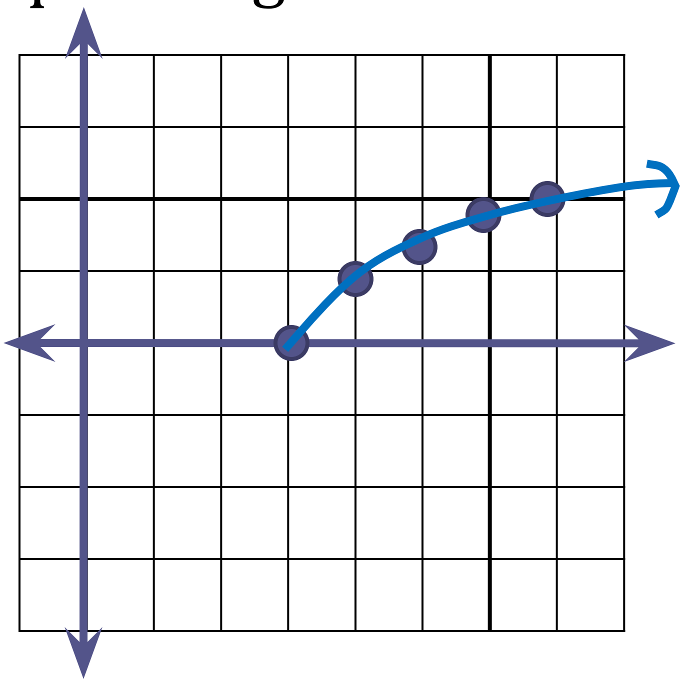
- $y = x^2$ means y is a function of x
- Y is not a function of x when a $\pm$ is in play
- Ex: Which of these has y as a function of x?
  - $x^2 - y = 7$
    - Solve for y first…
    - $-y = 7 - x^2$
    - $y = x^2 - 7$ … no $\pm$ means **YES!**
  - $x^2 + y^2 = 2x$
    - $y^2 = 2x - x^2$
    - $y = \pm\sqrt{2x - x^2}$ … so **NO!**

Finding Domain and Range

- The domain (set of all x's) is always assumed to be all real numbers unless some values cannot create outputs (y's).
- Ex: Find the domain of the following functions:
  - $y = 2x - 3$
    - Any x will produce a y, so the domain is **x∈ℝ** (all reals)
  - $y = \sqrt{x}$
    - The square root can't be negative, so the domain is **x≥0**
  - $y = \dfrac{3}{2x-4}$
    - The denominator can't be 0, so $2x - 4 \neq 0$…
    - …**x≠2**

Finding Domain and Range

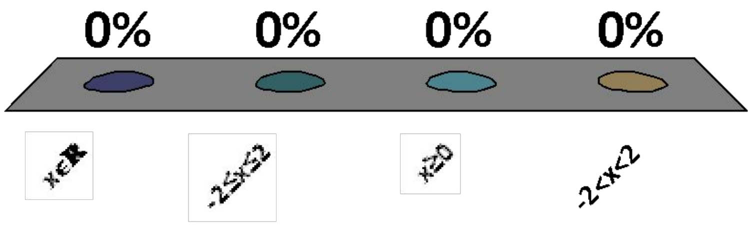
- To find range, graph the function and infer the range (set of all y's).
- Ex: Find the domain and range of the function $y = \sqrt{x-3}$
  - Graph the function first.
  - For the domain, we know from the equation given that **$x \geq 3$**. Our graph confirms that.
  - For the range, the graph shows us that there are no negative values for y, and the values will continue to increase as x increases.
  - Range: **$y \geq 0$**



$$f(x) = 4 - x^2$$

What is the domain?

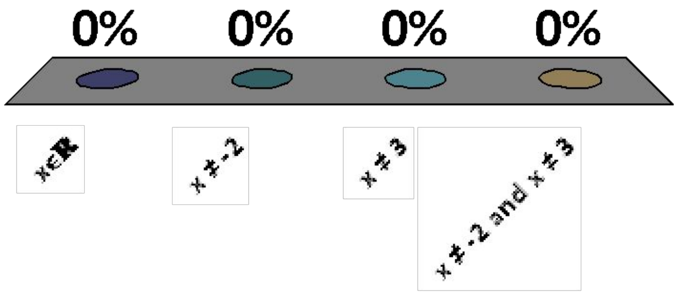
1. $x \in \mathbb{R}$
2. $-2 \leq x \leq 2$
3. $x \geq 0$
4. $-2 < x < 2$



$$f(x) = \frac{x+2}{x-3}$$

What is the domain?

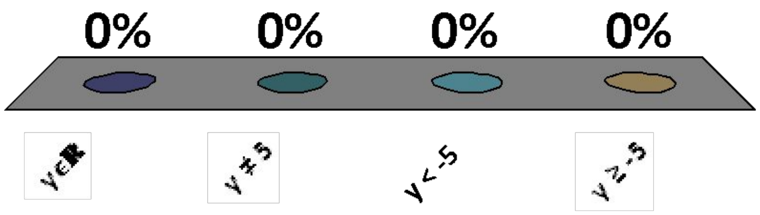
1. $x \in \mathbb{R}$
2. $x \neq -2$
3. $x \neq 3$
4. $x \neq -2$ and $x \neq 3$



$$f(x) = 2x^2 - 5$$

What is the range?

1. $y \in \mathbb{R}$
2. $y \neq 5$
3. $y < -5$
4. $y \geq -5$ ✓

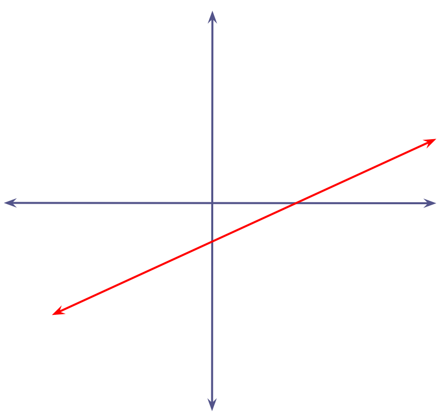


Ch. 1 – Functions and Their Graphs

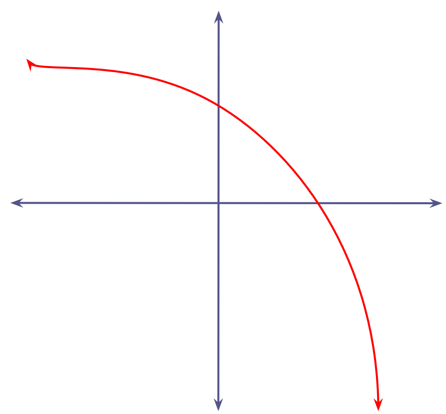
1.3 – More Functions

Vertical Line Test

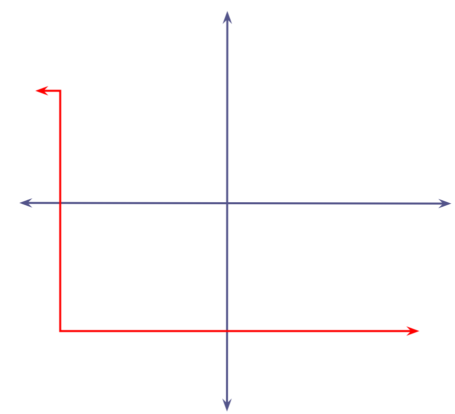
- Vertical is up and down!
- Vertical Line Test: If you can draw some vertical line on a graph and it goes through MORE THAN ONE point, the graph is NOT a function.
  - Ex: Are these graphs functions?



YES!



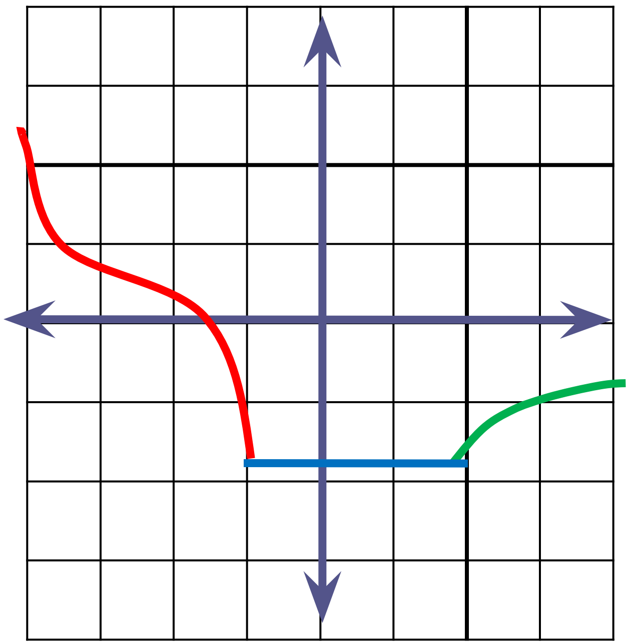
YES!



NO!

Vocab

- As we read left to right, the function to the right is…
  - …decreasing in the red region
    - Decreasing for x<-1, so we write $(-\infty, -1)$ to indicate that y decreases over that x interval
  - …constant in the blue region
    - Constant for -1≤x≤2, so we write $(-1, 2)$
  - …increasing in the green region
    - Increasing for x>2, so we write $(2, \infty)$

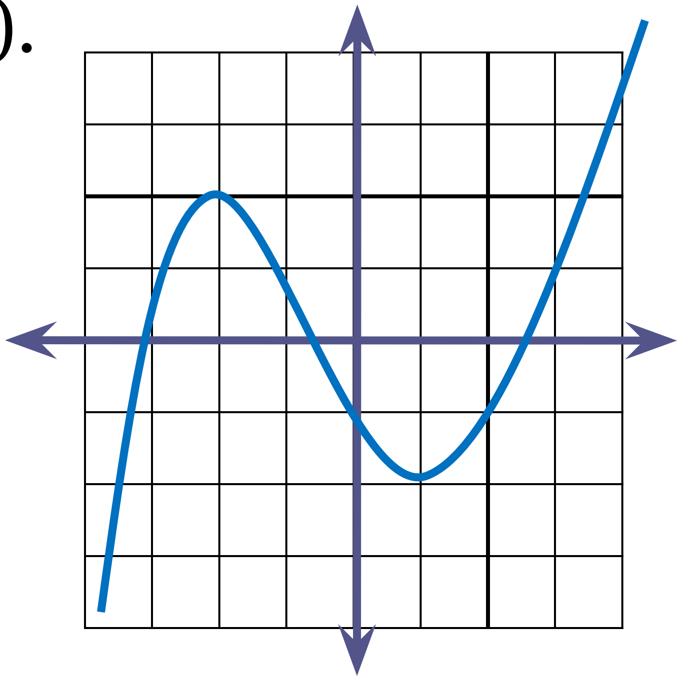


Vocab

- When a function goes from increasing to decreasing (or visa versa), it will have a relative minimum or a relative maximum.

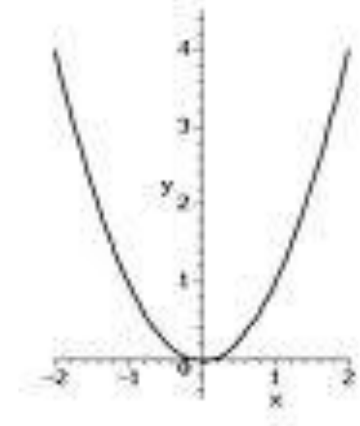
- The graph below has a relative maximum at (-2, 2) and a relative minimum at (1, -2).

- A graph can have any amount of relative minima or maxima.

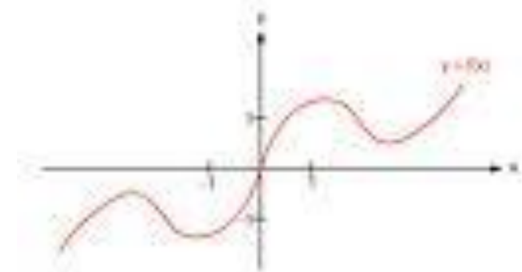


Functions

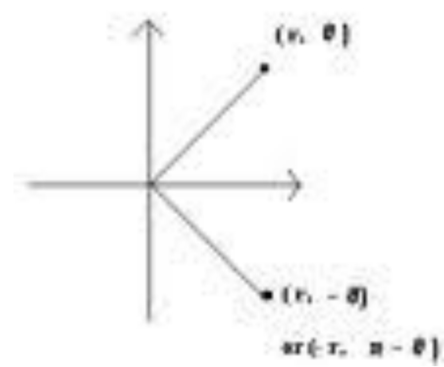
- A function is **even** if it is symmetric about the y-axis
  - $f(-x) = f(x)$



- A function is **odd** if it is symmetric about the origin
  - $f(-x) = -f(x)$

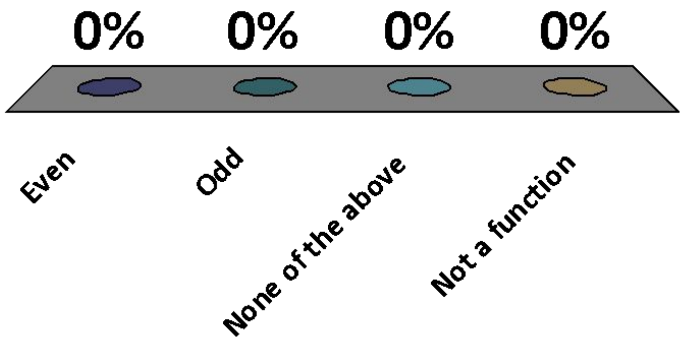


- A graph symmetric about the x-axis is...
  - ...not a function!



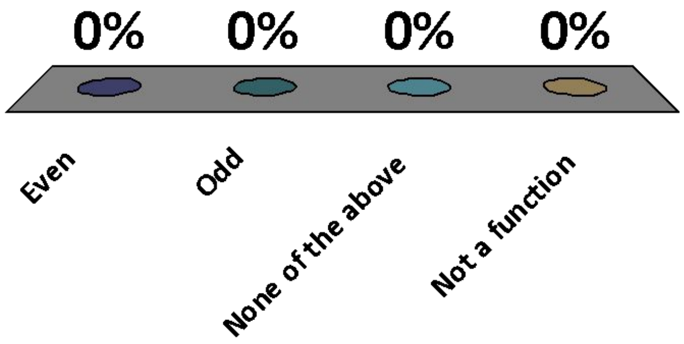
The function $y = 4x^2 - 2$ is…

1. Even
2. Odd
3. None of the above
4. Not a function



The function $y = 1/x$ is...

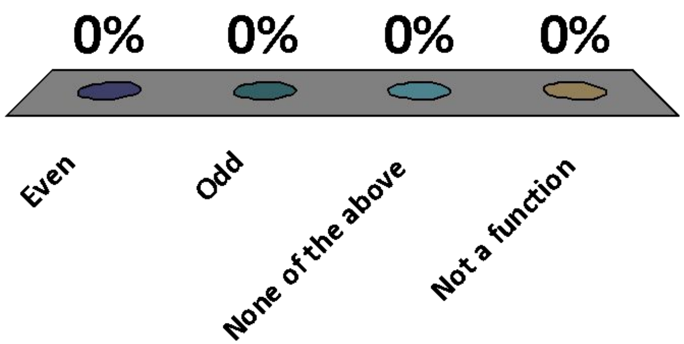
1. Even
2. Odd
3. None of the above
4. Not a function



The function $y = x^3 - x$ is...

1. Even
2. Odd
3. None of the above
4. Not a function

Figure it out algebraically – no graphing!!!



REPRESENTATIONS OF FUNCTIONS

There are four possible ways to represent a function:

- verbally (by a description in words)
- numerically (by a table of values)
- visually (by a graph)
- algebraically (by an explicit formula)