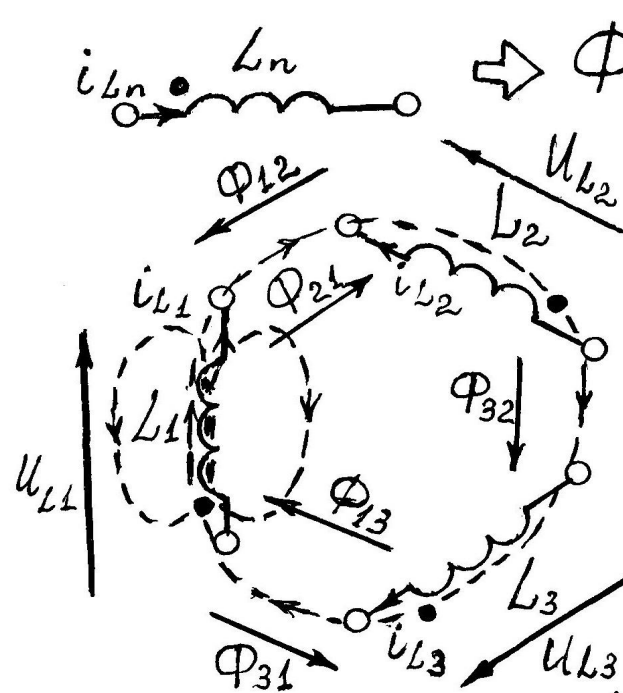


ЛІНІЙНІ КОЛА

Аналіз кіл з магнітним зв'язком при дії
гармонічних сигналів

Трифазні кола

Зв'язані індуктивні елементи



$\Rightarrow \Phi_{nn}, \Phi_{in} \rightarrow \text{додатні } (+)$

$$\Phi_{ik} = M_{ik} \cdot i_{Lk}, \quad M_{ik} = M_{ki}$$

M_{ik} - взаємна індуктивність

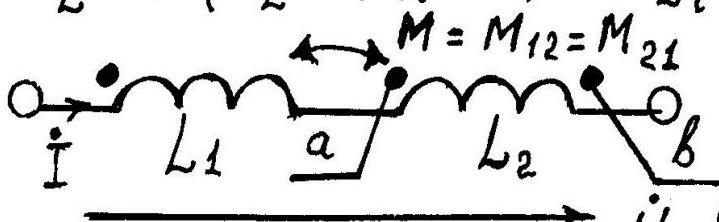
$$\Phi_{\Sigma, i} = \Phi_{ii} + \sum_{i \neq k}^n \Phi_{ik}$$

$$U_{Li} = L_i \frac{d(i_{Li})}{dt} + \sum_{i \neq k}^n M_{ik} \frac{d(i_{Lk})}{dt}$$

$$\Phi_{\Sigma, 1} = \Phi_{11} + \Phi_{12} - \Phi_{13}$$

$$U_{L1} = L_1 \frac{d(i_{L1})}{dt} + M_{12} \frac{d(i_{L2})}{dt} - M_{13} \frac{d(i_{L3})}{dt}$$

$$i_L = \text{Re}(\dot{I}_L \cdot \exp(j\omega t)); \quad \dot{U}_{Li} = j\omega L_i \dot{I}_i + \sum_{i \neq k}^K j\omega M_{ik} \cdot \dot{I}_{Lk}; \quad \dot{U}_{L1} = j\omega L_1 \cdot \dot{I}_1 \oplus$$



$$\text{a) } \dot{U} = j\omega (L_1 + L_2 + 2M) \dot{I} \quad \left[\oplus j\omega M_{12} \cdot \dot{I}_2 - j\omega M_{13} \dot{I}_3 \right]$$

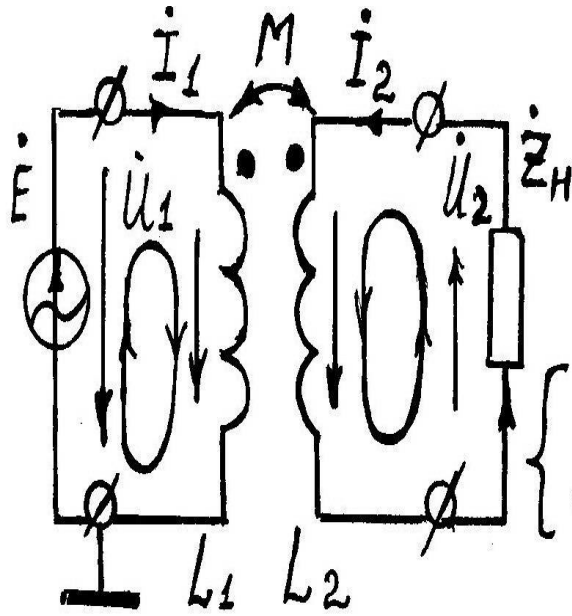
$$\text{б) } \dot{U} = j\omega (L_1 + L_2 - 2M) \dot{I}$$

$$\begin{cases} L_1 + L_2 - 2M \geq 0 \\ (L_1 + L_2)/2 \geq \sqrt{L_1 L_2} \end{cases} \Rightarrow$$

$$M \leq \sqrt{L_1 \cdot L_2} \Rightarrow M = k_{38}^m \cdot \sqrt{L_1 L_2}$$

$$k_{38}^m = 0 \dots 1 \quad k_{38} = \frac{M}{\sqrt{L_1 L_2}};$$

Передача потужності між зв'язаними індуктивностями



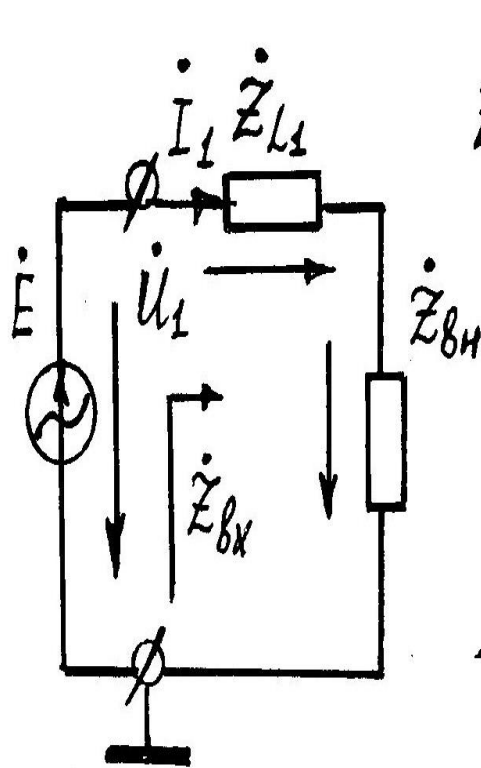
$$P_{12} = \operatorname{Re}\left(\frac{1}{2} \dot{U}_{12} \cdot \dot{I}_2^*\right) = \operatorname{Re}\left(\frac{1}{2} j\omega M \cdot \dot{I}_1 \cdot \dot{I}_2^*\right) \Rightarrow -P - jQ = \dot{S}$$

$$P_{21} = \operatorname{Re}\left(\frac{1}{2} \dot{U}_{21} \cdot \dot{I}_1^*\right) = \operatorname{Re}\left(\frac{1}{2} j\omega M \cdot \dot{I}_2 \cdot \dot{I}_1^*\right) \Rightarrow P + jQ = \dot{S}$$

$$\begin{cases} 1) \dot{U}_1 = j\omega L_1 \cdot \dot{I}_1 + j\omega M \cdot \dot{I}_2 \\ 2) 0 = j\omega L_2 \cdot \dot{I}_2 + j\omega M \cdot \dot{I}_1 + \dot{Z}_H \cdot \dot{I}_2 \end{cases} \Rightarrow \dot{I}_2 = \frac{-j\omega M}{j\omega L_2 + \dot{Z}_H} \cdot \dot{I}_1$$

$$\dot{U}_1 = \left(j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + \dot{Z}_H}\right) \cdot \dot{I}_1$$

Внесений опір та трансформація опору



$$\dot{Z}_{BX} = \frac{\dot{U}_1}{\dot{I}_1} = j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + \dot{Z}_H} \Rightarrow \dot{Z}_{L1} = j\omega L_1; \dot{Z}_{BH} = \frac{\omega^2 M^2}{j\omega L_2 + \dot{Z}_H}$$

При $|\dot{Z}_H| \rightarrow \infty$

$$\dot{Z}_{BX}^{xx} = j\omega L_1$$

$$\sqrt{-1} = j \Rightarrow -(j)^2 = 1$$

При $|\dot{Z}_H| \rightarrow 0$

$$\dot{Z}_{BX}^{K3}$$

$$= j\omega L_1 + \frac{-(j)^2 \omega^2 M^2 L_1}{j\omega L_2 \cdot L_1} = j\omega L_1 (1 - (k_{3B}^m)^2)$$

$$j\omega L_1 (1 - (k_{3B}^m)^2)$$

$$\dot{Z}_{BX}^{K3} = j\omega L_1 \dots 0$$

$$\Leftarrow$$

$$k_{3B}^m = 0 \dots 1$$

$$(k_{3B}^m)^2 = M^2 / L_1 L_2$$

а) Трансф. опоры:

$$\dot{Z}_{BX} = j\omega L_1 - \frac{(j)^2 \omega^2 M^2 L_1 / j\omega L_2 \cdot L_1}{(j\omega L_2 + \dot{Z}_H) j / j\omega L_2} = j\omega L_1 - \frac{j\omega L_1 (k_{3B}^m)^2}{1 - j\dot{Z}_H / \omega L_2} \left(\frac{1}{1 - X} \approx 1 + X \right)$$

$$\dot{Z}_{BX} = j\omega L_1 - (1 + j\dot{Z}_H / \omega L_2) \cdot (k_{3B}^m)^2 \cdot j\omega L_1 = j\omega L_1 (1 - (k_{3B}^m)^2) + (k_{3B}^m)^2 \cdot \dot{Z}_H \frac{L_1}{L_2}$$

При $k_{3B}^m = 1; n^2 = L_2 / L_1$ $\dot{Z}_{BX} = \dot{Z}_{BH} = \dot{Z}_H / n^2; n = \sqrt{L_2 / L_1}$ - коэффициент трансформации

Трансформація струму та напруги

в) Трансформація струму:

$$\frac{\dot{I}_2}{\dot{I}_1} = \frac{-j\omega M}{j\omega L_2 + \dot{Z}_H};$$

при: $\omega L_2 \gg |\dot{Z}_H|$ $\dot{I}_2 / \dot{I}_1 = -M / L_2$;

$$M = k_{38}^m \cdot \sqrt{L_1 \cdot L_2} \Rightarrow \dot{I}_2 / \dot{I}_1 = -k_{38}^m \cdot \sqrt{L_1 / L_2}$$

$$n = \sqrt{L_2 / L_1} \Rightarrow \boxed{\dot{I}_2 / \dot{I}_1 = -k_{38}^m \cdot \frac{1}{n}}$$

при: $k_{38}^m = 1 \Rightarrow \boxed{\dot{I}_2 / \dot{I}_1 = -1/n}$

г) Трансформація напруги:

$$\text{при } |\dot{Z}_H| \rightarrow \infty \quad \dot{I}_1 = \frac{\dot{U}_1}{j\omega L_1}, \quad \dot{U}_2 = -j\omega M \cdot \dot{I}_1$$

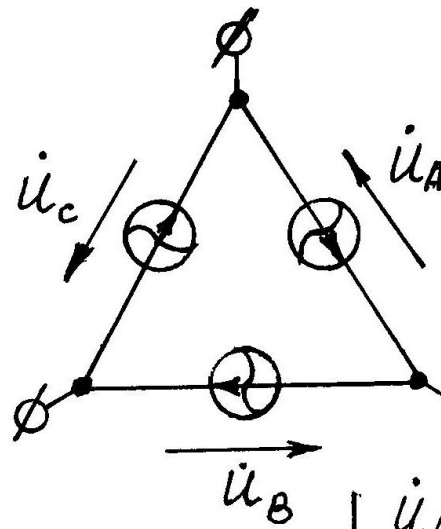
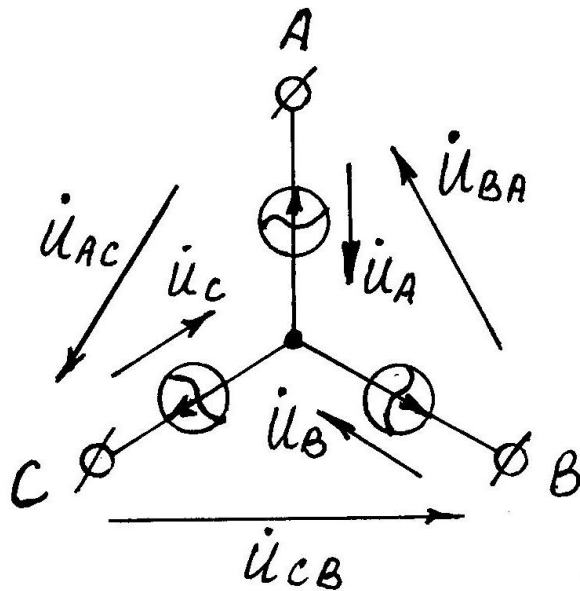
$$|\dot{I}_2| \rightarrow 0 \quad \dot{U}_1 = \dot{I}_1 j\omega L_1$$

$$M = k_{38}^m \cdot \sqrt{L_1 L_2} \quad \dot{U}_1 / \dot{U}_2 = -M / L_1$$

$$n = \sqrt{L_1 / L_2} \Rightarrow \boxed{\dot{U}_1 / \dot{U}_2 = -k_{38}^m \cdot n}$$

при $k_{38}^m = 1 \Rightarrow \boxed{\dot{U}_1 / \dot{U}_2 = -n}$

Трифазні кола



Трифазні системи
змінного струму

220/127(V); $f_c = 50 \text{ Гц}$
 220/127(V); $f_c = 60 \text{ Гц}$
 380/220(V); $f_c = 50 \text{ Гц}$
 660/380(V); $f_c = 50 \text{ Гц}$

$$\begin{aligned} \dot{U}_A &= U_g \exp(j0^\circ) \\ \dot{U}_B &= U_g \exp(-j120^\circ) \\ \dot{U}_C &= U_g \exp(+j120^\circ) \end{aligned}$$

$$\begin{aligned} \dot{U}_{AC} &= \dot{U}_A - \dot{U}_C = U_g \sqrt{3} \exp(-j30^\circ) \\ \dot{U}_{BC} &= \dot{U}_B - \dot{U}_C = U_g \sqrt{3} \exp(j90^\circ) \\ \dot{U}_{BA} &= \dot{U}_B - \dot{U}_A = U_g \sqrt{3} \exp(-j150^\circ) \end{aligned}$$

$$\dot{U}_{AC} = \dot{U}_A - \dot{U}_C = U_g \exp(j0^\circ) - U_g \exp(j120^\circ)$$

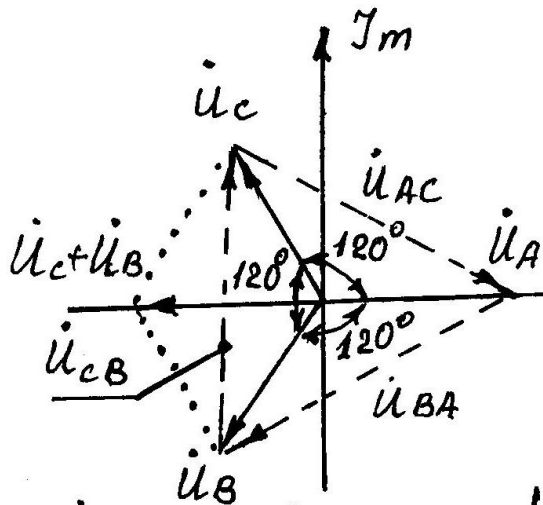
$$\text{Re } \dot{U}_{AC} = U_g (\cos 0^\circ + j \sin 0^\circ - \cos(120^\circ) - j \sin(120^\circ))$$

$$\dot{U}_{AC} = U_g \left(1 + \frac{1}{2} - j \frac{\sqrt{3}}{2} \right) = U_g \left(\frac{3}{2} - j \frac{\sqrt{3}}{2} \right);$$

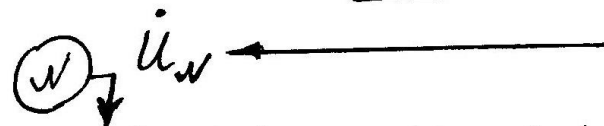
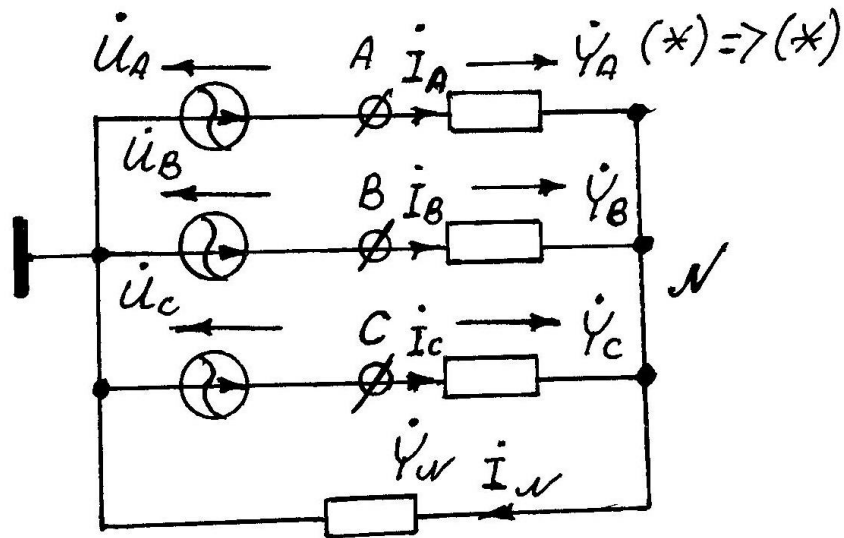
$$|\dot{U}_{AC}| = U_g \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = U_g \sqrt{3}; \quad \varphi_{AC} = \arctan\left(\frac{-\sqrt{3}/2}{3/2}\right) = -30^\circ$$

$$\dot{U}_A + \dot{U}_C + \dot{U}_B = 0$$

$$\dot{U}_{BA} + \dot{U}_{AC} + \dot{U}_{CB} = 0 \quad |\dot{U}_{AC}| = |\dot{U}_{BC}| = |\dot{U}_{BA}| = U_g \sqrt{3}; \quad \varphi_{CB} = 90^\circ; \quad \varphi_{BA} = -150^\circ$$



Розрахунок трифазних кіл



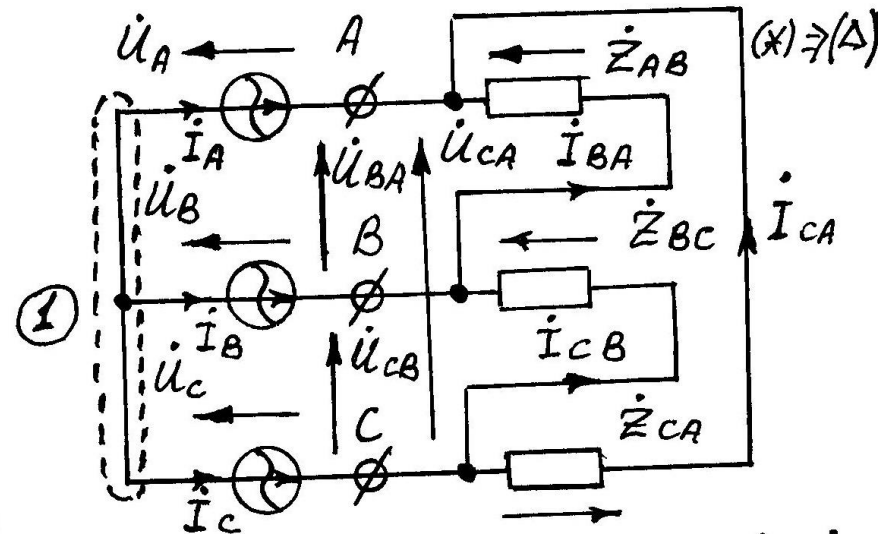
$$-\dot{Y}_A(\dot{U}_A - \dot{U}_N) - \dot{Y}_B(\dot{U}_B - \dot{U}_N) - \dot{Y}_C(\dot{U}_C - \dot{U}_N) \oplus$$

$$\oplus \dot{Y}_N \cdot \dot{U}_N = 0$$

$$\dot{U}_N = \frac{\dot{Y}_A \dot{U}_A + \dot{Y}_B \dot{U}_B + \dot{Y}_C \dot{U}_C}{\dot{Y}_A + \dot{Y}_B + \dot{Y}_C + \dot{Y}_N}$$

$$\text{При: } \dot{Y}_A = \dot{Y}_B = \dot{Y}_C \quad \left| \begin{array}{l} \dot{U}_A + \dot{U}_B + \dot{U}_C = 0 \end{array} \right. \Rightarrow \dot{U}_N = 0$$

$$\dot{I}_A = \dot{Y}_A(\dot{U}_A - \dot{U}_N); \dot{I}_B = \dot{Y}_B(\dot{U}_B - \dot{U}_N); \dot{I}_C = \dot{Y}_C(\dot{U}_C - \dot{U}_N)$$



$$\dot{U}_{BA} = \dot{U}_B - \dot{U}_A; \dot{U}_{CB} = \dot{U}_C - \dot{U}_B; \dot{U}_{CA} = \dot{U}_C - \dot{U}_A;$$

$$\textcircled{A} \begin{cases} -\dot{I}_A - \dot{I}_{BA} - \dot{I}_{CA} = 0 \\ \dot{I}_{BA} = \frac{\dot{U}_{BA}}{\dot{Z}_{AB}}; \dot{I}_{CA} = \frac{\dot{U}_{CA}}{\dot{Z}_{CA}}; \\ \dot{I}_A = -\dot{I}_{BA} - \dot{I}_{CA} = -\frac{\dot{U}_{BA}}{\dot{Z}_{AB}} - \frac{\dot{U}_{CA}}{\dot{Z}_{CA}}; \end{cases}$$

$$\textcircled{B} \begin{cases} -\dot{I}_B + \dot{I}_{BA} - \dot{I}_{CB} = 0 \\ \dot{I}_B = -\dot{I}_{BA} + \dot{I}_{CB} = -\frac{\dot{U}_{BA}}{\dot{Z}_{AB}} + \frac{\dot{U}_{CB}}{\dot{Z}_{BC}}; \end{cases}$$

$$\textcircled{C} \begin{cases} -\dot{I}_C + \dot{I}_{CA} + \dot{I}_{CB} = 0 \\ \dot{I}_C = \dot{I}_{CA} + \dot{I}_{CB} = \frac{\dot{U}_{CA}}{\dot{Z}_{CA}} + \frac{\dot{U}_{CB}}{\dot{Z}_{BC}}; \end{cases}$$

