LECTURE 4

IRob2305: Introduction to Robotics

AGENDA

- Homogenous Transformation Matrix
- Link Connections
- Denavit-Hartneberg Parameters
- DH-Parameters

WHAT DO WE KNOW FOR NOW?

- We can make a complete rotation matrix all the way from base to the end-effector frame by multiplying together each of the individual rotation matrices from one frame to the next frame:

$$R_2^0 = R_1^0 R_2^1$$

CAN WE DO IT WITH DISPLACEMENT VECTORS?

- $d_3 \neq d_1^0 d_2^1 d_3^2$
- What if we want to know things like the position of our end effector in the base frame?
- How could we find d03?

HOMOGENOUS TRANSFORMATION MATRIX

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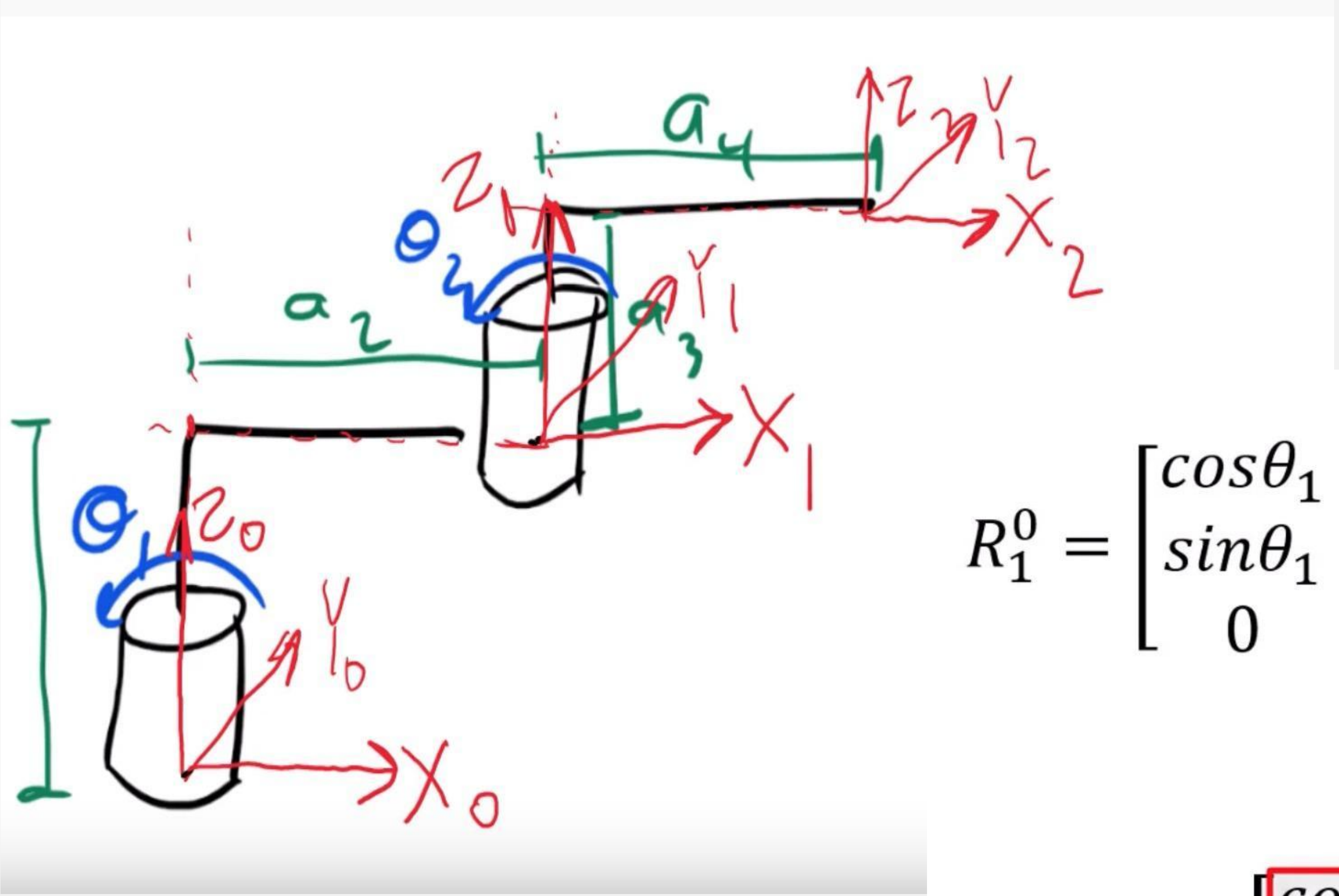
- H_n^m = rotation + position of one frame (n) relative to another frame (m)



We want to find the rotation matrix that tells us how the end effector frame is rotated relative to the base frame.

$$R_2^0 = R_1^0 R_2^1$$

$$R_1^0 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 \\ sin\theta_1 & cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^0 = \begin{bmatrix} a_2 cos\theta_1 \\ a_2 sin\theta_1 \\ a_1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & a_2 cos\theta_1 \\ sin\theta_1 & cos\theta_1 & 0 & a_2 sin\theta_1 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 \\ sin\theta_2 & cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^1 = \begin{bmatrix} a_4 cos\theta_2 \\ a_4 sin\theta_2 \\ a_3 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & a_4 cos\theta_2 \\ sin\theta_2 & cos\theta_2 & 0 & a_4 cos\theta_2 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = H_1^0 H_2^1$$

DENAVIT-HARTENBERG METHOD

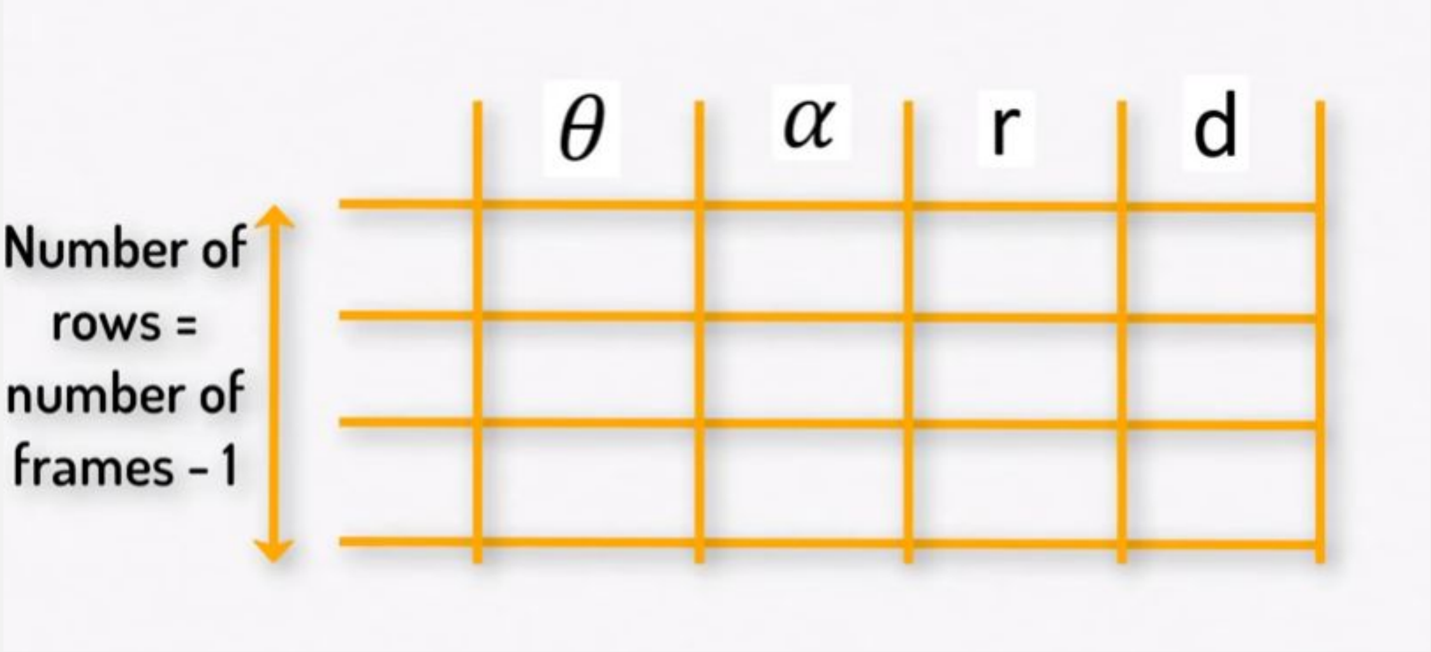
- Industry standard
- Faster
- Obscures the meaning behind the rotation matrix and displacement vector

STEP 1: ASSIGN FRAMES ACCORDING TO THE 4 DH RULES

STEP 2: FILL OUT THE DH PARAMETER TABLE

NOTES:

- Assigning coordinate systems: Assign Z_i along the axis of joint i.
  - For a revolute joint, the joint axis is along the axis of rotation.
  - For a prismatic joint, the joint axis is along the axis of translation.
- Choose X_i to point along the common perpendicular of Z_i and Z_{i+1} pointing towards the next joint.
  - if Z_i and Z_{i+1} intersect, then choose X_i to be normal to the plane of intersection.
- Choose Y_i to round out a right hand coordinate system.
  - The Y-axis is not used for Denavit Hartenberg so it is usually not drawn in the interest of less clutter.


θ
α
r
d
Number of rows = number of frames - 1

SYMBOL TERMINOLOGIES :

θ : A rotation about the *z*-axis.

d : The distance on the *z*-axis.

r : The length of each common normal (Joint offset).

α : The angle between two successive *z*-axes (Joint twist)

☐ **Only** *θ* and *d* are joint variables.

The necessary motions to transform **from one reference frame** to the next.

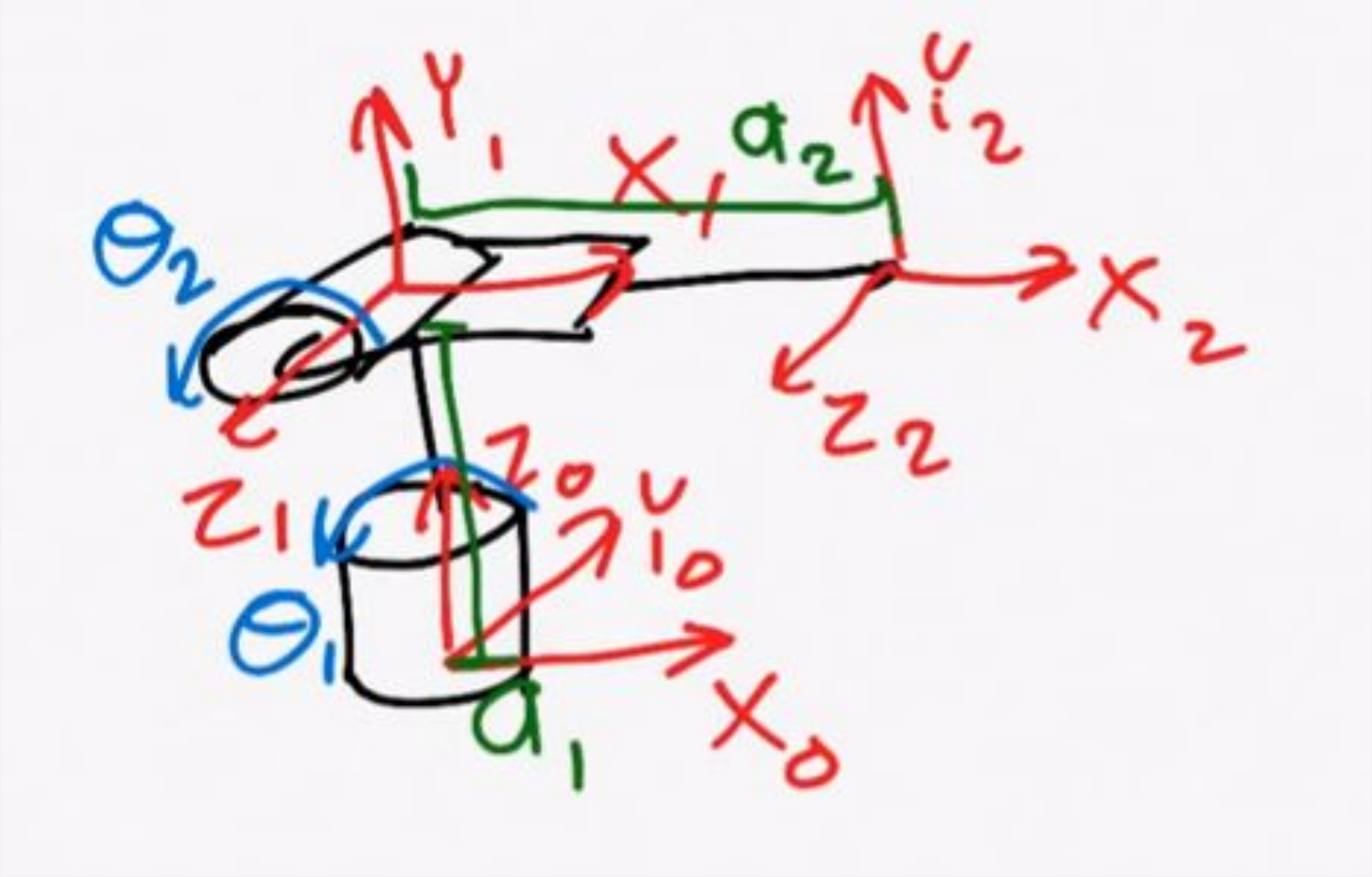
(I) Rotate about the z_n-axis an able of θ_{n+1}.

(II) Translate along z_n-axis a distance of d_{n+1} to make x_n and x_{n+1} colinear.

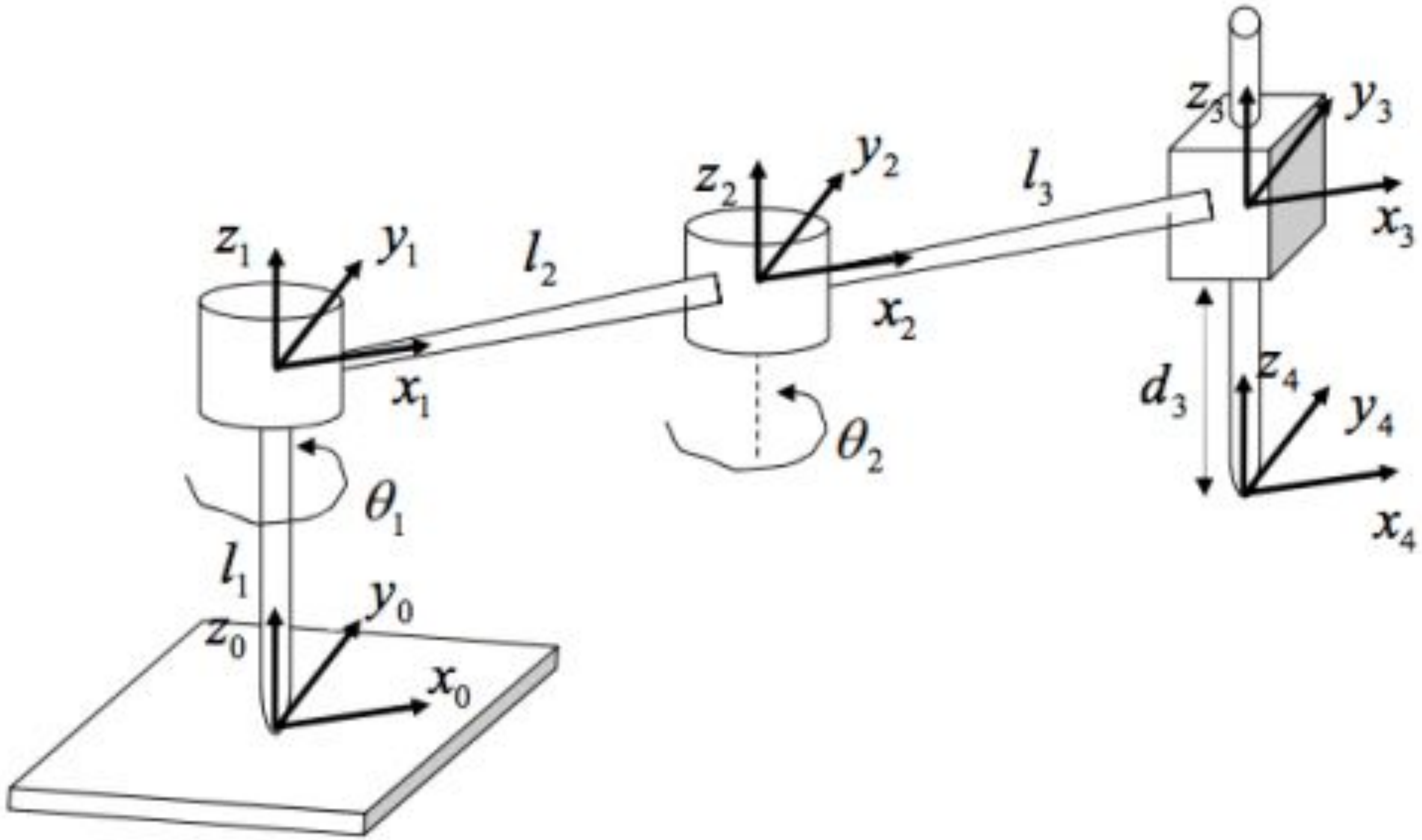
(III) Translate along the x_n-axis a distance of a_{n+1} to bring the origins of x_{n+1} together.

(IV) Rotate z_n-axis about x_{n+1} axis an angle of α_{n+1} to align z_n-axis with z_{n+1}-axis.

Example

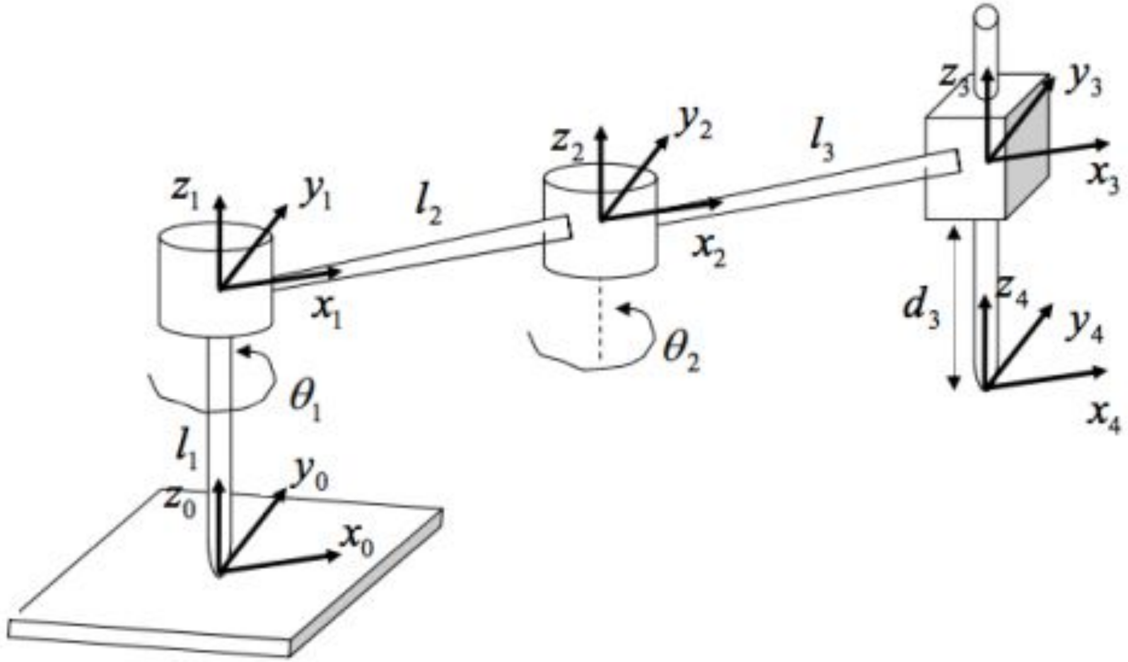


<u>Example</u>: Consider the SCARA robot arm and derive a geometric model of the robot using Denavit-Hartenberg convention



The DH-table

Link	a_i	α_i	d_i	θ_i
1	0	0	l_1	0
2	l_2	0	0	θ_1
3	l_3	0	0	θ_2
4	0	0	d_3	0



- **The homogeneous transformation matrices are**

$$\mathbf{H}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{H}_2 = \begin{bmatrix} c_1 & -s_1 & 0 & l_2 c_1 \\ s_1 & c_1 & 0 & l_2 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{H}_3 = \begin{bmatrix} c_2 & -s_2 & 0 & l_3 c_2 \\ s_2 & c_2 & 0 & l_3 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{H}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where,

$$c_1 = \cos\theta_1, \quad s_1 = \sin\theta_1$$
$$c_2 = \cos\theta_2, \quad s_2 = \sin\theta_2$$

- **Thus,**

$$
{}^{0}\mathbf{H}_4 = \mathbf{H}_1 \cdot \mathbf{H}_2 \cdot \mathbf{H}_3 \cdot \mathbf{H}_4
$$

$$
= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} c_1 & -s_1 & 0 & l_2 c_1 \\ s_1 & c_1 & 0 & l_2 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} c_2 & -s_2 & 0 & l_3 c_2 \\ s_2 & c_2 & 0 & l_3 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c_{12} & -s_{12} & 0 & l_3 c_{12} + l_2 c_1 \\ s_{12} & c_{12} & 0 & l_3 s_{12} + l_2 s_1 \\ 0 & 0 & 1 & l_1 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

where,

$$
\begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
c_{12} = \cos(\theta_1 + \theta_2), \quad s_{12} = \sin(\theta_1 + \theta_2)
$$