Математика

Занятие 106.
Таблица первообразных.
Правила интегрирования.

1. Таблица первообразных.
2. Правила интегрирования.

Таблица первообразных

$f(x)$	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
$F(x)$	kx	$\frac{x^{n+1}}{n+1}$	$\ln\lvert x\rvert$	$-\cos x$	$\sin x$

$f(x)$	e^x	a^x	$\frac{1}{\sqrt{x}}$	$\sqrt{x}$	$\frac{1}{\sin^2 x}$	$\frac{1}{\cos^2 x}$
$F(x)$	e^x	$\frac{a^x}{\ln a}$	$2\sqrt{x}$	$\frac{2}{3}\sqrt{x^3}$	$-ctgx$	tgx

Таблица первообразных

f(x)	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
F(x)	kx	$\frac{x^{n+1}}{n+1}$	$\ln\lvert x\rvert$	$-\cos x$	$\sin x$

f(x)	e^x	a^x	$\frac{1}{\sqrt{x}}$	$\sqrt{x}$	$\frac{1}{\sin^2 x}$	$\frac{1}{\cos^2 x}$
F(x)	e^x	$\frac{a^x}{\ln a}$	$2\sqrt{x}$	$\frac{2}{3}\sqrt{x^3}$	$-ctgx$	tgx

$$\int 9dx = 9x + C$$

$$\int dx = x + C$$

$$\int \sqrt{5}dx = x\sqrt{5} + C$$

$$\int 4^x dx = \frac{4^x}{\ln 4} + C$$

$$\int \frac{dx}{x} = \int \frac{1}{x}dx = \ln|x| + C$$

$$\int 7^x dx = \frac{7^x}{\ln 7} + C$$

$$\int \frac{dx}{\cos^2 x} = \int \frac{1}{\cos^2 x}dx = tgx + C$$

Таблица первообразных

$f(x)$	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
$F(x)$	kx	$\frac{x^{n+1}}{n+1}$	$\ln\lvert x\rvert$	$-\cos x$	$\sin x$

$$\int x^4 dx = \frac{x^5}{5} + C$$

$$\int x^{21} dx = \frac{x^{22}}{22} + C$$

$$\int x dx = \frac{x^2}{2} + C$$

$$\int \frac{dx}{x^4} = \int x^{-4} dx = \frac{x^{-3}}{-3} + C = -\frac{1}{3x^3} + C$$

$$\int \frac{dx}{x^6} = \int x^{-6} dx = \frac{x^{-5}}{-5} + C = -\frac{1}{5x^5} + C$$

Таблица первообразных

f(x)	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
F(x)	kx	$\frac{x^{n+1}}{n+1}$	$\ln\lvert x\rvert$	$-\cos x$	$\sin x$

$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}\sqrt{x^3} + C$$

$$\int \sqrt[5]{x^3}\,dx = \int x^{\frac{3}{5}}dx = \frac{x^{\frac{8}{5}}}{\frac{8}{5}} + C = \frac{5}{8}\sqrt[5]{x^8} + C$$

Таблица первообразных

$f(x)$	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
$F(x)$	kx	$\frac{x^{n+1}}{n+1}$	$\ln\lvert x\rvert$	$-\cos x$	$\sin x$

$$\int \frac{dx}{\sqrt{x}} = \int x^{-\frac{1}{2}} dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x} + C$$

$$\int \frac{dx}{\sqrt[5]{x^3}} = \int x^{-\frac{3}{5}} dx = \frac{x^{\frac{2}{5}}}{\frac{2}{5}} + C = \frac{5}{2}\sqrt[5]{x^2} + C$$

Правила интегрирования

$$\int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx$$

$$\int kf(x)dx = k\int f(x)dx$$

$$\int f(kx+b)dx = \frac{1}{k}F(kx+b)+C$$

Правила интегрирования

$$\int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx$$

$$\int kf(x)dx = k\int f(x)dx$$

$$\int (9x^2+2x-10)dx = \int 9x^2dx + \int 2xdx - \int 10dx =$$

$$= 9\int x^2dx + 2\int xdx - \int 10dx = 9*\frac{x^3}{3} + 2*\frac{x^2}{2} - 10x + C =$$

$$= 3x^3 + x^2 - 10x + C$$

$$\int (5e^x - 3\sin x + 1)dx = 5\int e^xdx - 3\int \sin xdx + \int dx =$$

$$= 5e^x - 3(-\cos x) + x + C = 5e^x + 3\cos x + x + C$$

Правила интегрирования

$$\int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx$$

$$\int kf(x)dx = k\int f(x)dx$$

$$\int\left(\frac{4}{x}+\frac{7}{\sqrt{x}}\right)dx = 4\int\frac{1}{x}dx + 7\int\frac{1}{\sqrt{x}}dx =$$

$$= 4\ln|x| + 7*2\sqrt{x} + C = 4\ln|x| + 14\sqrt{x} + C$$

$$\int\left(\frac{3}{x^4} - 10\sqrt[4]{x}\right)dx = 3\int x^{-4}dx - 10\int x^{\frac{1}{4}}dx = 3*\frac{x^{-3}}{-3} - 10*\frac{x^{\frac{5}{4}}}{\frac{5}{4}} + C =$$

$$= -x^{-3} - 10*\frac{4}{5}x^{\frac{5}{4}} + C = -\frac{1}{x^3} - 8\sqrt[4]{x^5} + C$$

Правила интегрирования

$$\int f(kx+b)dx = \frac{1}{k}F(kx+b)+C$$

$$\int (3x-1)^4 dx = \left| \begin{array}{l} kx+b=3x-1,\ k=3 \\ f(t)=t^4 \Rightarrow F(t)=\dfrac{t^5}{5} \end{array} \right| = \frac{1}{3} * \frac{(3x-1)^5}{5} + C = \frac{(3x-1)^5}{15} + C$$

$$\int \cos 4x dx = \left| \begin{array}{l} kx+b=4x,\ k=4 \\ f(t)=\cos t \Rightarrow F(t)=\sin t \end{array} \right| = \frac{1}{4} * \sin 4x + C$$

$$\int \sqrt{6x+5} dx = \left| \begin{array}{l} kx+b=6x+5,\ k=6 \\ f(t)=\sqrt{t} \Rightarrow F(t)=\dfrac{2}{3}\sqrt{t^3} \end{array} \right| = \frac{1}{6} * \frac{2}{3}\sqrt{(6x+5)^3} + C =$$

$$= \frac{1}{9}\sqrt{(6x+5)^3} + C$$

Правила интегрирования

$$\int f(kx+b)dx=\frac{1}{k}F(kx+b)+C$$

$$\int \sin\left(\frac{x}{2}-\frac{\pi}{5}\right)dx=\left|\begin{array}{l} kx+b=\frac{x}{2}-\frac{\pi}{5},\ k=\frac{1}{2} \\ f(t)=\sin t\Rightarrow F(t)=-\cos t \end{array}\right|=-2*\cos\left(\frac{x}{2}-\frac{\pi}{5}\right)+C$$

$$\int\frac{dx}{3-2x}=\left|\begin{array}{l} kx+b=-2x+3,\ k=-2 \\ f(t)=\frac{1}{t}\Rightarrow F(t)=\ln|t| \end{array}\right|=-\frac{1}{2}*\ln|3-2x|+C$$

$$\int\frac{dx}{(3-2x)^5}=\left|\begin{array}{l} kx+b=-2x+3,\ k=-2 \\ f(t)=\frac{1}{t^5}=t^{-5}\Rightarrow F(t)=\frac{t^{-4}}{-4}=-\frac{1}{4t^4} \end{array}\right|=$$

$$=-\frac{1}{2}*\left(-\frac{1}{4(3-2x)^4}\right)+C=\frac{1}{8(3-2x)^4}+C$$

Правила интегрирования

$$\int f(kx+b)dx = \frac{1}{k}F(kx+b)+C$$

$$\int e^{5x+3}dx = \left| \begin{array}{l} kx+b=5x+3,\ k=5 \\ f(t)=e^{t} \Rightarrow F(t)=e^{t} \end{array} \right| = \frac{1}{5}*e^{5x+3}+C$$

$$\int 8^{5x+3}dx = \left| \begin{array}{l} kx+b=5x+3,\ k=5 \\ f(t)=8^{t} \Rightarrow F(t)=\dfrac{8^{t}}{\ln 8} \end{array} \right| = \frac{1}{5}*\frac{8^{5x+3}}{\ln 8}+C = \frac{8^{5x+3}}{5\ln 8}+C$$

$$\int \frac{dx}{\cos^{2}\frac{x}{4}} = \left| \begin{array}{l} kx+b=\dfrac{x}{4},\ k=\dfrac{1}{4} \\ f(t)=\dfrac{1}{\cos^{2}t} \Rightarrow F(t)=tgt \end{array} \right| = 4*tg\frac{x}{4}+C$$