Уравнения умнее тех, кто их вывел.

Генрих Герц (1857-1894)

НЕГИПЕРБОЛИЧНОСТЬ СИСТЕМЫ ГИДРОДИНАМИЧЕСКИХ УРАВНЕНИЙ КЛИМАТИЧЕСКОЙ МОДЕЛИ АТМОСФЕРЫ

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Гидротермодинамические уравнения для атмосферы

$$\frac{d\rho}{dt} = -\frac{1}{\rho}\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$ – уравнение сохранения массы

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$$

$$\rho \frac{dv_x}{dt} = -\frac{\partial p}{\partial x} + \rho f_x^{(\mathrm{cor})} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}$$

$$\rho \frac{dv_y}{dt} = -\frac{\partial p}{\partial y} + \rho f_y^{(\mathrm{cor})} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}$$

$$\rho \frac{dv_z}{dt} = -\frac{\partial p}{\partial z} + \rho f_z^{(\mathrm{cor})} - \rho g + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

– уравнения сохранения импульса

– уравнения сохранения импульса

$$\rho \frac{dT}{dt} = \frac{Q^*}{c_v} + \frac{p}{\rho c_v}\frac{d\rho}{dt}$$ – уравнение притока тепла – $$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{\gamma p}\frac{dp}{dt} - \frac{\gamma - 1}{\gamma}\frac{Q^*}{p}$$

$$Q^* \equiv -\frac{\partial q}{\partial z} + Q - J^{(e)} l^{(e)} - J^{(m)} l^{(m)}$$

Уравнения состояния совершенного газа

$$p = R\rho T \qquad u = c_v T + \mathrm{const} \qquad c_v = R/(\gamma - 1)$$

ДЛЯ ЗАМЫКАНИЯ СИСТЕМЫ УРАВНЕНИЯ НЕОБХОДИМЫ УРАВНЕНИЯ:

- для вертикального теплового потока q (турбулентная теплопроводность).

- для выделения тепла из-за поглощения радиации Q, определяемого уравнениями для потоков коротковолновой радиации G, длинноволновой радиации U.

- для интенсивности испарения (конденсации) $J^{(e)}$, плавления льда или снега) $J^{(m)}$, и тепла $J^{(e)}l^{(e)} + J^{(m)}l^{(m)}$ фазовых переходов.

КЛИМАТИЧЕСКИЕ и МЕТЕОРОЛОГИЧЕСКИЕ МАСШТАБЫ

$\tau > 10^2$ s

$V_{hor} < 30$ m/s, $L_{hor} \sim V_{hor}\tau > 10^3$ m

$V_{ver} < 3$ m/s, $L_{ver} \sim V_{ver}\tau > 10^2$ m

КЛИМАТ, ПОГОДА

$\tau \sim 10^0$ s

$V_{hor} \sim 10^2$ m/s, $L_{hor} \sim 10^2$ m,

$V_{ver} \sim 30$ m/s, $L_{ver} \sim 10^2$ m,

ТАЙФУН, ШТОРМ, БОРА

$$\frac{\partial v_x}{\partial t} \sim \frac{\partial v_y}{\partial t} \sim \frac{V_{hor}}{\tau} \qquad \frac{\partial v_z}{\partial t} \sim \frac{V_{ver}}{\tau}$$

$$\frac{\partial v_x}{\partial x} \sim \frac{\partial v_x}{\partial y} \sim \frac{\partial v_x}{\partial y} \sim \frac{\partial v_y}{\partial x} \sim \frac{V_{hor}}{L_{hor}}$$

$$\frac{\partial v_z}{\partial x} \sim \frac{\partial v_z}{\partial y} \sim \frac{V_{ver}}{L_{hor}} \qquad \frac{\partial v_z}{\partial z} \sim \frac{V_{ver}}{L_{ver}}$$

$$\bar{t} \equiv \frac{t}{\tau}, \quad \bar{x} \equiv \frac{x}{L_{\text{hor}}}, \quad \bar{y} \equiv \frac{y}{L_{\text{hor}}}, \quad \bar{z} \equiv \frac{z}{L_{\text{ver}}},$$

$$\bar{\upsilon}_x \equiv \frac{\upsilon_x}{V_{\text{hor}}}, \quad \bar{\upsilon}_y \equiv \frac{\upsilon_x}{V_{\text{hor}}}, \quad \bar{\upsilon}_z \equiv \frac{\upsilon_z}{V_{\text{ver}}} \quad = \mathrm{O}(1)$$

$$\frac{\mathrm{d}\upsilon_x}{\mathrm{d}t} = \frac{V_{\text{hor}}}{\tau}\left(\frac{\partial\bar{\upsilon}_x}{\partial\bar{t}} + \bar{\upsilon}_x\frac{\partial\bar{\upsilon}_x}{\partial\bar{x}} + \bar{\upsilon}_y\frac{\partial\bar{\upsilon}_x}{\partial\bar{y}} + \bar{\upsilon}_z\frac{\partial\bar{\upsilon}_x}{\partial\bar{z}}\right) = A_{\text{hor}}\frac{\mathrm{d}\bar{\upsilon}_x}{\mathrm{d}\bar{t}} \quad = A_{\text{hor}}\mathrm{O}(1)$$

$$\frac{\mathrm{d}\upsilon_y}{\mathrm{d}t} = \frac{V_{\text{hor}}}{\tau}\left(\frac{\partial\bar{\upsilon}_y}{\partial\bar{t}} + \bar{\upsilon}_x\frac{\partial\bar{\upsilon}_y}{\partial\bar{x}} + \bar{\upsilon}_y\frac{\partial\bar{\upsilon}_y}{\partial\bar{y}} + \bar{\upsilon}_z\frac{\partial\bar{\upsilon}_y}{\partial\bar{z}}\right) = A_{\text{hor}}\frac{\mathrm{d}\bar{\upsilon}_y}{\mathrm{d}\bar{t}} \quad = A_{\text{hor}}\mathrm{O}(1)$$

$$\frac{\mathrm{d}\upsilon_z}{\mathrm{d}t} = \frac{V_{\text{ver}}}{\tau}\left(\frac{\partial\bar{\upsilon}_z}{\partial\bar{t}} + \bar{\upsilon}_x\frac{\partial\bar{\upsilon}_z}{\partial\bar{x}} + \bar{\upsilon}_y\frac{\partial\bar{\upsilon}_z}{\partial\bar{y}} + \bar{\upsilon}_z\frac{\partial\bar{\upsilon}_z}{\partial\bar{z}}\right) = A_{\text{ver}}\frac{\mathrm{d}\bar{\upsilon}_z}{\mathrm{d}\bar{t}} \quad = A_{\text{ver}}\mathrm{O}(1)$$

$$\left|-\mathbf{a}^{(\text{cor})}\right| = f^{(\text{cor})} = \left|\mathbf{f}^{(\text{cor})}\right| = \left|2\left[\vec{\mathbf{\Omega}} \times \vec{\mathbf{v}}\right]\right| = 2V_{\text{cor}}\Omega\left|\left[\vec{\mathbf{e}}_{\text{o}} \times \vec{\bar{\mathbf{v}}}\right]\right|$$

$$\Omega = \frac{2\pi}{24\ \text{час}} \approx 0{,}727\times10^{-4}\ \text{с}^{-1}, \qquad A_{\text{cor}} = V_{\text{cor}}\Omega \sim \frac{V_{\text{hor}}}{\tau_{\text{cor}}}$$

Безразмерные функции

$$\left(\bar{v}_i,\ \frac{d\bar{v}_i}{d\bar{t}}\right) = O(1)$$

Масштабы ускорений

$$A_{\text{hor}} = \frac{V_{\text{hor}}}{\tau} = \frac{V_{\text{hor}}^2}{L_{\text{hor}}} < 10^{-1}\,\text{м} / \text{с}^2,$$

$$A_{\text{ver}} = \frac{V_{\text{ver}}}{\tau} = \frac{V_{\text{ver}}^2}{L_{\text{verr}}} < 10^{-2}\,\text{м} / \text{с}^2,$$

$$A_{\text{cor}} = 2\Omega V_{\text{hor}} < 1{,}4\times 10^{-3}\,\text{м} / \text{с}^2$$

$$\varepsilon = \left(\frac{A_{\text{hor}}}{g},\ \frac{A_{\text{ver}}}{g},\ \frac{A_{\text{cor}}}{g}\right) < \left(10^{-2},\ 10^{-3},\ 10^{-4}\right)$$

$$\varepsilon = \left(\frac{A_{\text{hor}}}{g},\ \frac{A_{\text{ver}}}{g},\ \frac{A_{\text{cor}}}{g}\right) \rightarrow 0$$

$\tau > 10^2$ s

$V_{\text{hor}} < 30$ m/s, $L_{\text{hor}} \sim V_{\text{hor}}\tau > 10^3$ m

$V_{\text{ver}} < 3$ m/s, $L_{\text{ver}} \sim V_{\text{ver}}\,\tau > 10^2$ m

ТАЙФУН, ШТОРМ, БОРА

$\tau \sim 10^0$ s

$V_{hor} \sim 10^2$ m/s, $L_{hor} \sim 10^2$ m,

$V_{ver} \sim 10^1$ m/s, $L_{ver} \sim 10^2$ m,

$A_{ver} \sim \frac{V_{ver}}{\tau} \sim 10^1 \text{ m}/\text{s}^2 \sim g,$ $\frac{A_{ver}}{g} \sim 10^0$

$\tau > 10^2$ s

$V_{hor} < 30$ m/s, $L_{hor} \sim V_{hor}\tau > 10^3$ m

$V_{ver} < 3$ m/s, $L_{ver} \sim V_{ver}\tau > 10^2$ m

Акустика

$\tau \sim 10^{-4}$ s, $\delta \sim 10^{-5}$ m

$V \sim 10^{-1}$ m/s,

$L \sim C\tau \sim 3\times10^{-2}$ m,

$A \sim V/\tau \sim 10^3$ m/s^2 $>>$ g

Аэродинамика

$V_{hor} \sim 10^2$ m/s, $L_{hor} \sim 10^0$ m, $V_{ver} \sim 10^1$ m/s

$\tau \sim L_{hor}/V \sim 10^{-2}$ s,

$A \sim V_{ver}/\tau \sim 10^3$ m/s^2 $>>$ g

УРАВНЕНИЯ МПУЛЬСА

$$\rho\left(\frac{\mathrm{d}v_x}{\mathrm{d}t} + a_x^{(\mathrm{cor})}\right) = -\frac{\partial p}{\partial x}$$

$$\rho\left(\frac{\mathrm{d}v_y}{\mathrm{d}t} + a_y^{(\mathrm{cor})}\right) = -\frac{\partial p}{\partial y}$$

$$\rho\left(\frac{\mathrm{d}v_z}{\mathrm{d}t} - a_z^{(\mathrm{cor})}\right) = -\frac{\partial p}{\partial z} - \rho g$$

$$\frac{\partial p}{\partial x} = \rho O\left(A_{\mathrm{hor}} + A_{\mathrm{cor}}\right)$$

$$\frac{\partial p}{\partial y} = \rho O\left(A_{\mathrm{hor}} + A_{\mathrm{cor}}\right)$$

$$\frac{\partial p}{\partial z} = -\rho g\left(1 + O(\varepsilon)\right)$$

$$p(t,x,y,z) \underset{\varepsilon\to 0}{\longrightarrow} \int_z^{\infty} \rho g\, \mathrm{d}z' = g\int_z^{H} \rho\, \mathrm{d}z' = gM$$

$$\frac{\partial p}{\partial x} \underset{\varepsilon\to 0}{\longrightarrow} g\frac{\partial M}{\partial x} = g\int^{H} \frac{\partial \rho}{\partial x} dz'$$

$$\frac{\partial p}{\partial y} \underset{\varepsilon\to 0}{\longrightarrow} g\frac{\partial M}{\partial y} = g\int_z^{H} \frac{\partial \rho}{\partial y} dz'$$

$$M(t,x,y,z) = \int_z^{H} \rho(t,x,y,z')\, \mathrm{d}z'$$

Как рассчитать вертикальную

ВЕРТИКАЛЬНЫЕ ПОТОКИ. Дождь (фото с борта самолета)

ВЕРТИКАЛЬНЫЕ ПОТОКИ. Образование грозы (фото с борта самолета на высоте 11 000 м).

УРАВНЕНИЕ СОХРАНЕНИЯ МАССЫ

$$\frac{1}{\rho}\frac{d\rho}{dt} = \operatorname{div}\mathbf{v} \equiv -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) + \frac{1}{\rho}\frac{d\rho}{dt}$$

Уравнение для распределения вертикальной скорости по вертикали

ОКЕАН: $\Delta\rho/\rho \sim 10^{-4}$ - 10^{-3}

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)_{S,T}\frac{dp}{dt} + \frac{1}{\rho}\left(\frac{\partial\rho}{\partial S}\right)_{p,T}\frac{dS}{dt} + \frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_{S,T}\frac{dT}{dt} \ll \left|\frac{\partial v_x}{\partial x}, \frac{\partial v_y}{\partial y}, \frac{\partial v_z}{\partial z}\right|$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) + \frac{1}{\rho}\cdot\frac{d\rho}{dt}$$

$$\frac{\partial v_z}{\partial z} = -\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}$$ - Квазинесжимаемость

Обыкновенное дифференциальное уравнение для распределения вертикальной скорости по вертикали в океане

Хотя океанские течения происходят именно из-за переменности плотности: $\rho(p, T, S)$

Атмосфера: Δρ/ ρ ~ 10⁰

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{1}{\gamma p}\frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\gamma-1}{\gamma p}Q^*$$

- уравнение притока тепла для совершенного газа

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}$$

$$-\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\gamma p}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\gamma-1}{\gamma p}Q^*$$

$$p(t,x,y,z) = g\int_z^H \rho\,\mathrm{d}z' = Mg$$

$$\frac{dp}{dt}=\frac{\partial p}{\partial t}+v_z\frac{\partial p}{\partial z}+v_x\frac{\partial p}{\partial x}+v_y\frac{\partial p}{\partial y}=-g\int_z^H\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}\right)dz'+\left(v_x\frac{\partial p}{\partial x}+v_y\frac{\partial p}{\partial y}\right)$$

$g\dot{M}$

$O(\varepsilon)$

$$\frac{\partial v_z}{\partial z}=-\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}\right)-\frac{1}{\gamma p}\frac{dp}{dt}+\frac{\gamma-1}{\gamma p}Q^*$$

$$p=gM\equiv g\int_z^H\rho(t,x,y,z')\,dz'$$

$$\boldsymbol{\frac{\partial v_z}{\partial z}=-\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}\right)-\frac{1}{\gamma}\frac{\dot{M}}{M}+\frac{\gamma-1}{\gamma}\frac{Q^*}{gM}}$$

$$\left(M=\int_z^H\rho(t,x,y,z')\,dz',\quad \dot{M}\equiv-\int_z^H\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}\right)dz'=\frac{\partial M}{\partial t}-\rho v_z\right)$$

$$\frac{\partial M}{\partial z}=-\rho(z),\qquad \frac{\partial\dot{M}}{\partial z}=\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}\right)$$

Уравнения сохранения для атмосферы с вертикальной квазистатикой (относительно ρ, v_x, v_y)

$\frac{\partial}{\partial t}$:

$$\frac{\partial \rho}{\partial t} = -v_x \frac{\partial \rho}{\partial x} - v_y \frac{\partial \rho}{\partial y} - v_z \frac{\partial \rho}{\partial z} + \underbrace{\frac{1}{\gamma}\frac{\rho \dot{M}}{M} - \frac{\gamma - 1}{\gamma}\frac{\rho Q^*}{g}}_{-\rho\, \mathrm{div}\, \mathbf{v}}$$

$$Q^* \equiv -\frac{\partial q}{\partial z} + Q - J\, l$$

$$\frac{\partial v_x}{\partial t} = -v_x \frac{\partial v_x}{\partial x} - v_y \frac{\partial v_x}{\partial y} - v_z \frac{\partial v_x}{\partial z} - \frac{g}{\rho}\frac{\partial M}{\partial x} + f_x^{(\mathrm{cor})}$$

~~$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial \rho}\frac{\partial \rho}{\partial x} = C^2 \frac{\partial \rho}{\partial x}$~~

$$\frac{\partial v_y}{\partial t} = -v_x \frac{\partial v_y}{\partial x} - v_y \frac{\partial v_y}{\partial y} - v_z \frac{\partial v_y}{\partial z} - \frac{g}{\rho}\frac{\partial M}{\partial y} + f_y^{(\mathrm{cor})}$$

~~$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial \rho}\frac{\partial \rho}{\partial y} = C^2 \frac{\partial \rho}{\partial y}$~~

$\frac{\partial}{\partial z}$:

$$\frac{\partial M}{\partial z} = -\rho$$

$$\frac{\partial \dot{M}}{\partial z} = -\frac{\partial (\rho v_x)}{\partial x} - \frac{\partial (\rho v_y)}{\partial y}$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\gamma}\frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma}\frac{Q^*}{g\,M}$$

$$p = gM, \qquad T = \frac{g}{R}\frac{M}{\rho} \qquad \left(M = \int_z^H \rho(t, x, y, z')\, \mathrm{d}z', \qquad \dot{M} \equiv -\int_z^H \left(\frac{\partial (\rho v_x)}{\partial x} + \frac{\partial (\rho v_y)}{\partial y}\right) \mathrm{d}z' \right)$$

$(\otimes)$

$$\boldsymbol{\varepsilon} = \left(\varepsilon_{\text{ver}} = \frac{A_{\text{ver}}}{g}, \quad \varepsilon_{\text{hor}} = \frac{A_{\text{hor}}}{g}, \quad \varepsilon_{\text{cor}} = \frac{A_{\text{cor}}}{g}, \quad \varepsilon_{\text{C}} = \mathbf{M}^2 \right)$$

$$\left(A_{\text{ver}} = \frac{V_{\text{ver}}}{\tau} + \frac{V_{\text{ver}}^2}{L_{\text{ver}}}, \quad A_{\text{hor}} = \frac{V_{\text{hor}}}{\tau} + \frac{V_{\text{hor}}^2}{L_{\text{hor}}}, \quad A_{\text{cor}} = \frac{V_{\text{hor}}}{\tau_{\text{cor}}}, \quad \mathbf{M}^2 = \frac{V_{\text{ver}}^2}{C^2} \right)$$

Теорема. Уравнения (⊗) асимптотически точные уравнения для $\varepsilon_{\text{ver}} \to 0, \quad \varepsilon_{\text{hor}} \to 0, \quad \varepsilon_{\text{cor}} \to 0, \quad \varepsilon_C \to 0$

$$\frac{\partial \boldsymbol{v}_z}{\partial z} = -\left(\frac{\partial \boldsymbol{v}_x}{\partial x} + \frac{\partial \boldsymbol{v}_y}{\partial y} \right) - \frac{1}{\gamma} \frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma} \frac{Q^*}{g M}$$

Edward Lorenz (1967)

$$+ \left\{ \frac{1}{\gamma p} \left(\upsilon_x \frac{\partial p}{\partial x} + \upsilon_y \frac{\partial p}{\partial y} \right) \right\}$$

$$\mathrm{O}\left(\mathbf{M}^2\right) = \mathrm{O}(\varepsilon)$$

Edward Lorenz (1967)

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\gamma}\frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma}\frac{Q^*}{gM} + \left\{\frac{1}{\gamma p}\left(v_x\frac{\partial p}{\partial x} + v_y\frac{\partial p}{\partial y}\right)\right\}$$

$$\frac{\left\{v_x\left(\frac{\partial p}{\partial x}\right) + v_y\left(\frac{\partial p}{\partial y}\right)\right\}\frac{1}{\gamma p}}{\dot{M}/\gamma} =$$

$$\frac{1}{\gamma p}\left(v_x\frac{\partial p}{\partial x},\ v_y\frac{\partial p}{\partial y}\right) = \frac{V_{\text{hor}}}{\gamma p}\times \rho O\left(\frac{V_{\text{hor}}}{\tau} + \frac{V_{\text{hor}}^2}{L_{\text{hor}}} + \frac{V_{\text{cor}}^2}{L_{\text{cor}}}\right) = \frac{V_{\text{hor}}\,\rho}{\gamma p} O\left(A_{\text{hor}} + A_{\text{cor}}\right)$$

$$\frac{\dot{M}}{\gamma M} = \frac{\int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y}\right) dz'}{\gamma \int_z^H \rho(t,x,y,z')\, dz'} \sim \frac{\frac{\hat{\rho} V_{\text{hor}}(H-z)}{L_{\text{hor}}}}{\tilde{\gamma}\rho(H-z)} = O\left(\frac{V_{\text{hor}}}{L_{\text{hor}}}\right)$$

$$V_{\text{hor}}\,\rho\,\mathrm{O}\left(\frac{V_{\text{hor}}^2}{L_{\text{hor}}}+\frac{V_{\text{hor}}^2}{L_{\text{cor}}}\right)$$

$$\frac{\left\{v_x\left(\frac{\partial p}{\partial x}\right)+v_y\left(\frac{\partial p}{\partial y}\right)\right\}\frac{1}{\gamma p}}{\dot{M}/\gamma}=\frac{\frac{\rho V_{\text{hor}}}{\gamma p}\times \mathrm{O}\left(\frac{V_{\text{hor}}}{\tau}+\frac{V_{\text{hor}}^2}{L_{\text{hor}}}+\frac{V_{\text{hor}}^2}{L_{\text{cor}}}\right)}{\mathrm{O}\left(\frac{V_{\text{hor}}}{L_{\text{hor}}}\right)}=$$

$$\mathrm{O}\left(\frac{V_{\text{hor}}}{L_{\text{hor}}}\right)$$

$$=\frac{L_{\text{hor}}}{C^2}\mathrm{O}\left(\frac{V_{\text{hor}}^2}{L_{\text{hor}}}+\frac{V_{\text{hor}}^2}{L_{\text{cor}}}\right)=\frac{V_{\text{hor}}^2}{C^2}\mathrm{O}\left(\frac{L_{\text{hor}}}{V_{\text{hor}}\tau}+1+\frac{L_{\text{hor}}}{\tau_{\text{cor}}}\right)=\mathrm{O}\left(2+\frac{L_{\text{hor}}}{L_{\text{cor}}}\right)\mathrm{O}\left(\mathbf{M}^2\right)$$

$$\left(\mathbf{M}\equiv\frac{V_{\text{hor}}}{C}, \mathrm{M} \quad \mathrm{c}\ C=\left(\frac{\gamma p}{\rho}\right)^{1/2}=300\ -\ 350\quad /\right)$$

ε

$$\mathbf{M}^2 = \frac{V_{\mathrm{hor}}^2}{C^2} = \frac{L_{\mathrm{hor}}g}{C^2} \times \frac{V_{\mathrm{hor}}^2}{gL_{\mathrm{hor}}} = \frac{L_{\mathrm{hor}}g}{C^2} \times \varepsilon$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\,g\int_{z}^{H}\left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y}\right)\mathrm{d}z'$$

Часто использую
т

$$v_z = 0$$

Часто используют

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = 0$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Г.И. Марчук

$$\frac{\partial \rho}{\partial t} = 0$$

чтобы «отфильтровать акустику» ???

$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

Учебник Дж. Холтона

$$\frac{\partial p}{\partial t} = 0$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\,g\rho v_z$$

$$\frac{dp}{dt} = -g\int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} \right) dz'$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) - \frac{1}{\gamma}\frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma}\frac{Q^*}{g M}$$

Часто использую
т

$$v_z = 0$$

$$v_z = 0$$

Часто используют

$$\frac{d\rho}{dt} = 0$$

$$\operatorname{div} \mathbf{V} = 0$$

$$\frac{\partial v_z}{\partial z} = -\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y}$$

Г.И. Марчук

$$\frac{\partial \rho}{\partial t} = 0$$

чтобы «отфильтровать акустику» ???

$$\operatorname{div}(\rho \mathbf{V}) = 0$$

$$\frac{\partial(\rho v_z)}{\partial z} = -\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y}$$

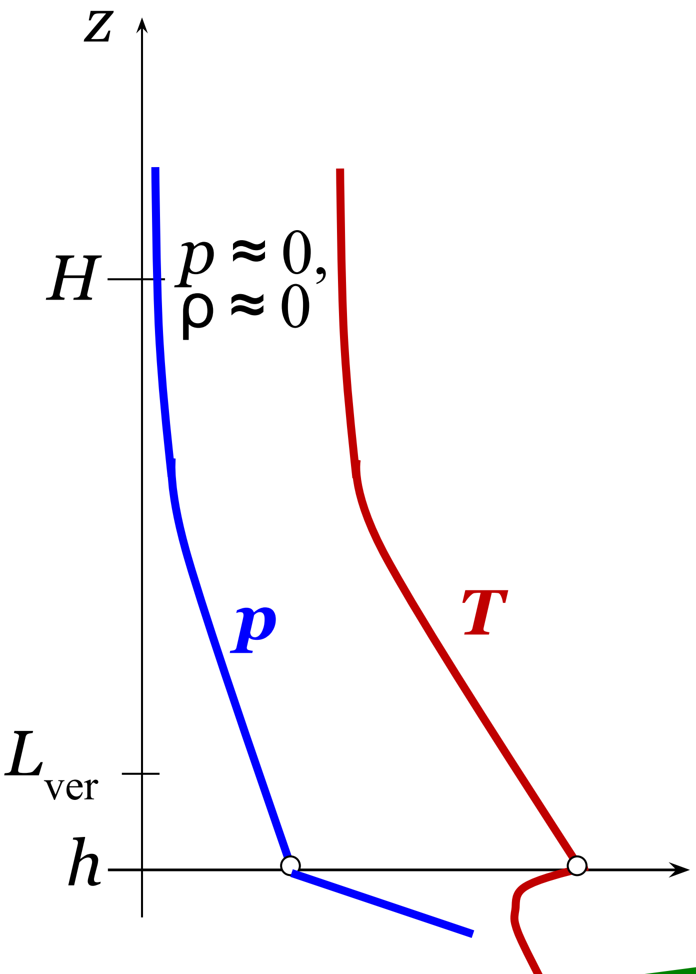
Дж. Холтон (учебник)

$$\frac{\partial \rho}{\partial t} = 0$$

$$\frac{dp}{dt} = -g\rho v_z$$

$$\rho v_z = \int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} \right) dz'$$

Схема распределения параметров в тропосфере над межфазной поверхностью



МЕЖФАЗНАЯ ГРАНИЦА

ξ - поток испаряющейся массы на нижней границе атмосферы

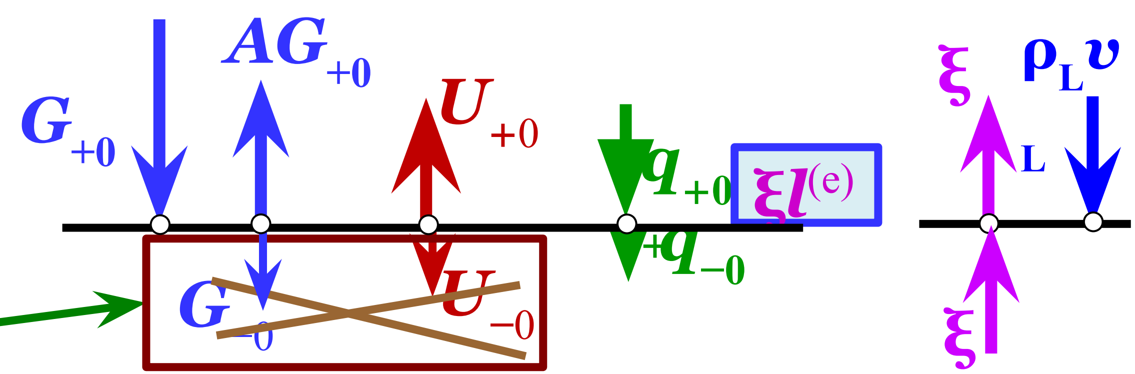
$\xi l^{(e)}$ - поток энергии испарения

$\rho_L \upsilon_L$ – дождевой (снежный) поток

q - поток тепла

G - коротковолновая солнечная радиация

U - длинноволновое тепловое излучение



НЕПРОЗРАЧНАЯ (твердая) фаза:
$G_{\_} = U_{\_} = 0$

$$q_{+0} - q_{-0} + (1-A)G_{+0} - G_{-0} - U_{+0} - U_{-0} = \xi_0^{(e)} l^{(e)} + \xi_0^{(m)} l^{(m)}$$

$$\mathbf{B}_t \frac{\partial \bar{\mathbf{U}}}{\partial \bar{t}} + \mathbf{B}_x \frac{\partial \bar{\mathbf{U}}}{\partial \bar{x}} + \mathbf{B}_y \frac{\partial \bar{\mathbf{U}}}{\partial \bar{y}} + \mathbf{B}_z \frac{\partial \bar{\mathbf{U}}}{\partial \bar{z}} + \mathbf{B} = 0,$$

$$\bar{\mathbf{U}} = \begin{pmatrix} \bar{\rho} \\ \bar{\upsilon}_x \\ \bar{\upsilon}_y \\ \bar{\upsilon}_z \\ \dot{\bar{M}} \\ \bar{M} \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} \bar{\rho}\left(\bar{Q}\gamma - \dot{\bar{M}}\right) / \left(\gamma \bar{M}\right) \\ -\bar{\upsilon}_y \tau f \\ \bar{\upsilon}_x \tau f \\ -\left(\bar{Q}\gamma - \dot{\bar{M}}\right) / \left(\gamma \bar{M}\right) \\ 0 \\ \bar{\rho} \end{pmatrix}$$

$$\mathbf{B}_t = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{B}_x = \begin{pmatrix} \overline{v}_x & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{v}_x & 0 & 0 & 0 & \overline{g}/\overline{\rho} \\ 0 & 0 & \overline{v}_x & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\overline{v}_x & -\overline{\rho} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{B}_z = \begin{pmatrix} \overline{v}_z & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{v}_z & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{v}_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_x = \begin{pmatrix} \overline{v}_y & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{v}_y & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{v}_y & 0 & 0 & \overline{g}/\overline{\rho} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\overline{v}_y & 0 & -\overline{\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$
\mathbf{B}' = \begin{pmatrix}
B_{11} & \dfrac{\partial \bar{\rho}}{\partial \bar{x}} & \dfrac{\partial \bar{\rho}}{\partial \bar{y}} & \dfrac{\partial \bar{\rho}}{\partial \bar{z}} & -\dfrac{\bar{\rho}}{\gamma \bar{M}} & -\dfrac{\bar{\rho} B_{11}}{\bar{M}} \\
B_{21} & \dfrac{\partial \bar{v}_x}{\partial \bar{x}} & \dfrac{\partial \bar{v}_x}{\partial \bar{y}} - \bar{f} & \dfrac{\partial \bar{v}_x}{\partial \bar{z}} & 0 & 0 \\
B_{31} & \dfrac{\partial \bar{v}_y}{\partial \bar{x}} + \bar{f} & \dfrac{\partial \bar{v}_y}{\partial \bar{y}} & \dfrac{\partial \bar{v}_y}{\partial \bar{z}} & 0 & 0 \\
0 & 0 & 0 & 0 & \dfrac{1}{\gamma \bar{M}} & \dfrac{B_{11}}{\bar{M}} \\
-\dfrac{\partial \bar{v}_x}{\partial \bar{x}} - \dfrac{\partial \bar{v}_y}{\partial \bar{y}} & -\dfrac{\partial \bar{\rho}}{\partial \bar{x}} & -\dfrac{\partial \bar{\rho}}{\partial \bar{y}} & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
B_{11} = \frac{\bar{Q}}{\bar{M}} - \frac{\dot{\bar{M}}}{\gamma \bar{M}}, \qquad B_{21} = \frac{1}{\bar{\rho}} \left(\frac{\mathrm{d} \bar{v}_x}{\mathrm{d} \bar{t}} - \bar{v}_y \tau f \right), \qquad B_{31} = \frac{1}{\bar{\rho}} \left(\frac{\mathrm{d} \bar{v}_y}{\mathrm{d} \bar{t}} + \bar{v}_x \tau f \right),
$$

1. **В дифференциальном операторе системы уравнений квазистатического по вертикали движения нет скорости звука C, даже в уравнениях горизонтального движения,**

т.е. уже «акустика отфильтрована» (выражение Г.И. Марчука).

2. Система уравнений только с вертикальной квазистатичностью негиперболична ($C \to \infty$) даже при отсутствии теплопроводности, т.е. при отсутствии параб о члена

$$\frac{\partial}{\partial x_k}\left(\lambda \frac{\partial T}{\partial x_k}\right)$$

Некоторое решение

$$\bar{\mathbf{U}} = \bar{\mathbf{U}}\left(\bar{t}, \bar{x}, \bar{y}, \bar{z}\right)$$

Другое решение, отличающееся от $\bar{\mathbf{U}}$ малым возмущением

$$\bar{\mathbf{U}}^{(\delta)} = \bar{\mathbf{U}} + \bar{\mathbf{U}}' \qquad \left(\left|\bar{\mathbf{U}}\right| \sim 1, \quad \left|\bar{\mathbf{U}}'\right| \ll 1\right).$$

$$\mathbf{B}_t \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{t}} + \mathbf{B}_x \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{x}} + \mathbf{B}_y \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{y}} + \mathbf{B}_z \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{z}} + \mathbf{B}'\bar{\mathbf{U}}' = \mathbf{F}'$$

$$\bar{\mathbf{U}} = \begin{pmatrix} \bar{\rho}' \\ \bar{v}_x' \\ \bar{v}_y' \\ \bar{v}_z' \\ \dot{\bar{M}}' \\ \bar{M}' \end{pmatrix}, \qquad \mathbf{F}' = \begin{pmatrix} \bar{\rho}\bar{Q}'/\bar{M} \\ 0 \\ 0 \\ \bar{\rho}\bar{Q}'/\bar{M} \\ 0 \\ \bar{\rho} \end{pmatrix}$$

Исходное гармоническое возмущение

$$t=0: \quad \bar{\mathbf{U}}' = \mathbf{A}\sin\left(\bar{k}_x\bar{x} + \bar{k}_y\bar{y} + \bar{k}_z\bar{z}\right)$$

$$\mathbf{A} = \begin{pmatrix} A^{(\rho)} \\ A^{(vx)} \\ A^{(vy)} \\ A^{(vz)} \\ A^{(\dot{M})} \\ A^{(M)} \end{pmatrix},$$

Коротковолновые возмущения с $Q' = 0$:

$$\bar{k}_x = k_x L_{\text{hor}},\ \bar{k}_y = k_y L_{\text{hor}},\ \bar{k}_z = k_z L_{\text{hor}} \gg 1,$$

$$l'_x = \frac{2\pi}{k_x} \ll L_{\text{hor}}, \quad l'_y = \frac{2\pi}{k_y} \ll L_{\text{hor}}, \quad l'_z = \frac{2\pi}{k_z} \ll L_{\text{ver}}$$

Коротковолновые возмущения

$$\bar{k}_x = k_x L_{\text{hor}},\ \bar{k}_y = k_y L_{\text{hor}},\ \bar{k}_z = k_z L_{\text{hor}} \gg 1,$$

$$l'_x = \frac{2\pi}{k_x} \ll L_{\text{hor}},\quad l'_y = \frac{2\pi}{k_y} \ll L_{\text{hor}},\quad l'_z = \frac{2\pi}{k_z} \ll L_{\text{ver}}$$

$$\overbrace{\mathbf{B}_t,\mathbf{B}_x,\mathbf{B}_y,\mathbf{B}_z,\mathbf{B}'}\approx \text{const},\ \ \mathbf{F}' = 0$$

$$\bar{v}'_x = A_*^{(vx)}E_*,\quad \bar{v}'_y = A_*^{(vy)}E_*,\quad \bar{v}'_z = A_*^{(vz)}E_*$$

$$\bar{\rho}' = A_*^{(\rho)}E_*,\quad \vec{\bar{M}}' = A_*^{(\vec{M})}E_*,\quad \bar{M}' = A_*^{(M)}E_*$$

$$E_* = \exp\left(i\left(\bar{k}_x\bar{x} + \bar{k}_y\bar{y} + \bar{k}_z\bar{z} - \bar{\omega}_*\bar{t}\right)\right) = \exp\left(\bar{\omega}_{**}\bar{t}\right)\cdot\exp\left(i\left(\bar{k}_x\bar{x} + \bar{k}_y\bar{y} + \bar{k}_z\bar{z} - \bar{\omega}\bar{t}\right)\right),$$

$$\bar{\omega}_* = \bar{\omega} + i\bar{\omega}_{**},\quad \bar{\omega}_* = \omega_*\cdot\tau$$

$\bar{\omega}_{**} > 0$ - неустойчивость (коротковолновая) решения $\bar{\mathrm{U}}\left(\bar{t},\bar{x},\bar{y},\bar{z}\right)$

$\bar{\omega}_{**} \underset{\bar{k}\to\infty}{\to} +\infty$ абсолютная неустойчивость решения $\bar{\mathrm{U}}\left(\bar{t},\bar{x},\bar{y},\bar{z}\right)$

некорректно поставленная задача

$$\mathbf{D}\mathbf{A}_* = 0, \qquad \mathbf{A}_* = \left(A_*^{(\rho)}, A_*^{(vx)}, A_*^{(vy)}, A_*^{(vz)}, A_*^{(\dot{M})}, A_*^{(M)}\right)^{\mathrm{T}}.$$

$$\mathbf{D} = i\left(-\bar{\omega}_*\mathbf{B}_t + \bar{k}_x\mathbf{B}_x + \bar{k}_y\mathbf{B}_y + \bar{k}_z\mathbf{B}_z\right) + \mathbf{B}'$$

$$\det \mathbf{D} = 0$$

$$\bar{\omega}_*^3 + b\bar{\omega}_*^2 + c\bar{\omega}_* + d = 0$$

b, c, d $-$ определяется значениями компонент $\mathbf{U}$,

а именно $\bar{\rho}, \bar{v}_x, \bar{v}_y, \bar{v}_z, \dfrac{\partial \bar{v}_x}{\partial x}, \dfrac{\partial \bar{v}_y}{\partial x}, \ldots$

Устойчивость покоя

$$\bar{\upsilon}_x = \bar{\upsilon}_y = \bar{\upsilon}_z = 0, \quad \frac{\partial \bar{\rho}}{\partial \bar{x}} = \frac{\partial \bar{\rho}}{\partial \bar{y}} = 0, \quad \dot{\bar{M}} = 0, \quad \bar{Q} = 0$$

$$\frac{\partial \bar{\upsilon}_x}{\partial \bar{t}} = \frac{\partial \bar{\upsilon}_y}{\partial \bar{t}} = 0, \quad \frac{\partial \bar{\upsilon}_x}{\partial \bar{x}} = \frac{\partial \bar{\upsilon}_x}{\partial \bar{y}} = \frac{\partial \bar{\upsilon}_x}{\partial \bar{z}} = \frac{\partial \bar{\upsilon}_y}{\partial \bar{x}} = \frac{\partial \bar{\upsilon}_y}{\partial \bar{y}} = \frac{\partial \bar{\upsilon}_y}{\partial \bar{z}} = \frac{\partial \bar{\upsilon}_z}{\partial \bar{z}} = 0,$$

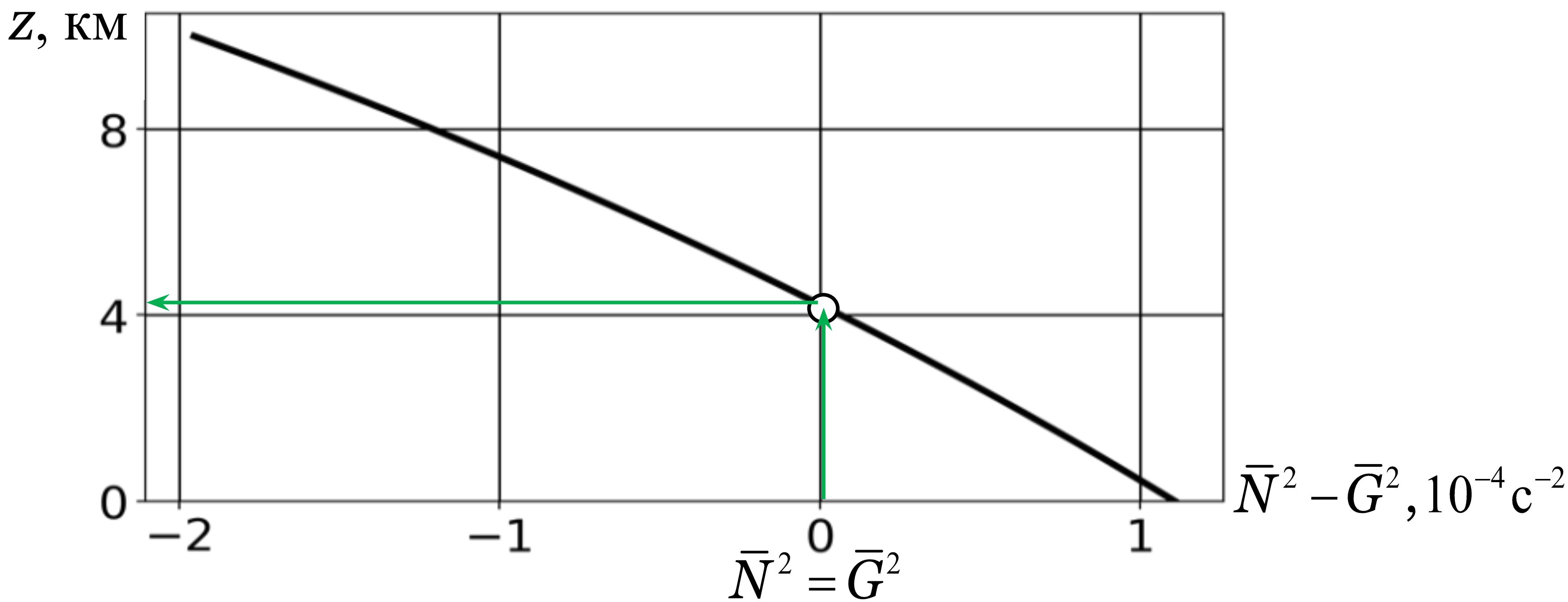
стандартная атмосфера $\rho(z)$, $\dfrac{\partial \bar{\rho}}{\partial \bar{z}}$ и $\dfrac{\rho}{p}$

$$\bar{\omega}_* \left[\bar{\omega}_*^2 - \left(\left(\bar{N}^2 - \bar{G}^2 \right) \frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^2} + \bar{f}^2 - \frac{\bar{N}^2 \bar{G}^2}{\bar{g}} \frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^3} \cdot i \right) \right] = 0,$$

$$\bar{N}^2 = -\frac{\bar{g}}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial \bar{z}} = \left(\frac{L_{\text{ver}} \tau}{L_{\text{hor}}} \right)^2 N^2 \qquad \left(N^2 = -\frac{g}{\rho} \frac{\partial \rho}{\partial z} \right),$$

$$\bar{G}^2 = \frac{\bar{\rho}\bar{g}}{\gamma \bar{M}} = \left(\frac{L_{\text{ver}} \tau}{L_{\text{hor}}} g \right)^2 \frac{\rho}{\gamma p} \qquad \left(G^2 = \left(\frac{g}{C} \right)^2 = \frac{g^2 \rho}{\gamma p} \right),$$

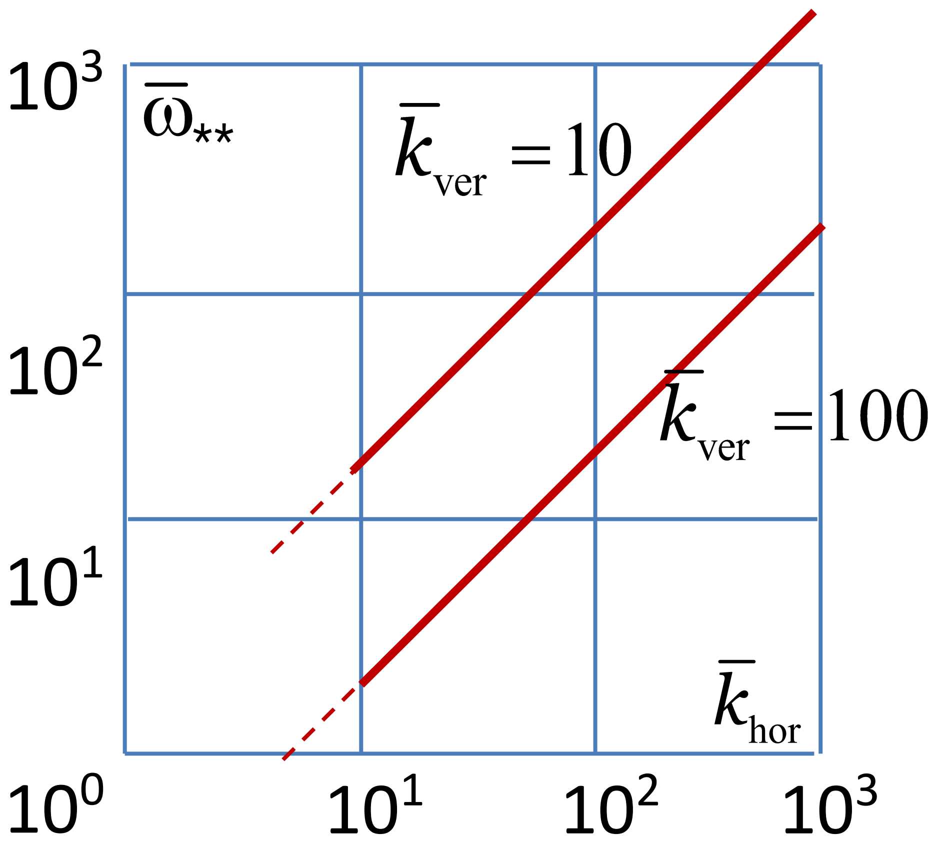
$$\bar{\omega}_*\left[\bar{\omega}_*^2-\left(\left(\bar{N}^2-\bar{G}^2\right)\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^2}+\bar{f}^2-\frac{\bar{N}^2\bar{G}^2}{\bar{g}}\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^3}\cdot i\right)\right]=0,$$

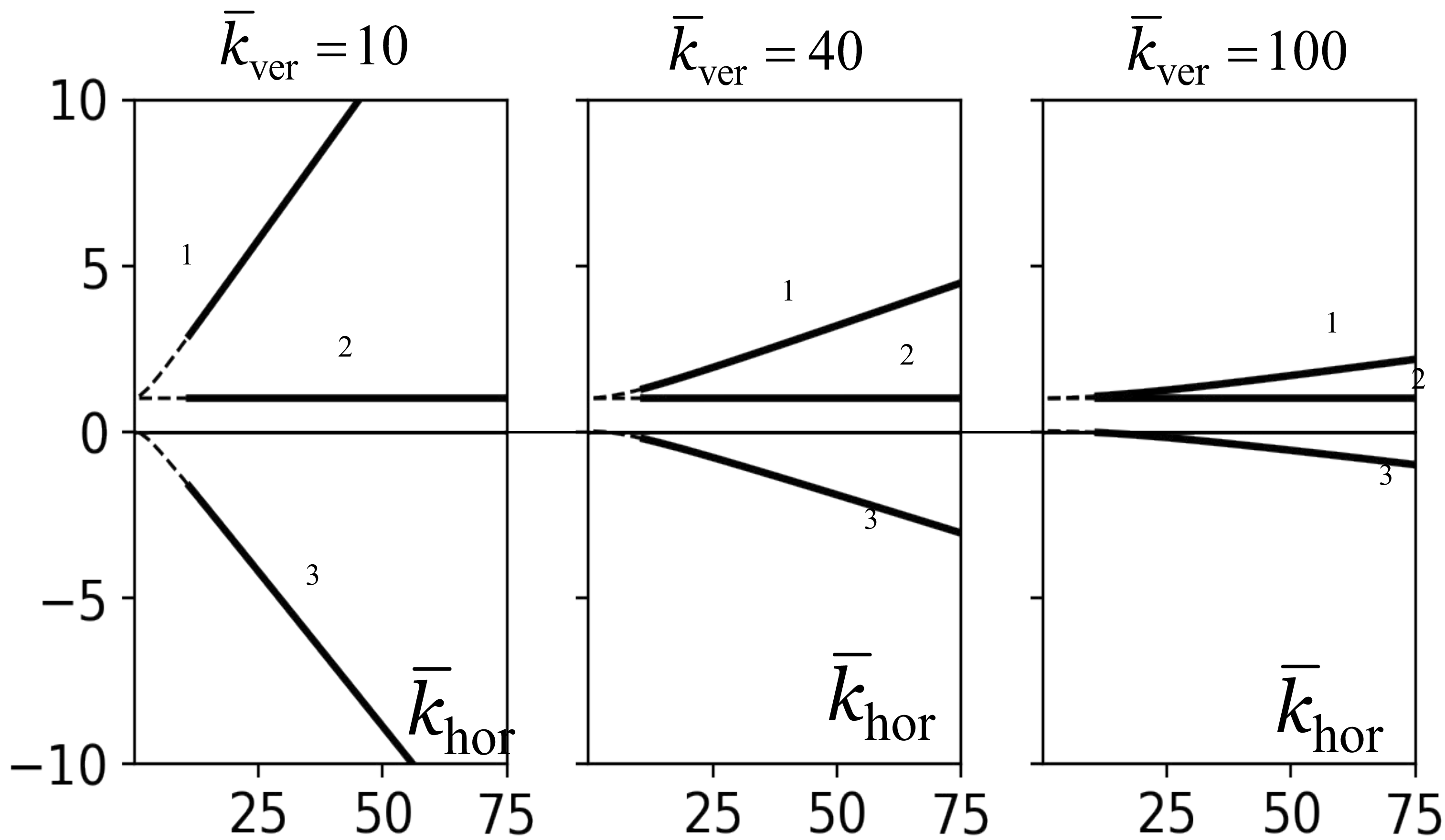


$$\bar{f}\ll\sqrt{\left|\bar{N}^2-\bar{G}^2\right|}\frac{\bar{k}_{\text{hor}}}{\bar{k}_{\text{ver}}},\qquad \bar{\omega}_{**}^{(2,3)}=\pm A_1\kappa_1\underset{\kappa_1\to\infty}{\longrightarrow}\pm\infty,\qquad \left(\kappa_1=\bar{k}_{\text{hor}}\big/\bar{k}_{\text{ver}}^2\right)$$

$$\bar{f}\ ?\ \sqrt{\left|\bar{N}^2-\bar{G}^2\right|}\frac{\bar{k}_{\text{hor}}}{\bar{k}_{\text{ver}}},\qquad \bar{\omega}_{**}^{(2,3)}=\pm A_2\kappa_2\underset{\kappa_2\to\infty}{\longrightarrow}\pm\infty,\qquad \left(\kappa_2=\bar{k}_{\text{hor}}^2\big/\bar{k}_{\text{ver}}^3\right)$$

Устойчивость покоя

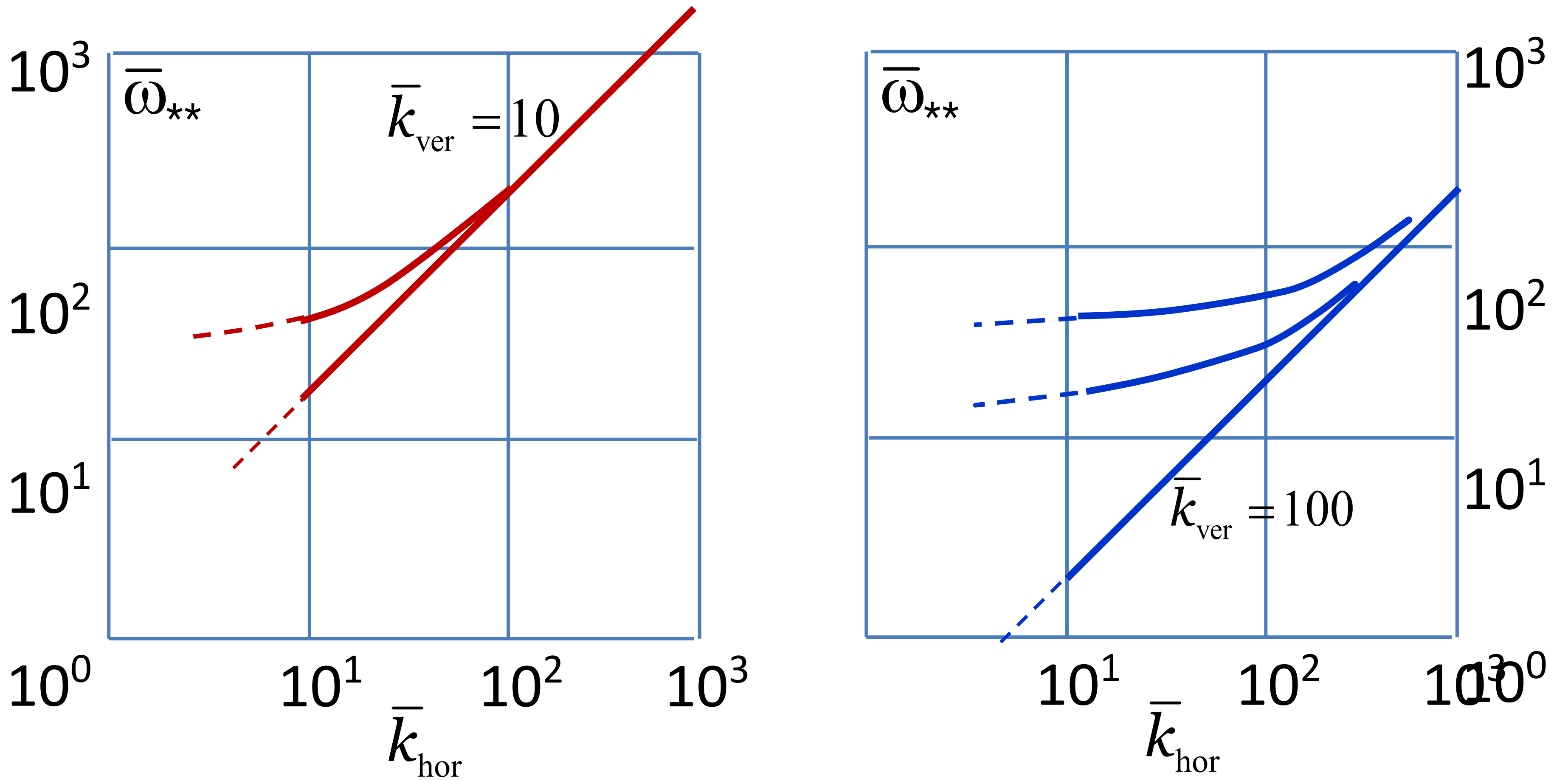




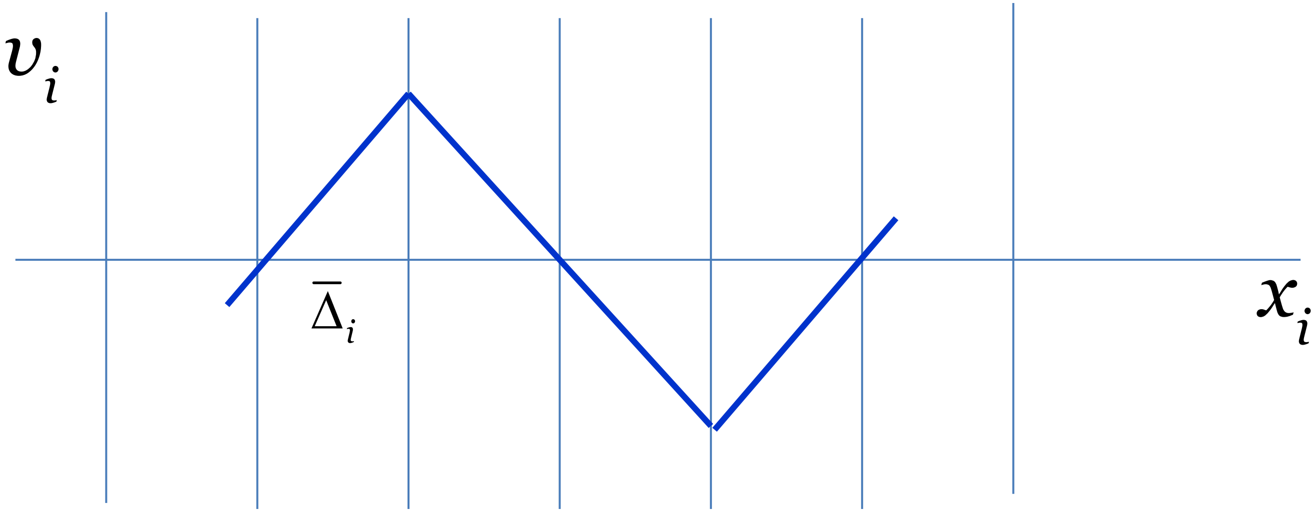
$$\bar{\upsilon}_x = \bar{\upsilon}_y = \bar{\upsilon}_z = 1, \quad \bar{Q} = 0,$$

$$\frac{\partial \bar{\upsilon}_x}{\partial \bar{z}} = \frac{\partial \bar{\upsilon}_y}{\partial \bar{z}} = \frac{\partial \bar{\upsilon}_z}{\partial \bar{z}} = 0, \quad \frac{\partial \bar{\rho}}{\partial \bar{x}} = \frac{\partial \bar{\rho}}{\partial \bar{y}} = 0,$$

$$\frac{\partial \bar{\upsilon}_y}{\partial \bar{x}} = \frac{\partial \bar{\upsilon}_x}{\partial \bar{y}} = 0.5, \quad \frac{\partial \bar{\upsilon}_x}{\partial \bar{x}} = \frac{\partial \bar{\upsilon}_y}{\partial \bar{y}} = -0.5,$$


10^3
$\overline{\omega}_{**}$
$\overline{k}_{\text{ver}} = 10$
10^2
10^1
10^0
10^1
10^2
10^3
$\overline{k}_{\text{hor}}$
$\overline{\omega}_{**}$
10^3
10^2
10^1
$\overline{k}_{\text{ver}} = 100$
10^1
10^2
10^3
10^0
$\overline{k}_{\text{hor}}$

РАЗНОСТНАЯ СЕТКА



$$\bar{l}_i^{\,\min} = 4\bar{\Delta}_i, \qquad \bar{k}_i^{\max}\bar{l}_i^{\,\min} = 2\pi, \qquad \bar{k}_i^{\max} = k_i^{\max} L_i = \tfrac{1}{2}\pi N_i$$

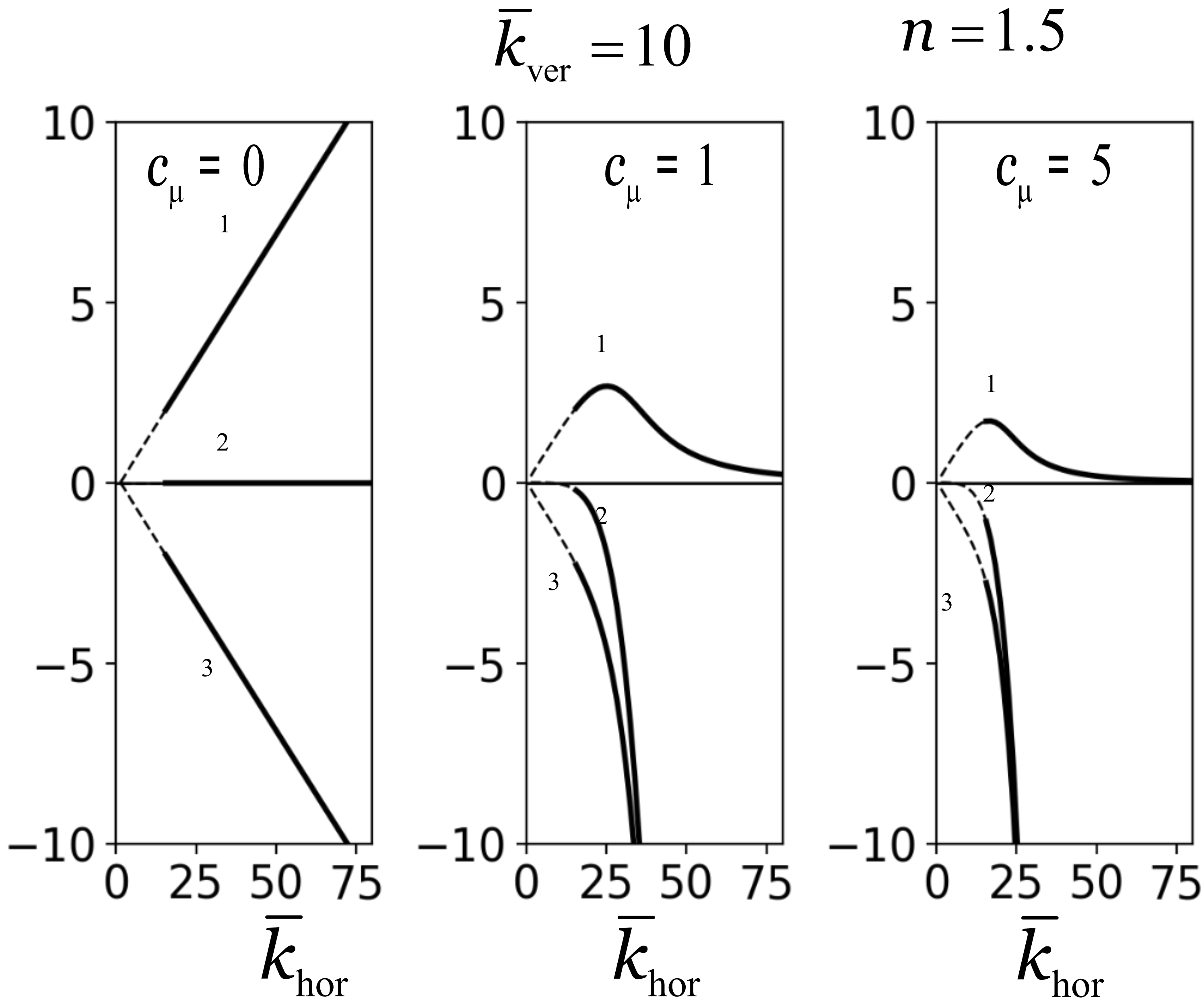
$$\left(\bar{\Delta}_i = \Delta\bar{x}, \Delta\bar{y}, \Delta\bar{z}, \qquad N_i = \frac{L_i}{\Delta_i}, \qquad i = x, y, z\right),$$

$$k_i < \bar{k}_i^{\max} = \tfrac{1}{2}\pi N_i$$

ИСКУССТВЕННАЯ ВЯЗКОСТЬ

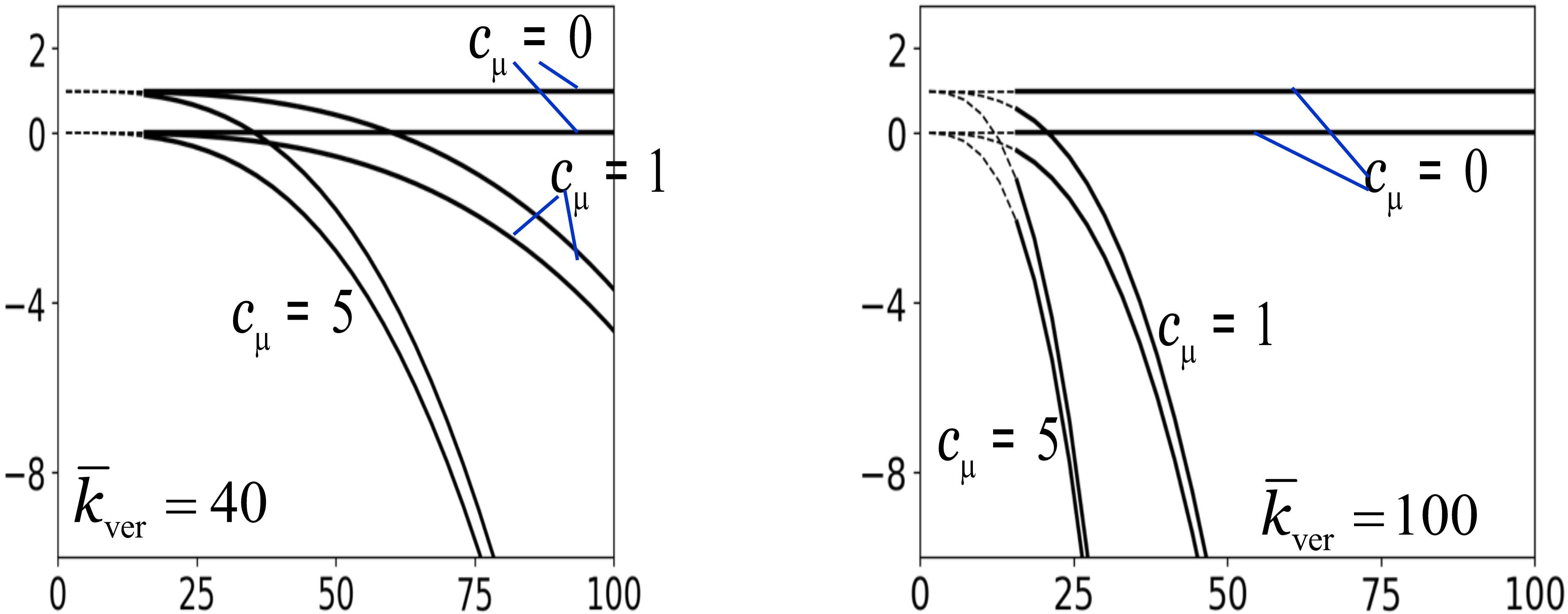
$$\bar{\mu} = c_{\mu}\bar{\mu}_{\mathrm{a}}\left(\bar{k}_x\bar{k}_y\bar{k}_z\right)^n, \qquad \bar{\lambda} = c_{\lambda}\bar{\mu}_{\mathrm{a}}\left(\bar{k}_x\bar{k}_y\bar{k}_z\right)^m,$$

$$\bar{\mu}_{\mathrm{a}} = \frac{\mu_{\mathrm{a}}}{\rho V_{\mathrm{ver}} L_{\mathrm{ver}}} \qquad \left(\mu_{\mathrm{a}} = 1.8\times10^{-5}\ \text{кг} / \text{м}\cdot\text{с}\right)$$



$\bar{k}_{ver} = 10$
$n = 1.5$
$c_\mu = 0$
$c_\mu = 1$
$c_\mu = 5$
1
2
3
10
5
0
−5
−10
0
25
50
75
$\bar{k}_{hor}$

$$\bar{\omega}_*\left(\bar{\omega}_*^2 + b_1(\bar{\mu})\bar{\omega}_* + c_1(\bar{\mu})\right) = 0 \qquad \left(\frac{\partial \rho}{\partial t} = 0 - \text{приближение Марчука}\right)$$



$$\bar{\upsilon}_x = \bar{\upsilon}_y = \bar{\upsilon}_z = 1, \quad \bar{Q} = 0, \qquad \frac{\partial \bar{\upsilon}_x}{\partial \bar{z}} = \frac{\partial \bar{\upsilon}_y}{\partial \bar{z}} = \frac{\partial \bar{\upsilon}_z}{\partial \bar{z}} = 0, \quad \frac{\partial \bar{\rho}}{\partial \bar{x}} = \frac{\partial \bar{\rho}}{\partial \bar{y}} = 0,$$

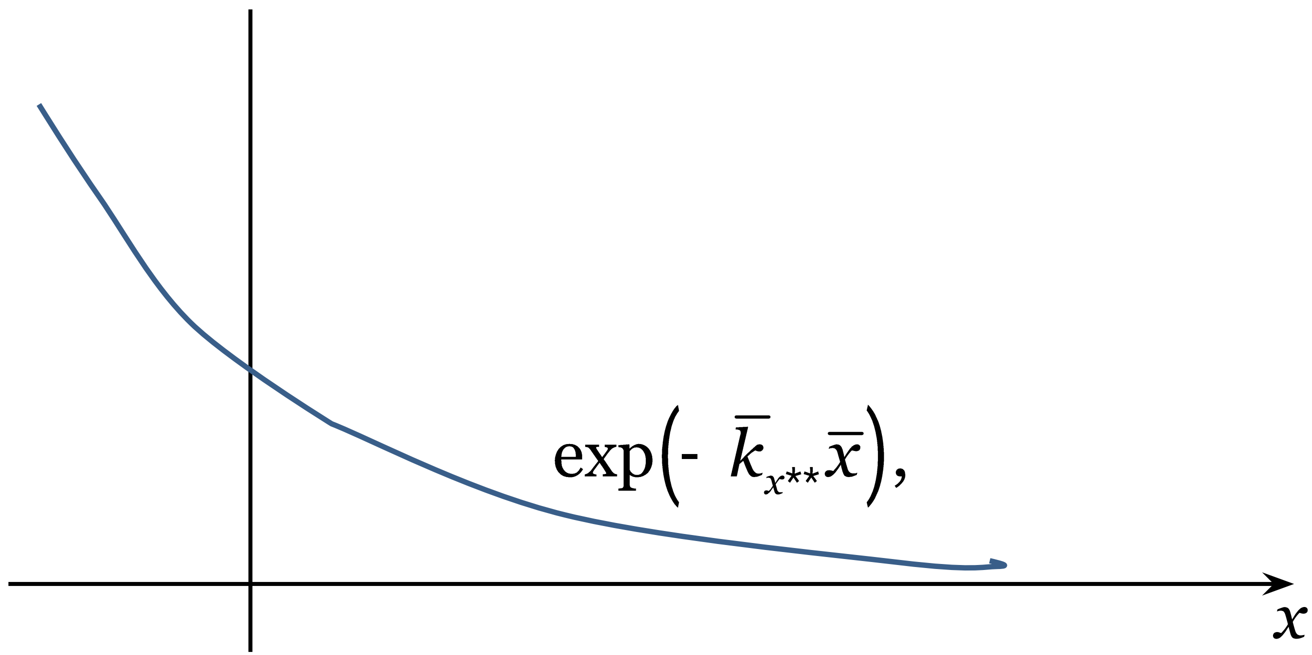
$$\frac{\partial \bar{\upsilon}_y}{\partial \bar{x}} = \frac{\partial \bar{\upsilon}_x}{\partial \bar{y}} = 0.5, \quad \frac{\partial \bar{\upsilon}_x}{\partial \bar{x}} = \frac{\partial \bar{\upsilon}_y}{\partial \bar{y}} = -0.5,$$

Локальное возмущение для $x, y, z > 0$

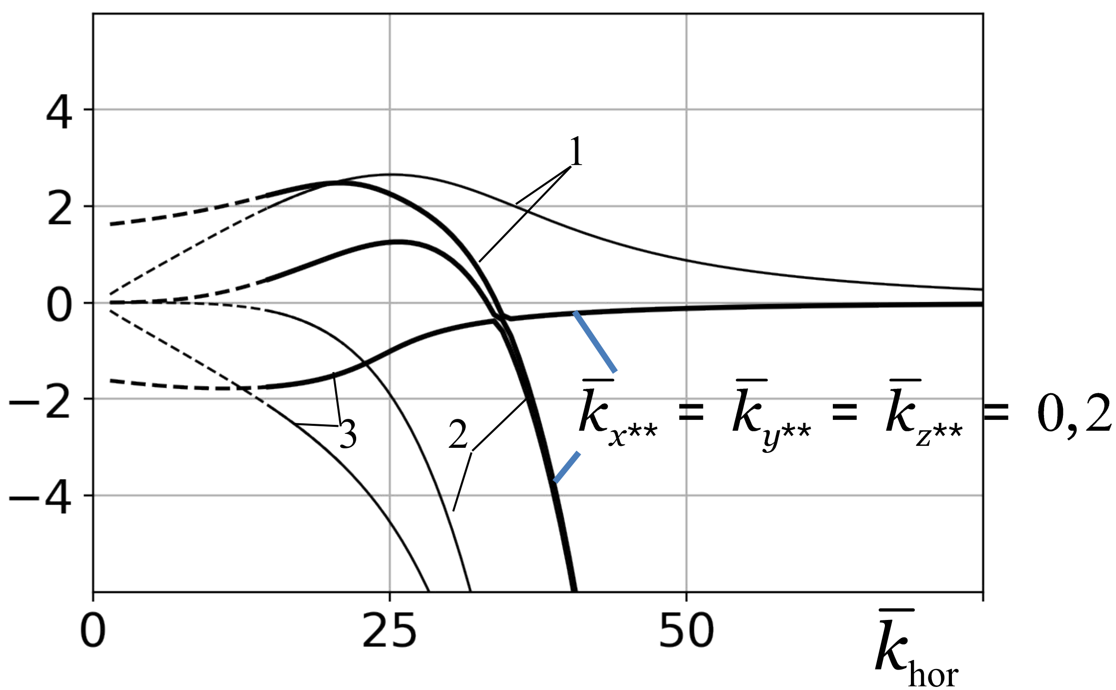
$$\bar{k}_{j*} = \bar{k}_j + i\bar{k}_{j**} \quad (j = x, y, z)$$

$$\boldsymbol{E}_* = \exp\left(i\left(\bar{k}_{x*}\bar{x} + \bar{k}_{y*}\bar{y} + \bar{k}_{z*}\bar{z} - \bar{\omega}_*\bar{t}\right)\right) =$$

$$= \exp\left(\bar{\omega}_{**}\bar{t} - \bar{k}_{x**}\bar{x} - \bar{k}_{y**}\bar{y} - \bar{k}_{z**}\bar{z}\right) \times \exp\left(i\left(\bar{k}_x\bar{x} + \bar{k}_y\bar{y} + \bar{k}_z\bar{z} - \bar{\omega}\bar{t}\right)\right),$$



$$\bar{k}_{\text{ver}} = 10, n = 1.5, c_\mu = 1$$



(т.е. $\varepsilon_2 = \bar{k}_{\text{hor}}^2 / \bar{k}_{\text{ver}}^3$? 1), $\bar{\Delta}_{\text{ver}}^3 / \bar{\Delta}_{\text{hor}}^2 =$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -g\int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} \right) \mathrm{d}z'$$

$$\frac{\partial \boldsymbol{v}_z}{\partial \boldsymbol{z}} = -\left(\frac{\partial \boldsymbol{v}_x}{\partial \boldsymbol{x}} + \frac{\partial \boldsymbol{v}_y}{\partial \boldsymbol{y}} \right) - \frac{\mathbf{1}}{\gamma}\frac{\dot{\boldsymbol{M}}}{\boldsymbol{M}} + \frac{\gamma - \mathbf{1}}{\gamma}\frac{\boldsymbol{Q}^*}{\boldsymbol{g}\boldsymbol{M}}$$

$$\left(\frac{\mathrm{d}p}{\mathrm{d}t} = -g\rho v_z \right)$$

Часто использую т

$$v_z = 0$$

$$v_z = 0$$

$\bar{\omega}_{**}$ ® $\bar{k}_{\text{hor}}$ ® ¥

® $\bar{G}\dfrac{\bar{k}_{\text{hor}}}{\bar{k}_{\text{ver}}}$ ® ¥

Часто используют

$$\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}\boldsymbol{t}} = \mathbf{0}$$

$$\frac{\partial v_z}{\partial z} = -\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y}$$

$$\bar{\omega}_*^2 = -\frac{\bar{g}}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial\bar{z}}\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^2}$$

$$\bar{\omega}_{**} = 0, \text{ if } \frac{\partial\bar{\rho}}{\partial\bar{z}} < 0$$

$$\bar{\omega}_{**} \xrightarrow[\bar{k}_{\text{hor}}\to\infty]{} \infty, \text{ if } \frac{\partial\bar{\rho}}{\partial\bar{z}} > 0$$

Г.И. Марчук

$$\frac{\partial\boldsymbol{\rho}}{\partial\boldsymbol{t}} = \mathbf{0}$$

чтобы «отфильтровать акустику» ???

$$\frac{\partial(\rho v_z)}{\partial z} = -\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y}$$

$$\bar{\boldsymbol{\omega}}_{**} = 0$$

Учебник Дж. Холтона

$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{t}} = 0$$

$$\rho v_z = -\dot{M} \equiv \int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} \right) \mathrm{d}z'$$

$$\bar{\omega}_*^2 = -\frac{\bar{g}}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial\bar{z}}\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^2},$$

$$\bar{\omega}_{**} = 0, \text{ if } \frac{\partial\bar{\rho}}{\partial\bar{z}} < 0,$$

$$\bar{\omega}_{**} \xrightarrow[\bar{k}_{\text{hor}}\to\infty]{} \infty, \text{ if } \frac{\partial\bar{\rho}}{\partial\bar{z}} > 0$$

Негиперболичность приводит к **некорректной** (ill posed) поста-новке задачи Коши при отсутствии диссипации, при которой **коротковолновые** возмущения

$$\delta W_k\big|_{t=0} = A(k)\left[\sin(kx)\right] \qquad \text{при } k \to \infty$$

растут неограниченно быстро:

$$\delta W = A(k)\times\exp\left(\omega_{**}(k)t\right)\left[\sin(kx)\right], \qquad \omega_{**}(k)\underset{k\to\infty}{\to}\infty$$

$L_* = \dfrac{p_0}{\rho_0 g} \sim H \sim 10^4$ м **- линейный размер, следующий из дифференциального оператора и начальных условий**

$k_* \approx \dfrac{2\pi}{L_*} = \dfrac{2\pi\rho_0 g}{p_0}$ **- характерное волновое число метеорологического процесса**

Конечно-разностная схема **генерирует «паразитные»** коротковолновые возмущения с длинами волн $l > 4\Delta x$ и волновым

$$k_x \approx \frac{2\pi}{l_x} \text{ и } k_z \approx \frac{2\pi}{l_z}$$

$$\text{если } k \approx \left(\frac{2\pi}{4\Delta z} \text{ или } \frac{2\pi}{4\Delta x}\right) >> k_* = \frac{2\pi\rho_0 g}{p_0}, \text{ то } \omega_{**}\tau >> 1, \quad \exp\left(\omega_{**}(k)\tau\right) >> 1$$

Необходим фильтр (численная диссипация) для возмущений типа

$$A(k)\times\sin(kx) \text{ при } k\Delta x > 1$$

$$A(k) \to 0 \qquad \text{при } k \to \infty$$

Луна и Земля

Снимок с борта Discovery 16.07.15 из точки Лагранжа (1,5 млн км от

СПАСИБО

ЗА ВНИМАНИЕ !