IRob2305: Introduction to Robotics

LECTURE 5

Manipulator Kinematics, Link Description, Link Connections, Denavit-Hartenberg Parameters, Summary - DH Parameters, Example - DH Table, Forward Kinematics

Link and Joint Parameters

- *Joint angle* θ_i: the angle of rotation from the X_{i-1} axis to the X_i axis about the Z_{i-1} axis. It is the joint variable if joint i is rotary.

- *Joint distance* d_i: the distance from the origin of the (i-1) coordinate system to the intersection of the Z_{i-1} axis and the X_i axis along the Z_{i-1} axis. It is the joint variable if joint i is prismatic.

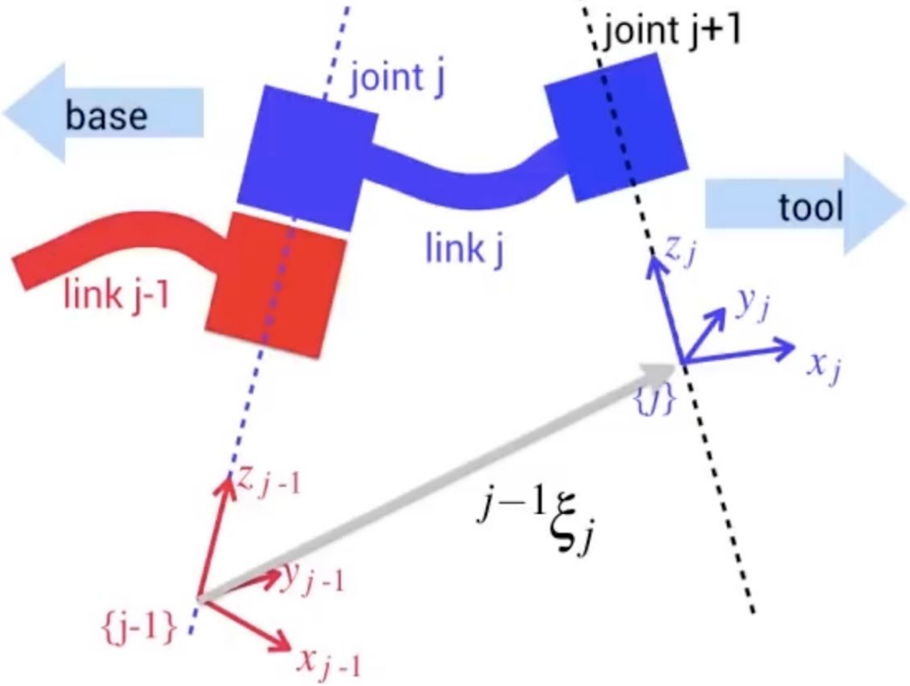
- *Link length* a_i: the distance from the intersection of the Z_{i-1} axis and the X_i axis to the origin of the ith coordinate system along the X_i axis.

- *Link twist angle* α_i: the angle of rotation from the Z_{i-1} axis to the Z_i axis about the X_i axis.

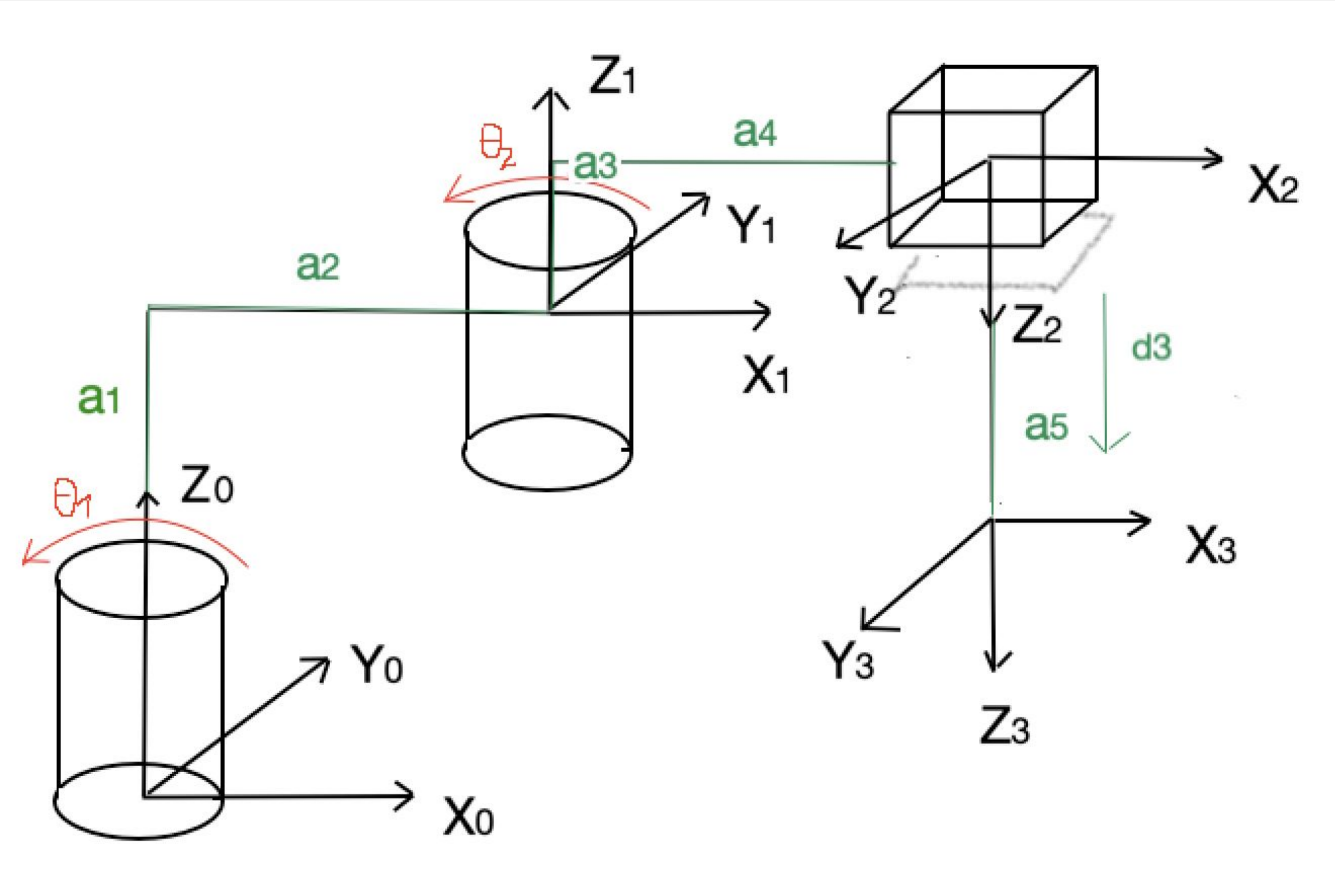
DH PARAMETERS

$$^{j-1}\xi_j \sim \mathbf{A} = \mathbf{R}_z(\theta_j)\mathbf{T}_z(d_j)\mathbf{T}_x(a_j)\mathbf{R}_x(\alpha_j)$$

- Attach a coordinate frame to the far (distal) end of every link
- Describe the pose of a link frame with respect to the previous link frame
- Very concise
  - ➥ Only four parameters: θ, d, a, α



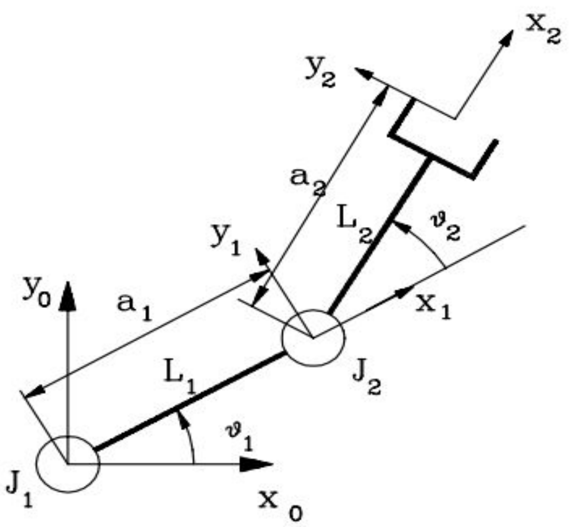
Example 1:



Link	Θ	α	a	d
1	Θ_1	0	a_2	a_1
2	Θ_2	180	a_4	a_3
3	0	0	0	$a_5 + d_3$

Example 2:

Let's consider a 2 dof planar manipulator:



Denavit-Hartenberg parameters

	d	θ	a	α
L1	0	θ_1	a_1	0^o
L2	0	θ_2	a_2	0^o

The ${}^{i-1}\mathbf{H}_i$ matrices result:

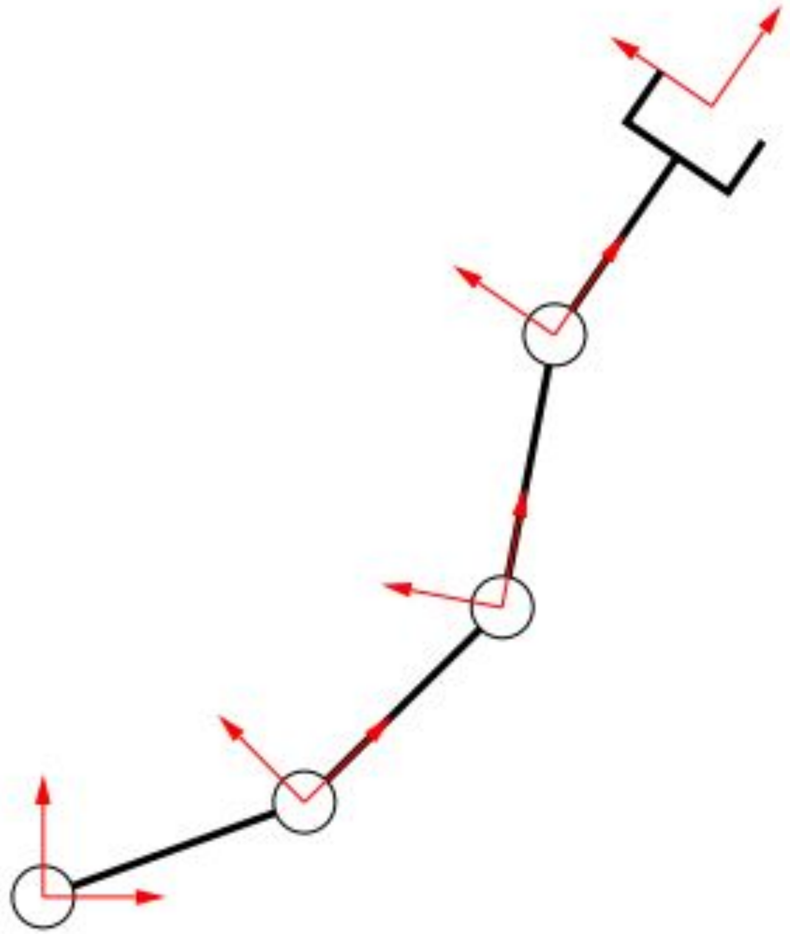
$$
{}^0\mathbf{H}_1 = \begin{bmatrix} C_1 & -S_1 & 0 & a_1 C_1 \\ S_1 & C_1 & 0 & a_1 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1\mathbf{H}_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Then

$$
{}^0\mathbf{T}_2 = {}^0\mathbf{H}_1 \, {}^1\mathbf{H}_2 = \begin{bmatrix} \mathbf{n} & \mathbf{s} & \mathbf{a} & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{12} & -S_{12} & 0 & a_1 C_1 + a_2 C_{12} \\ S_{12} & C_{12} & 0 & a_1 S_1 + a_2 S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Example 3:

Example: planar 4 dof manipulator (redundant)



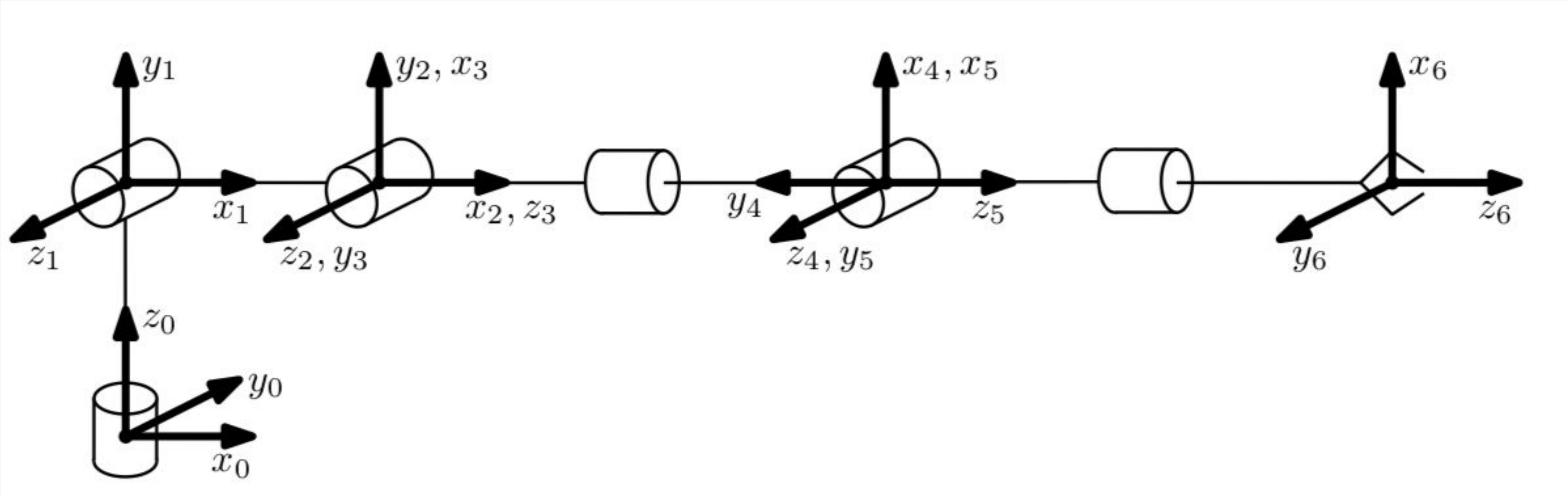
DH parameters

	d	θ	a	α
L1	0	θ_1	a_1	0^o
L2	0	θ_2	a_2	0^o
L3	0	θ_3	a_3	0^o
L4	0	θ_4	a_4	0^o

All the ${}^{i-1}\mathbf{H}_i$ matrices have the same structure

$$
{}^{i-1}\mathbf{H}_i = \begin{bmatrix} C_i & -S_i & 0 & a_i C_i \\ S_i & C_i & 0 & a_i S_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Example 4:



Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	$d1$	$\theta 1$
2	$a2$	0	0	$\theta 2$
3	0	$\pi/2$	0	$\theta 3 + \pi/2$
4	0	$-\pi/2$	$d4$	$\theta 4$
5	0	$\pi/2$	0	$\theta 5$
6	0	0	$d6$	$\theta 6$

$$
\begin{aligned}
T_i &= T_{z,\theta_i} T_{z,d_i} T_{x,a_i} T_{x,\alpha_i} = \\
&= \begin{bmatrix} R_{z,\theta_i} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & p_{d_i} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & p_{a_i} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{x,\alpha_i} & 0 \\ 0 & 1 \end{bmatrix} = \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},
\end{aligned}
$$

Where, the Rx, Rz – base rotation matrices

$$R_{z,\theta_i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$R_{x,\alpha_i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix},$$

Pd, Pa - vectors

$$p_{d_i} = \begin{bmatrix} 0 \\ 0 \\ d_i \end{bmatrix}, \quad p_{a_i} = \begin{bmatrix} a_i \\ 0 \\ 0 \end{bmatrix}.$$

Denavit Hartenberg matrix:

$$T_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_3^2 = \begin{bmatrix} \cos(\theta_3 + \frac{\pi}{2}) & 0 & \sin(\theta_3 + \frac{\pi}{2}) & 0 \\ \sin(\theta_3 + \frac{\pi}{2}) & 0 & -\cos(\theta_3 + \frac{\pi}{2}) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_4^3 = \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

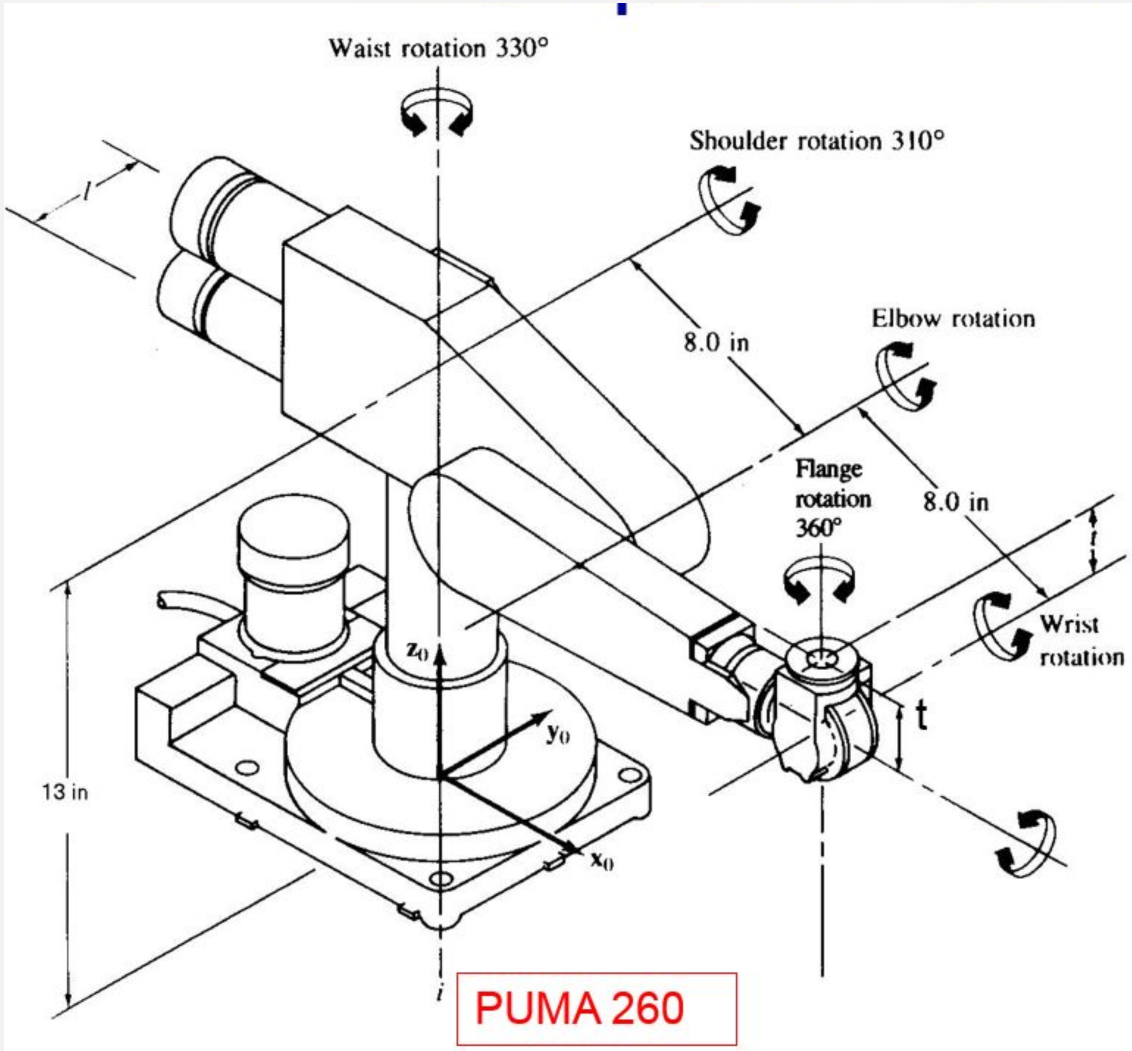
$$T_5^4 = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Final will be:

$$T_n^0(q) = T_1(q)T_2(q)\dots T_n(q) = \begin{bmatrix} R_n^0(q) & p_n^0(q) \\ 0 & 1 \end{bmatrix},$$

Example 5: PUMA 260



1. Number the joints
2. Establish base frame
3. Establish joint axis Z_i
4. Locate origin, (intersect. of Z_i & Z_{i-1}) OR (intersect of common normal & Z_i)
5. Establish X_i, Y_i

$$X_i = \pm(Z_{i-1} \times Z_i) / \|Z_{i-1} \times Z_i\|$$

$$Y_i = +(Z_i \times X_i) / \|Z_i \times X_i\|$$

J	θ_i	α_i	a_i	d_i
1	θ_1	-90	0	13
2	θ_2	0	8	0
3	θ_3	90	0	-l
4	θ_4	-90	0	8
5	θ_5	90	0	0
6	θ_6	0	0	t

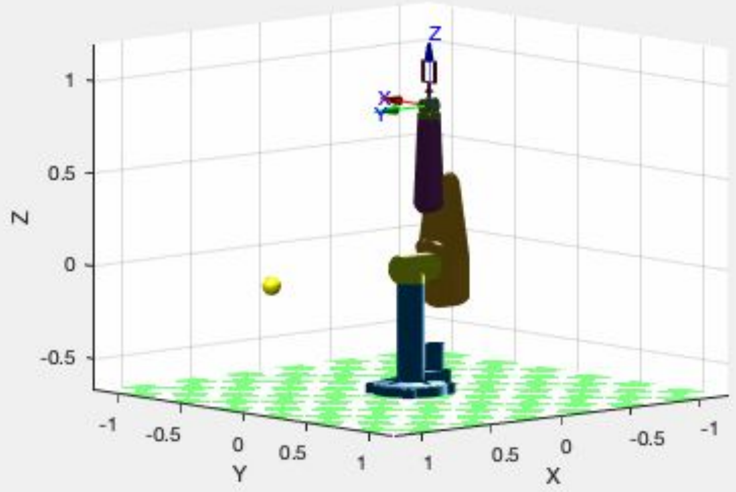
CODE EXAMPLE IN MATLAB

```
>> mdl_puma560
>> p560
p560 =

Puma 560 [Unimation]:: 6 axis, RRRRRR, stdDH, fastRNE
 - viscous friction; params of 8/95;
+---+-----------+-----------+-----------+-----------+-----------+
| j |     theta |         d |         a |     alpha |    offset |
+---+-----------+-----------+-----------+-----------+-----------+
|  1|         q1|          0|          0|     1.5708|          0|
|  2|         q2|          0|     0.4318|          0|          0|
|  3|         q3|    0.15005|     0.0203|    -1.5708|          0|
|  4|         q4|     0.4318|          0|     1.5708|          0|
|  5|         q5|          0|          0|    -1.5708|          0|
|  6|         q6|          0|          0|          0|          0|
+---+-----------+-----------+-----------+-----------+-----------+

>> p560.fkine([0 0 0 0 0 0])  % forward kinematics
ans =
         1         0         0    0.4521
         0         1         0     -0.15
         0         0         1    0.4318
         0         0         0         1
```

We can animate a path:



Best industrial robot systems

https://www.youtube.com/watch?v=neWc5I9IdQ4