

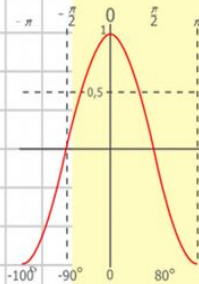
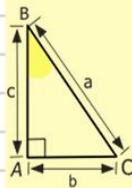
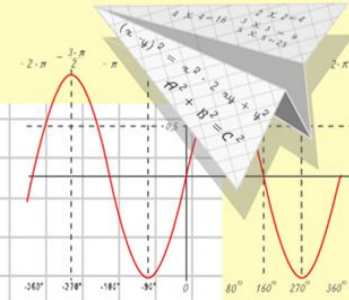
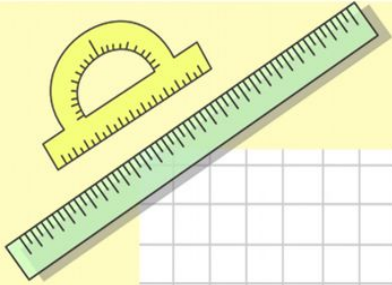
Математик

а

Занятие 106.

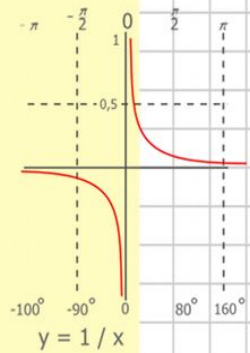
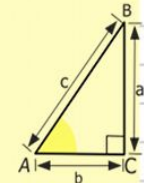
Таблица первообразных. Правила интегрирования.

1. Таблица первообразных.
2. Правила интегрирования.



$$y = \cos x$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

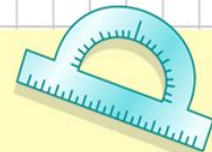


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

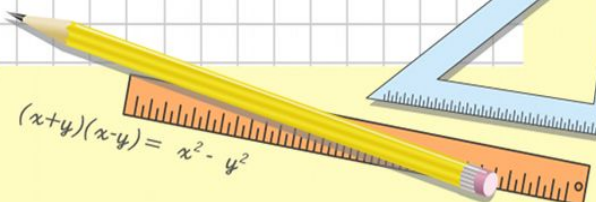
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$



$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

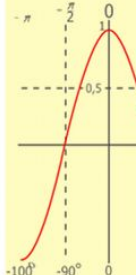
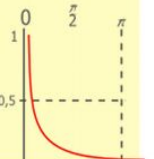
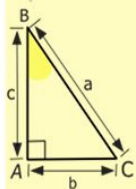
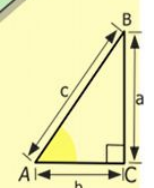
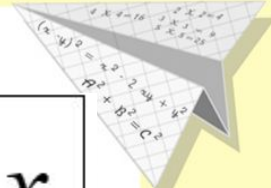
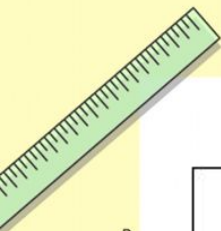


$$(x+y)(x-y) = x^2 - y^2$$

Таблица первообразных

$f(x)$	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
$F(x)$	kx	$\frac{x^{n+1}}{n+1}$	$\ln x $	$-\cos x$	$\sin x$

$f(x)$	e^x	a^x	$\frac{1}{\sqrt{x}}$	\sqrt{x}	$\frac{1}{\sin^2 x}$	$\frac{1}{\cos^2 x}$
$F(x)$	e^x	$\frac{a^x}{\ln a}$	$2\sqrt{x}$	$\frac{2}{3}\sqrt{x^3}$	$-ctgx$	tgx



$y = 1/x$

$y = \cos$

$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$

- 2 x 2 = 4
- 3 x 3 = 9
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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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$$(x+y)(x-y) = x^2 - y^2$$

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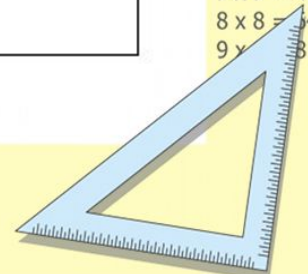


Таблица первообразных

$f(x)$	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
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$F(x)$	e^x	$\frac{a^x}{\ln a}$	$2\sqrt{x}$	$\frac{2}{3}\sqrt{x^3}$	$-\operatorname{ctgx}$	tgx

$$\int 9 dx = 9x + C$$

$$\int dx = x + C$$

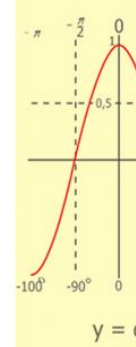
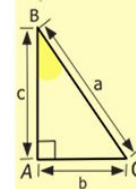
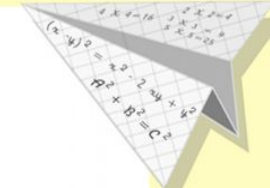
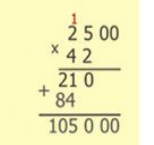
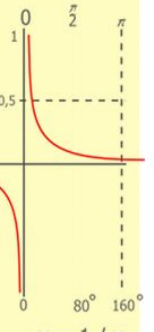
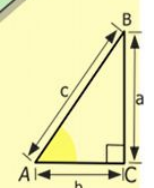
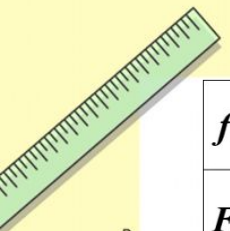
$$\int \sqrt{5} dx = x\sqrt{5} + C$$

$$\int 4^x dx = \frac{4^x}{\ln 4} + C$$

$$\int \frac{dx}{x} = \int \frac{1}{x} dx = \ln|x| + C$$

$$\int 7^x dx = \frac{7^x}{\ln 7} + C$$

$$\int \frac{dx}{\cos^2 x} = \int \frac{1}{\cos^2 x} dx = \operatorname{tgx} + C$$



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$$\frac{x}{70}$$

$$(x+y)(x-y) = x^2 - y^2$$

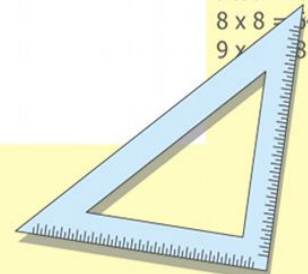


Таблица первообразных

$f(x)$	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
$F(x)$	kx	$\frac{x^{n+1}}{n+1}$	$\ln x $	$-\cos x$	$\sin x$

$$\int x^4 dx = \frac{x^5}{5} + C$$

$$\int x^{21} dx = \frac{x^{22}}{22} + C$$

$$\int x dx = \frac{x^2}{2} + C$$

$$\int \frac{dx}{x^4} = \int x^{-4} dx = \frac{x^{-3}}{-3} + C = -\frac{1}{3x^3} + C$$

$$\int \frac{dx}{x^6} = \int x^{-6} dx = \frac{x^{-5}}{-5} + C = -\frac{1}{5x^5} + C$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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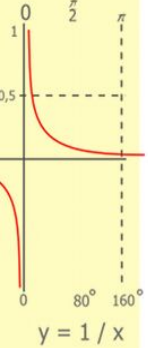
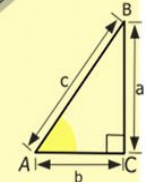
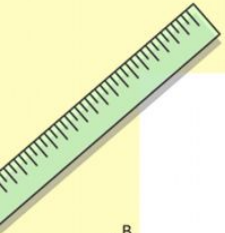
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

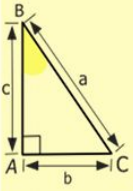
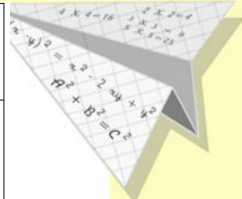
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



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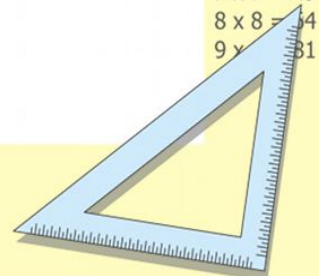
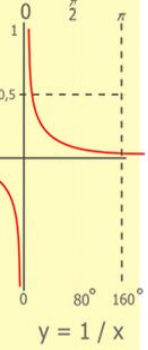
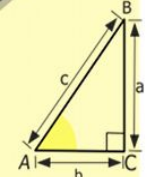
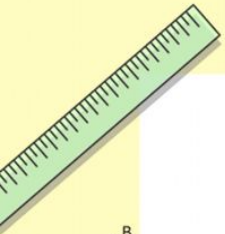


Таблица первообразных

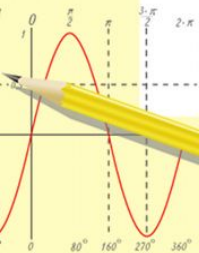
$f(x)$	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
$F(x)$	kx	$\frac{x^{n+1}}{n+1}$	$\ln x $	$-\cos x$	$\sin x$

$$\int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3} \sqrt{x^3} + C$$

$$\int \sqrt[5]{x^3} dx = \int x^{\frac{3}{5}} dx = \frac{x^{\frac{8}{5}}}{\frac{8}{5}} + C = \frac{5}{8} \sqrt[5]{x^8} + C$$



$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 42 \\ \hline 210 \\ + 840 \\ \hline 105\ 000 \end{array}$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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$$\sin 90^\circ = 1$$

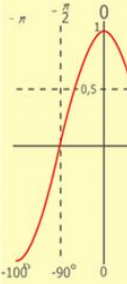
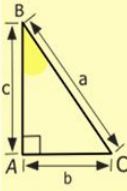
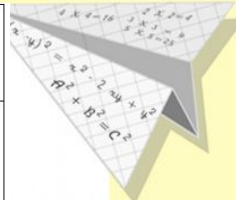


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

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$$(x+y)(x-y) = x^2 - y^2$$



$y = \cos$

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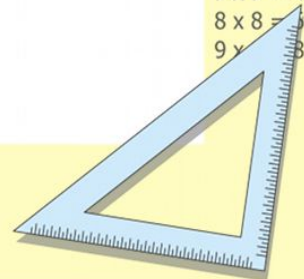
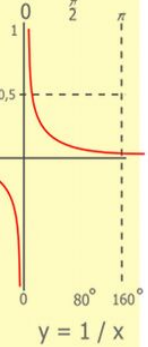
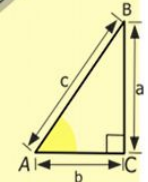
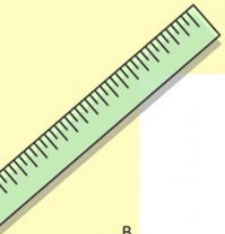


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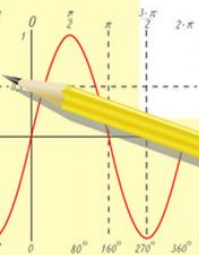
$f(x)$	k	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$
$F(x)$	kx	$\frac{x^{n+1}}{n+1}$	$\ln x $	$-\cos x$	$\sin x$

$$\int \frac{dx}{\sqrt{x}} = \int x^{-\frac{1}{2}} dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x} + C$$

$$\int \frac{dx}{\sqrt[5]{x^3}} = \int x^{-\frac{3}{5}} dx = \frac{x^{\frac{2}{5}}}{\frac{2}{5}} + C = \frac{5}{2} \sqrt[5]{x^2} + C$$



$$\begin{array}{r} 1\ 5\ 00 \\ \times 42 \\ \hline 21\ 0 \\ + 84\ 0 \\ \hline 105\ 00 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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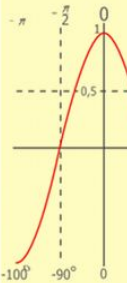
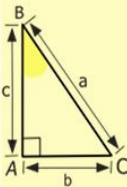
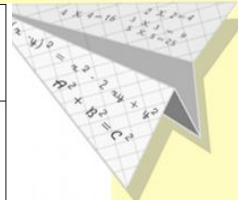


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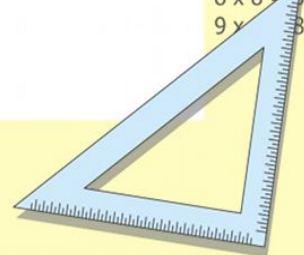
$$\frac{x}{70}$$

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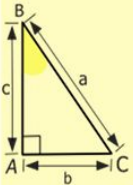
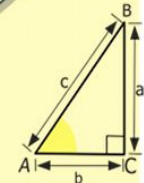
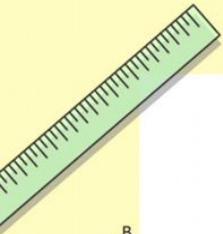


Правила интегрирования

$$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$$

$$\int kf(x)dx = k \int f(x)dx$$

$$\int f(kx + b)dx = \frac{1}{k} F(kx + b) + C$$



$$\begin{array}{r} \frac{1}{2} 500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

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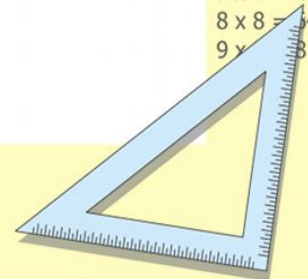


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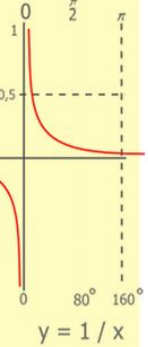
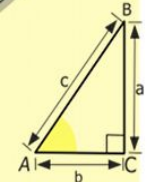
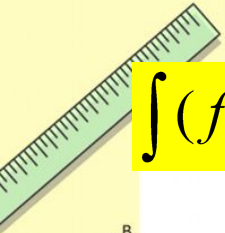
Правила интегрирования

$$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

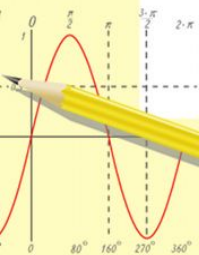
$$\int kf(x) dx = k \int f(x) dx$$

$$\begin{aligned} \int (9x^2 + 2x - 10) dx &= \int 9x^2 dx + \int 2x dx - \int 10 dx = \\ &= 9 \int x^2 dx + 2 \int x dx - \int 10 dx = 9 * \frac{x^3}{3} + 2 * \frac{x^2}{2} - 10x + C = \\ &= 3x^3 + x^2 - 10x + C \end{aligned}$$

$$\begin{aligned} \int (5e^x - 3 \sin x + 1) dx &= 5 \int e^x dx - 3 \int \sin x dx + \int dx = \\ &= 5e^x - 3(-\cos x) + x + C = 5e^x + 3 \cos x + x + C \end{aligned}$$



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 210 \\ \hline 10500 \end{array}$$



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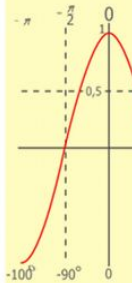
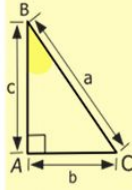
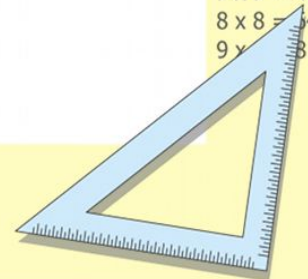


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$

Правила интегрирования

$$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

$$\int kf(x) dx = k \int f(x) dx$$

$$\int \left(\frac{4}{x} + \frac{7}{\sqrt{x}} \right) dx = 4 \int \frac{1}{x} dx + 7 \int \frac{1}{\sqrt{x}} dx =$$

$$= 4 \ln|x| + 7 * 2\sqrt{x} + C = 4 \ln|x| + 14\sqrt{x} + C$$

$$\int \left(\frac{3}{x^4} - 10\sqrt{x} \right) dx = 3 \int x^{-4} dx - 10 \int x^{\frac{1}{2}} dx = 3 * \frac{x^{-3}}{-3} - 10 * \frac{x^{\frac{5}{4}}}{\frac{5}{4}} + C =$$

$$= -x^{-3} - 10 * \frac{4}{5} x^{\frac{5}{4}} + C = -\frac{1}{x^3} - 8\sqrt{x^5} + C$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

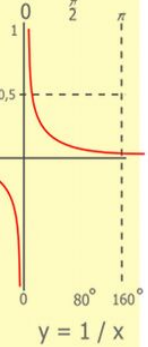
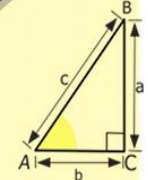
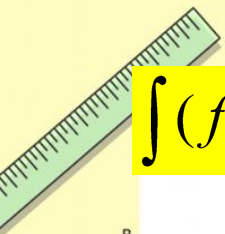
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

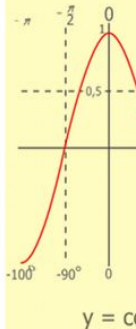
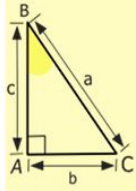
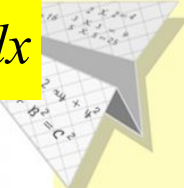
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

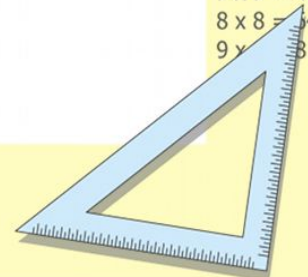
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
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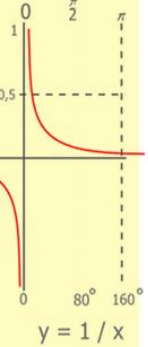
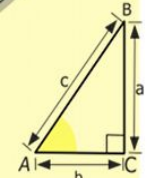
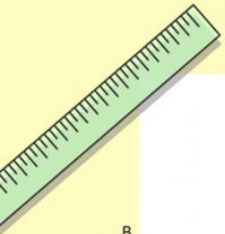
Правила интегрирования

$$\int f(kx + b) dx = \frac{1}{k} F(kx + b) + C$$

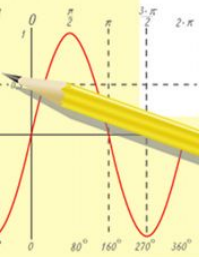
$$\int (3x - 1)^4 dx = \left| \begin{array}{l} kx + b = 3x - 1, k = 3 \\ f(t) = t^4 \Rightarrow F(t) = \frac{t^5}{5} \end{array} \right| = \frac{1}{3} * \frac{(3x - 1)^5}{5} + C = \frac{(3x - 1)^5}{15} + C$$

$$\int \cos 4x dx = \left| \begin{array}{l} kx + b = 4x, k = 4 \\ f(t) = \cos t \Rightarrow F(t) = \sin t \end{array} \right| = \frac{1}{4} * \sin 4x + C$$

$$\int \sqrt{6x + 5} dx = \left| \begin{array}{l} kx + b = 6x + 5, k = 6 \\ f(t) = \sqrt{t} \Rightarrow F(t) = \frac{2}{3} \sqrt{t^3} \end{array} \right| = \frac{1}{6} * \frac{2}{3} \sqrt{(6x + 5)^3} + C = \frac{1}{9} \sqrt{(6x + 5)^3} + C$$



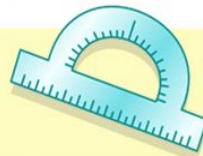
$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 2100 \\ + 8400 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

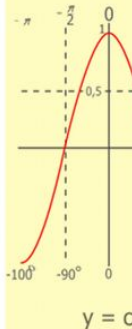
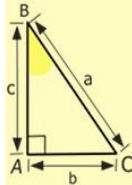


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

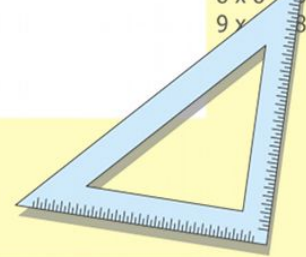
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

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Правила интегрирования

$$\int f(kx + b)dx = \frac{1}{k} F(kx + b) + C$$

$$\int \sin\left(\frac{x}{2} - \frac{\pi}{5}\right) dx = \left| \begin{array}{l} kx + b = \frac{x}{2} - \frac{\pi}{5}, k = \frac{1}{2} \\ f(t) = \sin t \Rightarrow F(t) = -\cos t \end{array} \right| = -2 * \cos\left(\frac{x}{2} - \frac{\pi}{5}\right) + C$$

$$\int \frac{dx}{3-2x} = \left| \begin{array}{l} kx + b = -2x + 3, k = -2 \\ f(t) = \frac{1}{t} \Rightarrow F(t) = \ln|t| \end{array} \right| = -\frac{1}{2} * \ln|3-2x| + C$$

$$\int \frac{dx}{(3-2x)^5} = \left| \begin{array}{l} kx + b = -2x + 3, k = -2 \\ f(t) = \frac{1}{t^5} = t^{-5} \Rightarrow F(t) = \frac{t^{-4}}{-4} = -\frac{1}{4t^4} \end{array} \right| =$$

$$= -\frac{1}{2} * \left(-\frac{1}{4(3-2x)^4} \right) + C = \frac{1}{8(3-2x)^4} + C$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

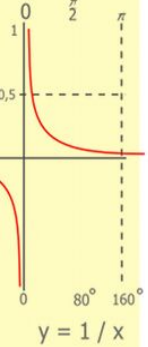
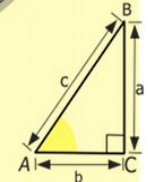
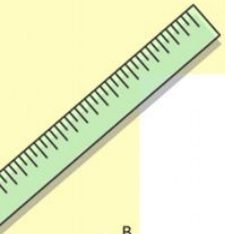
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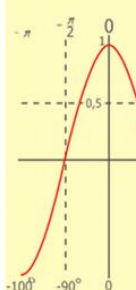
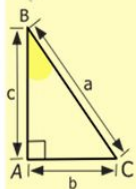
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

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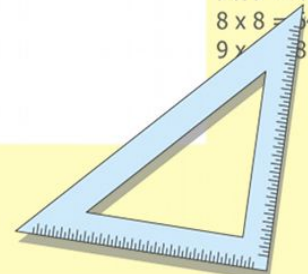
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



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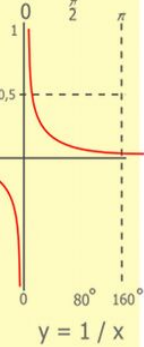
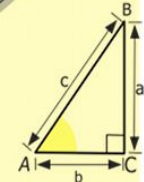
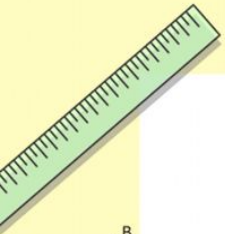
Правила интегрирования

$$\int f(kx + b) dx = \frac{1}{k} F(kx + b) + C$$

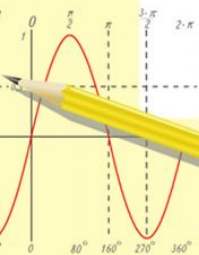
$$\int e^{5x+3} dx = \left| \begin{array}{l} kx + b = 5x + 3, k = 5 \\ f(t) = e^t \Rightarrow F(t) = e^t \end{array} \right| = \frac{1}{5} * e^{5x+3} + C$$

$$\int 8^{5x+3} dx = \left| \begin{array}{l} kx + b = 5x + 3, k = 5 \\ f(t) = 8^t \Rightarrow F(t) = \frac{8^t}{\ln 8} \end{array} \right| = \frac{1}{5} * \frac{8^{5x+3}}{\ln 8} + C = \frac{8^{5x+3}}{5 \ln 8} + C$$

$$\int \frac{dx}{\cos^2 \frac{x}{4}} = \left| \begin{array}{l} kx + b = \frac{x}{4}, k = \frac{1}{4} \\ f(t) = \frac{1}{\cos^2 t} \Rightarrow F(t) = \operatorname{tg} t \end{array} \right| = 4 * \operatorname{tg} \frac{x}{4} + C$$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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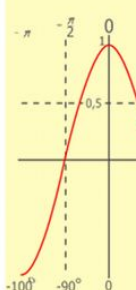
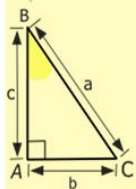


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