University Physics I

Forces

Review of Basic Concepts

Vectors and Scalars

All physical quantities (e.g. speed and force) are described by a magnitude and a unit.

VECTORS – also need to have their direction specified

examples: displacement, velocity, acceleration, force.

SCALARS – do not have a direction

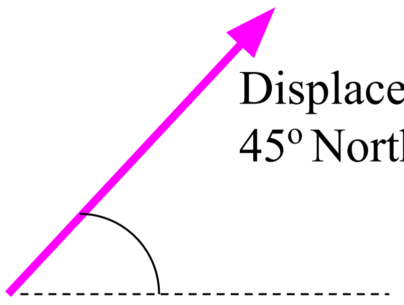
examples: distance, speed, mass, work, energy.

Representing Vectors

An arrowed straight line is used.

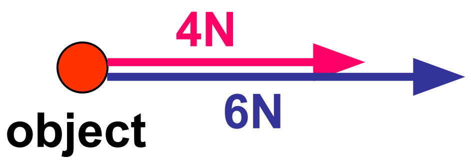
Displacement 50m EAST

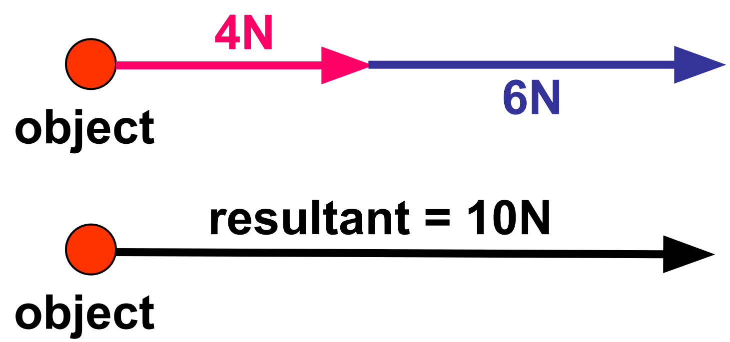
The arrow indicates the direction and the length of the line is proportional to the magnitude.



Displacement 25m at 45^{o} North of East

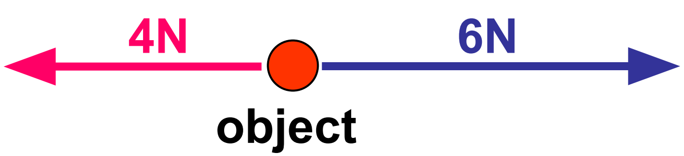
Addition of vectors 1

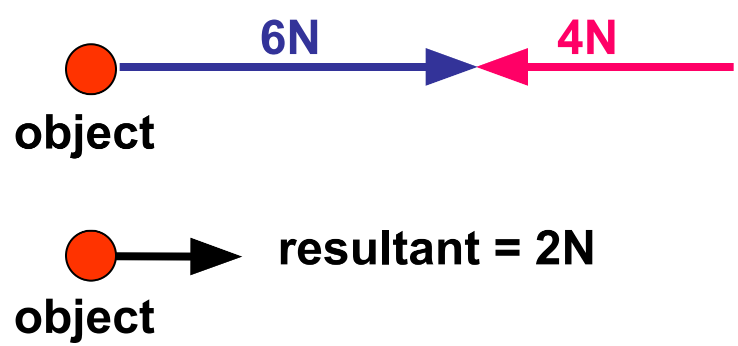




The original vectors are called **COMPONENT** vectors.

The final overall vector is called the **RESULTANT** vector.





Addition of vectors 2

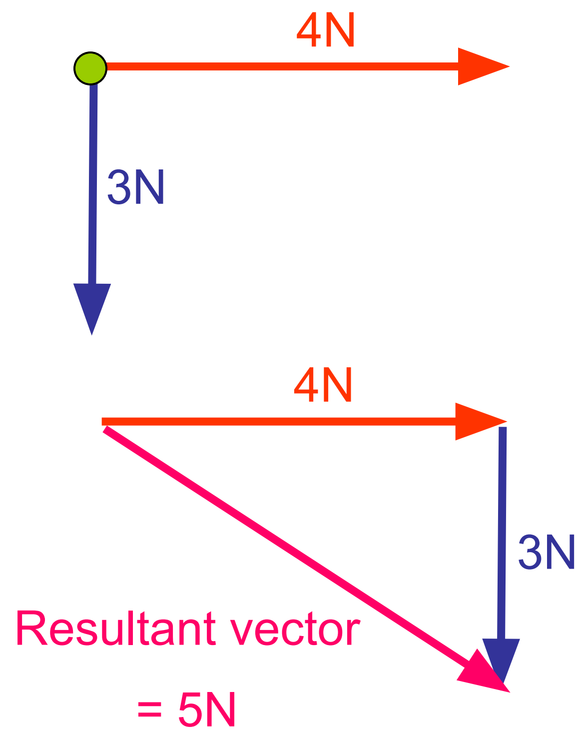
With two vectors acting at an angle to each other:

Draw the first vector.

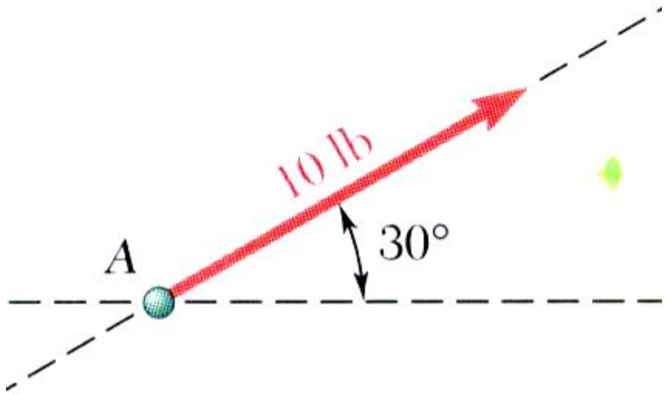
Draw the second vector with its tail end on the arrow of the first vector.

The resultant vector is the line drawn from the tail of the first vector to the arrow end of the second vector.

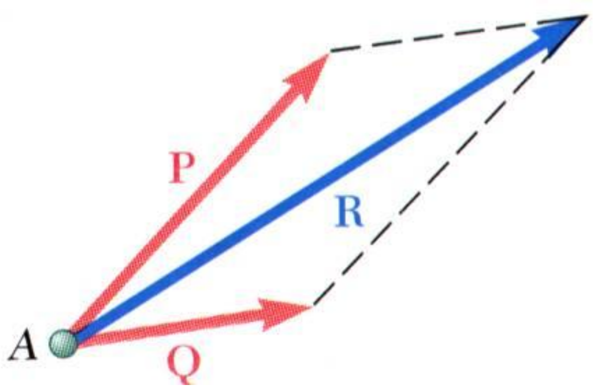
This method also works with three or more vectors.



Resultant of Two Forces

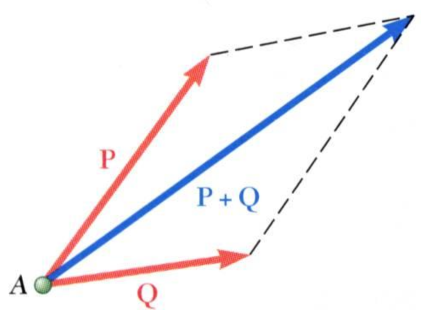


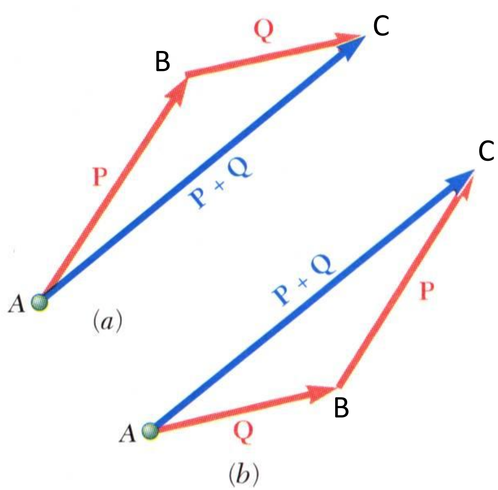
- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.

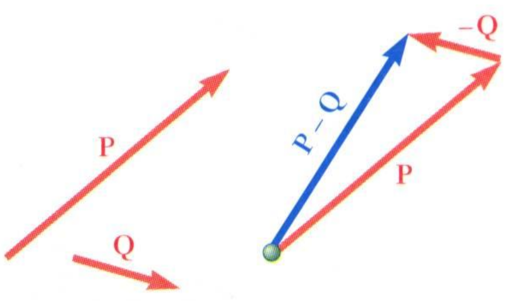


- Experimental evidence shows that the combined effect of two forces may be represented by a single *resultant* force.
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.
- Force is a *vector* quantity.

Addition of Vectors







(a) (b)

- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines,

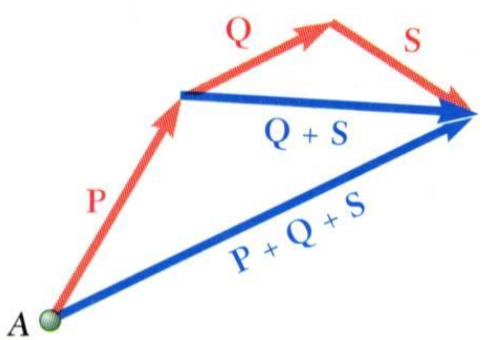
$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

- Vector addition is commutative,

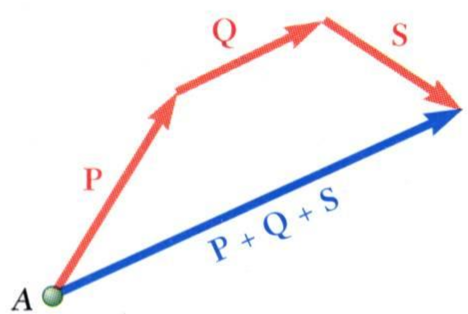
$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

Addition of Vectors

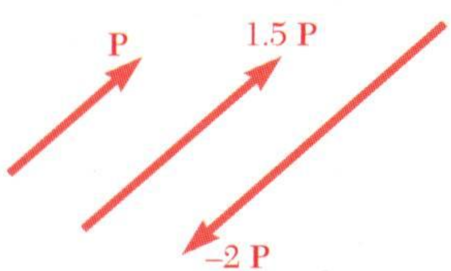


- Addition of three or more vectors through repeated application of the triangle rule



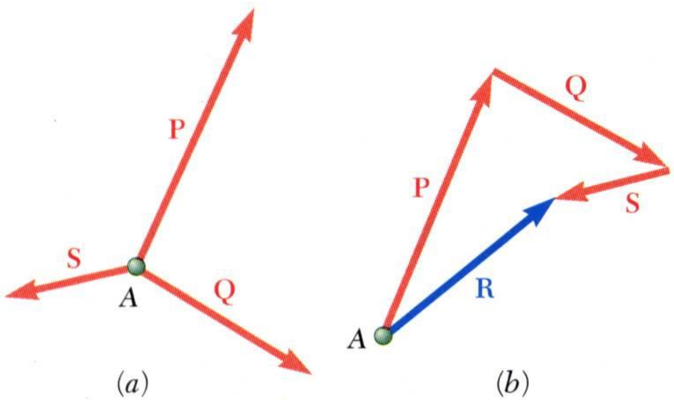
- The polygon rule for the addition of three or more vectors.
- Vector addition is associative,

$$\vec{P}+\vec{Q}+\vec{S}=\left(\vec{P}+\vec{Q}\right)+\vec{S}=\vec{P}+\left(\vec{Q}+\vec{S}\right)$$



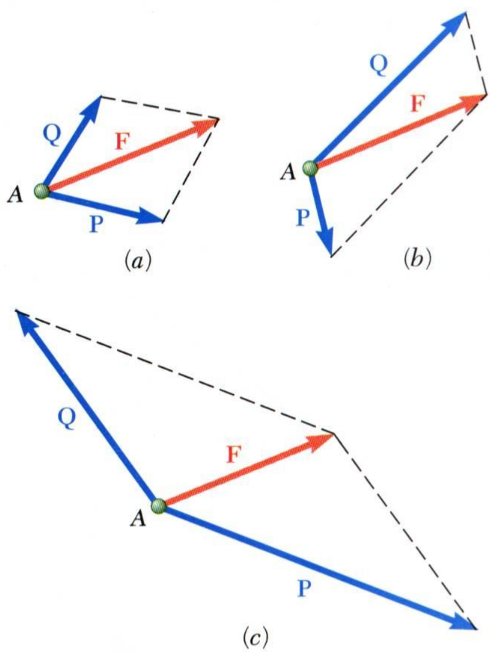
- Multiplication of a vector by a scalar

Resultant of Several Concurrent Forces



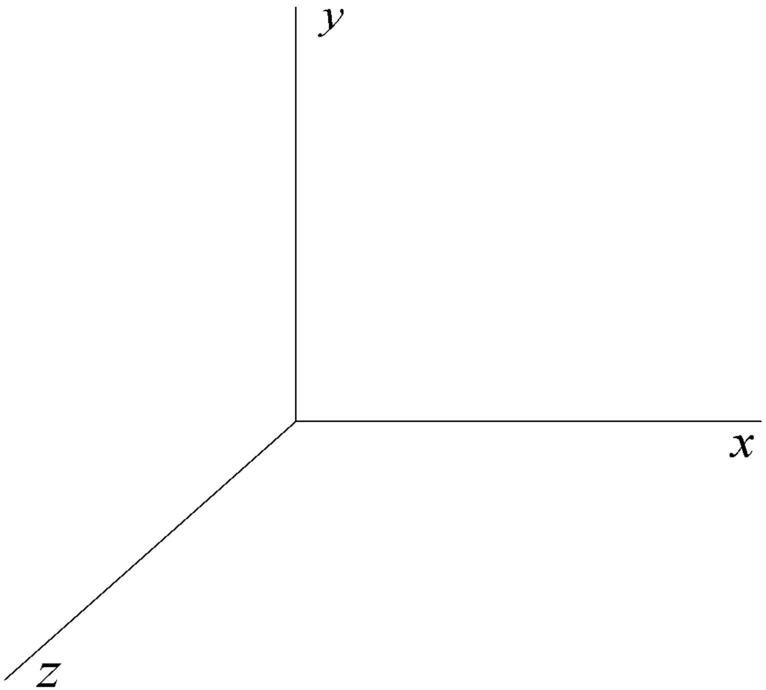
- *Concurrent forces*: set of forces which all pass through the same point.

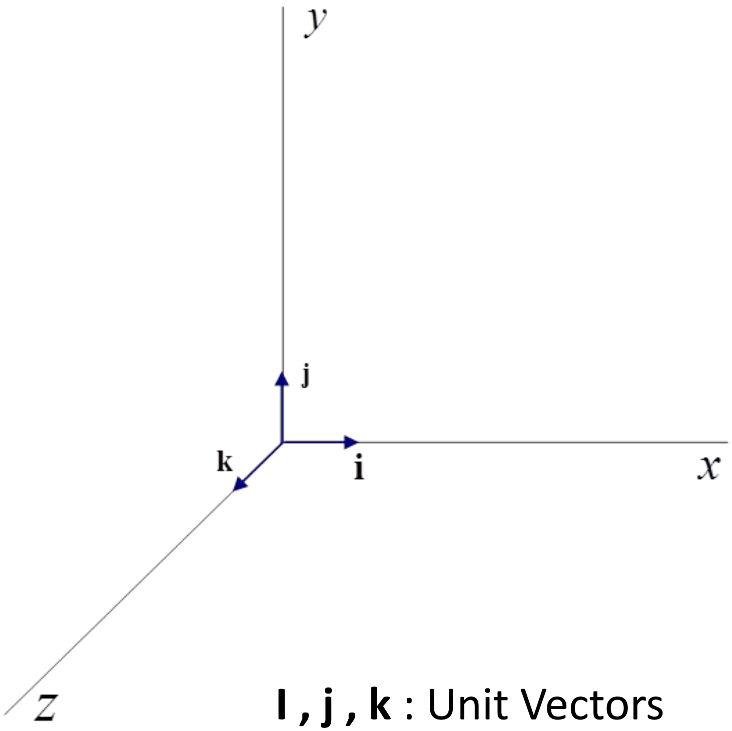
  A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.



- *Vector force components*: two or more force vectors which, together, have the same effect as a single force vector.

Rectangular Coordinate System





I , j , k : Unit Vectors

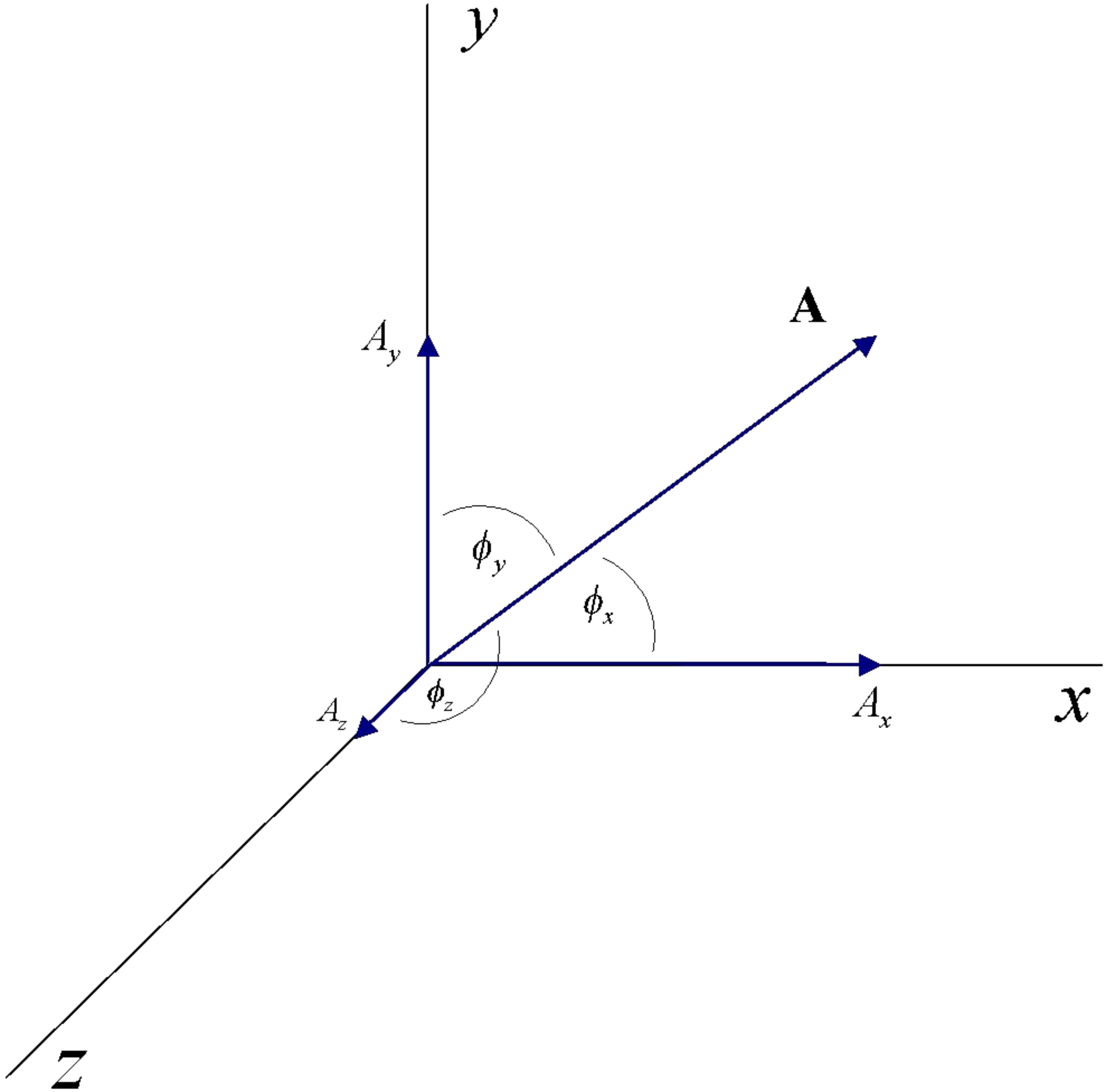
Vector Representation:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

Magnitude or Absolute Value:

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction Angles



Relationships for Direction Angles

$$\cos\phi_x = \frac{A_x}{A} = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\phi_y = \frac{A_y}{A} = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\phi_z = \frac{A_z}{A} = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Example 1.

A force has *x*, *y*, and *z* components of 3, 4, and –12 N, respectively.

Express the force as a vector in rectangular coordinates.

$$\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$$

Determine the magnitude of the force in previous example:

$$\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$$

$$F = \sqrt{(3)^2 + (4)^2 + (-12)^2}$$

$$= 13 \text{ N}$$

Determine the three direction angles for the force :

$$\cos\phi_x = \frac{A_x}{A} = \frac{3}{13} = 0.2308$$

$$\phi_x = \cos^{-1} 0.2308 = 76.66^\circ = 1.338 \text{ rad}$$

$$\cos\phi_y = \frac{A_y}{A} = \frac{4}{13} = 0.3077$$

$$\phi_y = \cos^{-1} 0.3077 = 72.08^\circ = 1.258 \text{ rad}$$

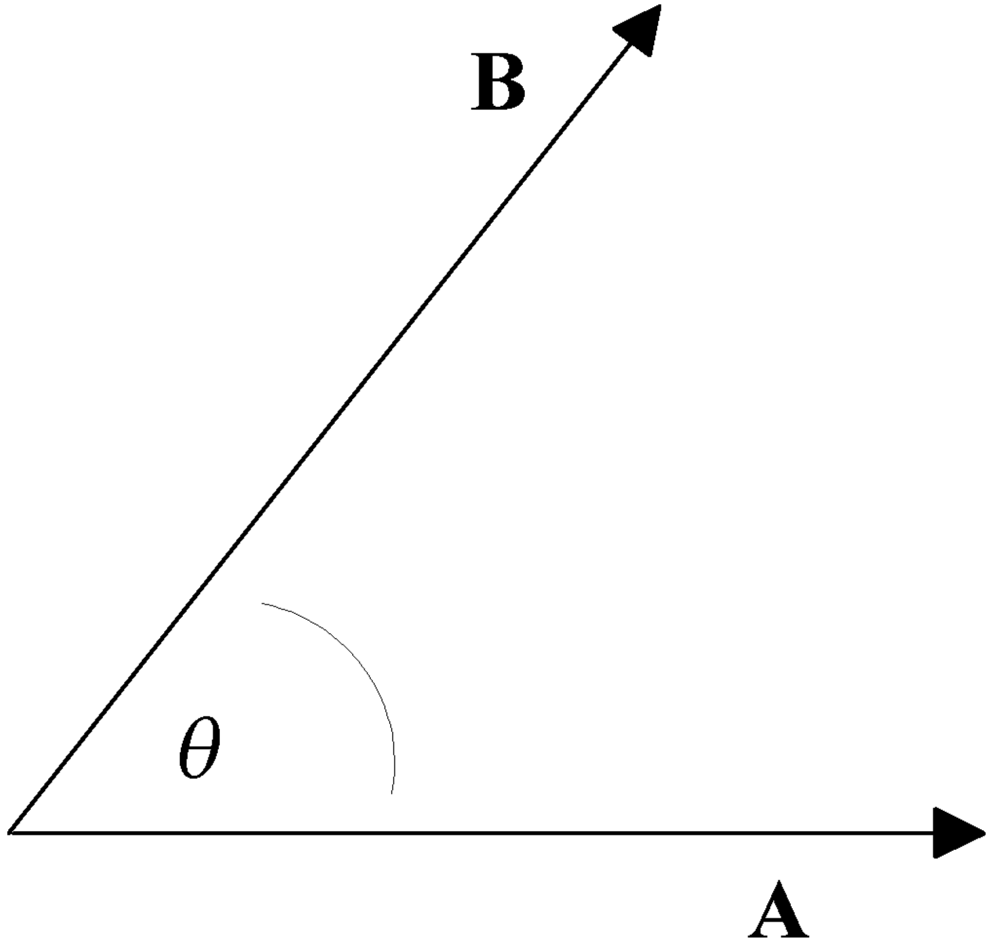
$$\cos\phi_z = \frac{A_z}{A} = \frac{-12}{13} = -0.9231$$

$$\phi_z = \cos^{-1}(-0.9231) = 157.4^{\circ} = 2.747 \text{ rad}$$

Vector Operations to be Considered

- Scalar or Dot Product: **A•B**
- Vector or Cross Product: **AxB**
- Triple Scalar Product: **(AxB)•C**

Consider two vectors **A** and **B** oriented in different directions.



Scalar or Dot Product

Definition:

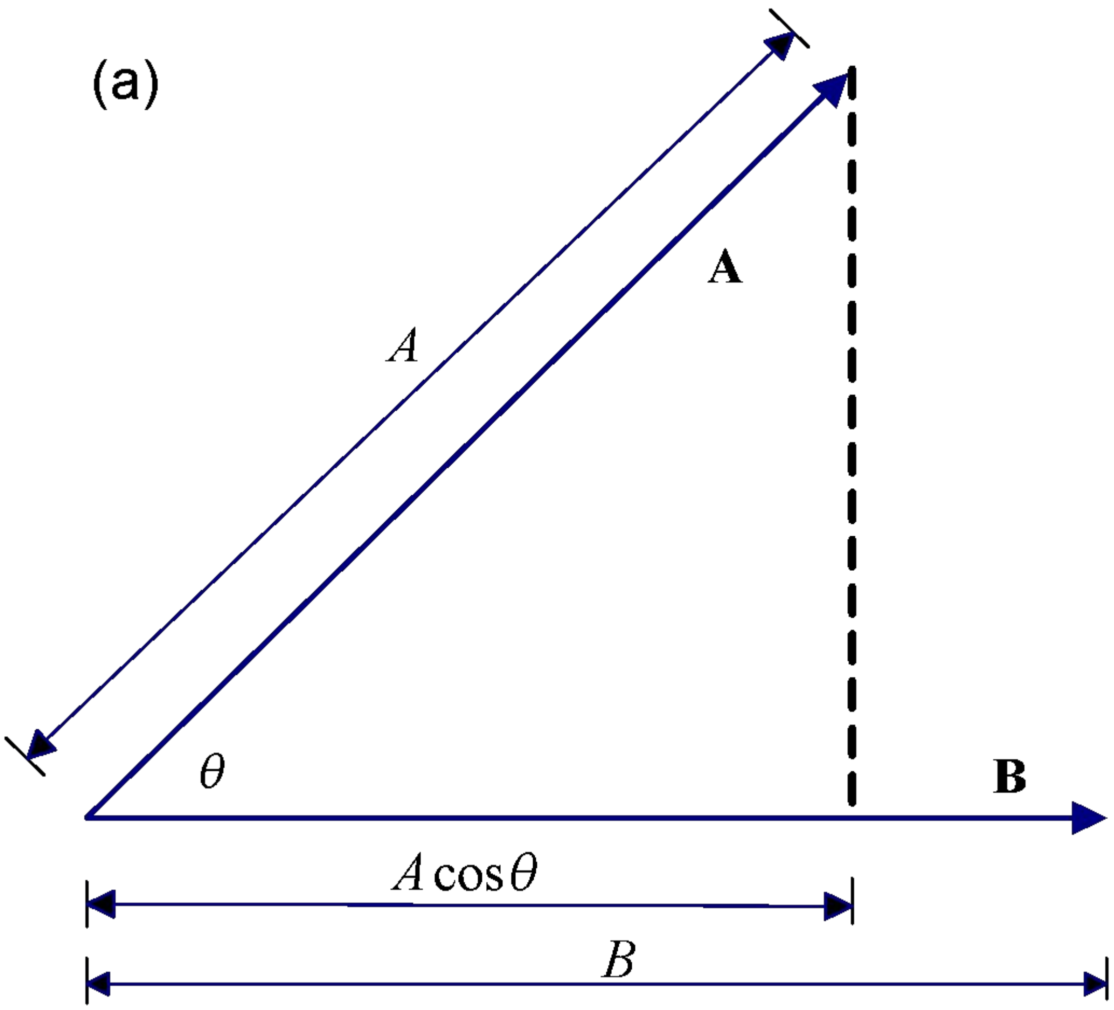
$$\mathbf{A} \bullet \mathbf{B} = AB\cos\theta$$

Computation:

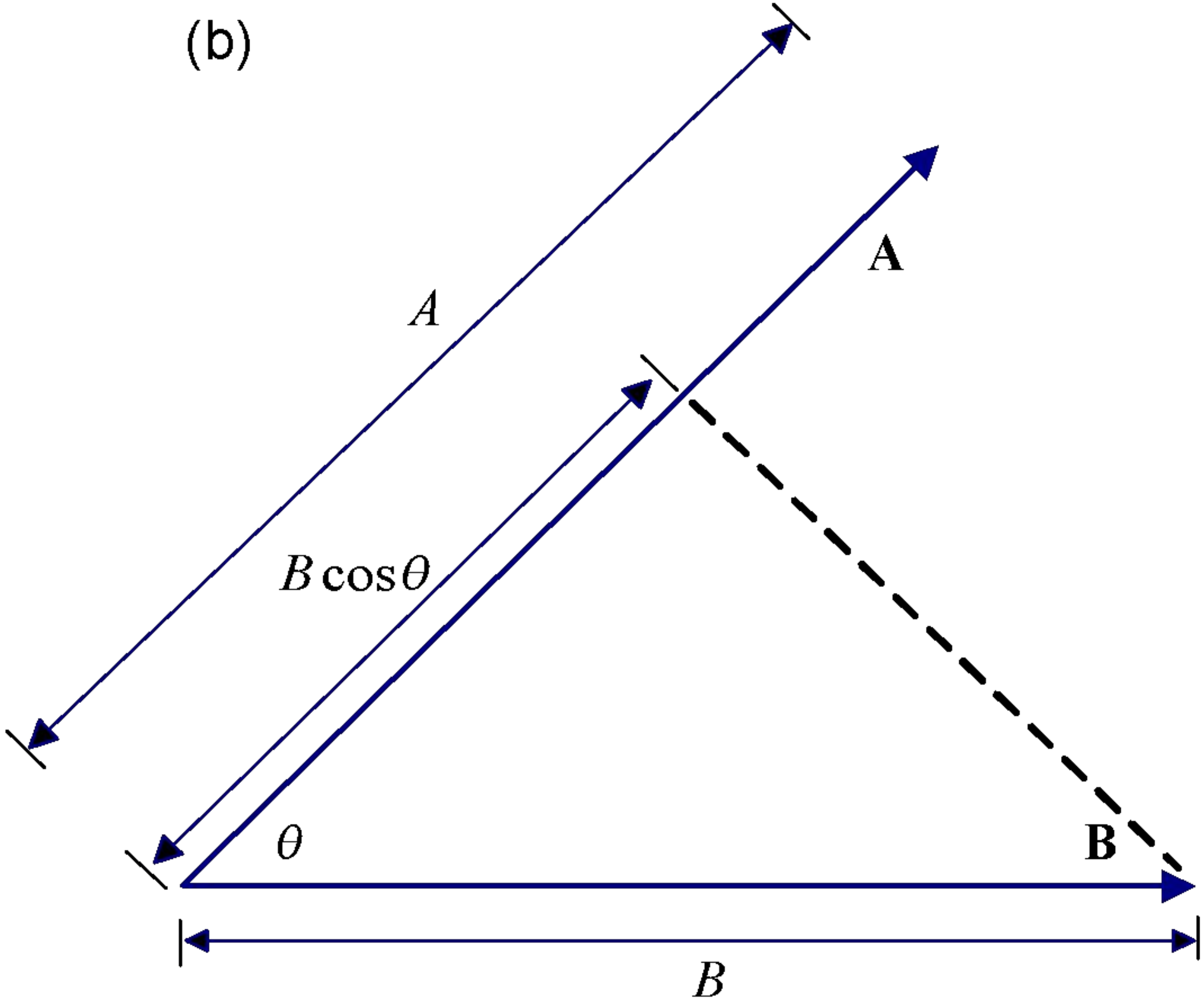
$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

Represents the Work done by the Force B during the displacement A for example.

First Interpretation of Dot Product:
Projection of A on B times the length of B.



Or alternatively:
Projection of **B** on **A** times the length of **A**.



Some Implications of Dot Product

$\theta = 0^\circ$

The vectors are parallel to each other and

$\mathbf{A} \cdot \mathbf{B} = AB$

$\theta = 90^\circ$

The vectors are $\perp$ to each other and

$\mathbf{A} \cdot \mathbf{B} = 0$

Example : Perform several scalar operations on the following vectors:

$$\mathbf{A} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} \qquad \mathbf{B} = 3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{(2)^2 + (-2)^2 + (1)^2} = 3$$

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$= \sqrt{(3)^2 + (4)^2 + (12)^2} = 13$$

$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= (2)(3) + (-2)(4) + (1)(12) = 10$$

$$\mathbf{A} \bullet \mathbf{B} = AB \cos\theta$$

$$\cos\theta = \frac{\mathbf{A} \bullet \mathbf{B}}{AB} = \frac{10}{3 \times 13} = \frac{10}{39} = 0.2564$$

$$\theta = \cos^{-1} 0.2564 = 75.14^{\circ} = 1.311 \text{ rad}$$

Vector or Cross Product

The Cross Product of 2 vectors A and B, is a vector C which is perpendicular to both A and B, and whose Amplitude is (AB sin(θ))

Definition:

$$\mathbf{A} \times \mathbf{B} = \left(AB \sin\theta \right) \mathbf{u_n}$$

Computation:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$