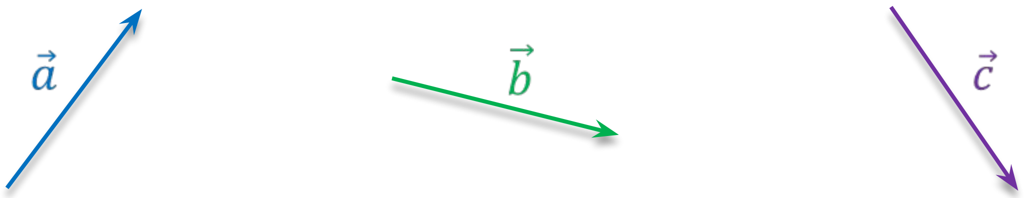
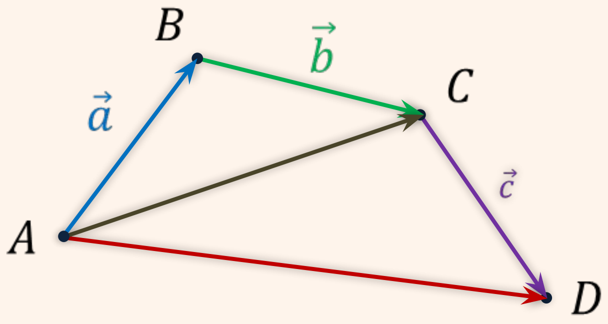
Сумма нескольких векторов

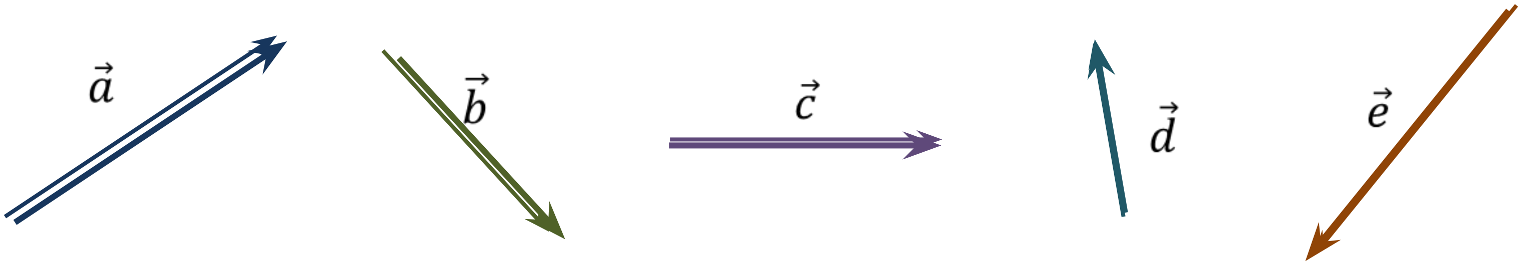


Правило *многоугольника*



1. $\overrightarrow{AB} = \vec{a}$
2. $\overrightarrow{BC} = \vec{b}$
3. $\overrightarrow{CD} = \vec{c}$
4. $\overrightarrow{AD} = \vec{a} + \vec{b} + \vec{c}$

Задача. Построить вектор суммы попарно неколлинеарных векторов $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ и $\vec{e}$.



Построение.

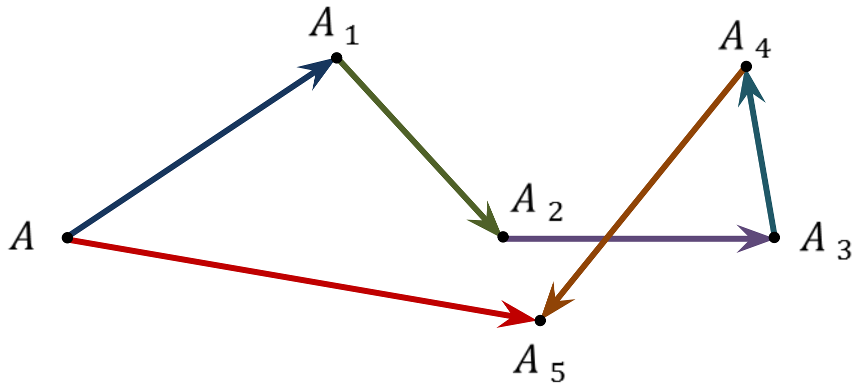
$\overrightarrow{AA_1} = \vec{a}$

$\overrightarrow{A_1A_2} = \vec{b}$

$\overrightarrow{A_2A_3} = \vec{c}$

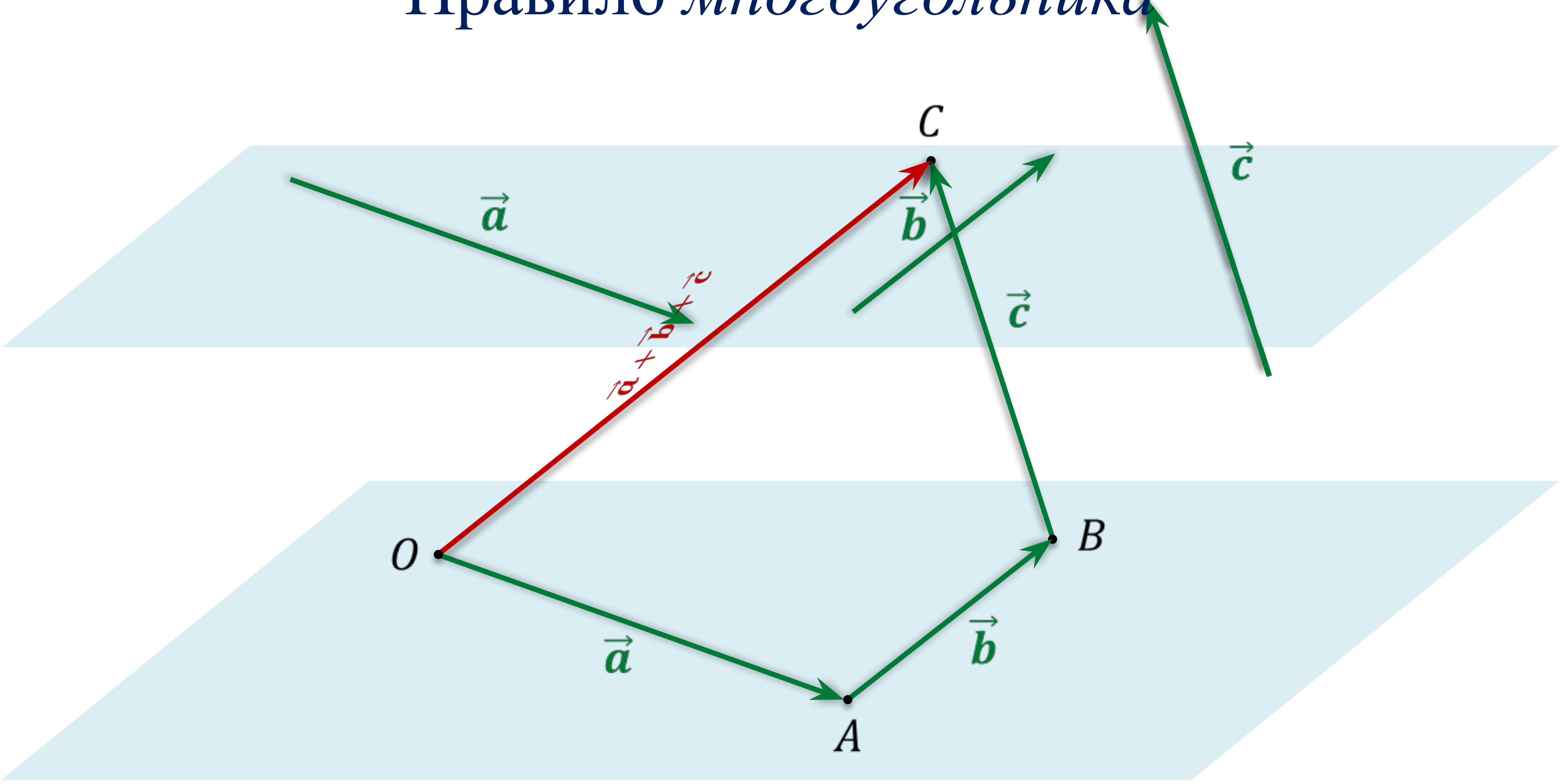
$\overrightarrow{A_3A_4} = \vec{d}$

$\overrightarrow{A_4A_5} = \vec{e}$

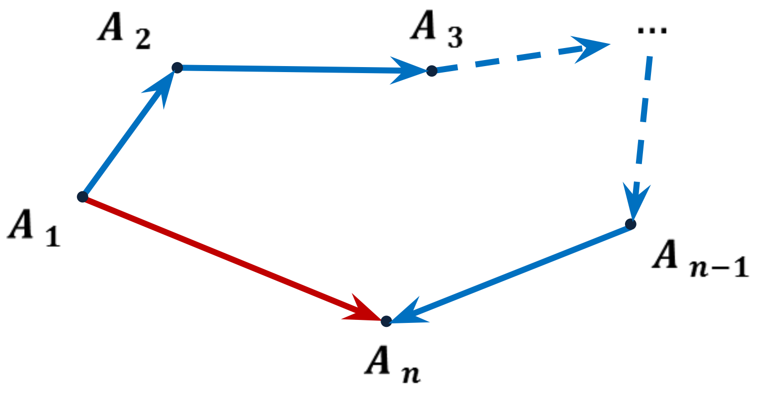


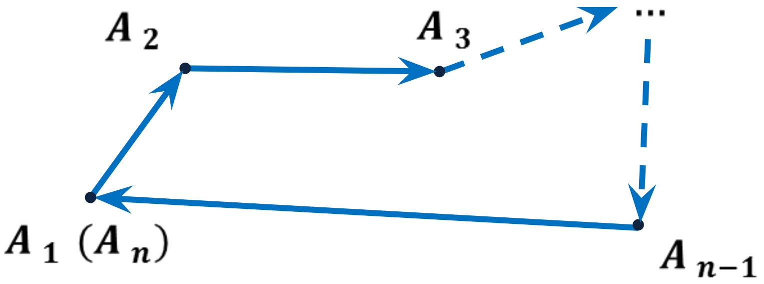
$$\overrightarrow{AA_1} + \overrightarrow{A_1A_2} + \overrightarrow{A_2A_3} + \overrightarrow{A_3A_4} + \overrightarrow{A_4A_5} = \overrightarrow{AA_5} \quad \Longrightarrow \quad \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} = \overrightarrow{AA_5}$$

Правило *многоугольника*

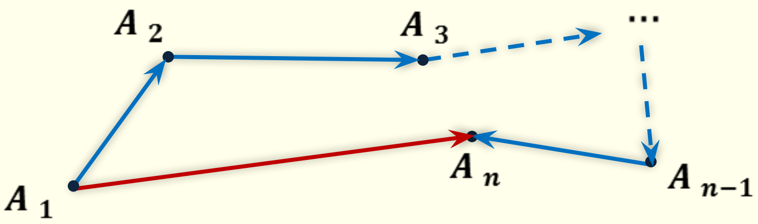


Правило *многоугольника*

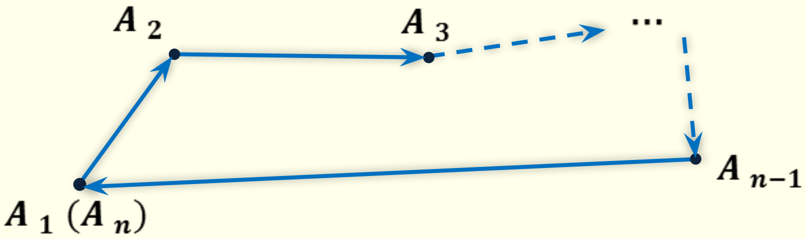




$$\overrightarrow{A_1A_2} + \overrightarrow{A_2A_3} + \cdots + \overrightarrow{A_{n-1}A_n} = \vec{0}$$



$$\overrightarrow{A_1A_2}+\overrightarrow{A_2A_3}+\dots+\overrightarrow{A_{n-1}A_n}=\overrightarrow{A_1A_n}$$



$$\overrightarrow{A_1A_2}+\overrightarrow{A_2A_3}+\dots+\overrightarrow{A_{n-1}A_n}=\vec{0}$$

Задача №1. Упростить выражения.

а) $\overrightarrow{AB}+\overrightarrow{BA}+\overrightarrow{CD}+\overrightarrow{MN}+\overrightarrow{DC}+\overrightarrow{NM}=\overrightarrow{AB}+\overrightarrow{BA}+\overrightarrow{CD}+\overrightarrow{DC}+\overrightarrow{MN}+\overrightarrow{NM}=\vec{0}+\vec{0}+\vec{0}=\vec{0}$

б) $\overrightarrow{KM}+\overrightarrow{DF}+\overrightarrow{AC}+\overrightarrow{FK}+\overrightarrow{CD}+\overrightarrow{CA}+\overrightarrow{MP}=\overrightarrow{CD}+\overrightarrow{DF}+\overrightarrow{FK}+\overrightarrow{KM}+\overrightarrow{MP}=\overrightarrow{CP}$

в) $\overrightarrow{OP}-\overrightarrow{EP}+\overrightarrow{KD}-\overrightarrow{KA}=\overrightarrow{OP}+\left(-\overrightarrow{EP}\right)+\overrightarrow{KD}+\left(-\overrightarrow{KA}\right)=\overrightarrow{OP}+\overrightarrow{PE}+\overrightarrow{AK}+\overrightarrow{KD}=\overrightarrow{OE}+\overrightarrow{AD}$

г) $\overrightarrow{AC}-\overrightarrow{BC}-\overrightarrow{PM}-\overrightarrow{AP}+\overrightarrow{BM}=\overrightarrow{AC}+\overrightarrow{CB}+\overrightarrow{MP}+\overrightarrow{PA}+\overrightarrow{BM}=\overrightarrow{AC}+\overrightarrow{CB}+\overrightarrow{BM}+\overrightarrow{MP}+\overrightarrow{PA}=\vec{0}$

Задача №2. A, B, C, D − произвольные точки пространства.
Представить вектор $\overrightarrow{AB}$ в виде алгебраической суммы векторов:

а) $\overrightarrow{AC}, \overrightarrow{DC}, \overrightarrow{BD}$ б) $\overrightarrow{DA}, \overrightarrow{DC}, \overrightarrow{CB}$ в) $\overrightarrow{DA}, \overrightarrow{CD}, \overrightarrow{BC}$

Решение.

а) $\overrightarrow{AC}, \overrightarrow{DC}, \overrightarrow{BD}$

$$\overrightarrow{\boldsymbol{AB}} = \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DB} = \overrightarrow{AC} + \left(-\overrightarrow{DC}\right) + \left(-\overrightarrow{BD}\right) = \overrightarrow{\boldsymbol{AC}} - \overrightarrow{\boldsymbol{DC}} - \overrightarrow{\boldsymbol{BD}}$$

б) $\overrightarrow{DA}, \overrightarrow{DC}, \overrightarrow{CB}$

$$\overrightarrow{\boldsymbol{AB}} = \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CB} = -\overrightarrow{\boldsymbol{DA}} + \overrightarrow{\boldsymbol{DC}} + \overrightarrow{\boldsymbol{CB}}$$

в) $\overrightarrow{DA}, \overrightarrow{CD}, \overrightarrow{BC}$

$$\overrightarrow{\boldsymbol{AB}} = \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CB} = -\overrightarrow{DA} + (-\overrightarrow{CD}) + (-\overrightarrow{BC}) = -\overrightarrow{\boldsymbol{DA}} - \overrightarrow{\boldsymbol{CD}} - \overrightarrow{\boldsymbol{BC}}$$

$ABCDA_1B_1C_1D_1$ – параллелепипед.

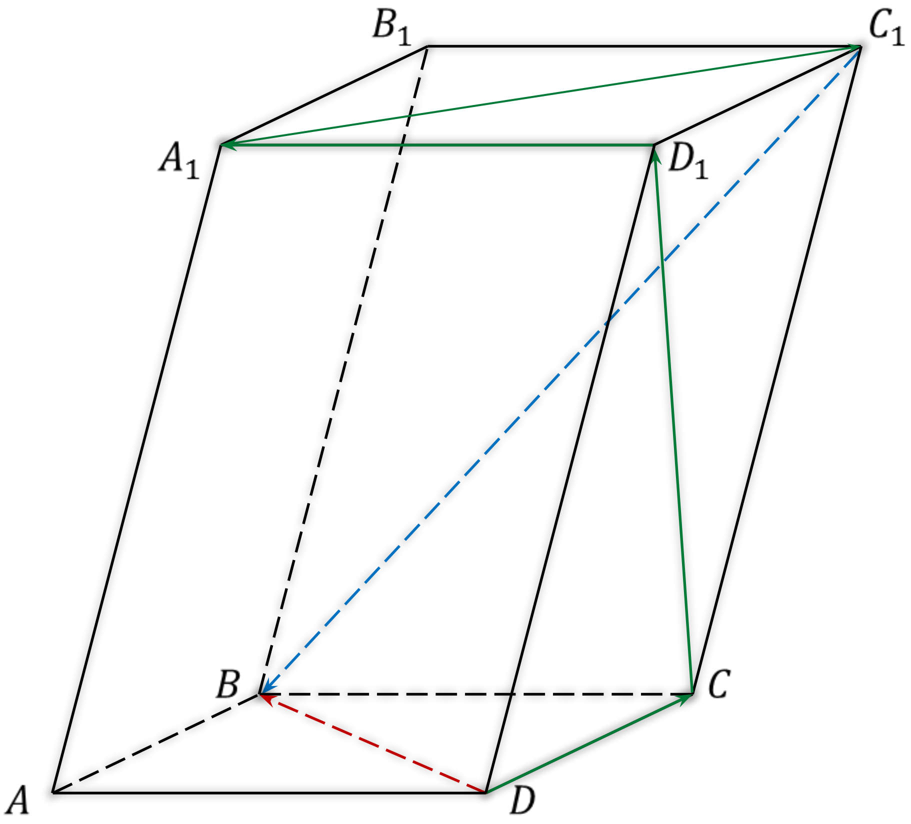
а) $\overrightarrow{DC} + \overrightarrow{D_1A_1} + \overrightarrow{CD_1} + \vec{x} + \overrightarrow{A_1C_1} = \overrightarrow{DB}$

$\boldsymbol{\vec{x} = \overrightarrow{C_1B}}$

б) $\overrightarrow{DA} + \vec{x} + \overrightarrow{D_1B} + \overrightarrow{AD_1} + \overrightarrow{BA} = \overrightarrow{DC}$

в) $\overrightarrow{AA_1} + \overrightarrow{B_1C} - \vec{x} = \overrightarrow{BA}$

г) $\overrightarrow{AC_1} - \overrightarrow{BB_1} + \vec{x} = \overrightarrow{AB}$



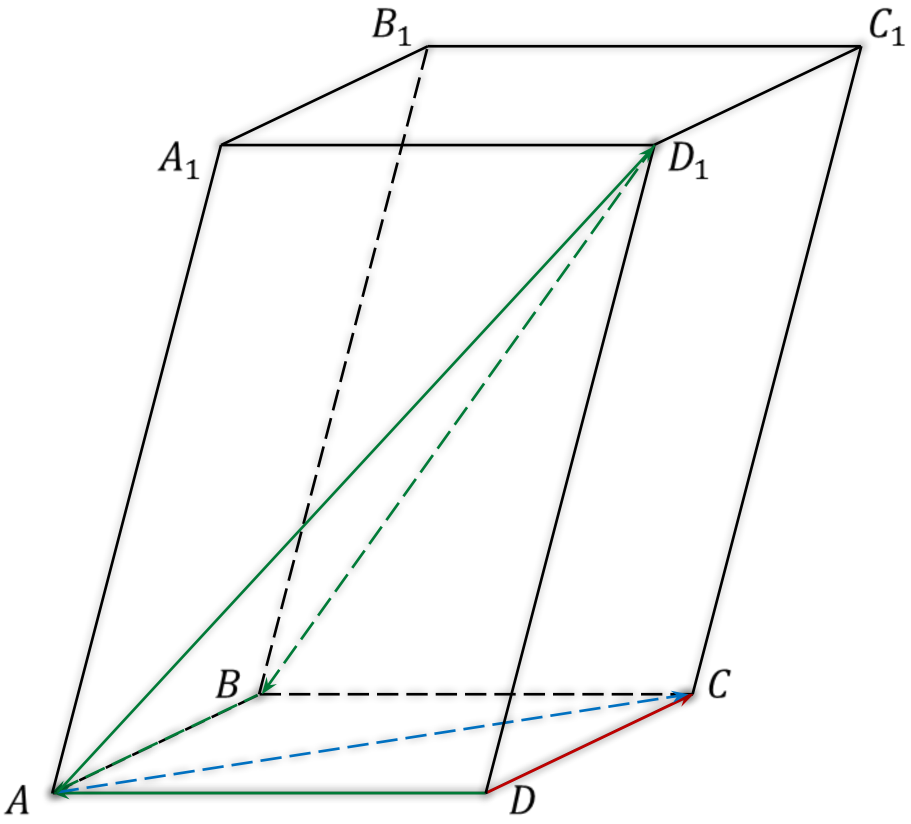
$ABCDA_1B_1C_1D_1$ – параллелепипед.

а) $\overrightarrow{DC} + \overrightarrow{D_1A_1} + \overrightarrow{CD_1} + \vec{x} + \overrightarrow{A_1C_1} = \overrightarrow{DB}$

$\boldsymbol{\vec{x} = \overrightarrow{C_1B}}$

б) $\overrightarrow{DA} + \vec{x} + \overrightarrow{D_1B} + \overrightarrow{AD_1} + \overrightarrow{BA} = \overrightarrow{DC}$

$\boldsymbol{\vec{x} = \overrightarrow{AC}}$



$ABCDA_1B_1C_1D_1$ – параллелепипед.

а) $\overrightarrow{DC} + \overrightarrow{D_1A_1} + \overrightarrow{CD_1} + \vec{x} + \overrightarrow{A_1C_1} = \overrightarrow{DB}$

$\boldsymbol{\vec{x} = \overrightarrow{C_1B}}$

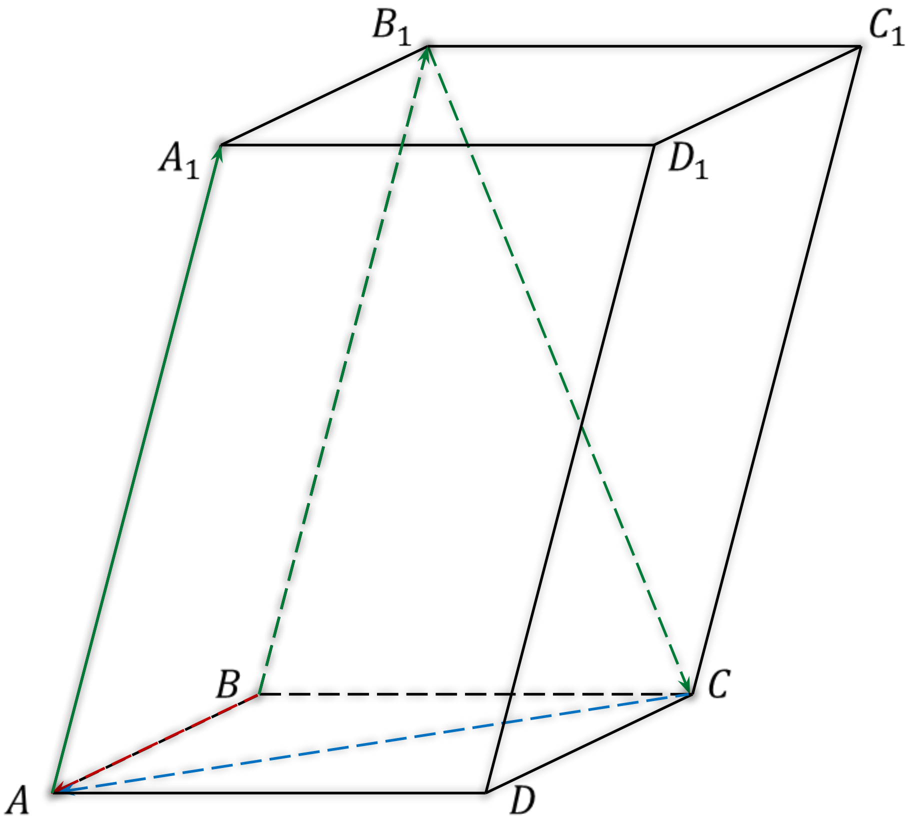
б) $\overrightarrow{DA} + \vec{x} + \overrightarrow{D_1B} + \overrightarrow{AD_1} + \overrightarrow{BA} = \overrightarrow{DC}$

$\boldsymbol{\vec{x} = \overrightarrow{AC}}$

в) $\overrightarrow{AA_1} + \overrightarrow{B_1C} - \vec{x} = \overrightarrow{BA}$

$\overrightarrow{BB_1} + \overrightarrow{B_1C} - \vec{x} = \overrightarrow{BA}$

$-\vec{x} = \overrightarrow{CA} \implies \boldsymbol{\vec{x} = \overrightarrow{AC}}$



$ABCDA_1B_1C_1D_1$ – параллелепипед.

а) $\overrightarrow{DC} + \overrightarrow{D_1A_1} + \overrightarrow{CD_1} + \vec{x} + \overrightarrow{A_1C_1} = \overrightarrow{DB}$

$\boldsymbol{\vec{x} = \overrightarrow{C_1B}}$

б) $\overrightarrow{DA} + \vec{x} + \overrightarrow{D_1B} + \overrightarrow{AD_1} + \overrightarrow{BA} = \overrightarrow{DC}$

$\boldsymbol{\vec{x} = \overrightarrow{DC}}$

в) $\overrightarrow{AA_1} + \overrightarrow{B_1C} - \vec{x} = \overrightarrow{BA}$

$\overrightarrow{BB_1} + \overrightarrow{B_1C} - \vec{x} = \overrightarrow{BA}$

$-\vec{x} = \overrightarrow{CA} \implies \boldsymbol{\vec{x} = \overrightarrow{AC}}$

г) $\overrightarrow{AC_1} - \overrightarrow{BB_1} + \vec{x} = \overrightarrow{AB}$

$\overrightarrow{AC_1} + (-\overrightarrow{BB_1}) + \vec{x} = \overrightarrow{AB}$

$\overrightarrow{AC_1} + \overrightarrow{B_1B} + \vec{x} = \overrightarrow{AB}$

$\boldsymbol{\vec{x} = \overrightarrow{C_1B_1}}$