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Mongeova projekcia

- **polohové úlohy**

Základné pojmy a obraz bodu v Mongeovej projekcii

Priemetne:

π – pôdorysňa, $^1s \perp \pi$,

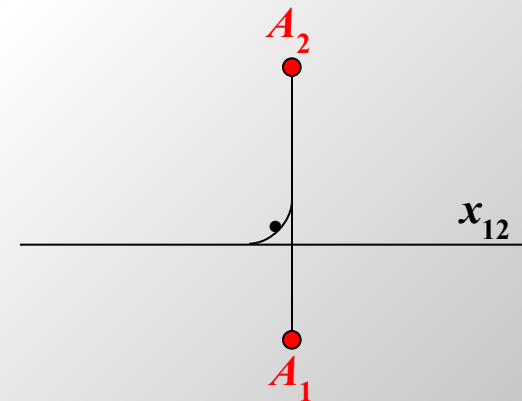
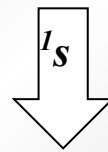
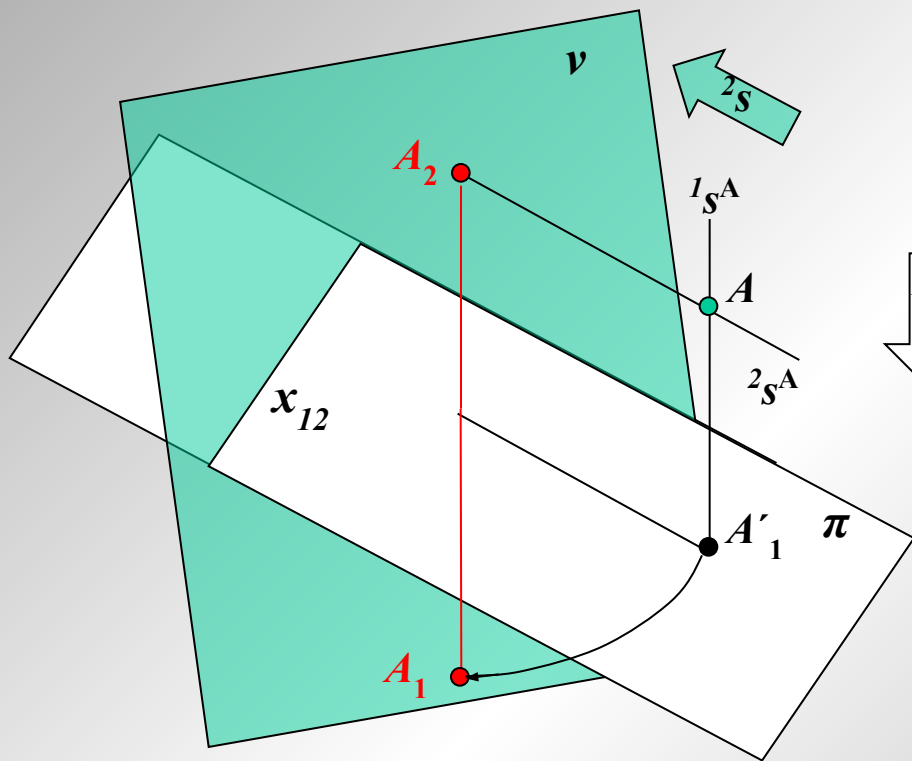
ν – nárysňa, $^2s \perp \nu$,

$\pi \perp \nu$, $\pi \cap \nu = x$, označujeme ju x_{12} – základnica.

Priemety bodu A :

$\pi \cap ^1s^A = A'_1$ – pôdorys bodu A , $^1s^A: A \in ^1s^A$, $^1s^A \perp \pi$,

$\nu \cap ^2s^A = A_2$ – nárys bodu A , $^2s^A: A \in ^2s^A$, $^2s^A \perp \nu$.



Združenie priemetní:

π otočíme do ν okolo x , A'_1 sa otočí do A_1 ,

A_1, A_2 – združené priemety bodu A ,

platí $A_1A_2 \perp x_{12}$,

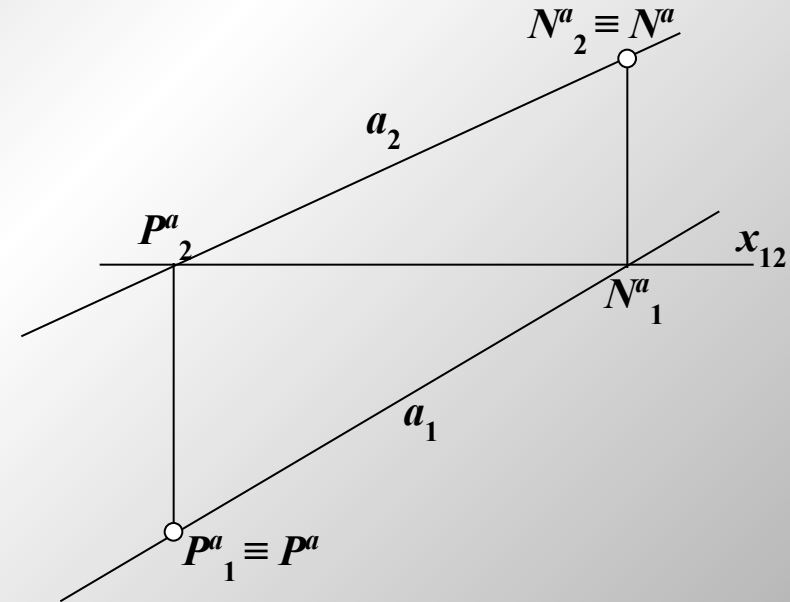
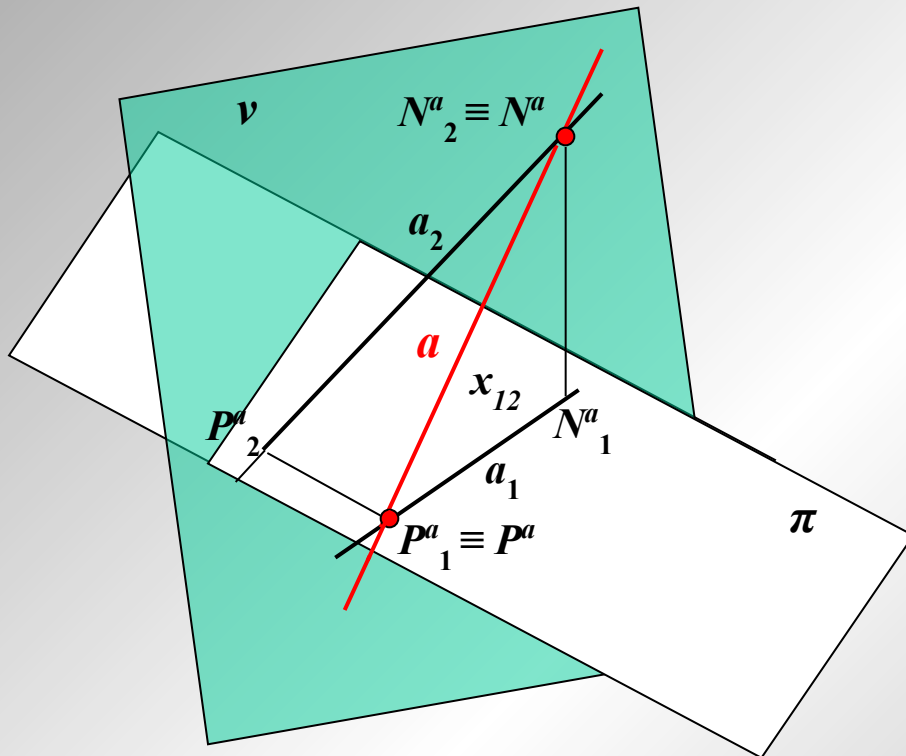
A_1A_2 – ordinála bodu A .

Definícia: Bijektívne zobrazenie, ktoré každému bodu $A \in E_3$ priradí združené priemety $[A_1, A_2]$, $A_1A_2 \perp x_{12}$, voláme **kolmé premietanie na dve navzájom kolmé priemetne – Mongeova projekcia**.

Obraz priamky v Mongeovej projekcii

Stopníky priamky: $a \cap \pi = P^a$ – pôdorysný stopník priamky a ,

$a \cap \nu = N^a$ – nárysný stopník priamky a .



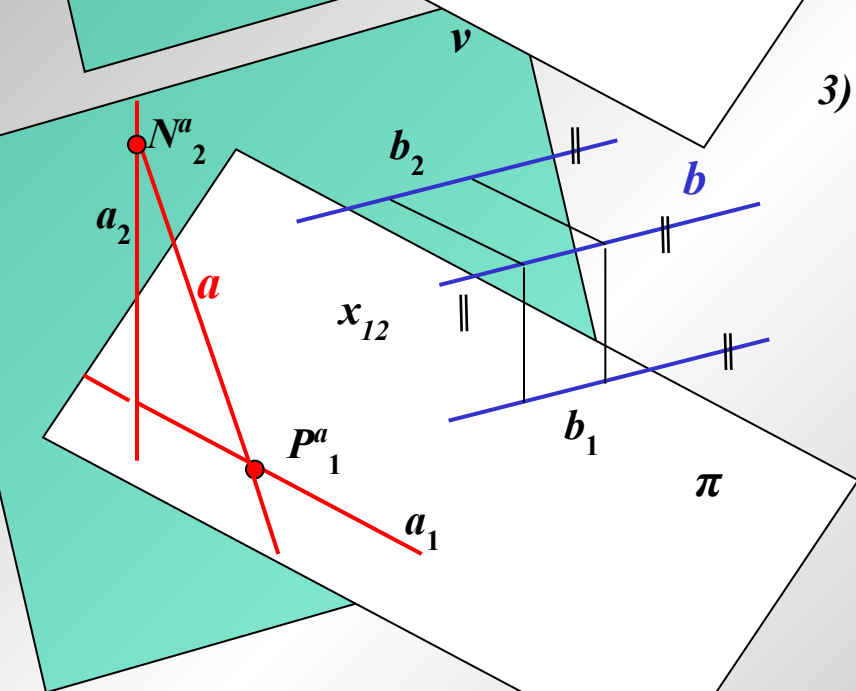
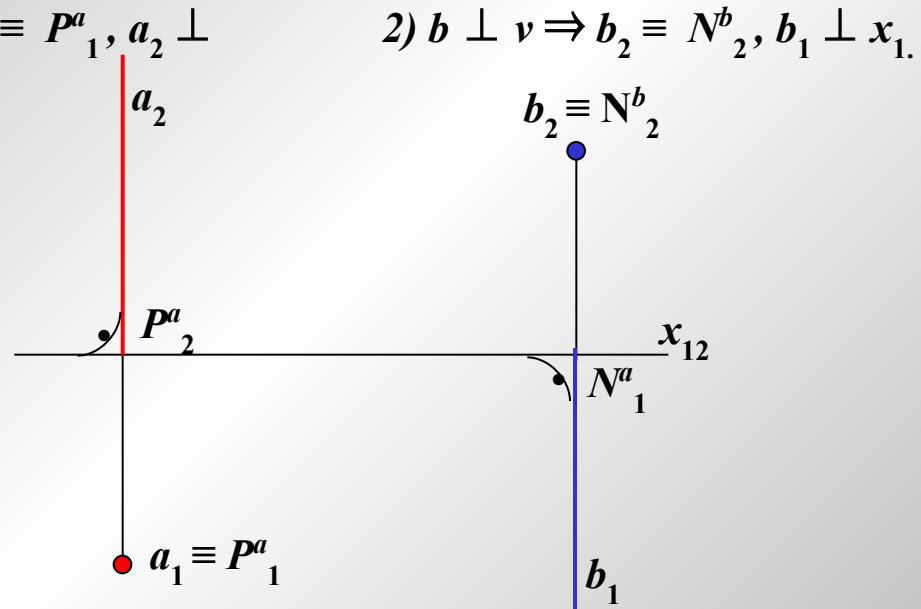
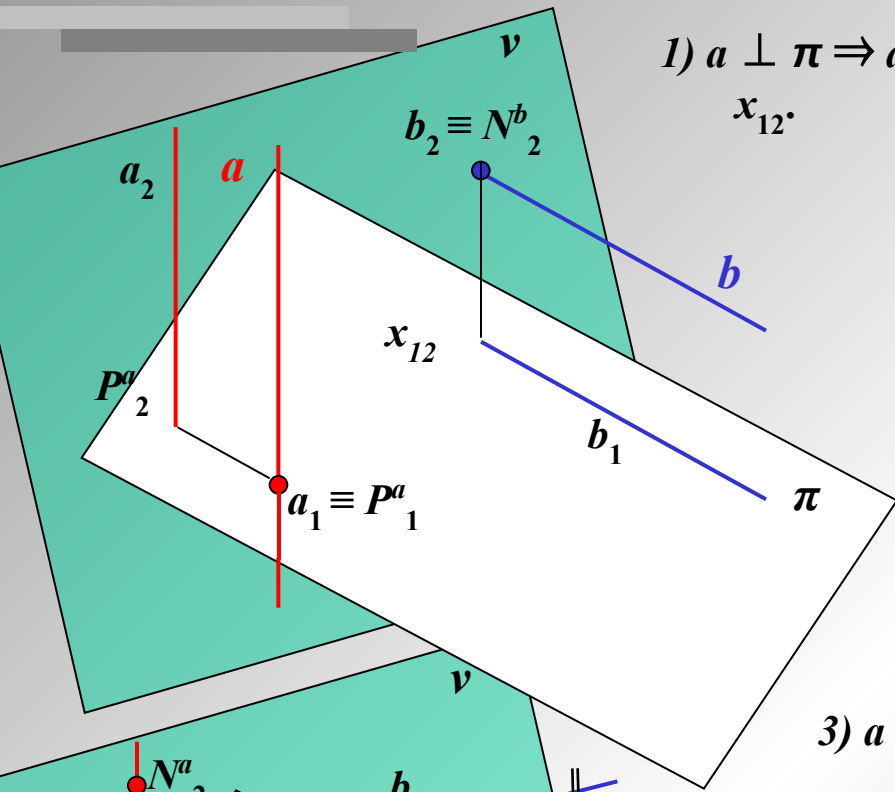
Konštrukcia pôdorysného stopníka:

$$a_2 \cap x_{12} = P_2^a, P_1 \in a_1$$

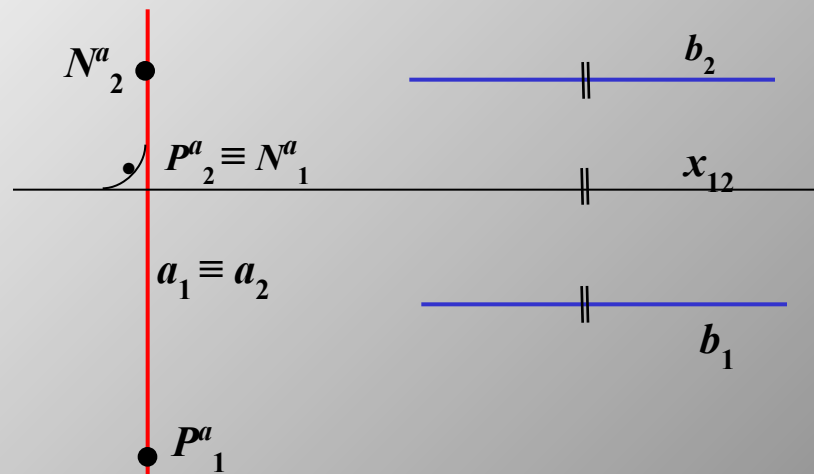
Konštrukcia nárysného stopníka:

$$a_1 \cap x_{12} = N_1^a, N_2 \in a_2.$$

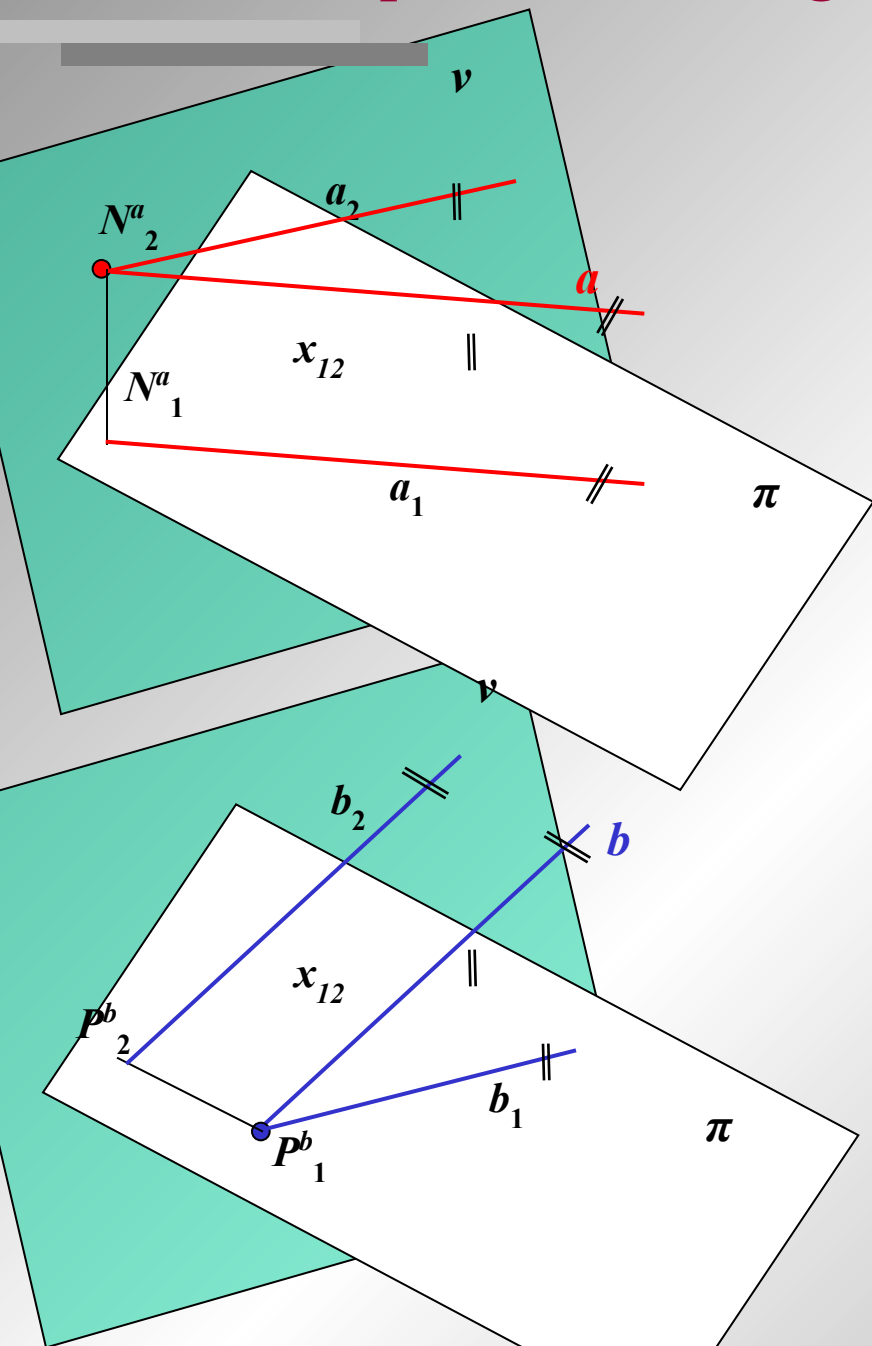
Obraz priamok v Mongeovej projekcii



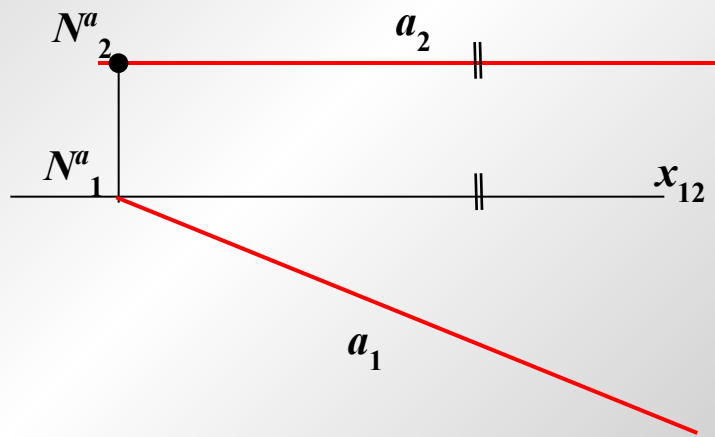
4) $b \parallel x \Rightarrow b_1 \parallel b_2 \parallel x_{12}$



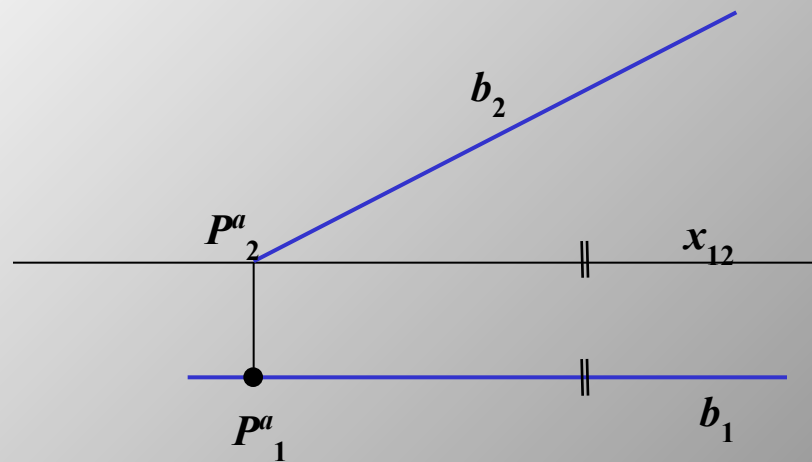
Obraz priamok v Mongeovej projekcii



5) $a \parallel \pi \Rightarrow a_2 \parallel x_{12}$



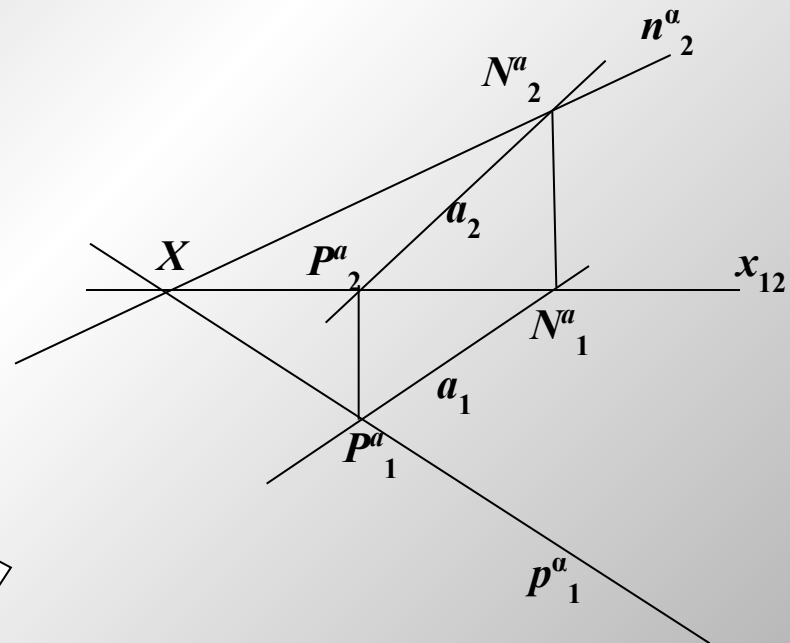
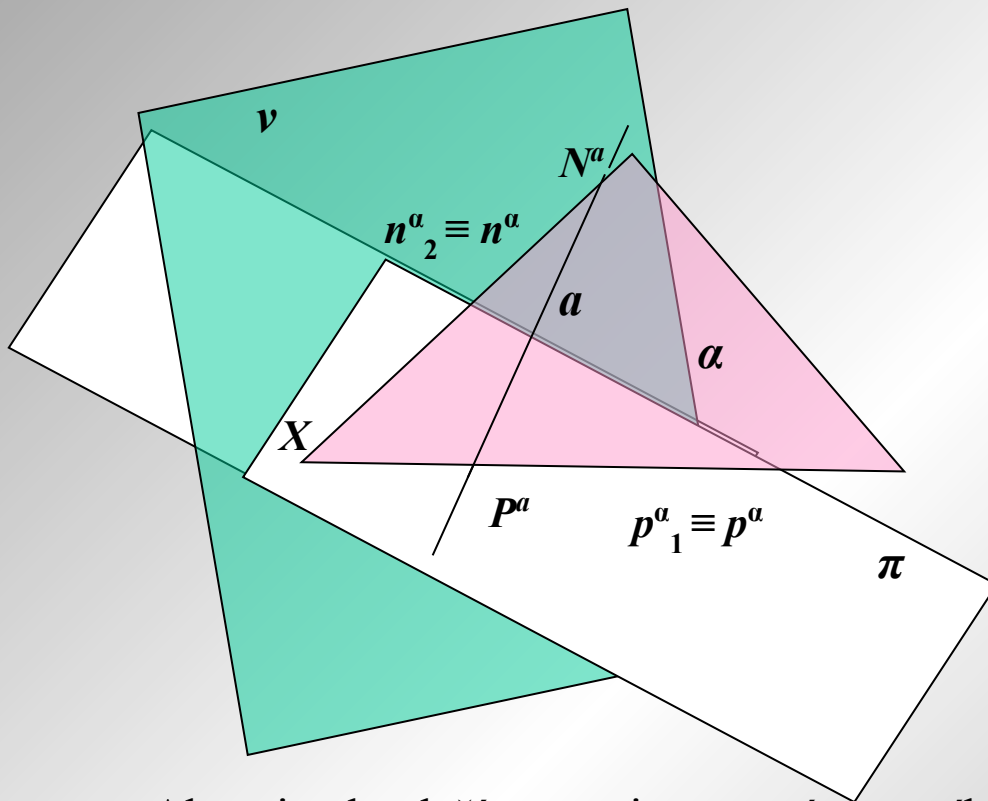
6) $b \parallel v \Rightarrow b_1 \parallel x_{12}$



Obraz roviny v Mongeovej projekcii

Stopy roviny: $\alpha \cap \pi = p^\alpha$ – pôdorysná stopa roviny α ,

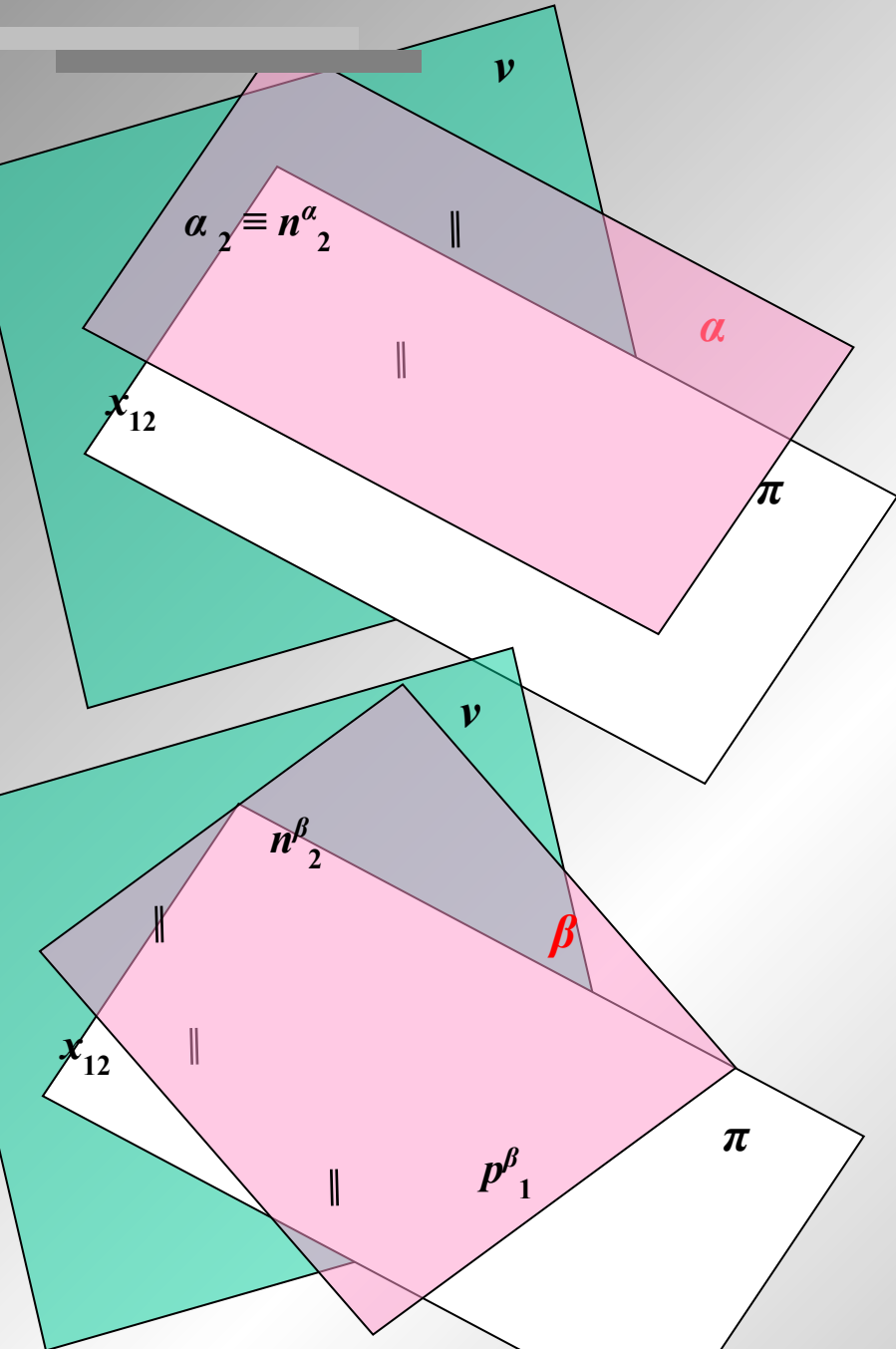
$\alpha \cap \nu = n^\alpha$ – nárysna stopa roviny α . Ak existuje $X = p^\alpha \cap n^\alpha$, potom $X \in x$.



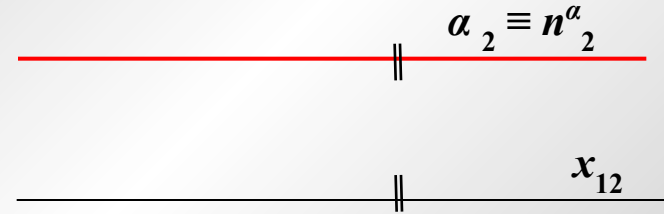
Ak priamka leží v rovine a má stopníky, potom jej pôdorysný stopník leží na pôdorysnej a nárysny na nárysnej stope roviny:

$$P^a_1 \in p^\alpha_1, N^a_2 \in n^\alpha_2$$

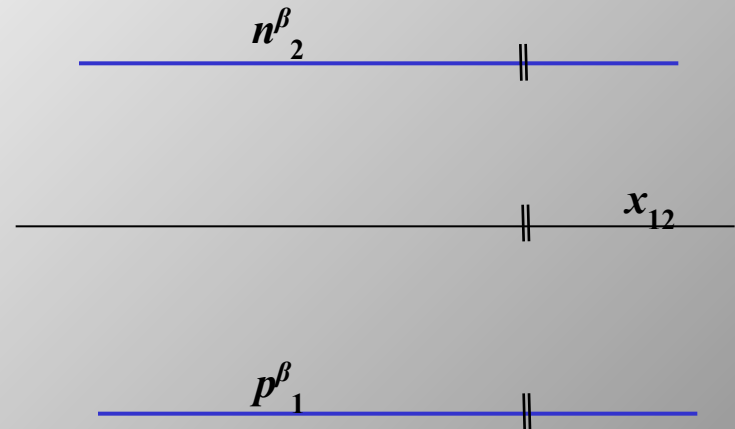
Roviny v Mongeovej projekcii



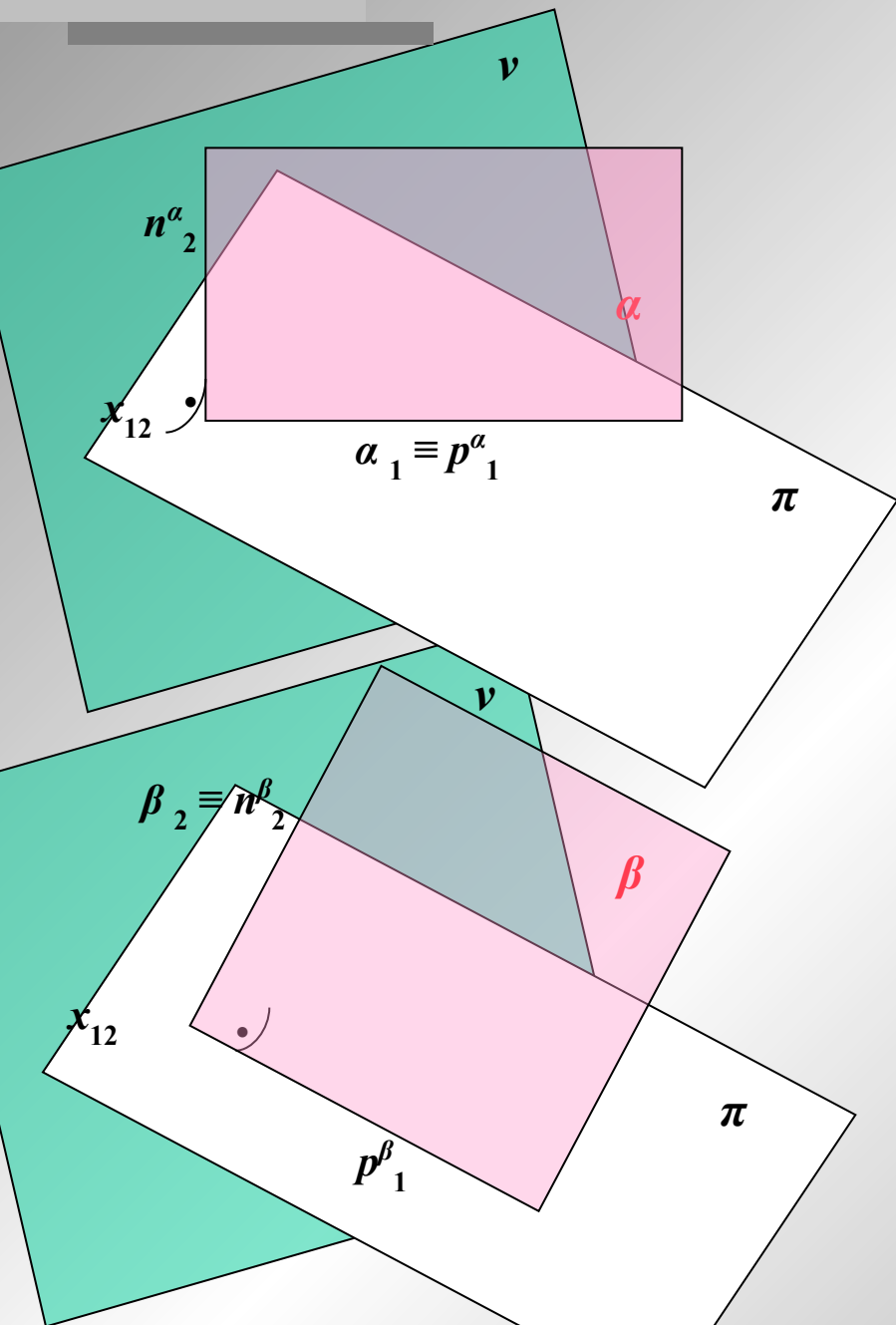
$$1) \alpha \parallel \pi \Rightarrow \alpha_2 \equiv n^{\alpha}_2 \parallel x_{12}$$



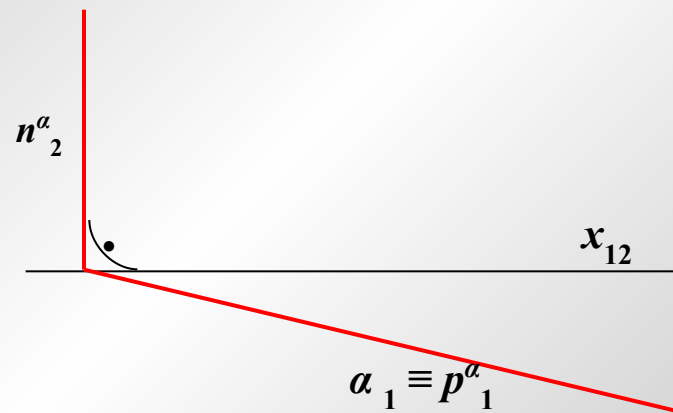
$$2) \beta \parallel \pi \Rightarrow p^{\beta}_1 \parallel n^{\beta}_2 \parallel x_{12}$$



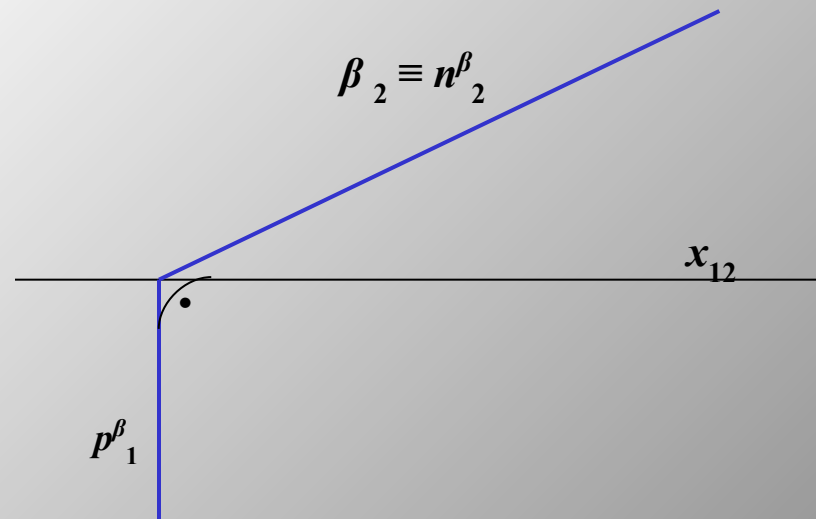
Roviny v Mongeovej projekcii



3) $\alpha \perp \pi \Rightarrow \alpha_1 \equiv p^{\alpha}_1, n^{\alpha}_2 \perp x_{12}$

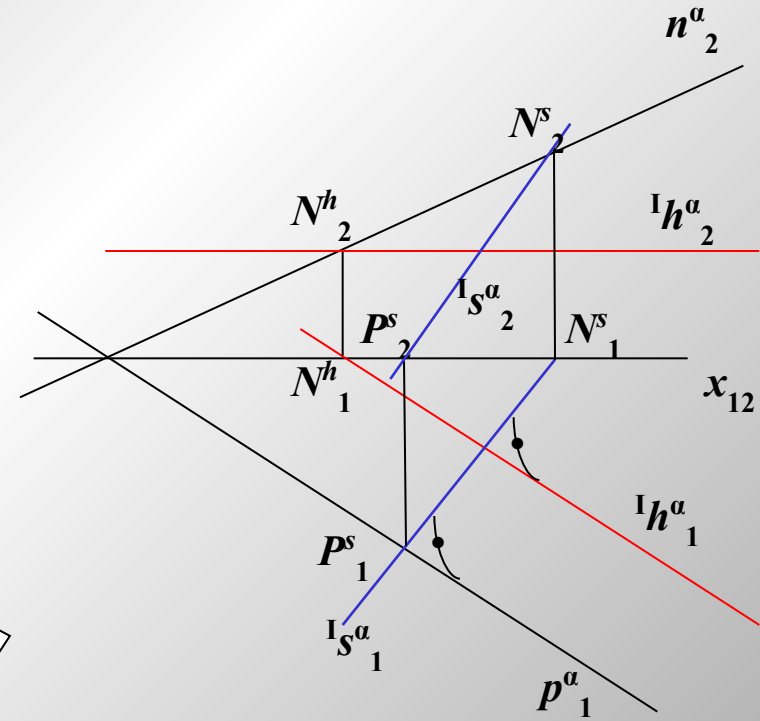
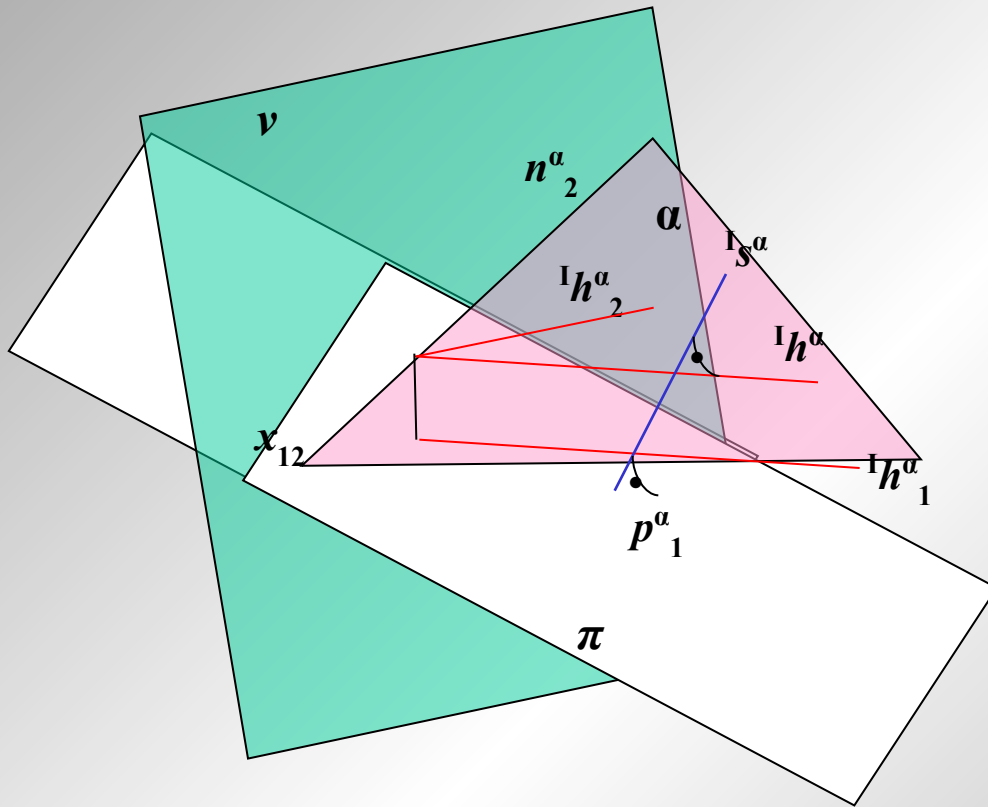


4) $\beta \perp \nu \Rightarrow \beta_2 \equiv n^{\beta}_2, p^{\beta}_1 \perp x_{12}$



Hlavné a spádové priamky roviny v Mongeovej projekcii

Hlavné priamky I. osnovy roviny α : ${}^I h^\alpha \parallel \pi$,
 ${}^I h^\alpha_1 \parallel p^\alpha_1, {}^I h^\alpha_2 \parallel x_{12}$.

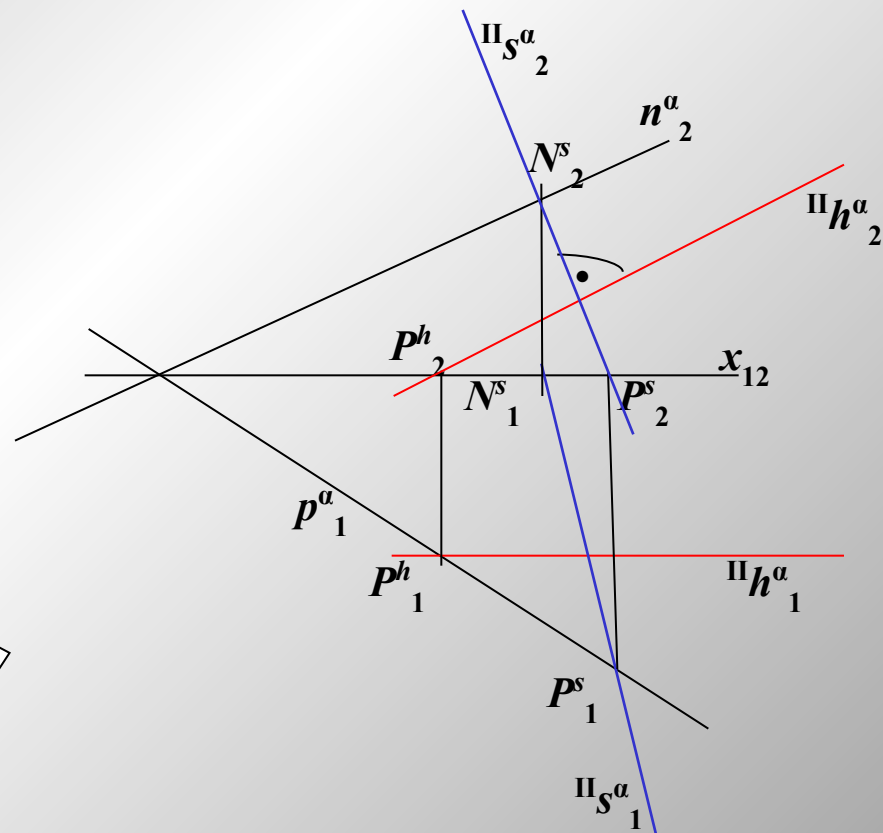
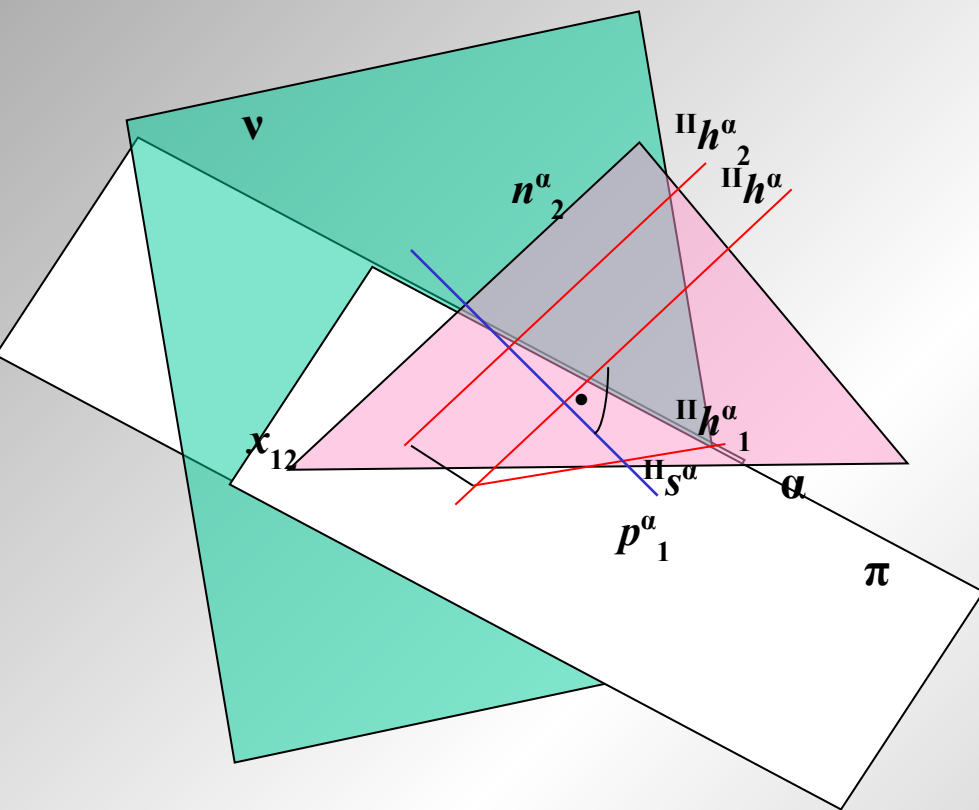


Spádové priamky I. osnovy roviny α : ${}^I s^\alpha \perp {}^I h^\alpha (p^\alpha)$,
 ${}^I s^\alpha_1 \perp p^\alpha_1, {}^I s^\alpha_2 = P^s_2 N^s_2$.

Hlavné a spádové priamky roviny v Mongeovej projekcii

Hlavné priamky II. osnovy roviny α : $\Pi h^\alpha \parallel \nu$

$$\Pi h_1^\alpha \parallel x_{12}, \Pi h_2^\alpha \parallel n_2^\alpha$$

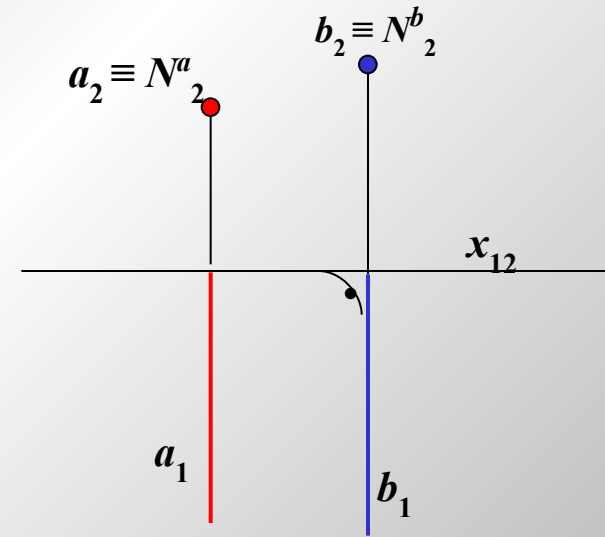
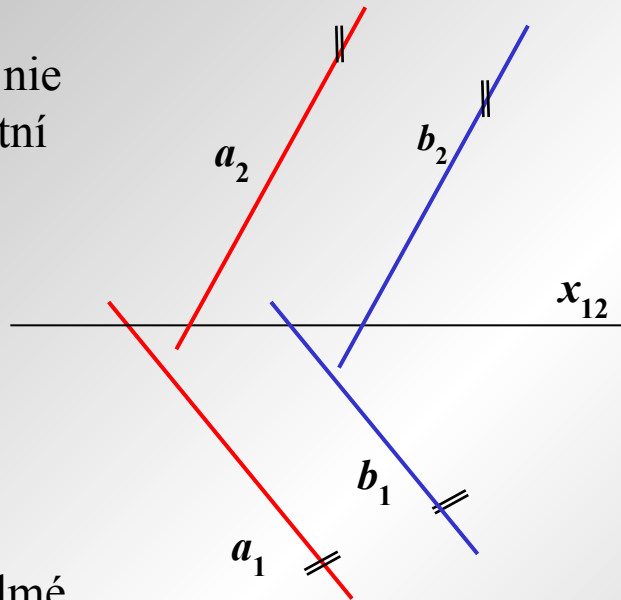


Spádové priamky II. osnovy roviny α : $\Pi S^\alpha \perp \Pi h^\alpha (n^\alpha)$

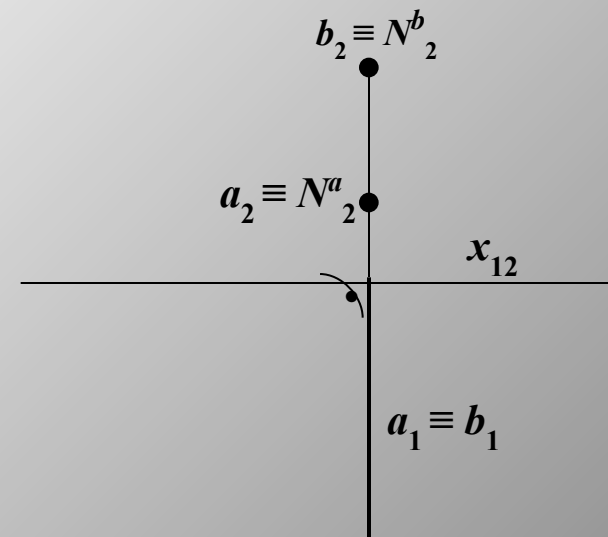
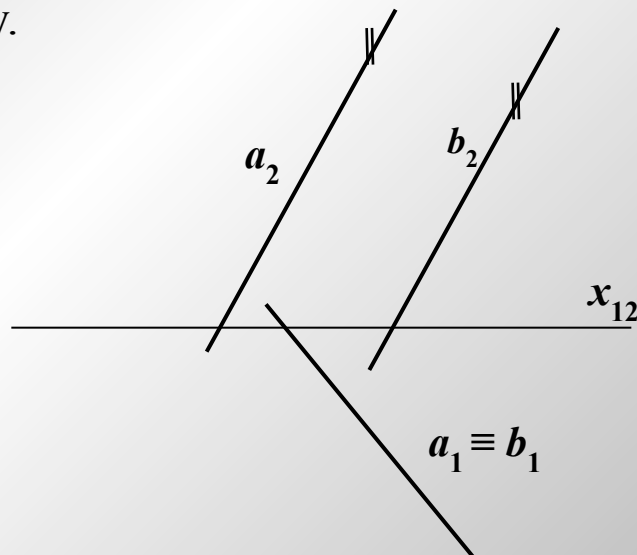
$$\Pi S_2^\alpha \perp n_2^\alpha, \Pi S_1^\alpha = P_1^s N_1^s$$

Vzájomná poloha 2 priamok v Mongeovej projekcii

- 1) **Rovnobežné priamky a, b** , ak nie sú kolmé na žiadnu z priemetní a $a \parallel b \Rightarrow a_1 \parallel b_1, a_2 \parallel b_2$.

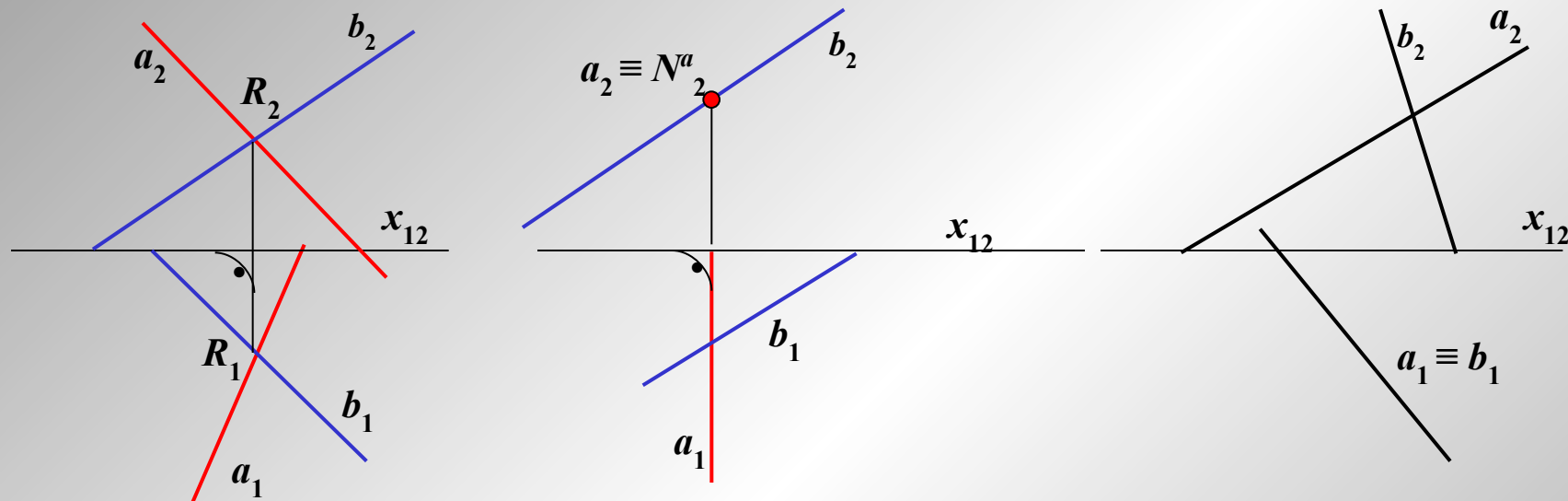


Ak sú rovnobežné priamky kolmé na niektorú z priemetní, ich priemetom v nej sú 2 body.

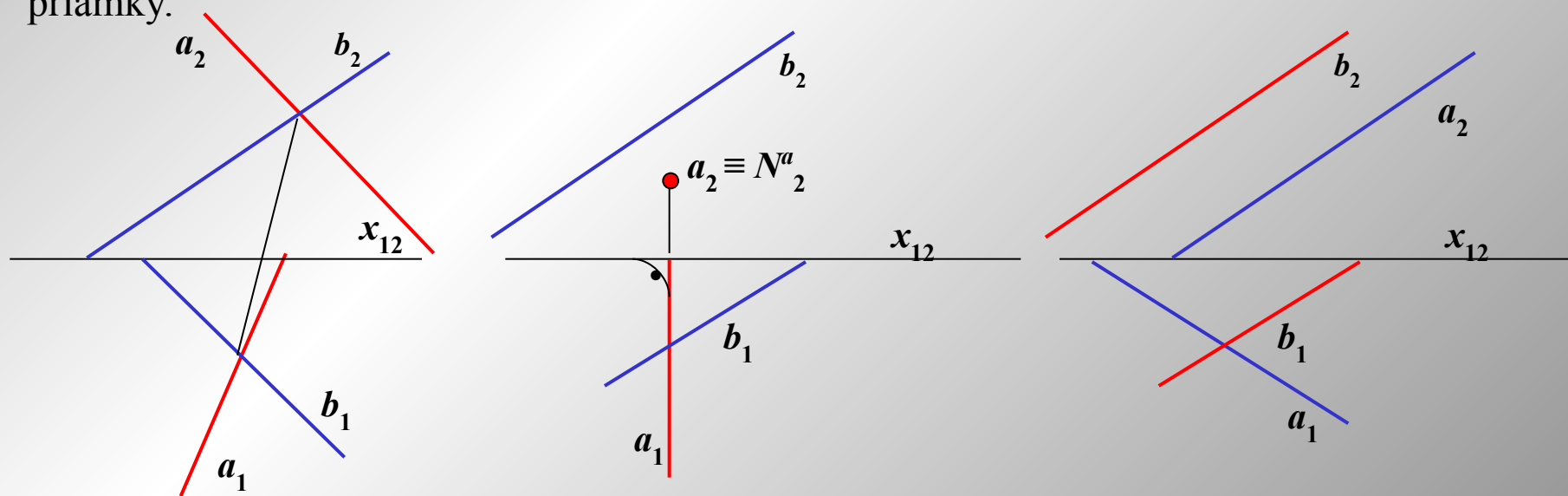


Vzájomná poloha 2 priamok v Mongeovej projekcii

2) **Rôznobežné priamky a, b :** $a \cap b = R \Rightarrow a_1 \cap b_1 = R_1, a_2 \cap b_2 = R_2$, potom $R_1 R_2 \perp x_{12}$.



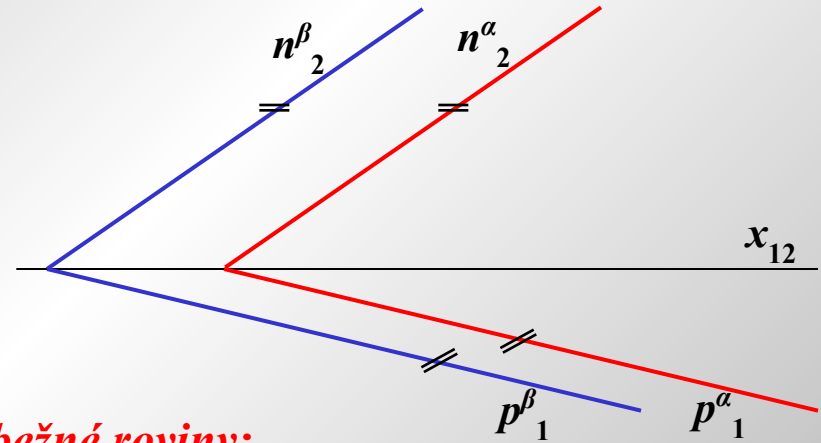
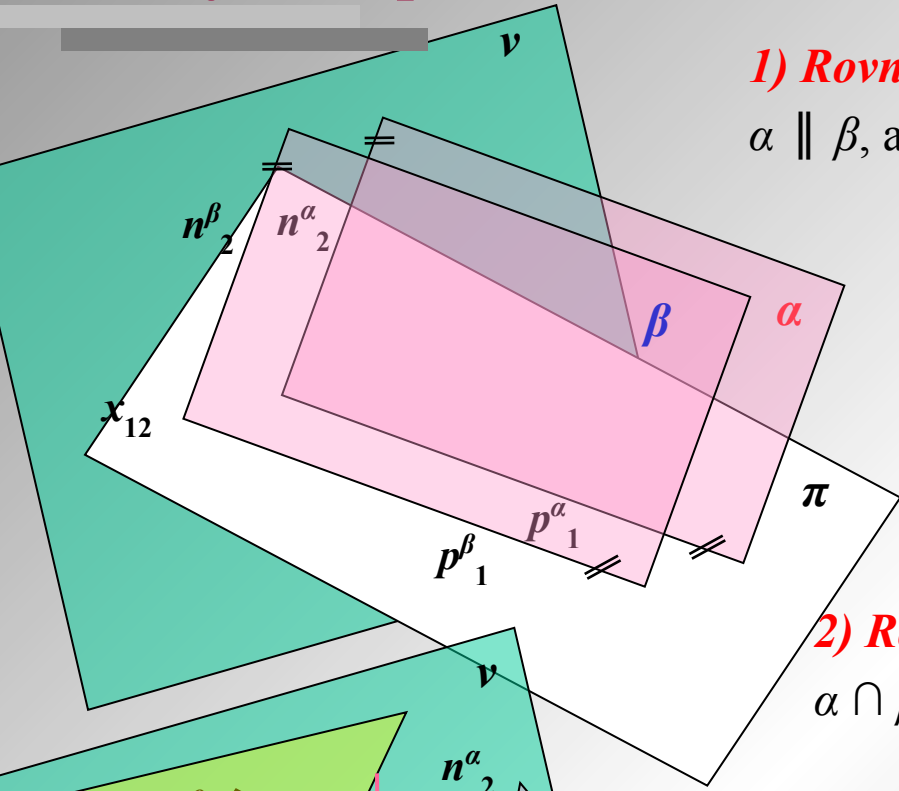
3) **Mimobežné priamky a, b :** neplatia predchádzajúce pravidlá pre rovnobežné, ani rôznobežné priamky.



Vzájomná poloha 2 rovín v Mongeovej projekcii

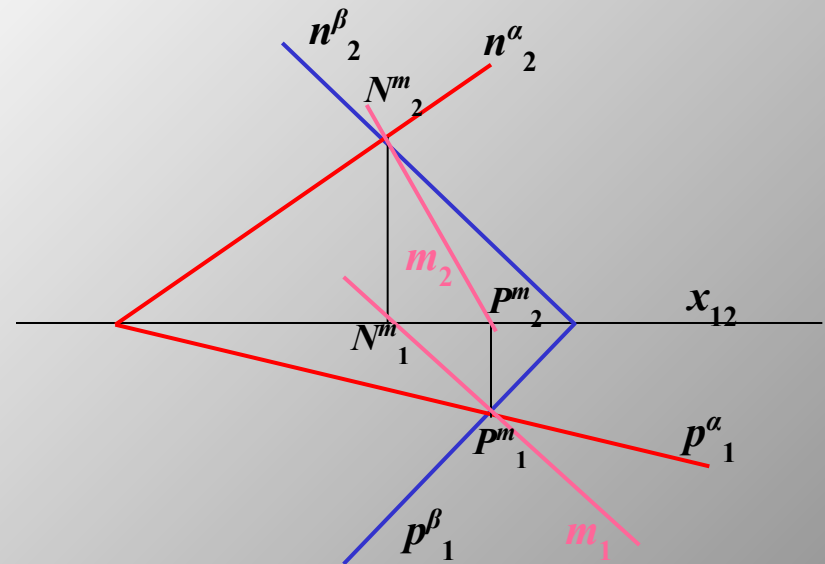
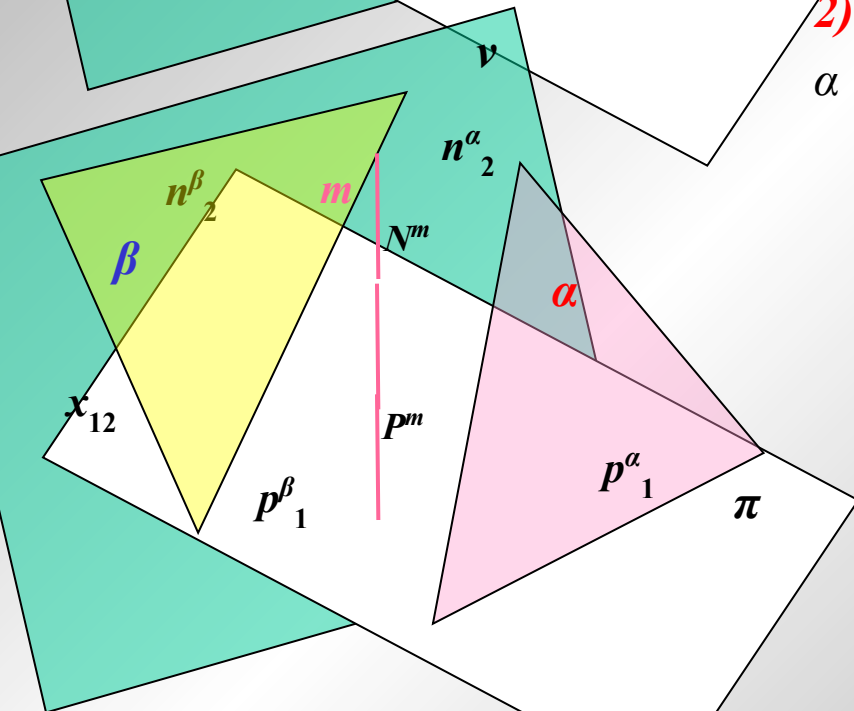
1) Rovnobežné roviny:

$\alpha \parallel \beta$, ak existujú ich stopy $\Rightarrow p_1^\alpha \parallel p_1^\beta, n_2^\alpha \parallel n_2^\beta$



2) Rôznobežné roviny:

$\alpha \cap \beta = m \Rightarrow P^m = p^\alpha \cap p^\beta, N^m = n^\alpha \cap n^\beta$



Vzájomná poloha priamky a roviny v Mongeovej projekcii

Všeobecný postup $a \cap \alpha$:

1. Priamkou a preložíme ľubovoľnú rovinu β : $a \subset \beta$.
2. Nech m je priesečnica rovín α a β : $\alpha \cap \beta = m$.
3. Podľa vzájomnej polohy priamok a a m určíme vzájomnú polohu priamky a a roviny α :

a, $a \equiv m \Rightarrow a \subset \alpha$

b, $a \parallel m \Rightarrow a \parallel \alpha$

c, $a \cap m = R \Rightarrow R = a \cap \alpha$

Postup v Mongeovej projekcii, dané je $a[a_1, a_1], \alpha(p^\alpha, n^\alpha)$, určte $a \cap \alpha$:

1. $\beta: a \subset \beta, \beta \perp \pi$

$a_1 \equiv p_1^\beta, n_2^\beta \perp x_{12}$

2. $\alpha \cap \beta = m : a_1 \equiv p_1^\beta \equiv m_1, m_2 = P_2^m N_2^m$

3. a, $a_2 \equiv m_2 \Rightarrow a \subset \alpha$

b, $a_2 \parallel m_2 \Rightarrow a \parallel \alpha$

c, $a_2 \cap m_2 = R_2 \Rightarrow R = a \cap \alpha$

