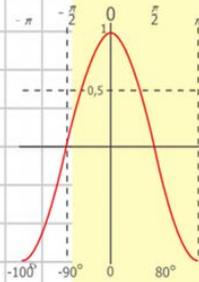
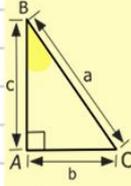
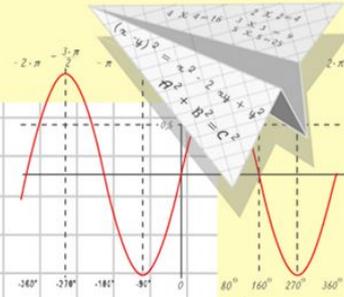
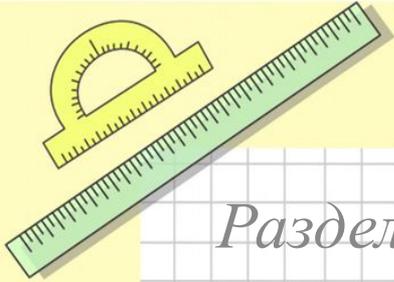


# Математик

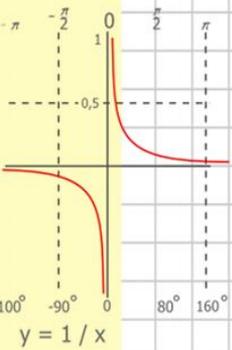
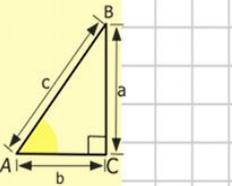
## Раздел 6. Метод <sup>а</sup> координат в пространстве

### Занятие 62.

# Векторы в пространстве



- $y = \cos x$
- $2 \times 2 = 4$
  - $3 \times 3 = 9$
  - $4 \times 4 = 16$
  - $5 \times 5 = 25$
  - $6 \times 6 = 36$
  - $7 \times 7 = 49$
  - $8 \times 8 = 64$



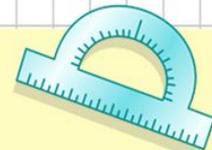
$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 2100 \\ + 8400 \\ \hline 105000 \end{array}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

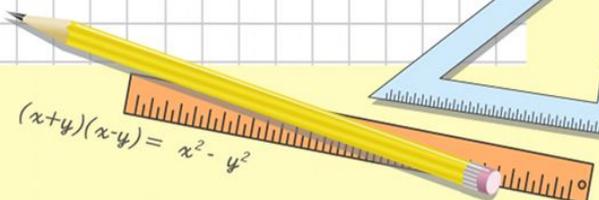


$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$



$$(x+y)(x-y) = x^2 - y^2$$

# Определение

Вектор – это направленный отрезок.



Вектор называют нулевым, если у него начало и конец совпадают.



Длиной (модулем) вектора называют длину отрезка, его изображающего.

$$|\vec{AB}| = AB$$

$$|\vec{0}| = 0$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

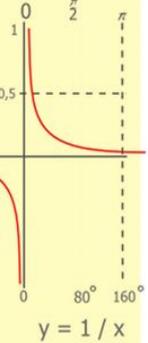
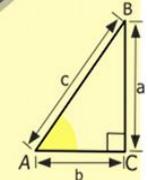
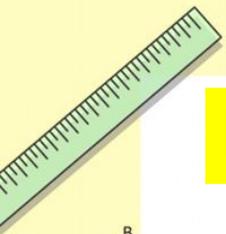
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

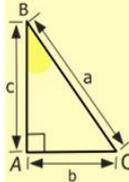
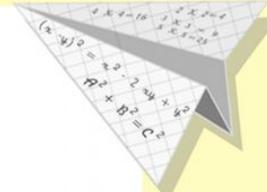
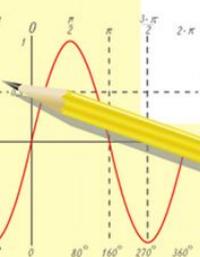
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

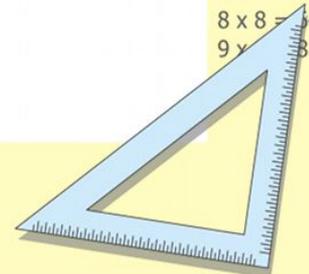
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

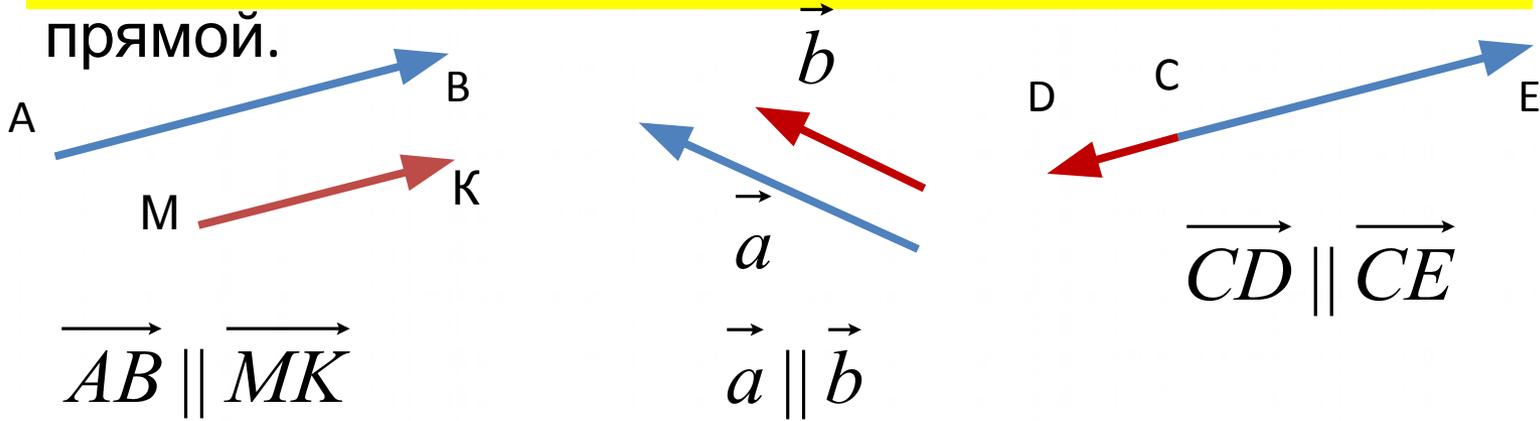


$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



# Коллинеарность векторов

Векторы называют коллинеарными, если они лежат на параллельных прямых или на одной прямой.



Среди коллинеарных векторов выделяют сонаправленные и противоположно направленные.

$$\vec{AB} \uparrow\uparrow \vec{MK}$$

$$\vec{a} \uparrow\uparrow \vec{b}$$

$$\vec{CD} \uparrow\downarrow \vec{CE}$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

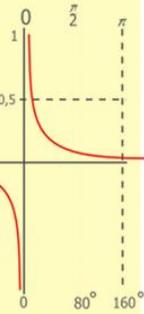
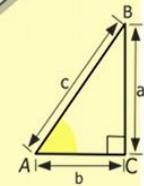
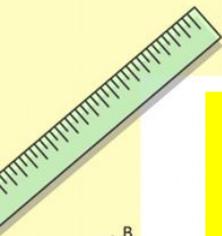
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

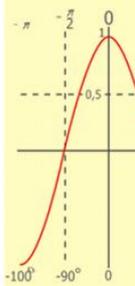
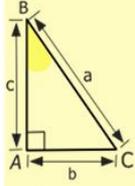
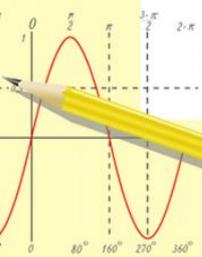
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$

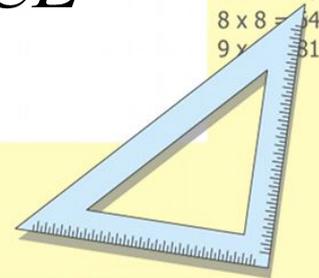
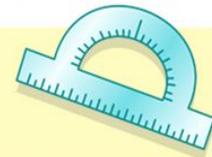


$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



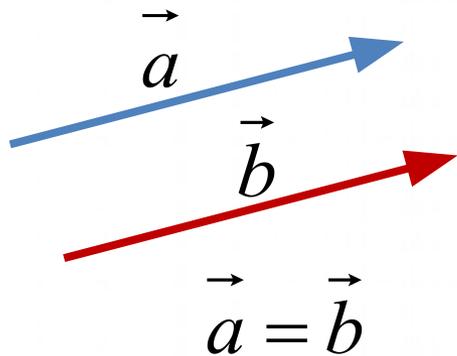
$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



# Равенство векторов

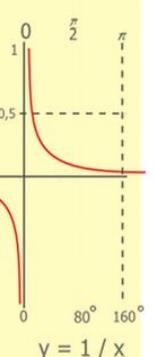
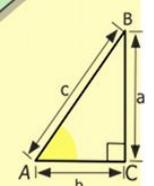
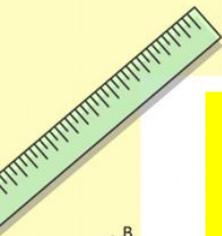
Векторы называют равными, если они сонаправленные и имеют одинаковую длину.



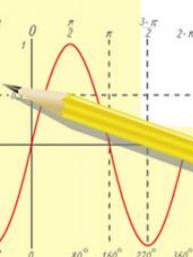
$$\vec{a} = \vec{b} \Leftrightarrow \begin{cases} \vec{a} \uparrow \uparrow \vec{b} \\ |\vec{a}| = |\vec{b}| \end{cases}$$



Укажите пары равных векторов:



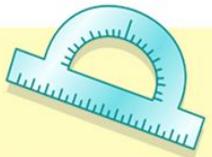
$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$\sin 90^\circ = 1$



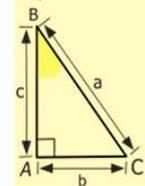
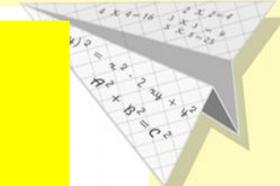
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

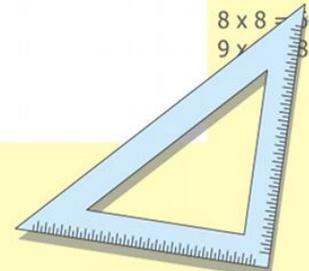

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$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$

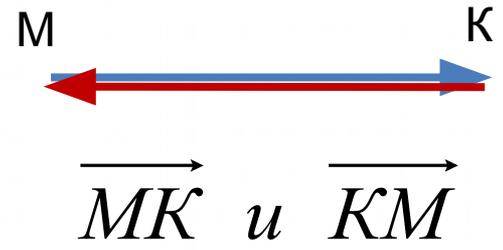
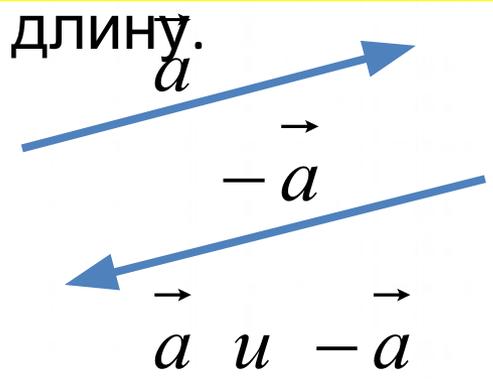


- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$

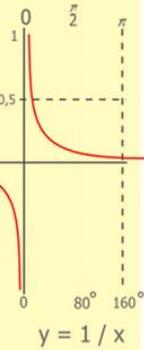
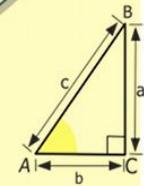
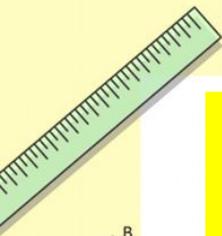


# Противоположные векторы

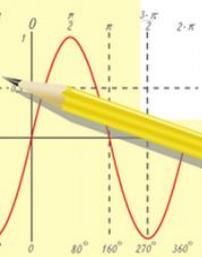
Векторы называют противоположными, если они противоположнонаправленные и имеют одинаковую длину.



ABCD-параллелограмм.  
Укажите пары противоположных векторов:



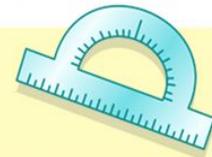
$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

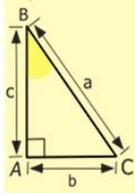
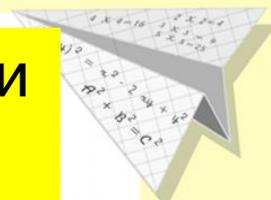
$\sin 90^\circ = 1$



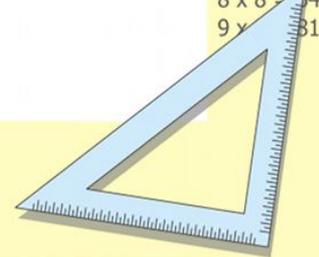
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

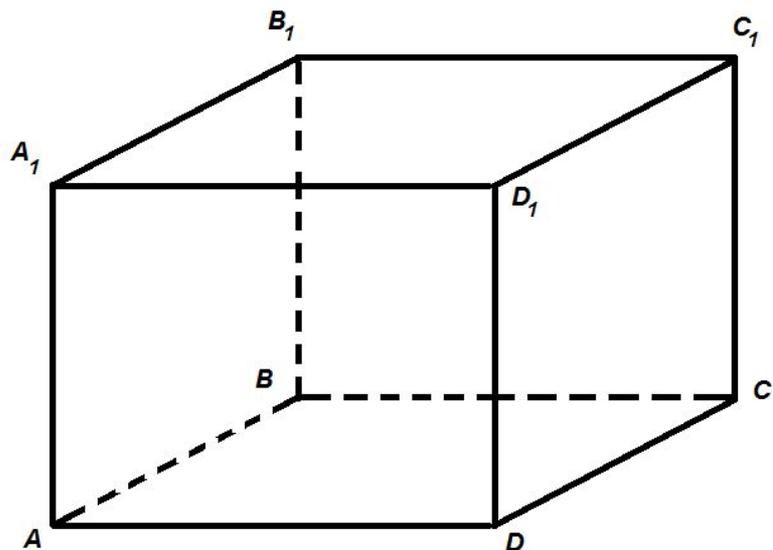
$$(x+y)(x-y) = x^2 - y^2$$



- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



# Равные и противоположные векторы



Равные векторы:

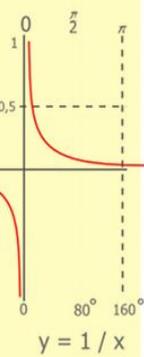
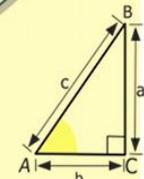
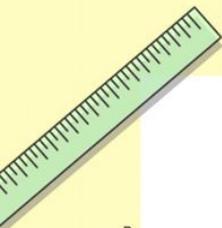
$$\vec{AB} = \vec{A_1B_1} = \boxtimes$$

$$\vec{BC} = \vec{B_1C_1} = \boxtimes$$

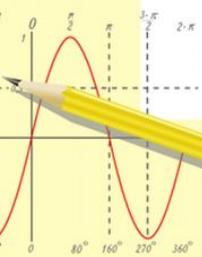
Векторы, противоположные данному вектору:

$$\vec{AD} : \vec{D_1A_1}; \boxtimes$$

$$\vec{B_1C} : \boxtimes$$



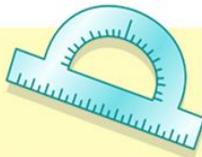
$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 4\ 2 \\ \hline 21\ 0 \\ + 84 \\ \hline 105\ 0\ 00 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

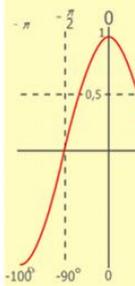
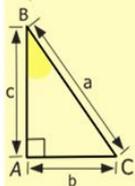
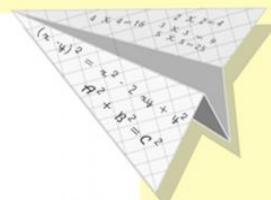
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

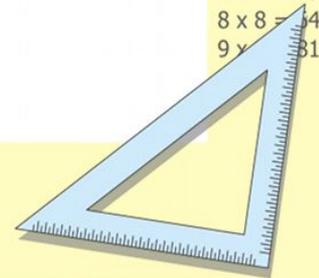
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

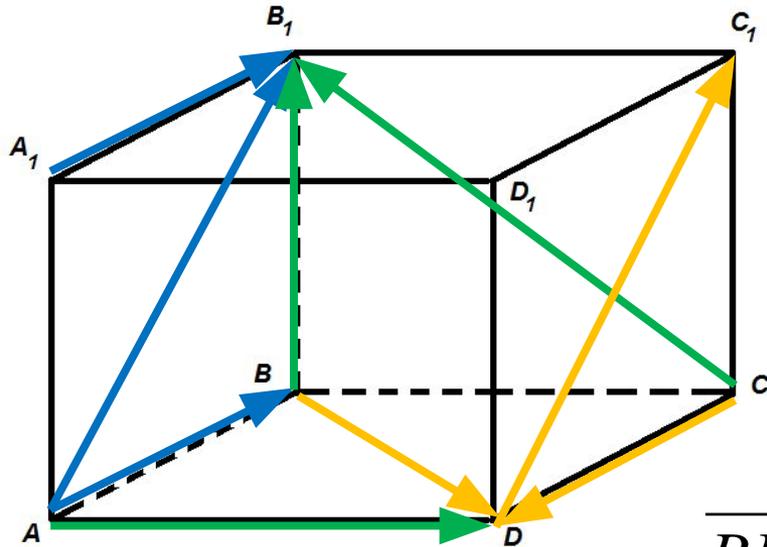
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



# Компланарность векторов

Три вектора называют компланарными, если они лежат в одной плоскости или в параллельных

плоскостях

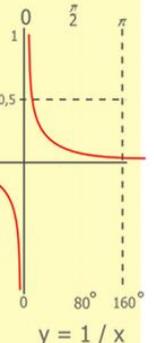
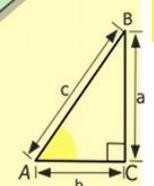


Например:

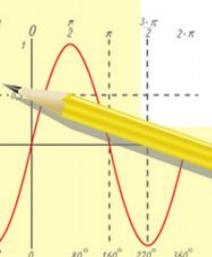
$\vec{AB}; \vec{A_1B_1}; \vec{AB_1}$  - компланарные

$\vec{AD}; \vec{BB_1}; \vec{CB_1}$  - компланарные

$\vec{BD}; \vec{DC_1}; \vec{CD}$  - некомпланарные



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

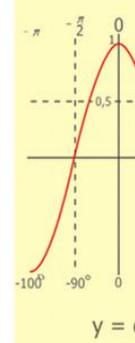
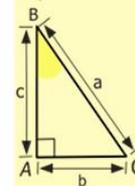
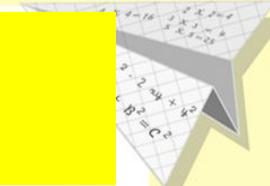
$\sin 90^\circ = 1$



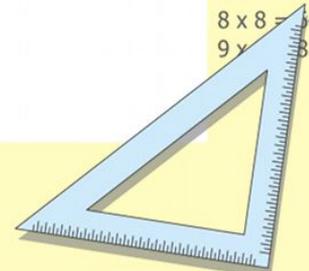
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

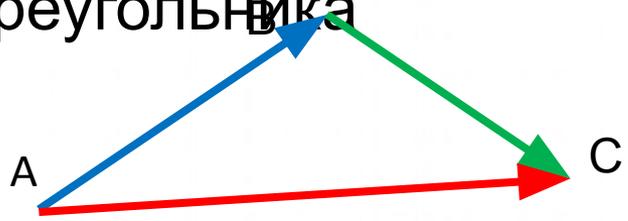


$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



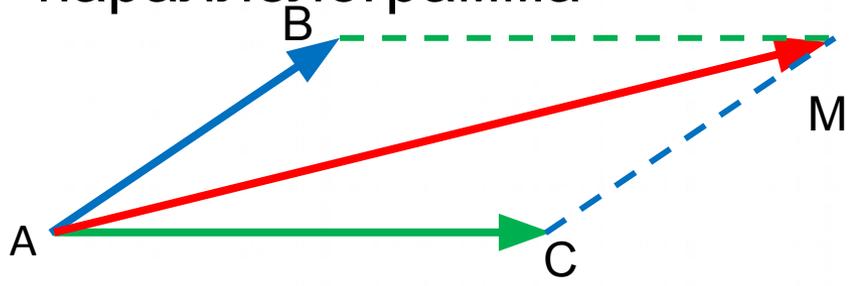
# Сложение векторов

Правило  
треугольника



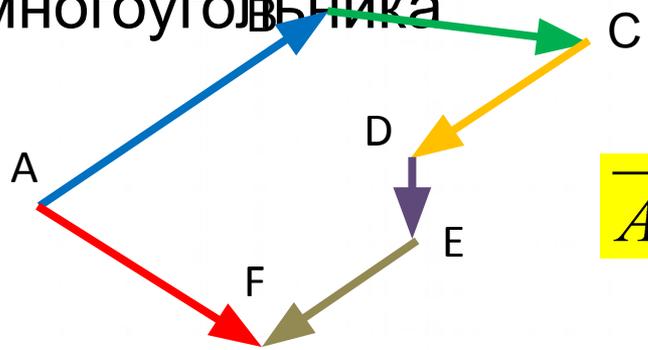
$$\vec{AB} + \vec{BC} = \vec{AC}$$

Правило  
параллелограмма



$$\vec{AB} + \vec{AC} = \vec{AM}$$

Правило  
многоугольника



$$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF} = \vec{AF}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

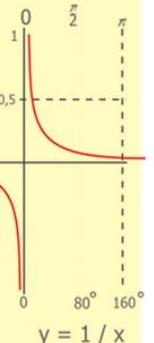
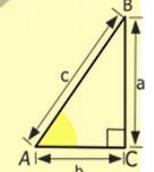
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

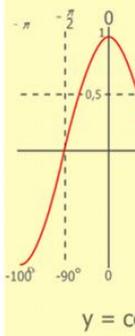
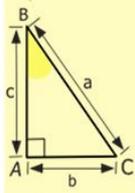
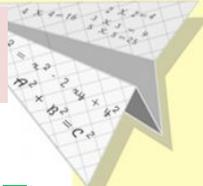
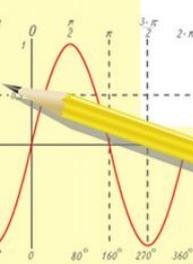
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

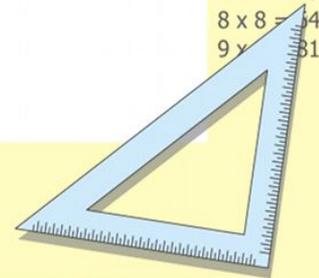
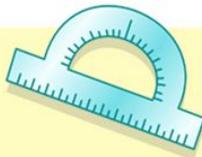
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

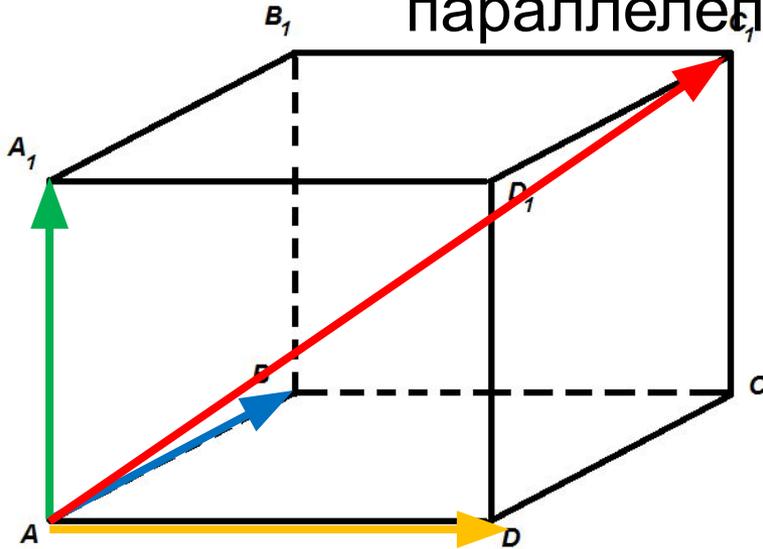


- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



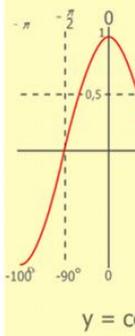
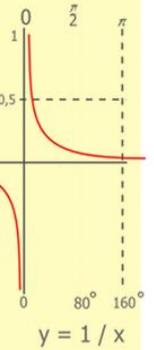
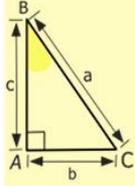
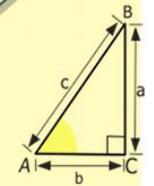
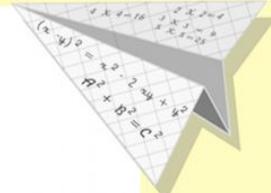
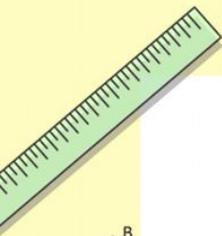
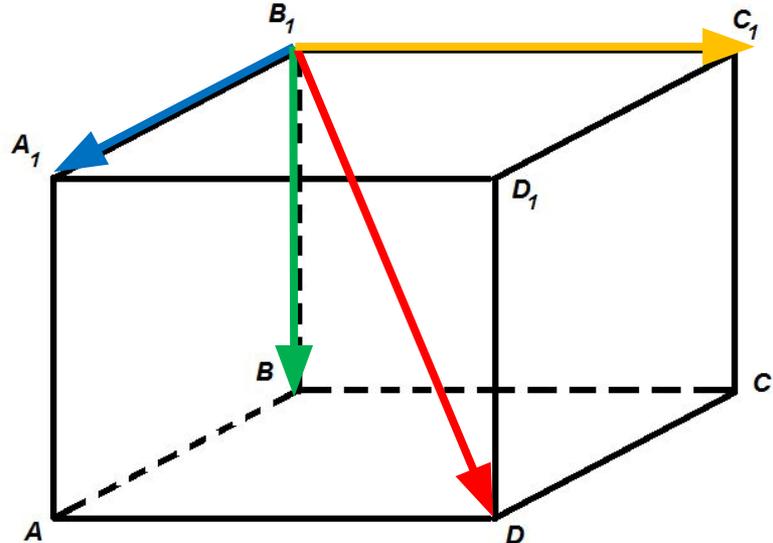
# Сложение векторов

## Правило параллелепипеда



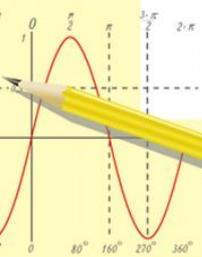
$$\vec{AB} + \vec{AD} + \vec{AA_1} = \vec{AC_1}$$

$$\vec{B_1C_1} + \vec{B_1A_1} + \vec{B_1B} = \vec{B_1D}$$



$\frac{1}{2} 500$   
 $\times 42$   
 $210$   
 $+ 84$   
 $\hline 105000$

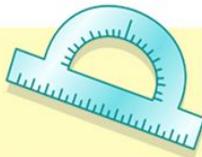
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

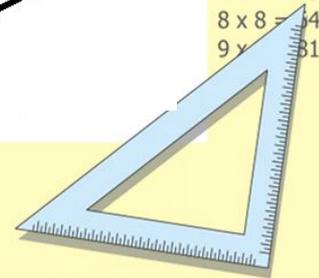
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin y \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

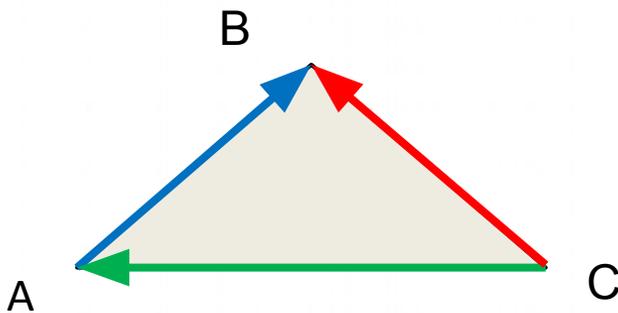
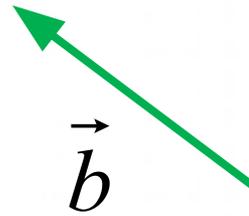
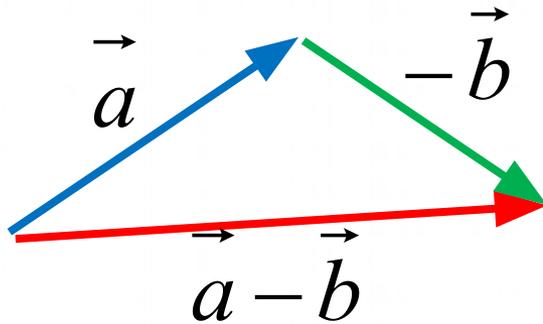
$$(x+y)(x-y) = x^2 - y^2$$



# Вычитание векторов

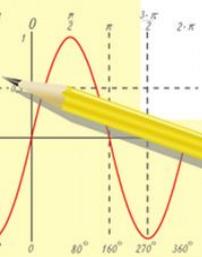
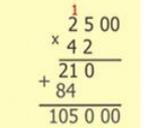
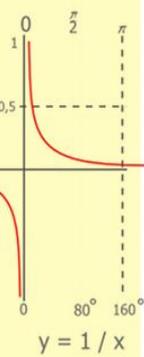
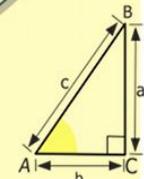
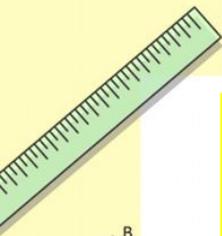
$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

Чтобы найти разность векторов, нужно к первому вектору прибавить вектор, противоположный второму



$$\vec{AB} - \vec{AC} = \vec{AB} + \vec{CA} =$$

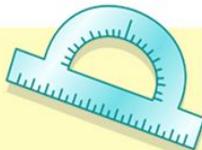
$$= \vec{CA} + \vec{AB} = \vec{CB}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

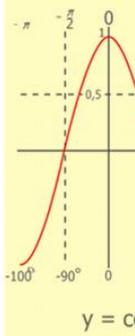
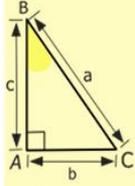
$$\sin 90^\circ = 1$$



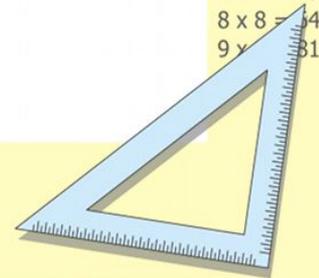
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

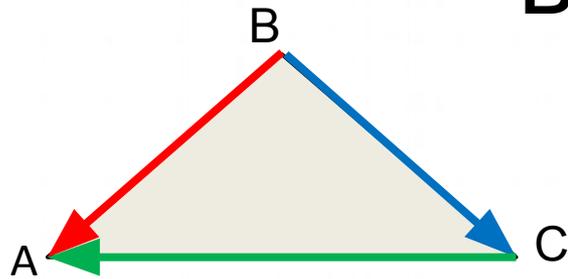


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- 6 x 6 = 36
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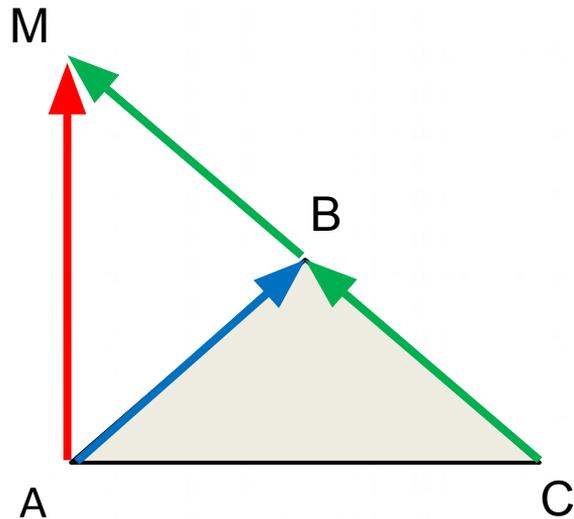


# ВЫЧИСЛИТЬ ВЕКТОРНОЕ

## ВЫРАЖЕНИЕ

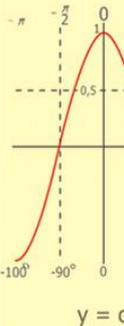
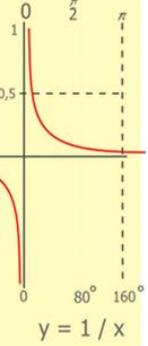
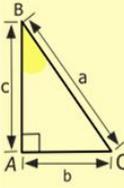
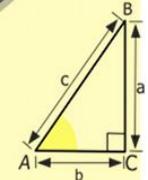
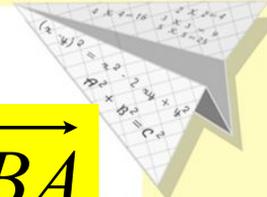
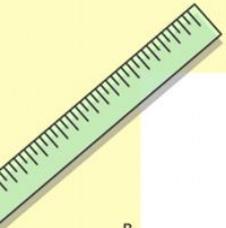


$$\vec{BC} - \vec{AC} = \vec{BC} + \vec{CA} = \vec{BA}$$



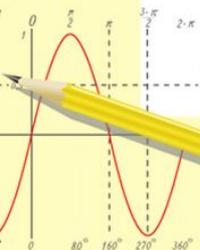
$$\vec{AB} - \vec{BC} = \vec{AB} + \vec{CB} =$$

$$= \vec{AB} + \vec{BM} = \vec{AM}$$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

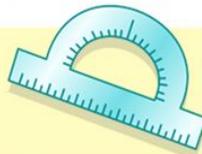
- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

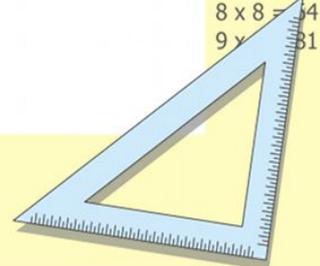
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



# ВЫЧИСЛИТЬ ВЕКТОРНОЕ

## ВЫРАЖЕНИЕ

1)

$$\overrightarrow{DM} - \overrightarrow{EK} - \overrightarrow{PM} + \overrightarrow{CD} - \overrightarrow{KP} =$$

$$= \overrightarrow{DM} + \overrightarrow{KE} + \overrightarrow{MP} + \overrightarrow{CD} + \overrightarrow{PK} =$$

$$= \overrightarrow{CD} + \overrightarrow{DM} + \overrightarrow{MP} + \overrightarrow{PK} + \overrightarrow{KE} = \overrightarrow{CE}$$

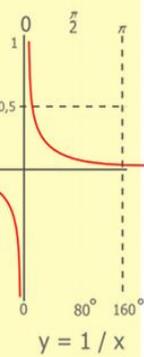
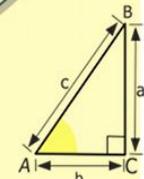
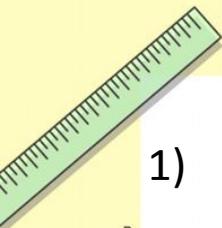
2)

$$\overrightarrow{TP} - \overrightarrow{KM} - \overrightarrow{CP} + \overrightarrow{BP} + \overrightarrow{KT} - \overrightarrow{MP} =$$

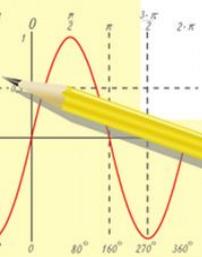
$$= \overrightarrow{TP} + \overrightarrow{MK} + \overrightarrow{PC} + \overrightarrow{BP} + \overrightarrow{KT} + \overrightarrow{PM} =$$

$$= (\overrightarrow{KT} + \overrightarrow{TP} + \overrightarrow{PM} + \overrightarrow{MK}) + (\overrightarrow{BP} + \overrightarrow{PC}) =$$

$$= \vec{0} + \overrightarrow{BC} = \overrightarrow{BC}$$



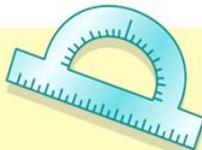
$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

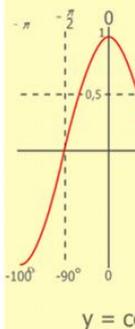
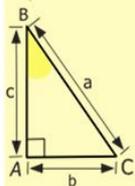
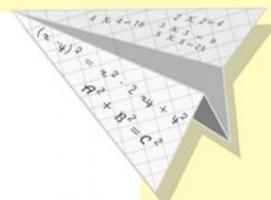
$$\sin 90^\circ = 1$$



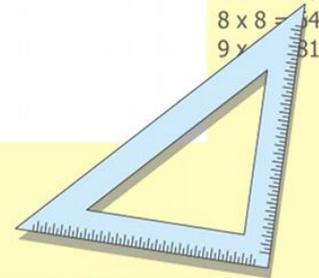
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

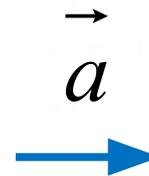


$2 \times 2 = 4$   
 $3 \times 3 = 9$   
 $4 \times 4 = 16$   
 $5 \times 5 = 25$   
 $6 \times 6 = 36$   
 $7 \times 7 = 49$   
 $8 \times 8 = 64$   
 $9 \times 9 = 81$

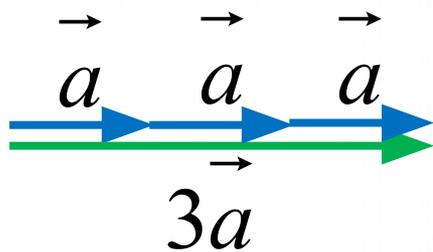


# Умножение вектора на число

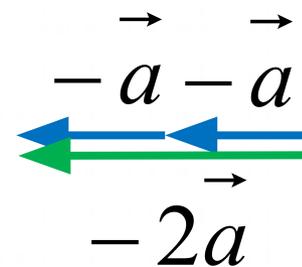
$$\vec{b} = k\vec{a} \Leftrightarrow \begin{cases} |\vec{b}| = |k| \cdot |\vec{a}| \\ \vec{a} \uparrow\uparrow \vec{b}, \text{ если } k > 0 \\ \vec{a} \uparrow\downarrow \vec{b}, \text{ если } k < 0 \end{cases}$$



$$3\vec{a} = \vec{a} + \vec{a} + \vec{a}$$



$$-2\vec{a} = 2 \cdot (-\vec{a}) = (-\vec{a}) + (-\vec{a})$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

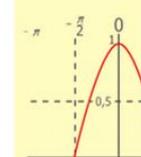
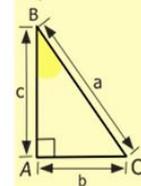
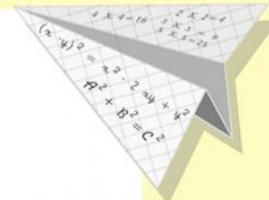
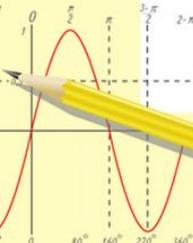
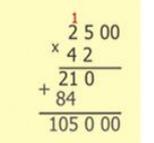
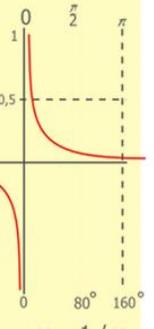
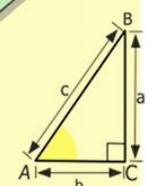
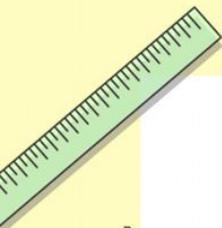
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

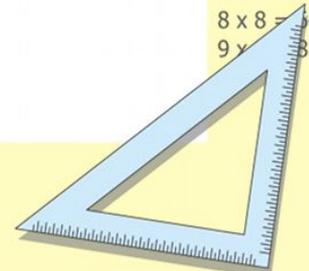
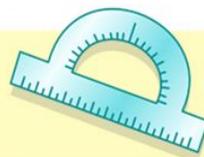
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81

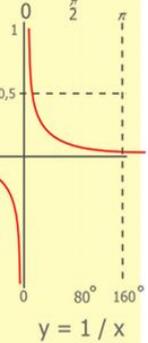
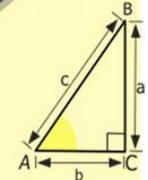
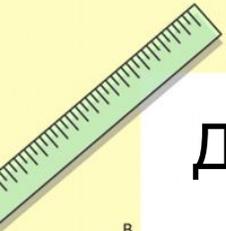
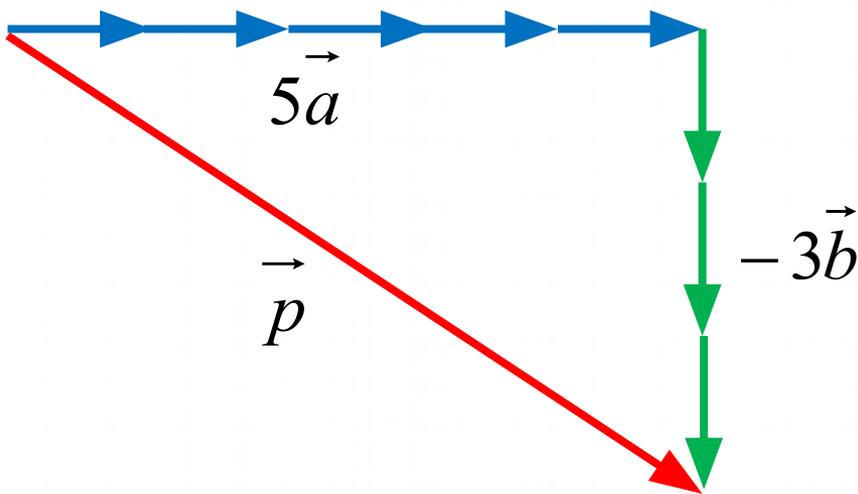


# Действия с векторами

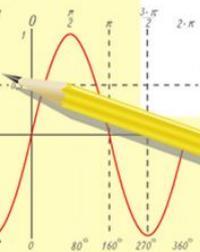
Даны  $\vec{a}$  и  $\vec{b}$ . Построить  $\vec{p} = 5\vec{a} - 3\vec{b}$



Построение  $\vec{p} = 5\vec{a} - 3\vec{b} = 5\vec{a} + (-3\vec{b})$



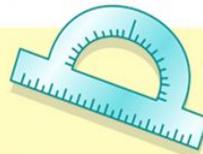
$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

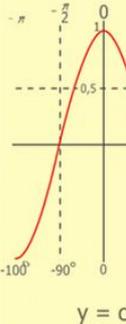
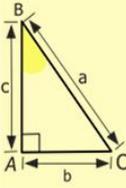
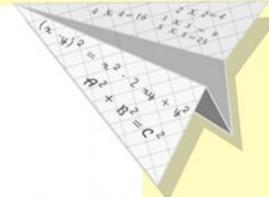
$$\sin 90^\circ = 1$$



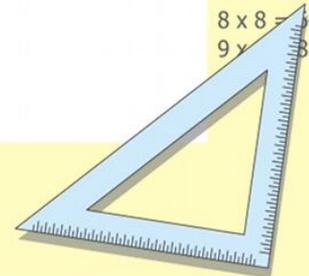
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

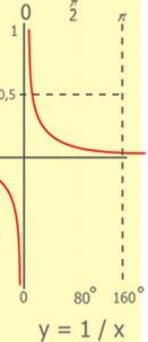
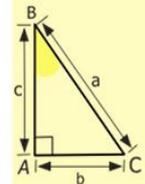
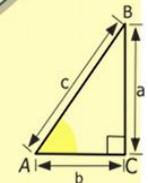
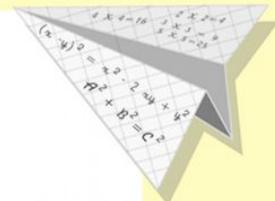
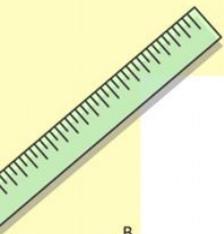
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$

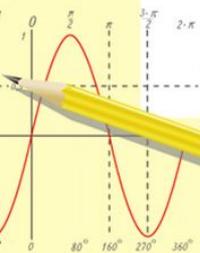


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ГБПОУ ЗКНО  
Москва, 2021г.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

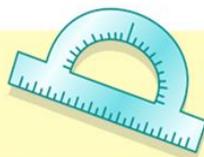
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