ECE 576 – Power System Dynamics and Stability

Lecture 10: Synchronous Machines Models

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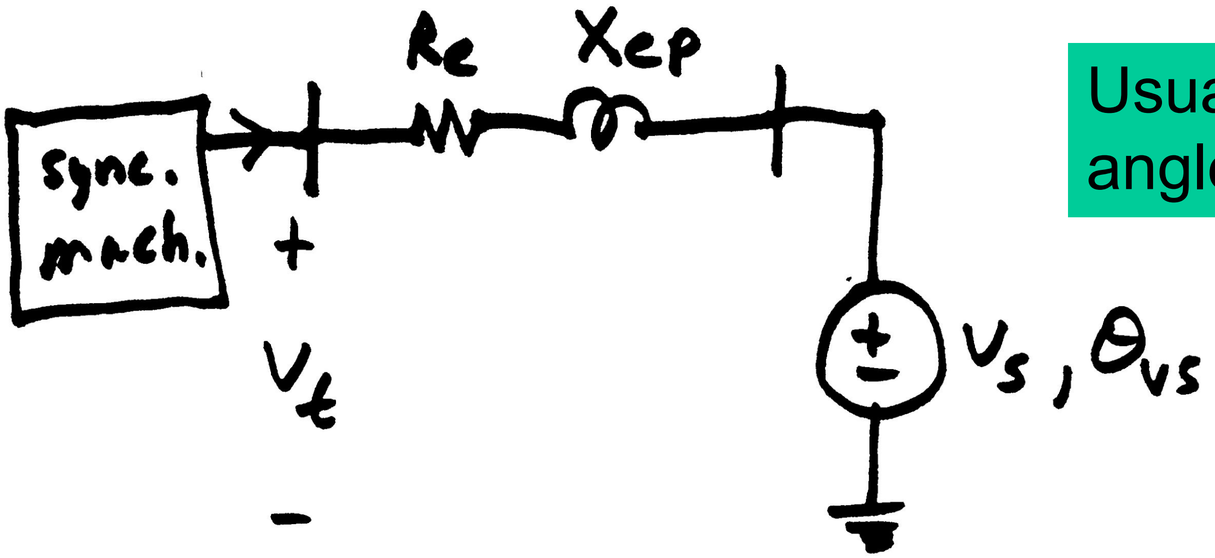
Announcements

- Homework 2 is due now
- Homework 3 is on the website and is due on Feb 27
- Read Chapters 6 and then 4

Single Machine, Infinite Bus System (SMIB)



Usually infinite bus angle, θ_{vs}, is zero

Book introduces new variables by combining machine values with line values

$$\psi_{de} = \psi_d + \psi_{ed}$$

$$X_{de} = X_d + X_{ep}$$

$$R_{se} = R_s + R_e$$

etc

Introduce New Constants

$$\omega_t = T_s(\omega - \omega_s)$$

"Transient Speed"

$$T_s = \sqrt{\frac{2H}{\omega_s}}$$

Mechanical time constant

$$\varepsilon = \frac{1}{\omega_s}$$

A small parameter

We are ignoring the exciter and governor for now; they will be covered in much more detail later

Stator Flux Differential Equations

$$\varepsilon \frac{d\psi_{de}}{dt} = R_{se} I_d + \left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_{qe} + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$\varepsilon \frac{d\psi_{qe}}{dt} = R_{se} I_q - \left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_{de} + V_s \cos\left(\delta - \theta_{vs}\right)$$

$$\varepsilon \frac{d\psi_{oe}}{dt} = R_{se} I_o$$

Special Case of Zero Resistance

$$\varepsilon \frac{d\psi_{de}}{dt} = \left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_{qe} + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$\varepsilon \frac{d\psi_{qe}}{dt} = -\left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_{de} + V_s \cos\left(\delta - \theta_{vs}\right)$$

Without resistance this is just an oscillator

An exact integral manifold (for any sized ε):

$$\psi_{de} = V_s \cos\left(\delta - \theta_{vs}\right)$$

$$\psi_{qe} = -V_s \sin\left(\delta - \theta_{vs}\right)$$

$$\left(Note: \quad T_s \frac{d\delta}{dt} = \omega_t\right)$$

Direct Axis Equations

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)$$

$$\left[I_d - \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2}\left(\psi_{1d} + \left(X'_d - X_{\ell s}\right)I_d - E'_q\right)\right] + E_{fd}$$

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right)I_d$$

Quadrature Axis Equations

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)\left[I_q - \frac{X'_q - X''_q}{\left(X'_q - X_{\ell s}\right)^2}\left(\psi_{2q} + \left(X'_q - X_{\ell s}\right)I_q + E'_d\right)\right]$$

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right)I_q$$

Swing Equations

$$T_s \frac{d\delta}{dt} = \omega_t \qquad \text{(recall } \omega_t = T_s(\omega - \omega_s) \text{ and } T_s = \sqrt{\frac{2H}{\omega_s}}\text{)}$$

$$T_s \frac{d\omega_t}{dt} = T_M - \left(\psi_{de} I_q - \psi_{qe} I_d\right) - T_{FW}$$

These are equivalent to the more traditional swing expressions

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_{de} I_q - \psi_{qe} I_d\right) - T_{FW}$$

Stator Flux Expressions

$$\psi_{de} = -X''_{de} I_d + \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$\psi_{qe} = -X''_{qe} I_q - \frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d + \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q}$$

$$\psi_{oe} = -X_{oe} I_o$$

Network Expressions

$$V_t = \sqrt{V_d^2 + V_q^2}$$

$$V_d = R_e I_d + \left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_{eq} - \varepsilon \frac{d\psi_{ed}}{dt} + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$V_q = R_e I_q - \left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_{ed} - \varepsilon \frac{d\psi_{eq}}{dt} + V_s \cos\left(\delta - \theta_{vs}\right)$$

$$\psi_{ed} = -X_{ep} I_d$$

$$\psi_{eq} = -X_{ep} I_q$$

Machine Variable Summary

<u>3</u> fast dynamic states $\psi_{de}, \psi_{qe}, \psi_{oe}$

<u>6</u> not so fast dynamic states

$$E'_q, \psi_{1d}, E'_d, \psi_{2q}, \delta, \omega_t$$

<u>8</u> algebraic states $I_d, I_q, I_o, V_d, V_q, V_t, \psi_{ed}, \psi_{eq}$

We'll get to the exciter and governor shortly; for now Efd is fixed

Elimination of Stator Transients

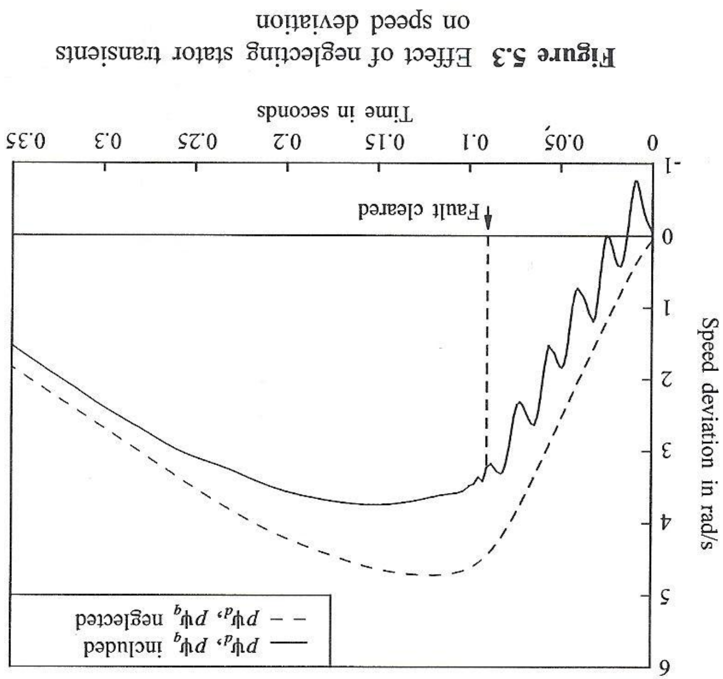
- If we assume the stator flux equations are much faster than the remaining equations, then letting ε go to zero creates an integral manifold with

$$0 = R_{se} I_d + \psi_{qe} + V_s \sin(\delta - \theta_{vs})$$

$$0 = R_{se} I_q - \psi_{de} + V_s \cos(\delta - \theta_{vs})$$

$$0 = R_{se} I_o$$

Impact on Studies



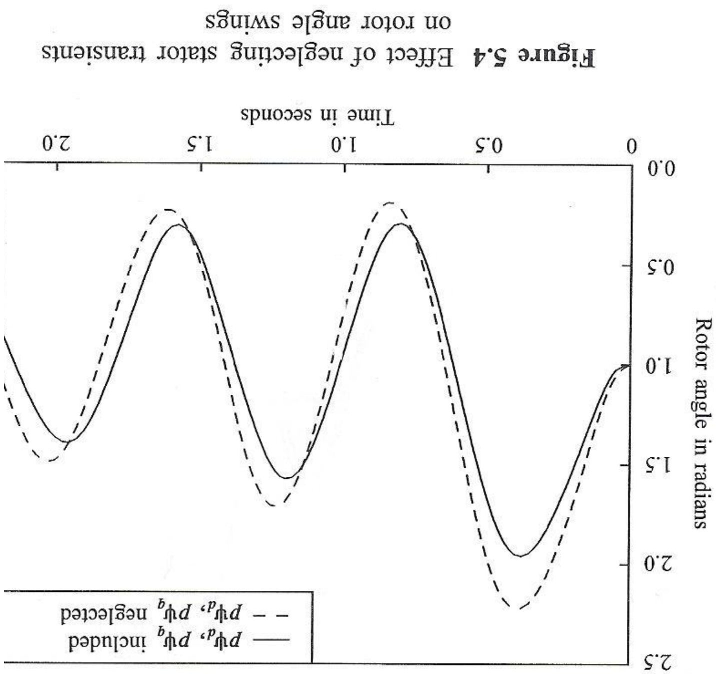


Image Source: P. Kundur, *Power System Stability and Control, EPRI, McGraw-Hill, 1994*

Stator Flux Expressions

$$\psi_{de} = -X''_{de} I_d + \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$\psi_{qe} = -X''_{qe} I_q - \frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d + \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q}$$

$$\psi_{oe} = -X_{oe} I_o$$

Network Constraints

$$0 = R_{se} I_d - X''_{qe} I_q - \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d + \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q}$$

$$+ V_s \sin\left(\delta - \theta_{vs}\right)$$

$$0 = R_{se} I_q + X''_{de} I_d - \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q - \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$+ V_s \cos\left(\delta - \theta_{vs}\right)$$

"Interesting" Dynamic Circuit

$$\left[\left(\frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d - \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q} + \left(X''_q - X''_d\right) I_q\right)\right.$$

$$\left. + j\left(\frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}\right)\right] e^{j(\delta - \pi/2)} =$$

$$\left(R_s + jX''_d\right)\left(I_d + jI_q\right) e^{j(\delta - \pi/2)}$$

$$+\left(R_e + jX_{ep}\right)\left(I_d + jI_q\right) e^{j(\delta - \pi/2)} + V_s e^{j\theta_{vs}}$$

"Interesting" Dynamic Circuit

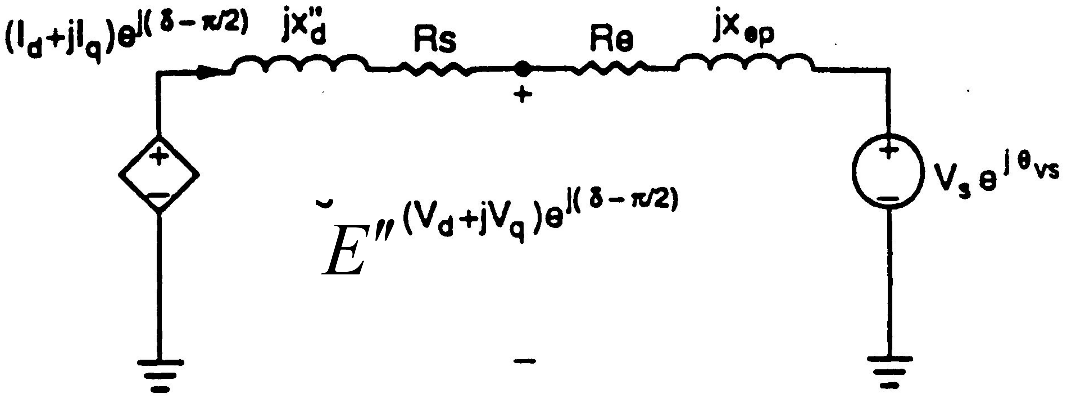
$$V_d = R_e I_d - X_{ep} I_q + V_s \sin(\delta - \theta_{vs})$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos(\delta - \theta_{vs})$$

These last two equations can be written as one complex equation.

$$(V_d + jV_q) e^{j(\delta - \pi/2)} = (R_e + jX_{ep})(I_d + jI_q) e^{j(\delta - \pi/2)} + V_s e^{j\theta_{vs}}$$

Subtransient Algebraic Circuit



$$\breve{E}'' = \left[\left(\frac{\left(X_q'' - X_{\ell s}\right)}{\left(X_q' - X_{\ell s}\right)} E_d' - \frac{\left(X_q' - X_q''\right)}{\left(X_q' - X_{\ell s}\right)} \psi_{2q} + \left(X_q'' - X_d''\right) I_q \right)\right.$$

$$\left. + j\left(\frac{\left(X_d'' - X_{\ell s}\right)}{\left(X_d' - X_{\ell s}\right)} E_q' + \frac{\left(X_d' - X_d''\right)}{\left(X_d' - X_{\ell s}\right)} \psi_{1d} \right)\right] e^{j(\delta - \pi/2)}$$

Subtransient Algebraic Circuit

$$\tilde{E}'' = \Bigg[\underbrace{(\,)E'_d + (\,)\psi_{2q}}_{\psi''_q} + \underbrace{\left(X''_q - X''_d\right) I_q}_{\text{often neglected}}$$

$$+ j \Bigg[\underbrace{(\,)E'_q + (\,)\psi_{1d}}_{\psi''_d} \Bigg] \Bigg] e^{j(\delta - \pi/2)}$$

Subtransient saliency use to be ignored (i.e., assuming $X''_q = X''_d$). However that is increasingly no longer the case

Simplified Machine Models

- Often more simplified models were used to represent synchronous machines
- These simplifications are becoming much less common
- Next several slides go through how these models can be simplified, then we'll cover the standard industrial models

Two-Axis Model

- If we assume the damper winding dynamics are sufficiently fast, then T''_{do} and T''_{qo} go to zero, so there is an integral manifold for their dynamic states

$$\left.\begin{aligned} \psi_{1d} &= E'_q - \left(X'_q - X_{\ell s}\right) I_d \\ \psi_{2q} &= -E'_d - \left(X'_q - X_{\ell s}\right) I_q \end{aligned}\right]$$

Two-Axis Model

- Then

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right)I_d = 0$$

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)\times$$

$$\left[I_d - \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2}\left(\psi_{1d} + \left(X'_d - X_{\ell s}\right)I_d - E'_q\right)\right] + E_{fd}$$

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)I_d + E_{fd}$$

Two-Axis Model

- And

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right)I_q = 0$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)\times$$

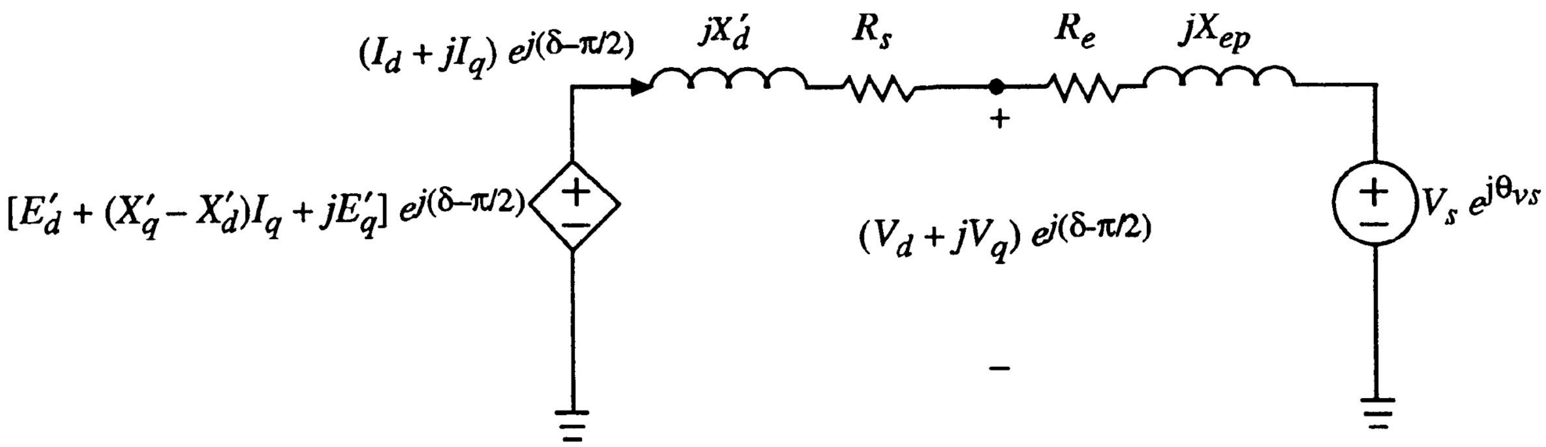
$$\left[I_q - \frac{X'_q - X''_q}{\left(X'_q - X_{\ell s}\right)^2}\left(\psi_{2q} + \left(X'_q - X_{\ell s}\right)I_q + E'_d\right)\right]$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)I_q$$

Two-Axis Model

$$0 = (R_s + R_e)I_d - (X'_q + X_{ep})I_q - E'_d + V_s \sin(\delta - \theta_{vs})$$

$$0 = (R_s + R_e)I_q + (X'_d + X_{ep})I_d - E'_q + V_s \cos(\delta - \theta_{vs})$$



Two-Axis Model

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - (X'_q - X'_d)I_d I_q - T_{FW}$$

No saturation effects are included with this model

Two-Axis Model

$$0 = \left(R_s + R_e\right) I_d - \left(X'_q + X_{ep}\right) I_q - E'_d + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$0 = \left(R_s + R_e\right) I_q + \left(X'_d + X_{ep}\right) I_d - E'_q + V_s \cos\left(\delta - \theta_{vs}\right)$$

$$V_d = R_e I_d - X_{ep} I_q + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos\left(\delta - \theta_{vs}\right)$$

$$V_t = \sqrt{V_d^2 + V_q^2}$$

Flux Decay Model

- If we assume T'qo is sufficiently fast then

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)I_q = 0$$

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)I_d + E_{fd}$$

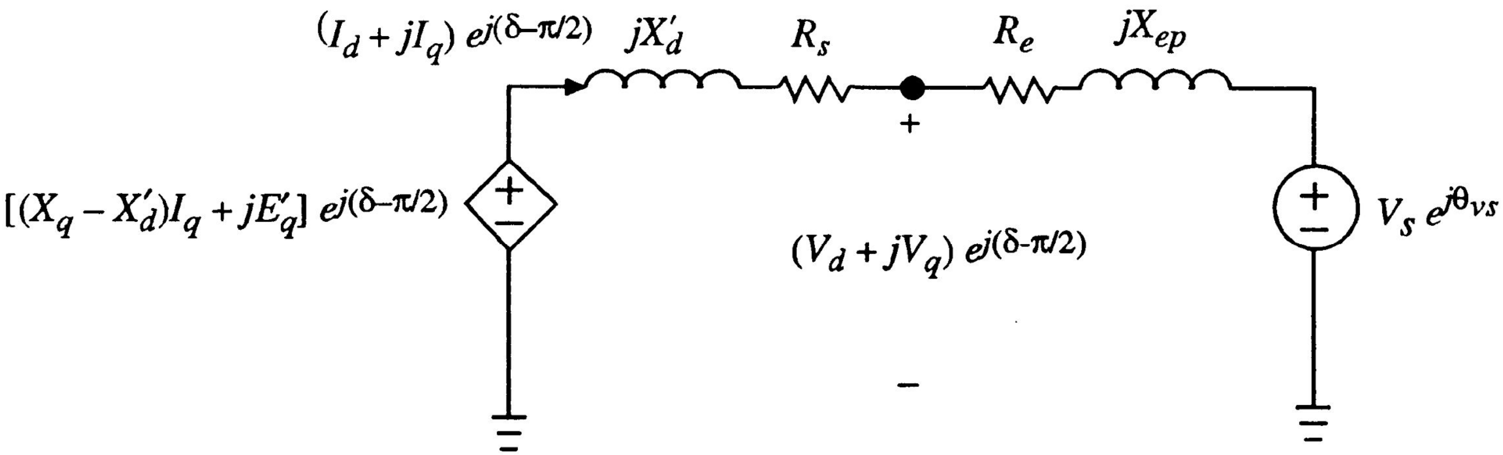
$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

$$= T_M - \left(X_q - X'_q\right)I_q I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

$$= T_M - E'_q I_q - \left(X_q - X'_d\right)I_d I_q - T_{FW}$$

Flux Decay Model



This model is no longer common

Classical Model

- Has been widely used, but most difficult to justify
- From flux decay model

$$X_q = X'_d \qquad T'_{do} = \infty$$

$$E' = E'_q \qquad \delta'^0 = 0$$

- Or go back to the two-axis model and assume

$$X'_q = X'_d \qquad T'_{do} = \infty \qquad T'_{qo} = \infty$$

$$(E'_q = \text{const} \quad E'_d = \text{const})$$

$$E' = \sqrt{E'^{0^2}_q + E'^{0^2}_d}$$

$$\delta'^0 = \tan^{-1}\left(E'^0_q \big/ E'^0_d \right) - \pi/2$$

Classical Model

Or, argue that an integral manifold exists for

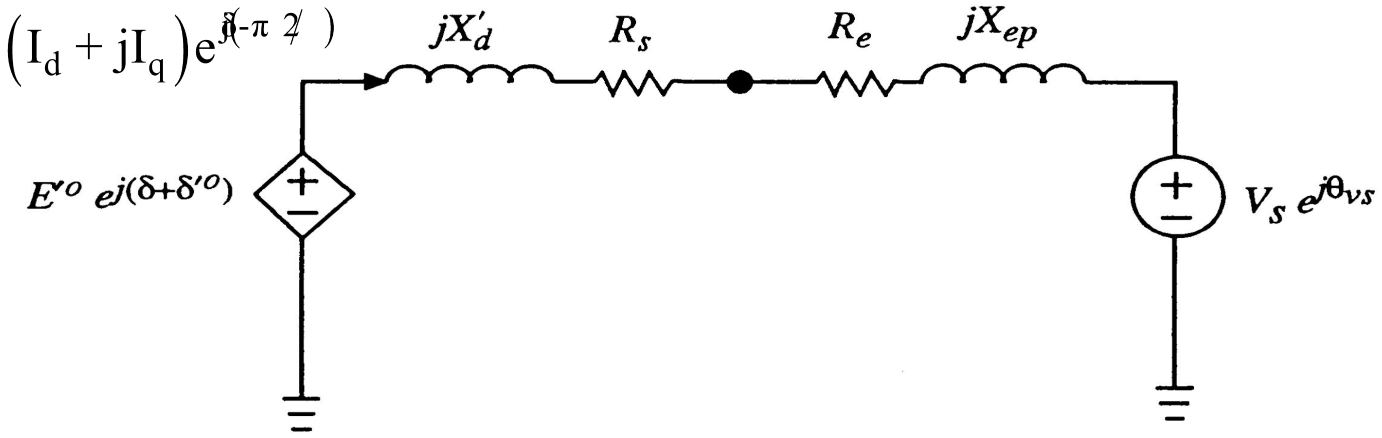
$E'_q, E'_d, E_{fd}, R_f, V_R$ such that $E'_q = \text{const.}$

$$E'_d + \left(X'_q - X'_d\right) I_q = \text{const}$$

$$E'^0 = \sqrt{\left(E'^0_d + \left(X'_q - X'_d\right) I^0_q\right)^2 + {E'^0_q}^2}$$

$$\delta'^0 = \tan^{-1}(\;) - \pi/2$$

Classical Model



$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_0}\frac{d\omega}{dt} = T_M^0 - \frac{E'^0 V_s}{X'_d + X_{ep}}\sin\left(\delta - \theta_{vs}\right) - T_{FW}$$

This is a pendulum model

Summary of Five Book Models

a) Full model with stator transients

b) Sub-transient model $\left(\frac{1}{\omega_s} \to 0\right)$

c) Two-axis model $\left(T''_{qo} = T''_{do} = 0\right)$

d) One-axis model $\left(T'_{qo} = 0\right)$

e) Classical model (const. E behind X'_d)

Damping Torques

- Friction and windage
  - Usually small
- Stator currents (load)
  - Usually represented in the load models
- Damper windings
  - Directly included in the detailed machine models
  - Can be added to classical model as $D(\omega-\omega_s)$

Industrial Models

- There are just a handful of synchronous machine models used in North America
  - GENSAL
    - Salient pole model
  - GENROU
    - Round rotor model that has $X''_d = X''_q$
  - GENTPF
    - Round or salient pole model that allows $X''_d \neq X''_q$
  - GENTPJ
    - Just a slight variation on GENTPF

Network Reference Frame

- In transient stability the initial generator values are set from a power flow solution, which has the terminal voltage and power injection
  - Current injection is just conjugate of Power/Voltage
- These values are on the network reference frame, with the angle given by the slack bus angle

$\bar{V}_j = V_{r,j} + jV_{i,j}$ In book $\bar{V}_i = V_{Di} + jV_{Qi}$

- Voltages at bus j converted to d-q reference by

$$\begin{bmatrix} V_{d,j} \\ V_{q,j} \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_{r,j} \\ V_{i,j} \end{bmatrix} \quad \begin{bmatrix} V_{r,j} \\ V_{i,j} \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_{d,j} \\ V_{q,j} \end{bmatrix}$$

Similar for current; see book 7.24, 7.25

Network Reference Frame

- Issue of calculating δ, which is key, will be considered for each model
- Starting point is the per unit stator voltages (3.215 and 3.216 from the book)

$$V_d = -\psi_q \omega - R_s I_d$$

$$V_q = \psi_d \omega - R_s I_q$$

Equivalently, $\left(\mathrm{V_d} + j\mathrm{V_q}\right) + R_s\left(I_d + j\mathrm{I_q}\right) = \omega\left(-\psi_q + j\psi_d\right)$

- Sometimes the scaling of the flux by the speed is neglected, but this can have a major impact on the solution

Two-Axis Model

- We'll start with the PowerWorld two-axis model (two-axis models are not common commercially, but they match the book on 6.110 to 6.113
- Represented by two algebraic equations and four differential equations

The bus number subscript is omitted since it is not used in commercial block diagrams

$$E'_q = V_q + R_s I_q + X'_d I_d$$

$$E'_d = V_d + R_s I_d - X'_q I_q$$

$$\frac{dE'_q}{dt} = \frac{1}{T'_{do}}\left(-E'_q - (X_d - X'_d)I_d + E_{fd}\right), \quad \frac{dE'_d}{dt} = \frac{1}{T'_{qo}}\left(-E'_d + (X_q - X'_q)I_q\right)$$

$$\frac{d\delta}{dt} = \omega - \omega_s, \quad \frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - (X'_q - X'_d)I_d I_q - T_{FW}$$

Two-Axis Model

- Value of δ is determined from (3.229 from book)

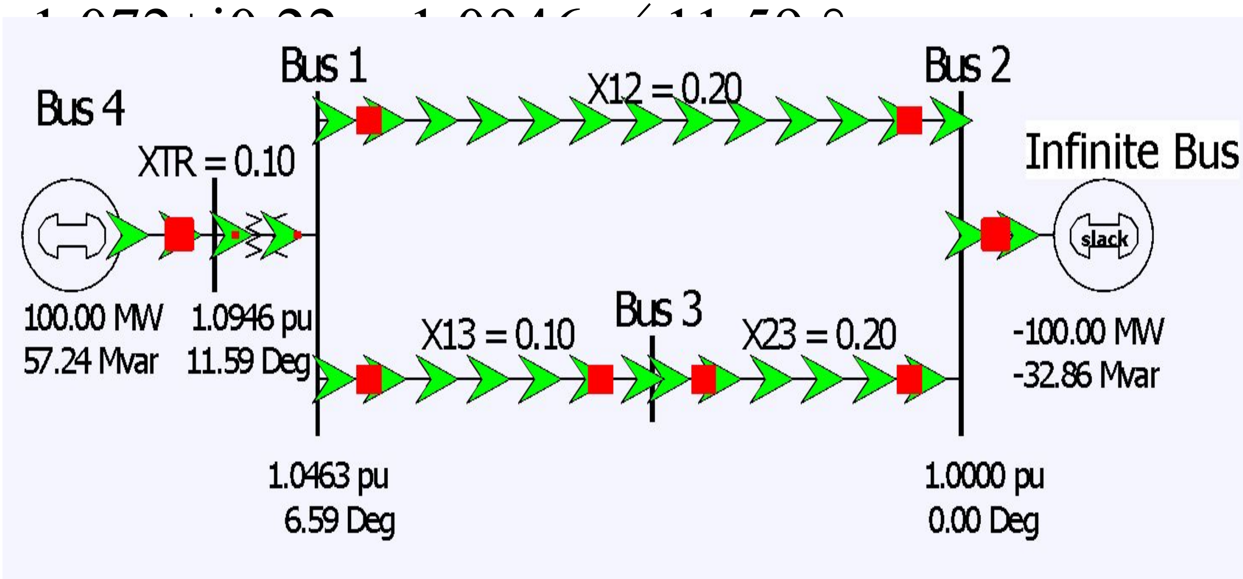
$$\left|E\right|\angle\delta = \bar{V} + \left(R_s + jX_q\right)\bar{I}$$

Sign convention on current is out of the generator is positive

- Once δ is determined then we can directly solve for E'_q and E'_d

Example (Used for All Models)

- Below example will be used with all models. Assume a 100 MVA base, with gen supplying 1.0+j0.3286 power into infinite bus with unity voltage through network impedance of j0.22
  - Gives current of 1.0-j0.3286 and generator terminal voltage of 1.072+j0.22 = 1.0946 $\angle$ 11.59 °



Two-Axis Example

- For the two-axis model assume H = 3.0 per unit-seconds, R_s=0, X_d = 2.1, X_q = 2.0, X'_d = 0.3, X'_q = 0.5, T'_{do} = 7.0, T'_{qo} = 0.75 per unit using the 100 MVA base.
- Solving we get

$$\bar{E} = 1.0946\angle 11.59^\circ + (j2.0)(1.052\angle -18.2^\circ) = 2.814\angle 52.1^\circ$$

$$\rightarrow \delta = 52.1^\circ$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

Two-Axis Example

- And

$$E'_q = 0.8326 + (0.3)(0.9909) = 1.1299$$

$$E'_d = 0.7107 - (0.5)(0.3553) = 0.5330$$

$$E_{fd} = 1.1299 + (2.1 - 0.3)(0.9909) = 2.9135$$

Saved as case B4_TwoAxis

Subtransient Models

- The two-axis model is a transient model
- Essentially all commercial studies now use subtransient models
- First models considered are GENSAL and GENROU, which require $X''_d = X''_q$
- This allows the internal, subtransient voltage to be represented as

$$\bar{E}'' = \bar{V} + (R_s + jX'')\bar{I}$$

$$E''_d + jE''_q = \left(-\psi''_q + j\psi''_d\right)\omega$$

Subtransient Models

- Usually represented by a Norton Injection with

$$I_d + jI_q = \frac{E''_d + jE''_q}{R_s + jX''} = \frac{\left(-\psi''_q + j\psi''_d\right)\omega}{R_s + jX''}$$

- May also be shown as

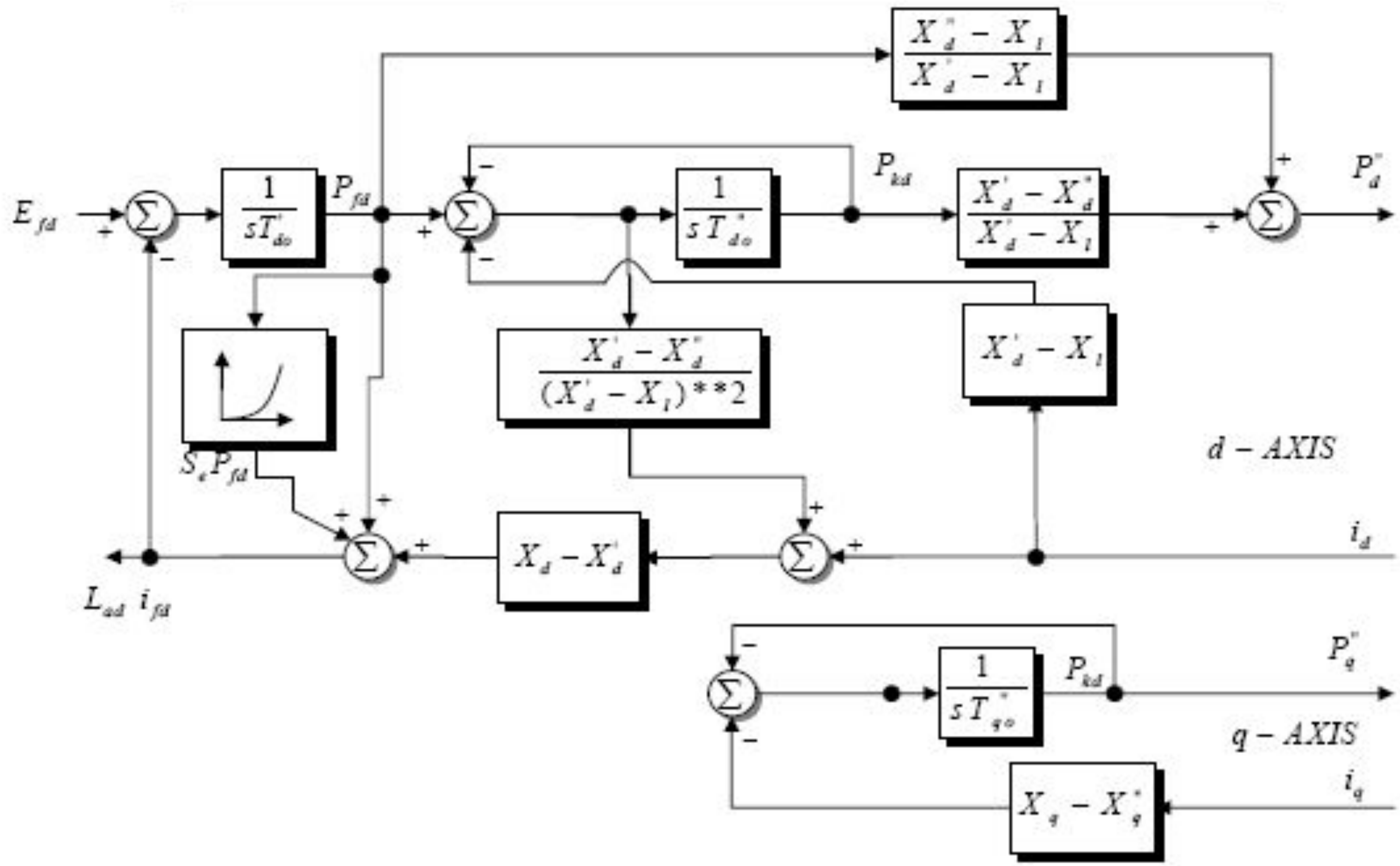
$$-j\left(I_d + jI_q\right) = I_q - jI_d = \frac{-j\left(-\psi''_q + j\psi''_d\right)\omega}{R_s + jX''} = \frac{\left(\psi''_d + j\psi''_q\right)\omega}{R_s + jX''}$$

In steady-state $\omega = 1.0$

GENSAL

- The GENSAL model has been widely used to model salient pole synchronous generators
  - In the 2010 WECC cases about 1/3 of machine models were GENSAL; in 2013 essentially none are, being replaced by GENTPF or GENTPJ
- In salient pole models saturation is only assumed to affect the d-axis

GENSAL Block Diagram (PSLF)



A quadratic saturation function is used. For initialization it only impacts the Efd value

GENSAL Initialization

- To initialize this model

1. Use S(1.0) and S(1.2) to solve for the saturation coefficients
2. Determine the initial value of δ with
   $$|E|\angle\delta = \bar{V} + \left(R_s + jX_q\right)\bar{I}$$
3. Transform current into dq reference frame, giving i_d and i_q
4. Calculate the internal subtransient voltage as
   $$\bar{E}'' = \bar{V} + \left(R_s + jX''\right)\bar{I}$$
5. Convert to dq reference, giving $P''_d + jP''_q = \Psi''_d + \Psi''_q$
6. Determine remaining elements from block diagram by

GENSAL Example

- Assume same system as before, but with the generator parameters as H=3.0, D=0, R_a = 0.01, X_d = 1.1, X_q = 0.82, X'_d = 0.5, X''_d=X''_q=0.28, X_l = 0.13, T'_{do} = 8.2, T''_{do} = 0.073, T''_{qo} =0.07, S(1.0) = 0.05, and S(1.2) = 0.2.
- Same terminal conditions as before
  - Current of 1.0-j0.3286 and generator terminal voltage of 1.072+j0.22 = 1.0946 ∠11.59 °
- Use same equation to get initial δ

$$|E|\angle\delta = \bar{V} + \left(R_s + jX_q\right)\bar{I}$$

$$= 1.072 + j0.22 + (0.01 + j0.82)(1.0 - j0.3286)$$

$$= 1.35 + j1.037 = 1.70\angle 37.5^\circ$$

GENSAL Example

- Then $$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_r \\ I_i \end{bmatrix}$$

$$= \begin{bmatrix} 0.609 & -0.793 \\ 0.793 & 0.609 \end{bmatrix} \begin{bmatrix} 1.0 \\ -0.3286 \end{bmatrix} = \begin{bmatrix} 0.869 \\ 0.593 \end{bmatrix}$$

And

$$\bar{V} + (R_s + jX'')\bar{I}$$

$$= 1.072 + j0.22 + (0.01 + j0.28)(1.0 - j0.3286)$$

$$= 1.174 + j0.497$$

GENSAL Example

- Giving the initial fluxes (with ω = 1.0)

$$\begin{bmatrix} -\psi_q'' \\ \psi_d'' \end{bmatrix} = \begin{bmatrix} 0.609 & -0.793 \\ 0.793 & 0.609 \end{bmatrix} \begin{bmatrix} 1.174 \\ 0.497 \end{bmatrix} = \begin{bmatrix} 0.321 \\ 1.233 \end{bmatrix}$$

- To get the remaining variables set the differential equations equal to zero, e.g.,

$$\psi_q'' = -\left(X_q - X_q''\right) I_q = -(0.82 - 0.28)(0.593) = -0.321$$

$$E_q' = 1.425, \quad \psi_d' = 1.104$$

Solving the d-axis requires solving two linear equations for two unknowns

GENSAL Example

- Once E'_q has been determined, the initial field current (and hence field voltage) are easily determined by recognizing in steady-state the E'_q is zero

$$E_{fd} = E'_q\left(1+Sat(E'_q)\right)+\left(X_d - X'_d\right)I_D$$

$$=1.425\left(1+B\left(E'_q - A\right)^2\right)+\left(1.1-0.5\right)(0.869)$$

$$=1.425\left(1+1.25\left(1.425-0.8\right)^2\right)+0.521=2.64$$

Saturation coefficients were determined from the two initial values

Saved as case B4_GENSAL