LECTURE 5
SEQUENTIAL GAMES: EMPIRICAL EVIDENCE AND BARGAINING

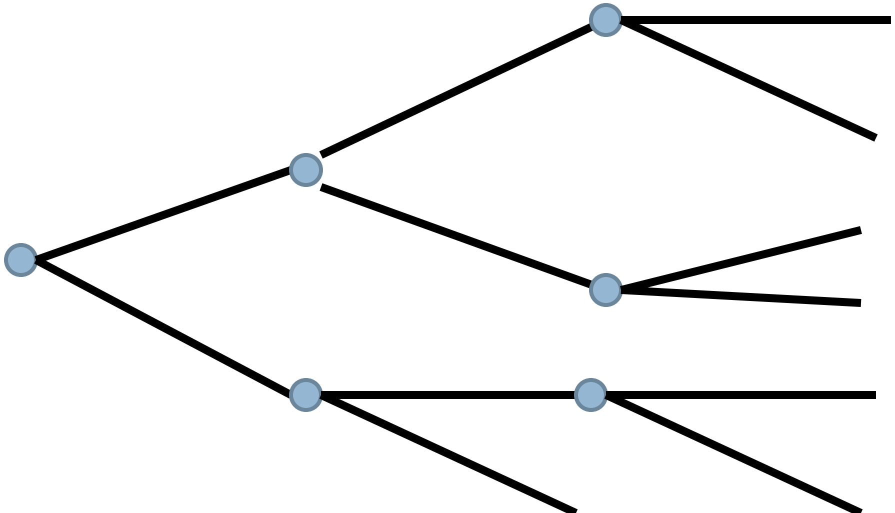
Introduction

- Sequential games require players to look forward and reason backward SPE
- Order of play matters.
  - First-mover advantage: Stackelberg game, Entry game.
- Strategic moves may be used to obtain an advantageous position credibility problem
- Outline:
  1. Empirical evidence on how individuals play sequential games
  2. Application to bargaining

Game complexity

- **Games differ with respect to their complexity**
  - very simple: Stackelberg.
  - moderately complex: connect four
  - very complex: chess
- **Chess**
  - problem with backward induction: game tree way too large, even for computers.
  - first two moves: 20×20

= 400 possible games.

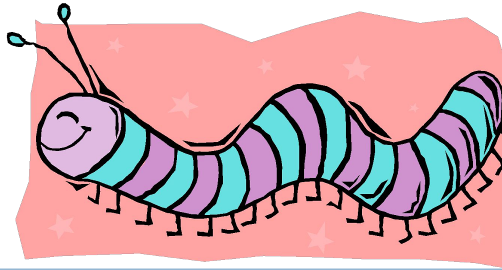


Game complexity

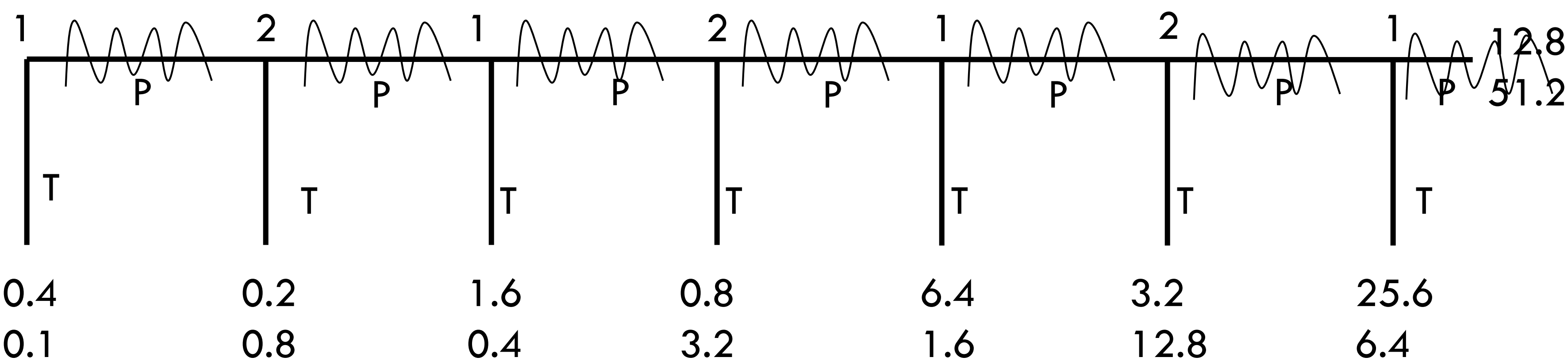
- Number of board positions in Chess:

  app. 10^{46} = 10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,0 00000,000,000
- Sequential games can be incredibly complex, and backward induction may not be feasible
- What about less complex games?
  - do players use backward induction?
  - if not, what rules do they use?

Centipede game



- Each node a player can take (T) or pass (P)
  - Pass: let the other player move, the pie gets bigger
  - Take: take 80% of the growing pie



- SPE: Using rollback: Player 1 chooses T in the last period... player 1 plays T in period 1

Centipede game

- In a six-move centipede game played with students, economist McKelvey found that:
  - 0% choose take at the first node (theory predicts 100%)
  - 6% choose take at the second node
  - 18% choose take at the third node
  - 43% choose take at the fourth node
  - 75% choose take at the fifth node
- Players rarely take in early nodes, and the likelihood of Take increases at each node
- SPE is inconsistent with the way people behave in (complicated) games.

Centipede game

What does it tell us about players' rationality?

- **Limited ability to use rollback over many steps**
  - People only think a few steps ahead not fully rational!
  - Explains why Probability(Take) increases as the end of the game approaches.
- **Alternatively, players may be rational and believe that the other players are not rational**
  - If a player believes that the other player will choose "Pass", it is his best interest to also choose "Pass" this period.
  - Maybe players have developed a mutual understanding that neither of them will choose Take too soon.

Centipede game

Discussion

- **Players use rules of thumb that work well in certain situations.**
  - I pass as long as the other player passes. As we get close the end of the game, I may choose Take.
  - This rule of thumb contributes to higher payoffs

- Backward induction is used to some extent, but not to the extent predicted by game theory.

BARGAINING GAMES

An Application of Sequential Move Games

What is bargaining?

- Economic markets
  - Many buyers & many sellers → ***traditional market***
  - Many buyers & one seller → ***auction***
  - One buyer & one seller → ***bargaining***

- Bargaining problems arise when the size of the market is small. There are no obvious price standards because the good is unique.
- Foundations of bargaining theory: John NASH: The bargaining problem. Econometrica, 1950.

What is bargaining?

A seller and a buyer bargain over the price of a house

Two countries bargain over the terms of a trade agreement

Labor unions and manager bargain over wages

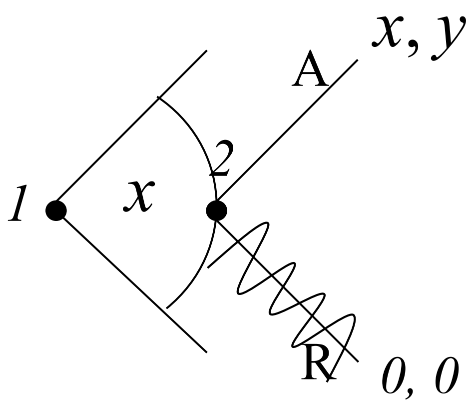
Haggling at informal market

What is bargaining?

- The "Bargaining Problem" arises in economic situations where there are **gains from trade**
  - The problem is how to divide the gains (or surplus) generated from trade.
  - E.g. the buyer values the good higher than the seller.
- The gains from trade are represented by a sum of money, v, that is "on the table."
- Players move sequentially, making alternating offers.

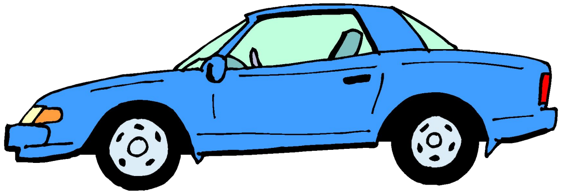
Ultimatum games

- 2 players. Divide a sum of money of v=1.
- Player 1 proposes a division.
  - x for player 1 and y for player 2, such that x+y=1.
- Player 2: accept or reject Player 1's proposal.
- If Player 2 accepts, the proposal is implemented. If he rejects, both receive 0.



Ultimatum games

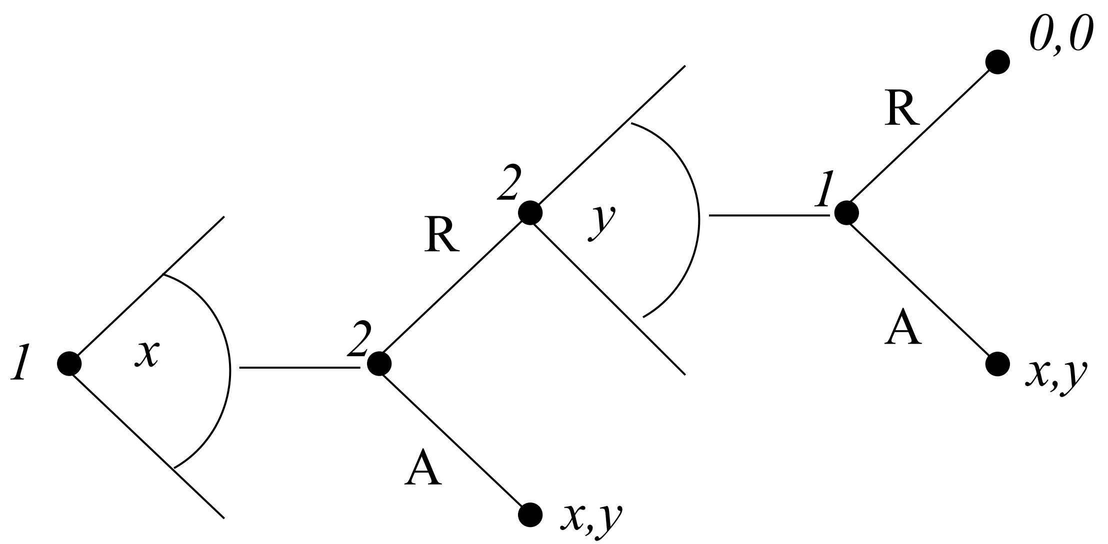
- Backward induction
  - Player 2 receives 0 if he rejects.
  - Player 2 will accept any amount y>0
- **Player 1 will keep "almost all", and player 2 accepts the offer. SPE: x=1; y=0. (first-mover advantage)**



- Second-hand car example
  - Buyer is willing to pay up to $10,500.
  - Seller will not sell for less than $10,000. (v=$500)
  - The seller knows the buyer will accept any price p<$10,500.
  - The seller maximizes his gain by proposing a price just below $10,500 (say, $10,499). His gain from trade is almost $500.

Alternating Offers (2 rounds)

- Take-it-or-leave-it games are too trivial; there is no back-and-forth bargaining..
  - If the offer is rejected, is it really believable that both players walk away? Or do they continue bargaining?
- Suppose that if Player 2 rejects the offer, he can make a counteroffer. If Player 1 rejects the counteroffer, both get 0.



Alternating Offers (2 rounds)

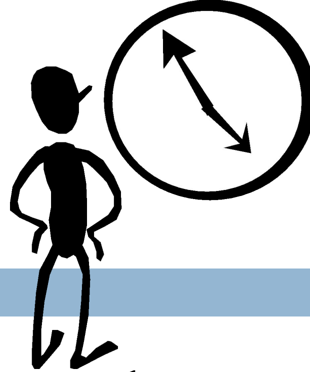
- Reasoning backwards:
  - Player 1 will accept any positive counteroffer from player 2.
  - Player 2 will then propose to keep "almost all".
  - Player 1 is in no position to make an offer that player 2 will accept, unless he proposes player 2 to keep almost all.
- SPE: Player 2 gains (almost) the whole surplus.

Lesson: Put yourself into a position to make a take-it-or leave-it offer. (last-mover advantage)

When does it end??

- Alternating offers bargaining games could continue indefinitely. In reality they do not.
  - The gains from trade ***diminish*** in value over time, and may disappear. – e.g. Labor negotiations –
    - Later agreements come at a price of strikes, work stoppages.
  - The players are ***impatient*** (time is money!).
    - If time has value, both parties would prefer to come to an agreement today rather than tomorrow.

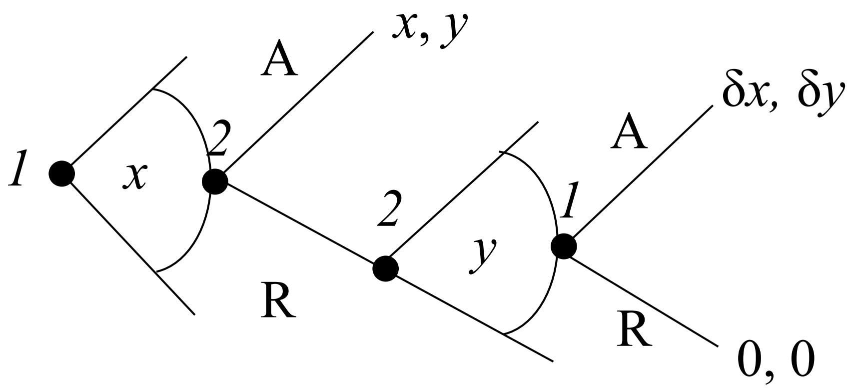
Impatience



- Suppose players value \$1 now as equivalent to \$1(1+r) one round later.
  - Discount factor is $\delta = 1/(1+r)$. Indeed \$1/(1+r) now= \$1 later, or \$$\delta$ now = \$1 later.
- If r is high, then δ is low: players discount future money amounts heavily, and are therefore very impatient.
  - E.g. r=0.6 □ δ =0.62
- If r is low, then δ is high; players regard future money almost the same as current amounts of money and are more patient.
  - E.g r=0.05 □ δ =0.95

Impatience

- Game representation:



Alternating offers (2 rounds) with impatience

- In round 2, only δ remains.
- Player 2 proposes to split δ as **{0, δ}** and player 1 accepts. Player 2 obtains everything: δ.
- In round 1, players offers just enough for player 2 to accept:
  - Player 1 offers δ, and keeps $1-\delta$.
  - Thus, player 1 proposes $\{x, y\} = \{1-\delta, \delta\}$, which is accepted.

First- or second-mover advantage?

- Are you better off being the first to make an offer, or the second? It depends on δ, (δ between 0 and 1).
- If $\delta=0.8$
  - SPE: $\{1-\delta, \delta\} = \{0.2, 0.8\}$. □ second-mover advantage
  - When players are slightly impatient, the second-mover is better off. Low cost for player 2 of rejecting the first offer.
- If $\delta=0.2$
  - SPE: $\{1-\delta, \delta\} = \{0.8, 0.2\}$. □ first-mover advantage
  - When players are very impatient, the first-mover is better off. High cost of rejecting the first offer.

Example: Bargaining over a House

Seller want
$150,000

Surplus: $10,000

Buyer will pay
maximum
$160,000

- $\delta = 0.8$
- There are two rounds of bargaining.
  - The Seller has to sell by a certain date
  - The Buyer has to start a new job and needs a house.
- The buyer makes a proposal first.
- Equilibrium: $\{1-\delta, \delta\} = \{0.2, 0.8\}$ □ $8,000 for the seller; $2,000 for the buyer.
  - The sale price of the house is $150,000+$8,000=$158,000.

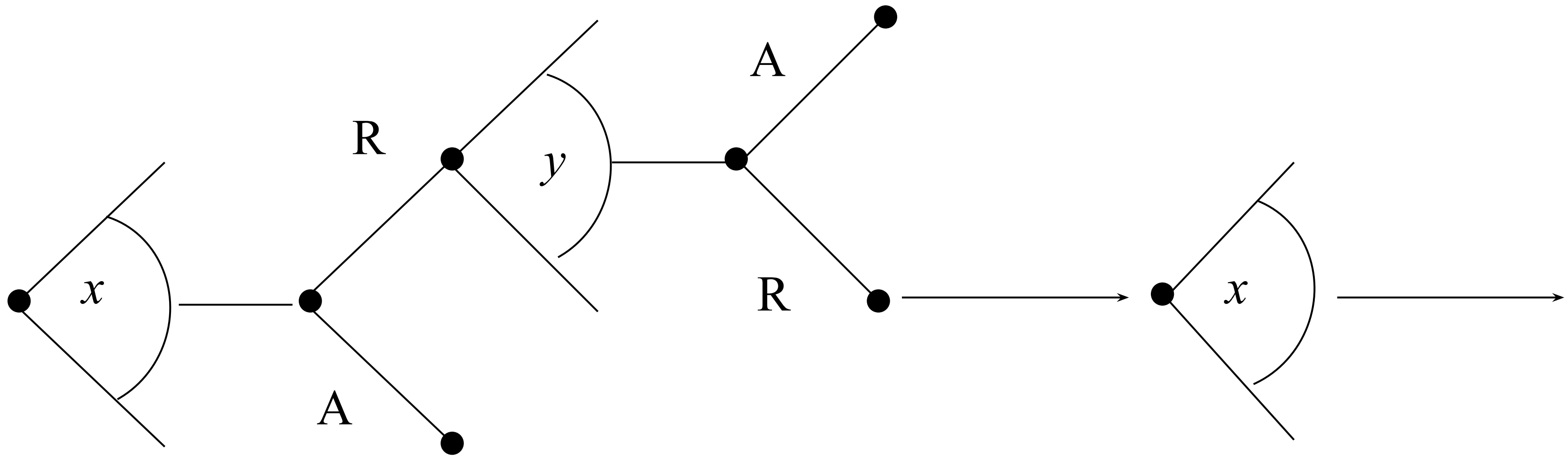
Don't Waste

In any bargaining setting,
strike a deal as early as possible!

- In reality, bargaining sometimes drags on. Why doesn't this always happen?
  - Reputation building: Showing toughness can help in future bargaining situations.
  - Lack of information: Seller overestimates the buyer's willingness to pay.

Infinitely Repeated Analysis

- What if the game is repeated infinitely and players are impatient? No limit to the number of counteroffers.



- To solve, note that: If player 1 offer is rejected, player 2 will be in the same position player 1 faced.

Infinitely Repeated Analysis

- Player 1 knows that player 2 can get share x in round 2.
- Thus player 1 must offer δx for player 2 to accept it. (δx today is equivalent to x tomorrow)
- Player 1 is left with 1- δx.
- But since the game is the same each round, if player 2 can get x next round, player 1 can also get x this round.
- Thus, x= 1- δx, or:

$$x = \frac{1}{1+\delta}$$

$$1 - x = \frac{\delta}{1+\delta}$$

→ **Player 1 gets more than player 2**

Infinitely Repeated Analysis

- In our example of bargaining over a house, the buyer was the first to make an offer:

$$x = \frac{1}{1+\delta} = \frac{1}{1.8} = 0.56$$

$$1 - x = \frac{\delta}{1+\delta} = \frac{0.8}{1.8} = 0.44$$

  - The buyer keeps 56% of the surplus; the seller gets 44%
  - The price of the house is $154,440
    - $150,000+0.44*10,000

Unequal Discount Factors

- Now suppose that the two players are not equally impatient, i.e. $\delta_1 \neq \delta_2$
  - For instance, δ is 0.9 for player 1; and 0.95 for player 2.
- Denote by x the amount that player 1 gets when he starts the process, and y the amount that player 2 gets when he starts the process.
- Player 1 knows that he must give $\delta_2 y$ to player 2.
- Thus, player 1 gets $x = 1 - \delta_2 y$
- Similarly, when player 2 starts the process, we must offer $\delta_1 x$, and keeps $y = 1 - \delta_1 x$

Unequal Discount Factors

- By substitution player 1 keeps:

$$x = 1 - \delta_2 y = 1 - \delta_2 (1 - \delta_1 x)$$

$$\Rightarrow x = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}$$

- ...and offers $1 - x = \dfrac{\delta_2 (1 - \delta_1)}{1 - \delta_1 \delta_2}$

- The more impatient is a player, the less he receives in equilibrium...
- First-/second-mover advantage depends on the relative levels of impatience.

Unequal Discount Factors

- In the Dixit and Skeath textbook (pp.710-711):

$$\delta_1 = \frac{1}{1+r}$$

$$\delta_2 = \frac{1}{1+s}$$

- It follows that: $x = \dfrac{1-\delta_2}{1-\delta_1\delta_2} = \dfrac{s+rs}{r+s+rs}$

e.g. $\delta_1 = 0.5; \delta_2 = 0.9 \Rightarrow x = 0.18$

Outside options

- In some situations, a bargaining party has the option of breaking off negotiations
  - A buyer negotiating with a seller may decide to start bargaining with another seller
  - A firm negotiating with a union may have the option of closing down and selling its assets
- The outside options are called the BATNAs (best alternative to a negotiated agreement)
  - BATNAs show what players would get if bargaining fails.
- The higher is a player's outside option, the more he can claim. ("bargaining power")

Outside options

Strategic moves to manipulate BATNAs

- A player can try to improve his BATNA to be stronger in the bargaining.
  - For instance, before asking for a raise, try to get an offer from another employer. Your BATNA is higher, and your employer may not be in a position to refuse.
- A player can also try to reduce the BATNA of the other player.
  - If you want to ask for a raise, make yourself indispensable. The employer would lose if you leave.
- A final option is to lower both players' BATNAs, but decrease it more for the other player.
  - "This will hurt you more than it hurts me".

Practical Lessons I

- In reality, bargainers do not know one another's levels of patience or BATNAs, but may try to guess these values.
- Signal that you are patient, even if you are not. For example, do not respond with counteroffers right away. Act unconcerned that time is passing. Have a "poker face."
- Remember that the bargaining model indicates that the more patient player gets the higher fraction of the amount that is on the table.

Practical Lessons II

- How to find out the other player BATNA and level of impatience?
  - Suppose you consider buying a house.
  - Is the house on the market for a long time? □ low BATNA for the seller (no one wants to buy).
  - If the owner moving to another city.
    - □ low δ, or highly impatient

Summary

- Bargaining as sequential games. Use rollback to find the SPE.
- Split of surplus depends on the number of rounds, and relative patience.
- BATNAs affect the outcome
  - Better have good outside options
- Potential for strategic moves to increased your BATNA or perceived patience