Faculty of Information Technology

Fall 2020

Modelling and Simulation

IS 331

Lec (6)

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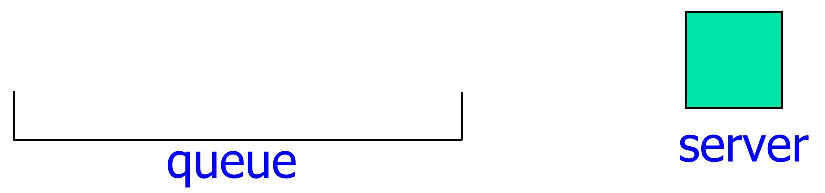
Single server Queuing system modeling

Discrete event Simulation (DES)

Representation of the system schematically



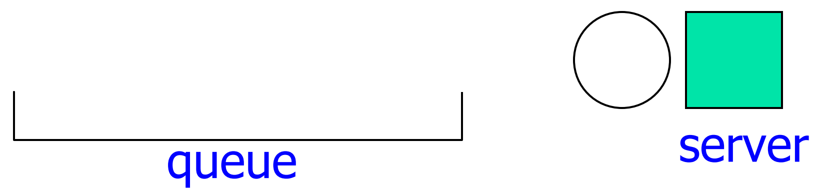
Server=idle

queue=0

e_0 =0

Representation of the system schematically

First customer arrival time :
A_1 =1min , T1=1 min



Generate service time :
S1=0.4 min
C1=1.4 min

Server=BUSY

queue=0

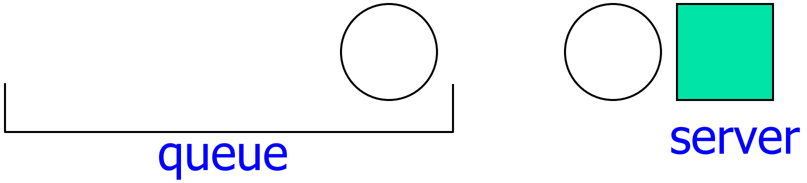
$e_1 = T_1$

Representation of the system schematically

second customer arrival time :

A_2 =0.2 min , T2=1.2 min

T2<C1 □ e2=T2



Server=busy

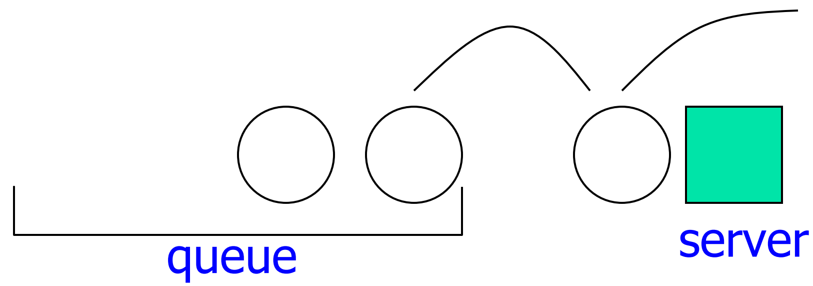
queue=1

$e_2 = T_2$

Representation of the system schematically

third customer arrival time :
A_3 =0.4 min , T3=1.6 min

T3>C1 □ e3=C1



Generate service time
For second customer :
S2=0.2 min
C2=C1+S2=1.6 min

Server=busy

queue=1

$e_3 = C_1$

Therefore first customer departs after 1.4 min with no
Delay in the system (D=0)

Second customer now **enter service** :
Delay in system = C1-T2=1.4-1.2=0.2 min.

1. Simulate M/M/1 queuing system. Assume the queue is empty at the beginning and server is idle. The inter-arrival and service times are:

A_i	0.4	1.2	0.5	1.7	0.2	1.6	0.2	1.4	1.9
S_i	2	0.7	0.2	1.1	3.7	0.6	-	-	-

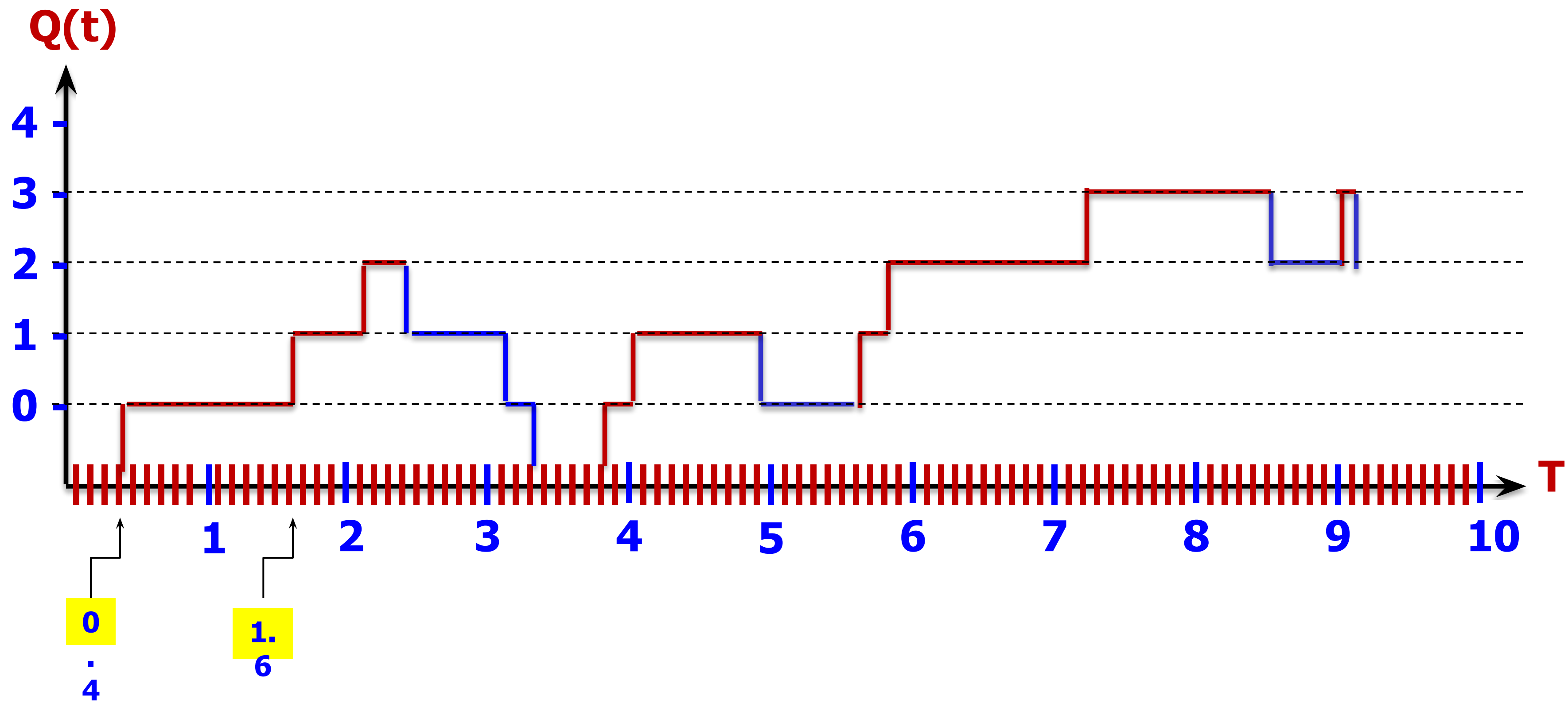
Assume the simulation run is until 6th customer leaves the facility.

- Trace the simulation by following the events.
- Sketch Q(t) and B(t) of the queue and estimate the following parameters
  - i. Average delays in the queue.
  - ii. Time-average number of customers in the queue.
  - iii. Utilization of the server.

A_i	0.4	1.2	0.5	1.7	0.2	1.6	0.2	1.4	1.9
S_i	2	0.7	0.2	1.1	3.7	0.6	-	-	-

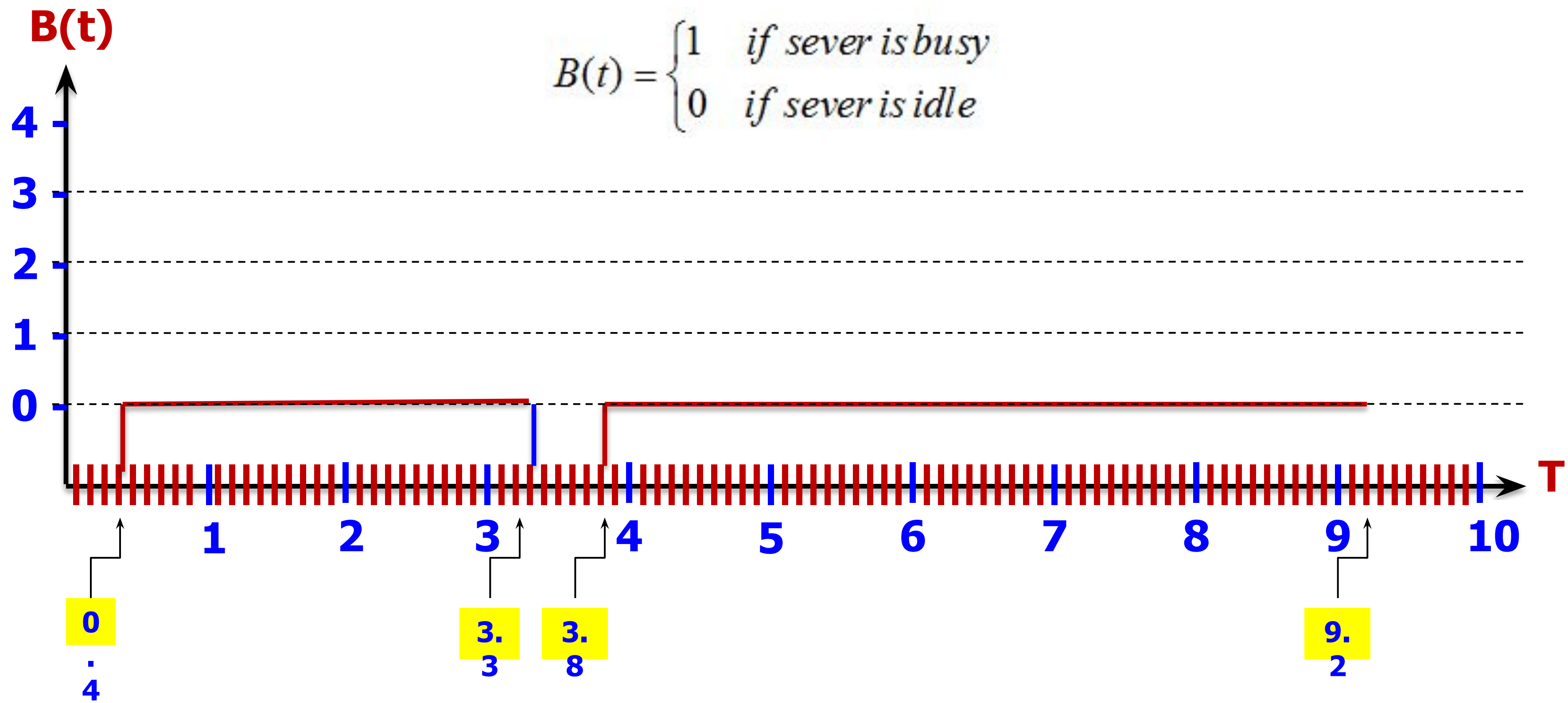
9	8	7	6	5	4	3	2	1	Customers
1.9	**1.4**	**0.2**	**1.6**	**0.2**	**1.7**	**0.5**	**1.2**	**0.4**	Inter-arrivals Ai
-	**-**	**-**	**0.6**	**3.7**	**1.1**	**0.2**	**0.7**	**2**	Service Si
9.1	**7.2**	**5.8**	**5.6**	**4**	**3.8**	**2.1**	**1.6**	**0.4**	Arrival Times
			9.2	**8.6**	**4.9**	**3.3**	**3.1**	**2.4**	End Service Time
		3.4	**3**	**0.9**	**-**	**1**	**0.8**	**-**	Delays

9	8	7	6	5	4	3	2	1	Customers
1.9	1.4	0.2	1.6	0.2	1.7	0.5	1.2	0.4	Inter-arrivals Ai
-	-	-	0.6	3.7	1.1	0.2	0.7	2	Service Si
9.1	7.2	5.8	5.6	4	3.8	2.1	1.6	0.4	Arrival Times
			9.2	8.6	4.9	3.3	3.1	2.4	End Service Time
		3.4	3	0.9	-	1	0.8	-	Delays

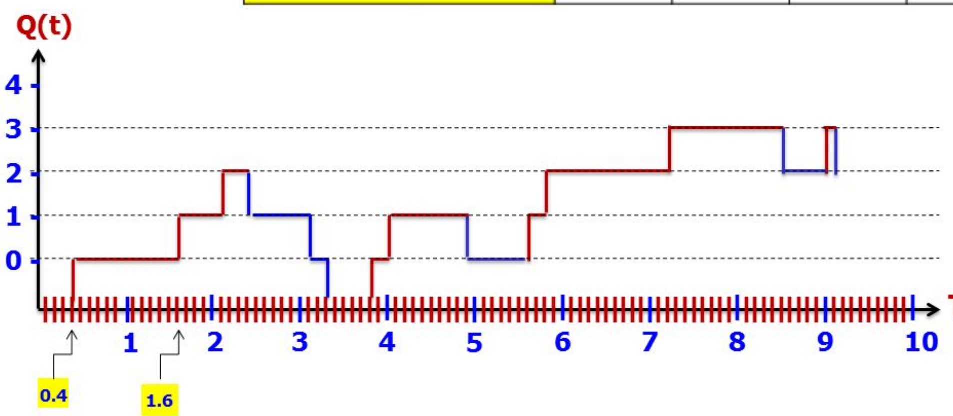


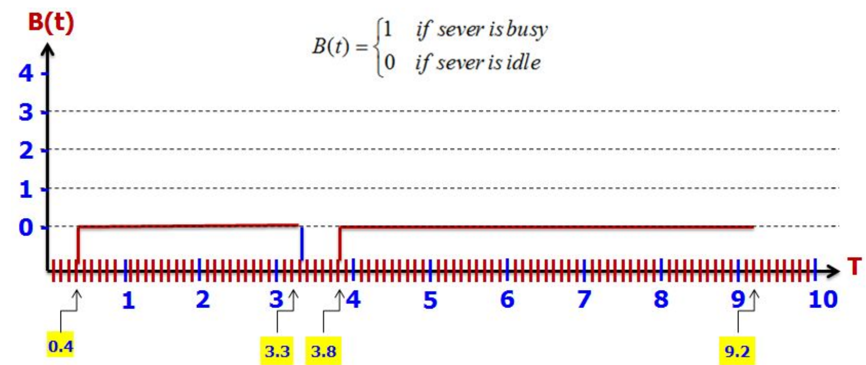
9	8	7	6	5	4	3	2	1	Customers
1.9	1.4	0.2	1.6	0.2	1.7	0.5	1.2	0.4	Inter-arrivals Ai
-	-	-	0.6	3.7	1.1	0.2	0.7	2	Service Si
9.1	7.2	5.8	5.6	4	3.8	2.1	1.6	0.4	Arrival Times
			9.2	8.6	4.9	3.3	3.1	2.4	End Service Time
		3.4	3	0.9	-	1	0.8	-	Delays

$$B(t) = \begin{cases} 1 & \text{if sever is busy} \\ 0 & \text{if sever is idle} \end{cases}$$



Customers	1	2	3	4	5	6	7	8	9
Inter-arrivals Ai	0.4	1.2	0.5	1.7	0.2	1.6	0.2	1.4	1.9
Service Si	2	0.7	0.2	1.1	3.7	0.6	-	-	-
Arrival Times	0.4	1.6	2.1	3.8	4	5.6	5.8	7.2	9.1
End Service Time	2.4	3.1	3.3	4.9	8.6	9.2			
Delays	-	0.8	1	-	0.9	3	3.4		

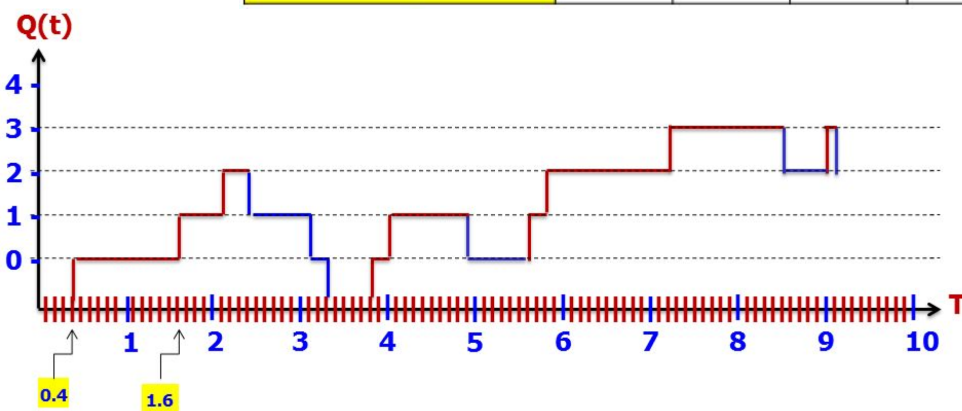


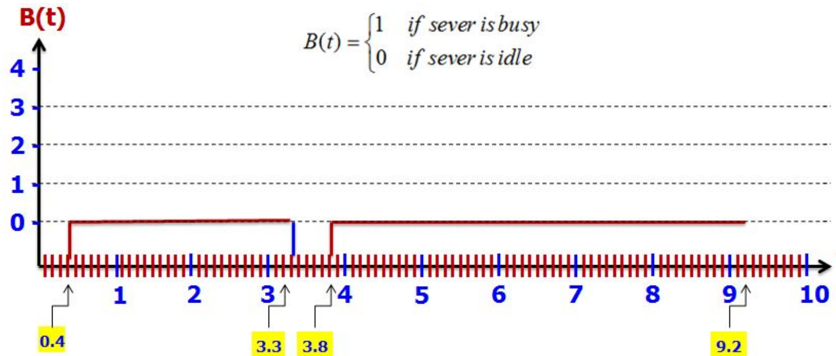


Average delays in the queue.

$$= \frac{\text{Sum of all delays}}{\text{No of customers}} = \frac{0.8 + 1 + 0.9 + 3 + 3.4}{7} = 9.1 \text{ minute/customer}$$

Customers	1	2	3	4	5	6	7	8	9
Inter-arrivals Ai	0.4	1.2	0.5	1.7	0.2	1.6	0.2	1.4	1.9
Service Si	2	0.7	0.2	1.1	3.7	0.6	-	-	-
Arrival Times	0.4	1.6	2.1	3.8	4	5.6	5.8	7.2	9.1
End Service Time	2.4	3.1	3.3	4.9	8.6	9.2			
Delays	-	0.8	1	-	0.9	3	3.4		





Time-average number of customers in the queue

T_0 = (1.6-0.4)+(3.3-3.1)+(4-3.8)+(5.6-4.9) = 1.2+0.2+0.2+0.7 = 2.3

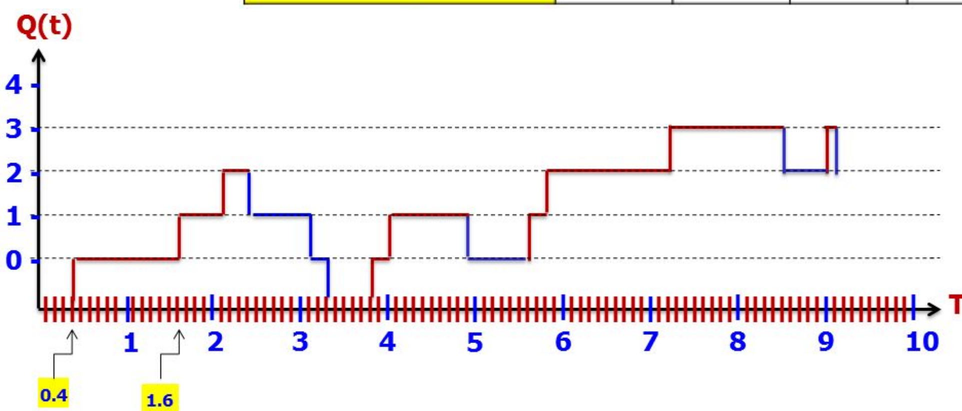
T_1 = (2.1-1.6)+(3.1-2.4)+(4.9-4)+(5.8-5.6) = 0.5+0.7+0.9+0.2 = 2.3

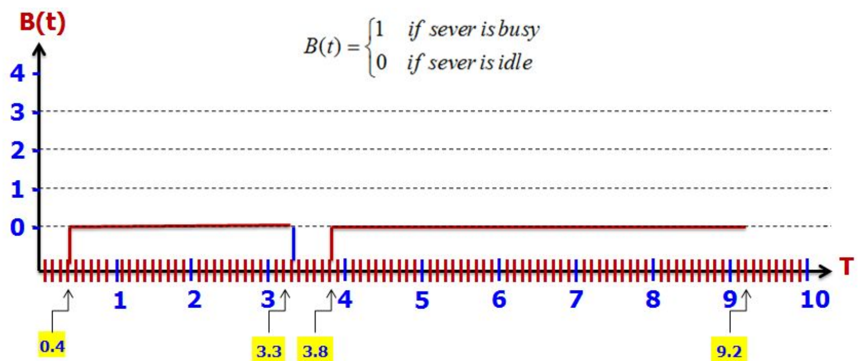
T_2 = (2.4-2.1)+(7.2-5.8)+(9.1-8.6) = 0.3+1.4+0.5 = 2.2

T_3 = (8.6-7.2)+(9.2-9.1) = 1.4+0.1 = 1.5

$$q(n)=\frac{\sum_{i=0}^{\infty} iT_i}{T(n)} = \frac{2.3\,(0) + 2.3\,(1) + 2.2\,(2) + 1.5\,(3)}{9.2} = 1.21 \text{ Customer}$$

Customers	1	2	3	4	5	6	7	8	9
Inter-arrivals Ai	0.4	1.2	0.5	1.7	0.2	1.6	0.2	1.4	1.9
Service Si	2	0.7	0.2	1.1	3.7	0.6	-	-	-
Arrival Times	0.4	1.6	2.1	3.8	4	5.6	5.8	7.2	9.1
End Service Time	2.4	3.1	3.3	4.9	8.6	9.2			
Delays	-	0.8	1	-	0.9	3	3.4		





Utilization of the server

$$u(n) = \frac{T_{total}[B(t)=1]}{T(n)} = \frac{\text{Server on Times}}{\text{Total Time}} = \frac{(3.3 - 0.4) + (9.2 - 3.8)}{9.2}$$

$$= \frac{2.9 + 5.4}{9.2} = 0.9021$$

2. A service facility consists of two servers in series(tandem), each with its own FIFO queue (see Fig.1), A customer completing service at server 1 proceeds to server 2, while a customer completing service at server 2 leaves the facility. Assume that the inter-arrival times of customers to server 1 are IID exponential random variables with mean 1 minute. Service times of customers at server 1 are IID exponential random variables with mean 0.8 minute, and at server 2 are IID exponential random variables with mean 0.9 minute.

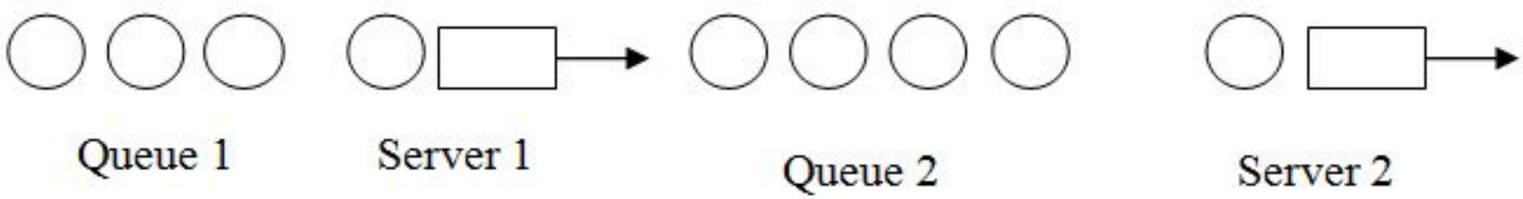


Fig.1. A tandem queuing system.

The inter-arrival and service times are:

A_i	1.445	1.215	0.241	1.565	0.126
S_i1	0.071	0.590	0.624	0.351	0.713
S_i2	2.082	0.904	1.406	0.566	-

Assume the simulation run is until 4th customer leaves the facility.

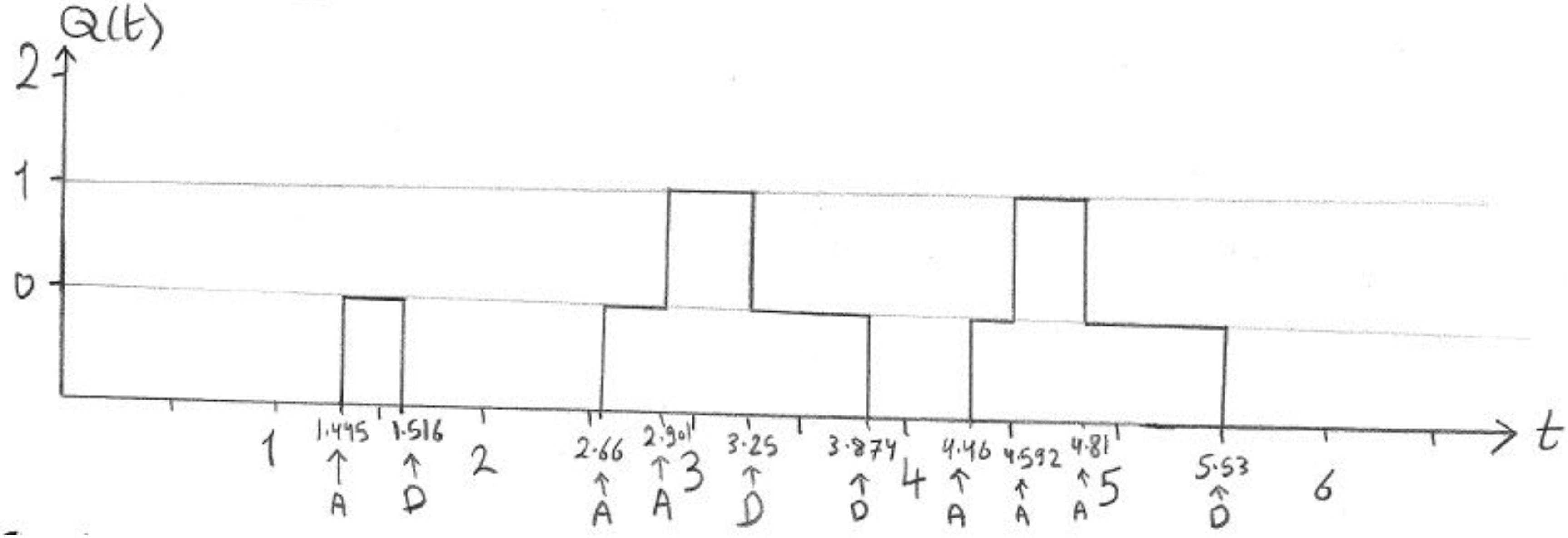
- Trace the simulation by following the events.
- Sketch Q(t) and B(t) of both queues and estimate the following parameters
  i. Average delays in every queue.
  ii. Time-average number of customers in every queue.
  iii. Utilization of both servers.

A_i	1.445	1.215	0.241	1.565	0.126
S_i1	0.071	0.590	0.624	0.351	0.713
S_i2	2.082	0.904	1.406	0.566	-

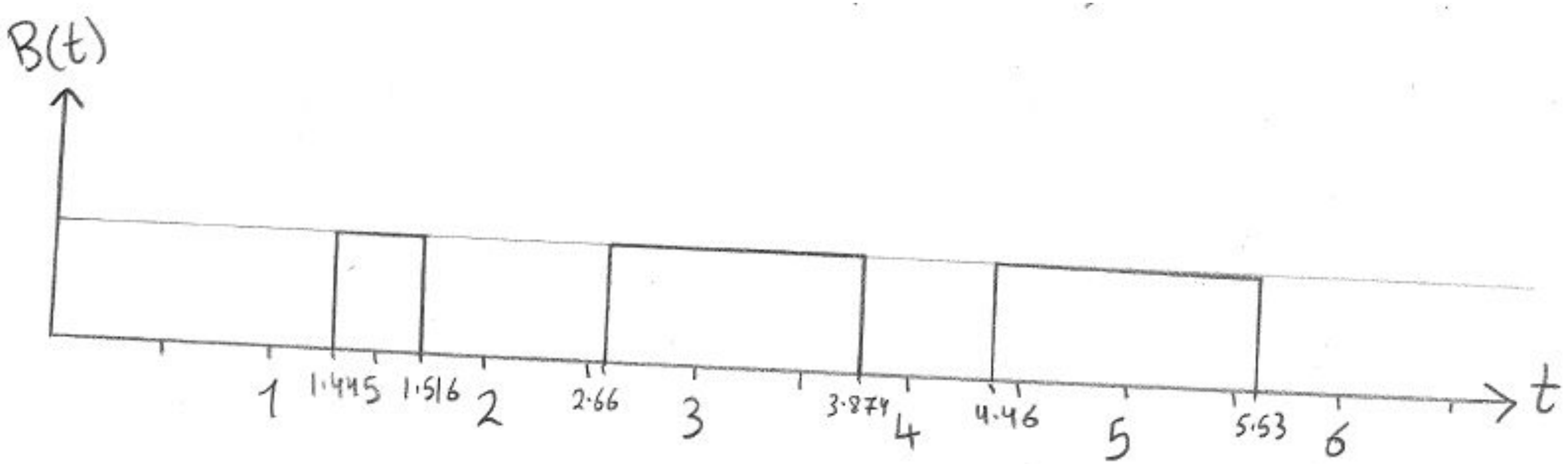
5	4	3	2	1	Customers
0.126	**1.565**	**0.241**	**1.215**	**1.445**	Inter-arrivals Ai
0.713	**0.351**	**0.624**	**0.59**	**0.071**	Service Si 1
-	**0.566**	**1.406**	**0.904**	**2.082**	Service Si 2
4.592	**4.466**	**2.901**	**2.66**	**1.445**	Arrival Times
5.53	**4.817**	**3.874**	**3.25**	**1.516**	End Service Time 1
	6.474	**5.908**	**4.502**	**3.598**	End Service Time 2
0.225	**-**	**0.349**	**-**	**-**	Delays 1
0.944	**1.091**	**0.628**	**0.348**	**-**	Delays 2

Customers	1	2	3	4	5
Inter-arrivals Ai	1.445	1.215	0.241	1.565	0.126
Service Si 1	0.071	0.59	0.624	0.351	0.713
Arrival Times	1.445	2.66	2.901	4.466	4.592
End Service Time 1	1.516	3.25	3.874	4.817	5.53
Delays 1	-	-	0.349	-	0.225

For server 1:



Customers	1	2	3	4	5
Inter-arrivals Ai	1.445	1.215	0.241	1.565	0.126
Service Si 1	0.071	0.59	0.624	0.351	0.713
Arrival Times	1.445	2.66	2.901	4.466	4.592
End Service Time 1	1.516	3.25	3.874	4.817	5.53
Delays 1	-	-	0.349	-	0.225



Customers	1	2	3	4	5
Inter-arrivals Ai	1.445	1.215	0.241	1.565	0.126
Service Si 1	0.071	0.59	0.624	0.351	0.713
Arrival Times	1.445	2.66	2.901	4.466	4.592
End Service Time 1	1.516	3.25	3.874	4.817	5.53
Delays 1	-	-	0.349	-	0.225

i) Average delay $= \frac{0.349 + 0.225}{5} = 0.1148$ min/customer

ii) Time-average number of customers

$T_1 = (3.25 - 2.901) + (4.817 - 4.592) = 0.574$

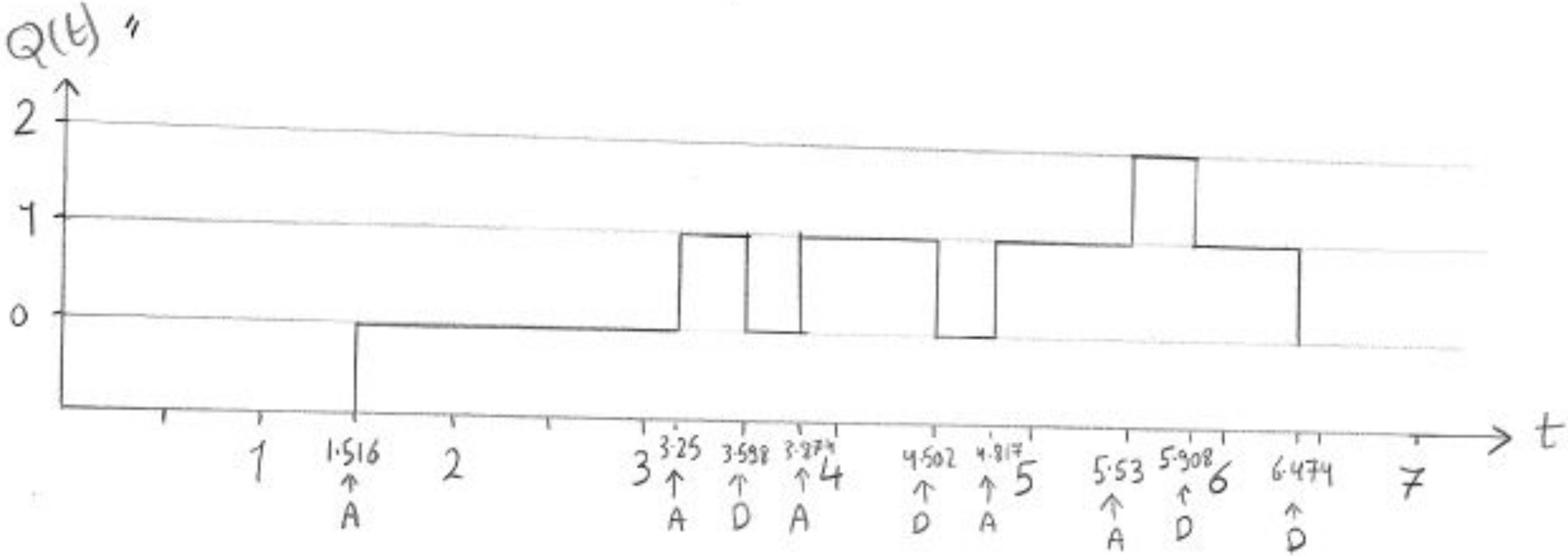
$Av. = \frac{T_1(1)}{T(n)} = \frac{0.574(1)}{6.474} = 0.0886$ customer.

iii) Utilization of server 1 $= \frac{(1.516 - 1.445) + (3.874 - 2.66) + (5.53 - 4.46)}{6.474}$

$= 0.3637$

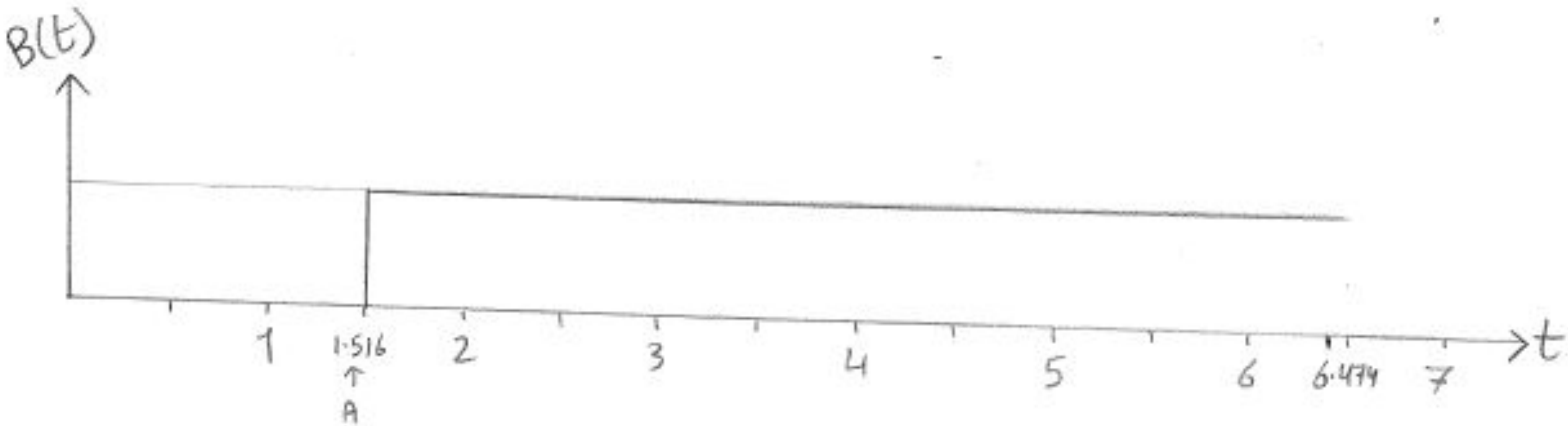
Customers	1	2	3	4	5
Inter-arrivals Ai	1.445	1.215	0.241	1.565	0.126
Service Si 2	2.082	0.904	1.406	0.566	-
End Service Time 1	1.516	3.25	3.874	4.817	5.53
End Service Time 2	3.598	4.502	5.908	6.474	
Delays 2	-	0.348	0.628	1.091	0.944

For server 2:



Customers	1	2	3	4	5
Inter-arrivals Ai	1.445	1.215	0.241	1.565	0.126
Service Si 2	2.082	0.904	1.406	0.566	-
End Service Time 1	1.516	3.25	3.874	4.817	5.53
End Service Time 2	3.598	4.502	5.908	6.474	
Delays 2	-	0.348	0.628	1.091	0.944

For server 2:



Customers	1	2	3	4	5
Inter-arrivals Ai	1.445	1.215	0.241	1.565	0.126
Service Si 2	2.082	0.904	1.406	0.566	-
End Service Time 1	1.516	3.25	3.874	4.817	5.53
End Service Time 2	3.598	4.502	5.908	6.474	
Delays 2	-	0.348	0.628	1.091	0.944

For server 2:

i) Average delay $= \frac{0.348 + 0.628 + 1.091 + 0.944}{5} = 0.6022$ min/customer

ii) $T_1 = (3.598 - 3.25) + (4.502 - 3.874) + (5.53 - 4.817) + (6.474 - 5.908) = 2.255$

$T_2 = 5.908 - 5.53 = 0.378$

Average Customers $= \frac{2.255(1) + 0.378(2)}{6.474} = 0.465$ customer

iii) Utilization of server 2 $= \frac{6.474 - 1.516}{6.474} = 0.7658$

END