# ECONOMICS OF PRICING AND DECISION MAKING

Lecture 1

# What makes a business successful?



- Providing a service that customers like
- Building partnerships
- Being ahead of competitors
- Building brand value

...“**Interactions**”

with customers, suppliers, competitors, regulators, people within the firm...

# What is game theory?



- ...a collection of tools for predicting outcomes of a group of interacting agents
- ... a bag of analytical tools designed to help us understand the phenomena that we observe when decision makers interact (Osborne and Rubinstein)
- ...the study of mathematical models of conflict and cooperation between intelligent rational decision makers (Myerson)

# What is game theory?



- Study of interactions between parties (e.g. individuals, firms)
- Helps us understand situations in which decision makers interact: strategies & likely outcome
- Game theory consists of a series of models, often technical as well as intuitive
- The models predict how parties are likely to behave in certain situations

# The Game:
# Strategic Environment



- Players
  - Everyone who has an effect on your earnings (payoff)
- Actions:
  - Choices available to the players
- Strategies
  - Define a plan of action for every contingency
- Payoffs
  - Numbers associated with each outcome
  - Reflect the interests of the players

# Strategic Thinking



**Example: Apple vs. Samsung**

- Apple's action depends on how Apple predicts Samsung's action.
- Apple's action depends on how Apple predicts how Samsung predicts the Apple's action.
- Apple's action depends on how Apple predicts how Samsung predicts how Apple predicts the Samsung's action.

etc…

# The Assumptions



- Rationality
  - Players aim to maximize their payoffs, and are self-interested.
  - Players are perfect calculators
  - Players consider the responses/reactions of other players
- Common Knowledge
  - Each player knows the rules of the game
  - Each player knows that each player knows the rules
  - Each player knows that each player knows that each player knows the rules
  - Each player knows that each player knows that each player knows that each player knows the rules
  - Each player knows that each player knows that each player knows that each player knows that each player knows the rules
  - ...

# History of game theory



- 1928, 1944: John von Neumann
- 1950: John Nash
- 1960s: Game theory used to simulate thermonuclear war between the USA and the USSR
- 1970s: Oligopoly theory
- 1980s: Game theory used
  - Evolutionary biology
  - Political science
- More recent applications: Philosophy, computer science
- 1994, 2005, 2007, 2012: Economics Nobel prize

# Lectures



- 1-3: Simultaneous games
  - Nash equilibrium
  - Oligopoly
  - Mixed strategies
- 4-5: Sequential games
  - Subgame perfect equilibrium
  - Bargaining
- 6: Repeated games
  - Two firms interacting repeatedly

# Lectures



- 7: Evolutionary games
  - How do players “learn” to play the Nash equilibrium
- 8-9: Incomplete information
  - Cooperation and coordination with incomplete information
  - Signaling, and moral hazard.
- 10: Auctions
  - Strategies for bidders and sellers

# Assessment



- Assessment consist is a final exam:
  - 100% exam
  - 2-hour
- Section A: 5 compulsory questions, at most 3 "mathematical/analytical" questions. (10 marks each)
- Section B: choose 1 essay question from a list of 2. (50 marks)

# SIMULTANEOUS GAMES WITH DISCRETE CHOICES

# PURE STRATEGY NASH EQUILIBRIUM

# Simultaneous games with discrete choices



- A game is <u>simultaneous</u> when players
  - choose their actions at the same time
  - or, choose their actions in isolation, without knowing what the other players do
- <u>Discrete choices</u>: the set of possible actions is finite
  - e.g. {yes,no}; {a,b,c}.
  - Opposite of continuous choices: e.g. choose any number between 0 and 1.

# Strategic Interaction



- Players: Reynolds and Philip Morris
- Payoffs: Companies' profits
- Strategies: Advertise or Not Advertise

- Strategic Landscape:
  - Each firm initially earns $50 million from its existing customers
  - Advertising costs a firm $20 million
  - Advertising captures $30 million from competitor
- Simultaneous game with discrete choices

# Representing a Game
## (strategic form / normal form)



| | | Philip Morris | |
|---|---|---|---|
| | | No Ad | Ad |
| Reynolds | No Ad | 50 , 50 | 20 , 60 |
| | Ad | 60 , 20 | 30 , 30 |

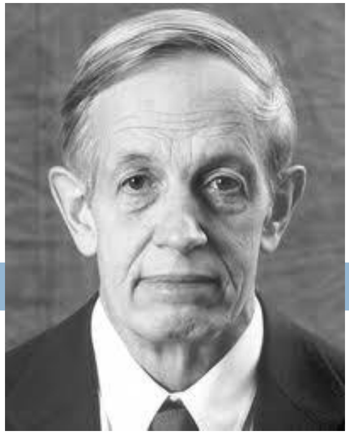
50-20+30 50-30 50-20+30-30

**What is the likely outcome?**

**We want a “stable”, “rational” outcome.**

# Solving the game: Nash equilibrium





- The Nash equilibrium, is a set of strategies, one for each player, such that no player has incentive to unilaterally change his action
  - The NE describes a stable situation.
- Nash equilibrium: likely outcome of the game when players are rational
  - Each player is playing his/her best strategy given the strategy choices of all other players
  - No player has an incentive to change his or her action unilaterally

# Solving the Game



| | | Philip Morris | |
|---|---|---|---|
| | | No Ad | Ad |
| Reynolds | No Ad | 50 , 50 | 20 , 60 |
| | Ad | 60 , 20 | 30 , 30 |

- Can (No Ad,No Ad) be a Nash equilibrium?
  - No, 60>50
- Can (No Ad,Ad) be a Nash equilibrium?
  - No: 30>20
- Can (Ad,No Ad) be a Nash equilibrium?
  - No: 30>20

# Solving the Game



| | | Philip Morris | |
|---|---|---|---|
| | | No Ad | Ad |
| Reynolds | No Ad | 50 , 50 | 20 , 60 |
| | Ad | 60 , 20 | 30 , 30 |

Equilibrium

- Can (Ad,Ad) be a Nash equilibrium?
  - YES: 30>20
  - If Philip Morris "believes" that Reynolds will choose Ad, it will also choose Ad.
  - If Reynolds "believes" that Philip Morris will choose Ad, it will also choose Ad.
  - (Ad, Ad) is a "stable" outcome, neither player will want to change action unilaterally.

# Equilibrium vs. optimal outcome



"Optimal"

| | No Ad | Ad |
|---|---|---|
| No Ad | 50 , 50 | 20 , 60 |
| Ad | 60 , 20 | 30 , 30 |

Equilibrium

- The optimal outcome is the one that maximizes the sum of all players' payoffs. (No Ad, No Ad)
- The NE does not necessarily maximize total payoff. (Ad,Ad). The NE is individually rational, but not always collectively rational.

# Game of cooperation (prisoner's dilemma)



| | | Player 2 Cooperate | Player 2 Defect |
|---|---|---|---|
| Player 1 | Cooperate | 50 , 50 | 20 , 60 |
| | Defect | 60 , 20 | 30 , 30 |

Players can choose between cooperate and defect. The NE is that both players defect. But the optimal outcome is that both cooperate.

In this example: Cooperate = No Ad ; Defect = Ad

# Nash equilibrium existence



- Q: Does a NE always exist?
  - A: Yes (in almost every cases). [If there is no equilibrium with pure strategies, there will be one with mixed strategies.]
- **Theorem (Nash, 1950)**

  "There exists at least one Nash equilibrium in any finite games in which the numbers of players and strategies are both finite."

# Nash equilibrium

## A formal definition

- Any social problem can be formalized as a "game," consisting of three elements:

Players: i=1,2,…,N

i's Strategy: $s_i \in S_i$

i's Payoff: $\pi_i(s_1, \ldots, s_N)$

# Nash equilibrium

## A formal definition



- Definition: A Nash Equilibrium is a profile of strategies $(s_i^*, s_{-i}^*)$ such that each player's strategy is an optimal response to the other players strategies:

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*)$$

- If all players play according to the NE, no player has any incentive to change his action <u>unilaterally</u>.
- Why is the NE the most likely outcome:
  - Any other outcome is not "stable".
  - In the long term, players learn how to play and always select the NE

# How to find the Nash equilibrium?



- There are two techniques to find the NE
  1. Successive elimination of dominated strategies
  2. Best response analysis

# Elimination of dominated strategies (1st method)



- Procedure: eliminate, one by one, the strategies that are strictly dominated by at least one other strategy.
- Consider two strategies, A and B. Strategy A strictly dominates Strategy B if the payoff of Strategy A is strictly higher than the payoff of Strategy B no matter what opposing players do.
  - For Philip Morris, Ad dominates No Ad: $\pi(\text{Ad,any}) > \pi(\text{No Ad,any})$. For Reynolds Ad also dominates No Ad.
- Strictly dominated strategies can be eliminated, they would not be chosen by rational players.
  - □ No Ad can be eliminated for both players.

# Elimination of dominated strategies



| | | Philip Morris: No Ad | Philip Morris: Ad |
|---|---|---|---|
| Reynolds | No Ad | ~~50 , 50~~ | ~~20 , 60~~ |
| | Ad | 60 , 20 | 30 , 30 |

# Elimination of dominated strategies



- The order in which strategies are eliminated does not matter. Select any player, any strategy, and check whether it is <u>strictly</u> dominated by any other strategy. If it is strictly dominated, eliminate it.
- When several strategies are strictly dominated, it does not matter which one you eliminate first.

# Elimination of dominated strategies

| | Left | Middle | Right |
|---|---|---|---|
| Up | 5, 2 | 2, 3 | 3, 4 |
| Medium | 4, 1 | 3, 2 | 4, 0 |
| Down | 3, 3 | 1, 2 | 2, 2 |

# Elimination of dominated strategies

| | ~~Left~~ | Middle | Right |
|---|---|---|---|
| Up | ~~5, 2~~ | ~~2, 3~~ | ~~3, 4~~ |
| Medium | ~~4, 1~~ | 3, 2 | ~~4, 0~~ |
| ~~Down~~ | ~~3, 3~~ | ~~1, 2~~ | ~~2, 2~~ |

Up dominates (>)Down.
Now that Down is out, Middle>Left.
Now that Left is out, Medium>Up.
Middle>Right
□ The NE is {Medium,Middle}

# Weak dominance

- Strategy A weakly dominates strategy B if its strategy A's payoff is in some cases higher (>) and in some cases equal (≥) to strategy B's payoff.
- Alternative scenario:

| | |
|---|---|
| 50 , 50 | 30 , 60 |
| 60 , 30 | 30 , 30 |

- One strategy <u>weakly</u> dominates the other
  - 60>50
  - 30=30

# Weak dominance

- Weakly dominated strategies <u>cannot</u> be eliminated.
- In some cases, when strategies are only weakly dominated, successive elimination can get eliminate some Nash equilibria.

# Best response analysis (2nd method)



| | | Philip Morris: No Ad | Philip Morris: Ad |
|---|---|---|---|
| Reynolds | No Ad | 50 , 50 | 20 , 60 |
| | Ad | 60 , 20 | 30 , 30 |

Procedure: For each possible strategy, draw a circle around the best response of the other player.

The NE is where there is a joint best response.

# Best response analysis

| | Left | Middle | Right |
|---|---|---|---|
| Up | 5, 2 | 2, 3 | 3, 4 |
| Medium | 4, 1 | 3, 2 | 4, 0 |
| Down | 3, 3 | 1, 2 | 2, 2 |

# Exercise

| | | Column | | |
|---|---|---|---|---|
| | | Left | Middle | Right |
| Row | Top | 3, 1 | 2, 3 | 10, 2 |
| | High | 4, 5 | 3, 0 | 6, 4 |
| | Low | 2, 2 | 5, 4 | 12, 3 |
| | Bottom | 5, 6 | 4, 5 | 9, 7 |

# Comparing the two methods



- The two methods for finding the NE are NOT equivalent.
- The best response analysis is fully reliable, and always finds the NE.
- Sometimes, the elimination of dominated strategies will fail to find the NE. This may happen when that are more than one NE.

# Comparing the two methods



- Example of an entry game:
  - Two businesses must choose which market to enter.

| | Market A | Market B |
|---|---|---|
| Market A | 0,0 | 2,2 |
| Market B | 2,2 | 0,0 |

- This is a game of <u>coordination</u> (not cooperation!): class of games with multiple NE (two in this case).

# Comparing the two methods

- 1st method: The game is not dominance solvable, there are no dominated strategies.
- 2nd method: With best response analysis, both equilibria are found.

**When best-response analysis of a discrete strategy game does not find a Nash equilibrium, then the game has no equilibrium in pure strategies.**

# Summary

- What is game theory
- Game representation
- Nash equilibrium as the likely outcome of the game
- Finding the NE: dominance vs. best response