CALCULAS studies the relationships that exists between one collection of objects and another

Theme: The function of one variable

Literature:

[1], v. I, p. 174-180,
[2], part I, p. 40-57,
[3], p. 174-181.

1. The main definitions
2. The different ways of representing of the functions
3. The main characteristics of behavior of the function
4. The basic elementary functions
5. The composite function
6. The elementary functions

Given two sets X and Y

Definition. A <u>function</u> is a rule which assigns to each element x of X one and only one element y of Y.

Notation: $y=f(x)$

x - the <u>independent variable</u>

y - the <u>dependent variable</u>

The set X - <u>the domain of the function (D(y))</u>

The set of all corresponding values of y - <u>the range of the function (E(y))</u>

Examples. $D(y)$ - ?

1) $f(x)=x^3-4x+2$ (polynomial of the third power)

2) $y=\dfrac{1}{x-2}$

3) $y=\sqrt[3]{x-2}$

4) $y=\dfrac{1}{\sqrt{x-2}}$

5) $y=\ln\left(\dfrac{x+2}{x-4}+1\right)$

6) $y=\sqrt{\dfrac{x^2-4x+4}{x^3-2x^2-3x}}$

The most important ways of representing of the functions:

- the analytic method;

- the tabular method;

- the graphical method

The analytic method:

The function $y=f(x)$ is represented analytically if the variables x and y are connected with each other by equations

Examples

1) $y = x^2 + 1$ - an explicit function

2) $y = \begin{cases} x^2 + 1, & x < 2, \\ x + 4, & x \geq 2 \end{cases}$

3) $y^2 - 4x = 0$ - an implicit function

4) $y^2 \sin x - 4xe^y + 1 = 0$ - an implicit function

5) The demand function:

$$p = \frac{400}{q+3}$$

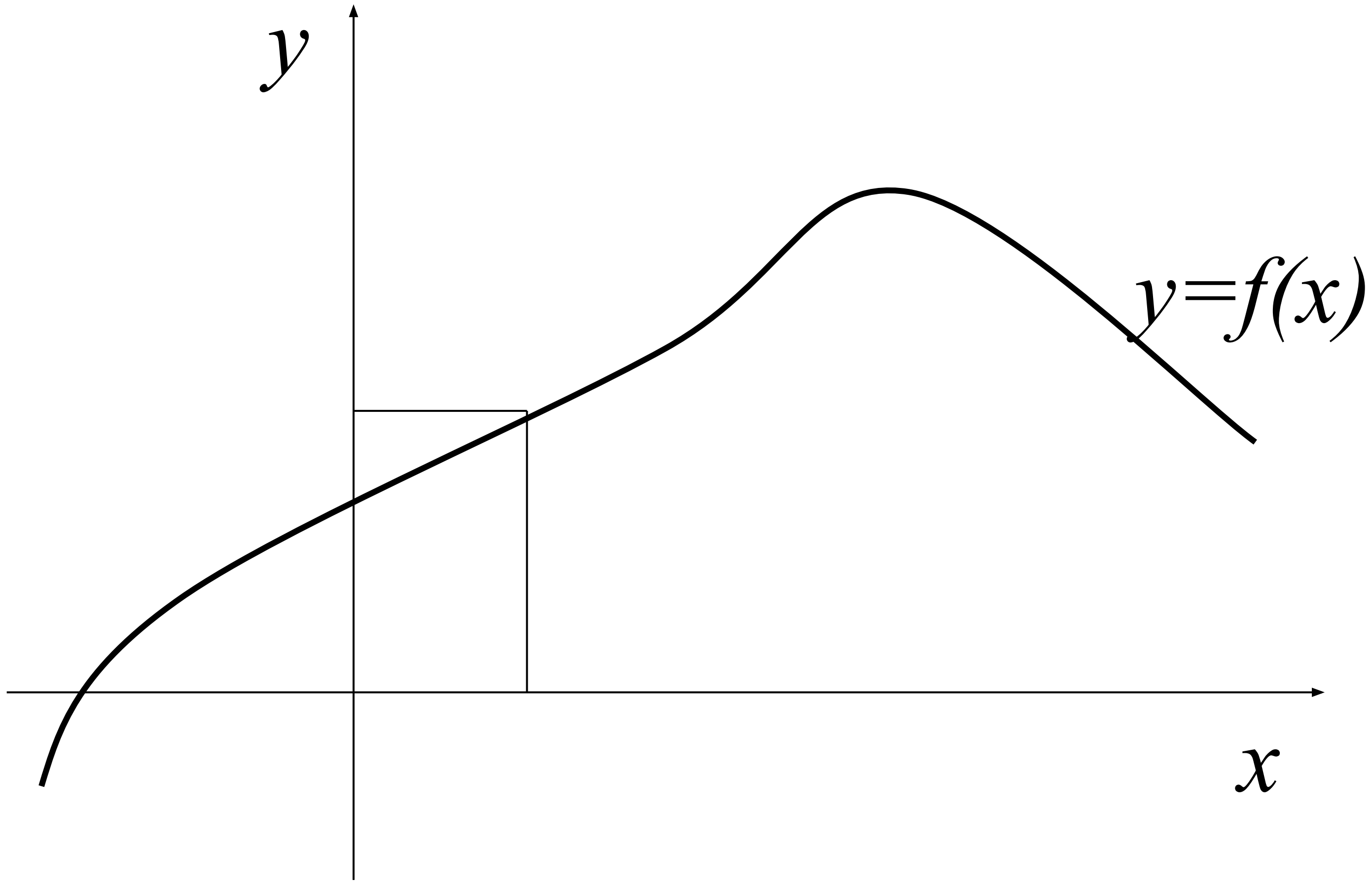
q - price, p – demand

6) Cost function *V(x)*, income function *D(x)*, profit function *P(x)*, where *x* – the volume of production

The tabular method:

x	x_1	x_2	$\cdots$	x_n
y	y_1	y_2	$\cdots$	y_n

The graphical method:



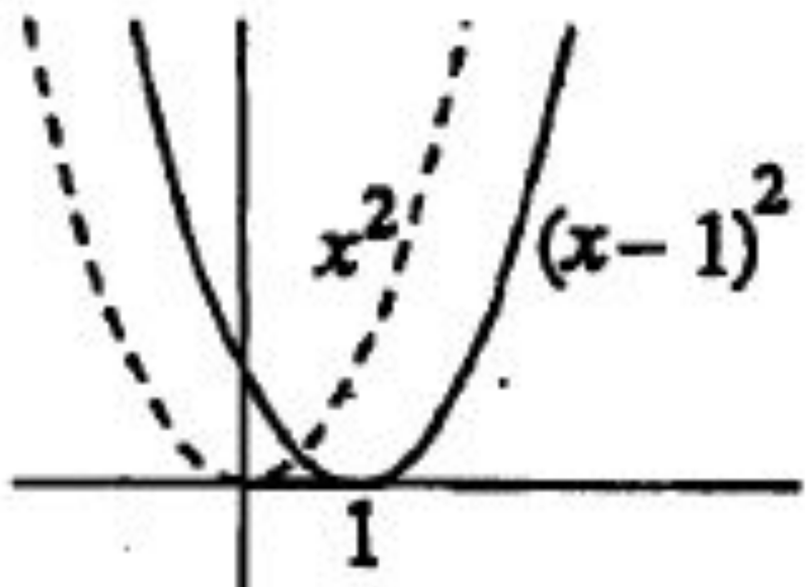
The main ways of the graph transformations

1) Right-left translation:

Moving the $f(x)$ graph c units to the $\begin{cases} \textit{right} \\ \textit{left} \end{cases}$

gives the graph of $\begin{cases} f(x-c) \\ f(x+c) \end{cases}$

Example: $y=(x-1)^2$



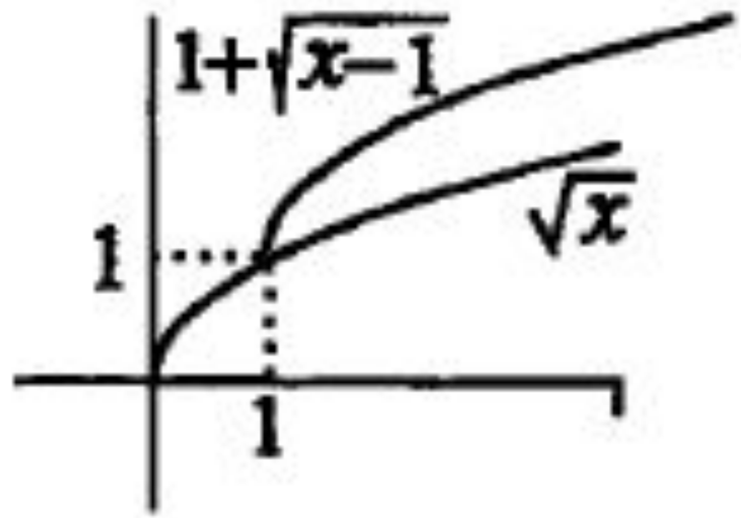
2) Up-down translation:

Moving the $f(x)$ graph c units $\begin{cases} up \\ down \end{cases}$

gives the graph of $\begin{cases} f(x)+c \\ f(x)-c \end{cases}$

Example: Sketch the graph

$$y = 1 + \sqrt{x-1}$$



y=x^2+4x+1 - ?

3) Changing scale: stretching and shrinking

$$\left\{\begin{array}{l}\textit{Stretching} \\ \textit{Shrinking}\end{array}\right. \quad \textit{the } x\textit{-axis by } c$$

$$\textit{changes the graph of } f(x) \textit{ into that of } \left\{\begin{array}{l} f(x/c) \\ f(cx)\end{array}\right.$$

Example: Sketch the graph

$$y = \sin 2x$$

$$\left\{ \begin{array}{l} \textit{Stretching} \\ \textit{Shrinking} \end{array} \right. \quad \textit{the y-axis by } c$$

changes the graph of $f(x)$ into that of $\left\{ \begin{array}{l} c\,f(x) \\ f(x)/c \end{array} \right.$

Example: Sketch the graph:

$$y = 3\sin 2x$$

$$y = -2e^{x-1}$$

The main characteristics of behavior of the function

- monotonic function (increasing or decreasing):
  - increasing: $x_1 < x_2 \quad \Rightarrow \quad f(x_1) < f(x_2)$
  - decreasing: $x_1 < x_2 \quad \Rightarrow \quad f(x_1) > f(x_2)$
- even or odd:
  - even: $f(-x) = f(x), \; x \in D(x)$
  - odd: $f(-x) = -f(x), \; x \in D(x)$

$$y = \frac{x}{x^2 - 1} \qquad y = \frac{2x^2}{x^2 + 1} \qquad y = \frac{x^3 - 1}{2x^2 + 1}$$

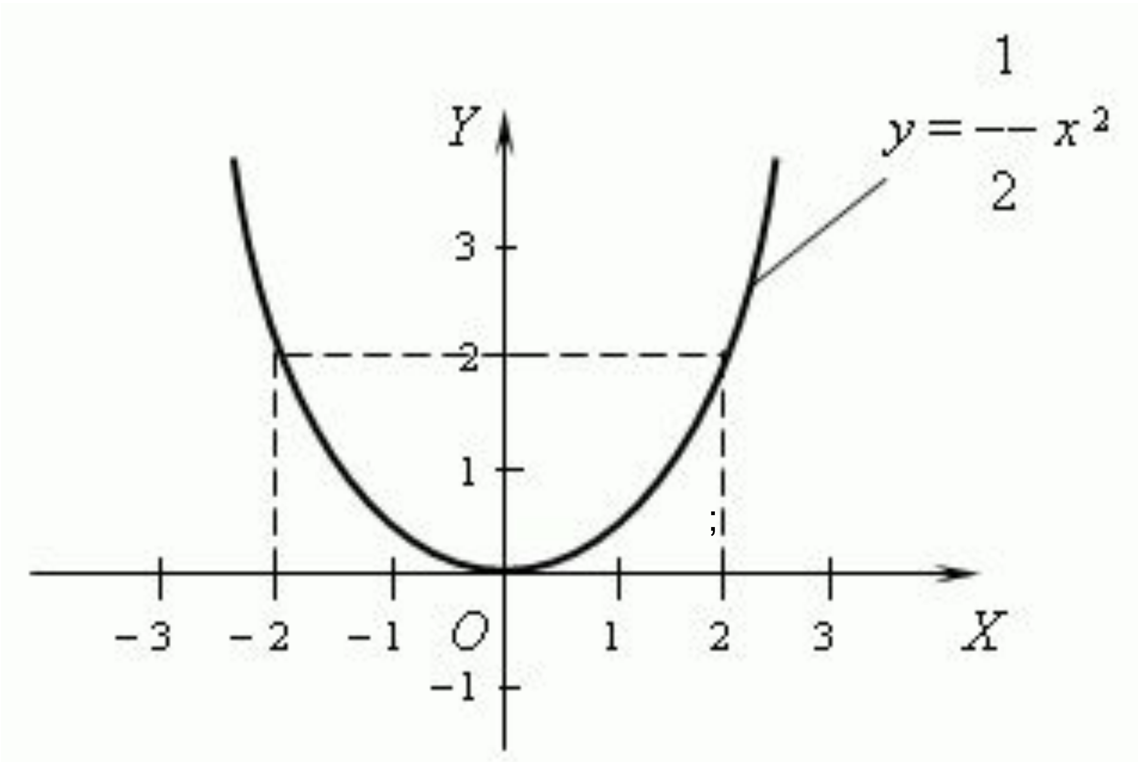
- periodicity: The periodic function is a function that repeats its values in regular intervals

The basic elementary functions:

- The power function;
- The exponential function;
- The logarithmic function;
- The trigonometric functions (4);
- The inverse trigonometric functions (4).

1) The power function: $y = x^{\alpha}, \quad \alpha \in R$

Some particular cases: $a) \quad \alpha = 2n, \quad n \in N: \quad y = x^{2n}$

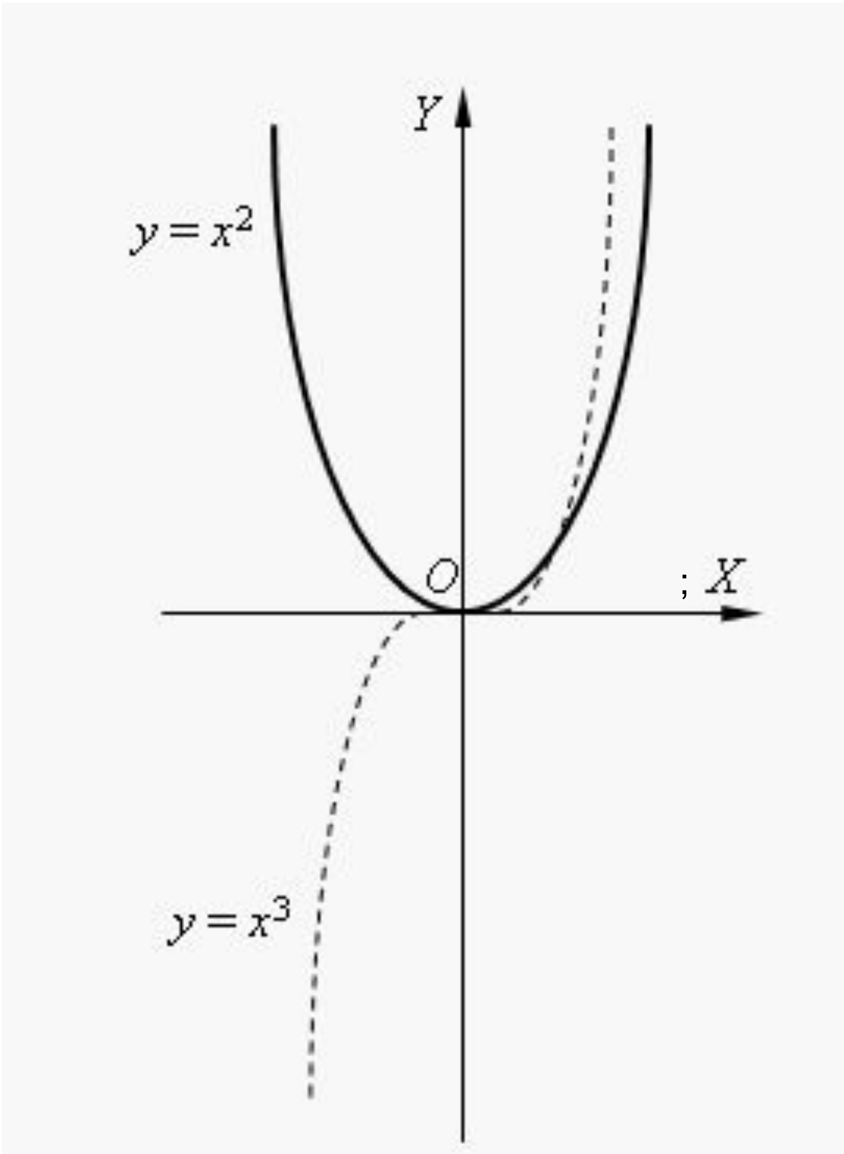


$D(f) = R,$

$E(f) = [0; +\infty[$

- even;
- decreasing on [-∞;0];
- increasing on [0;+∞]

$$b)\ \alpha = 2n-1, \quad n \in N: \quad y = x^{2n-1}$$



:

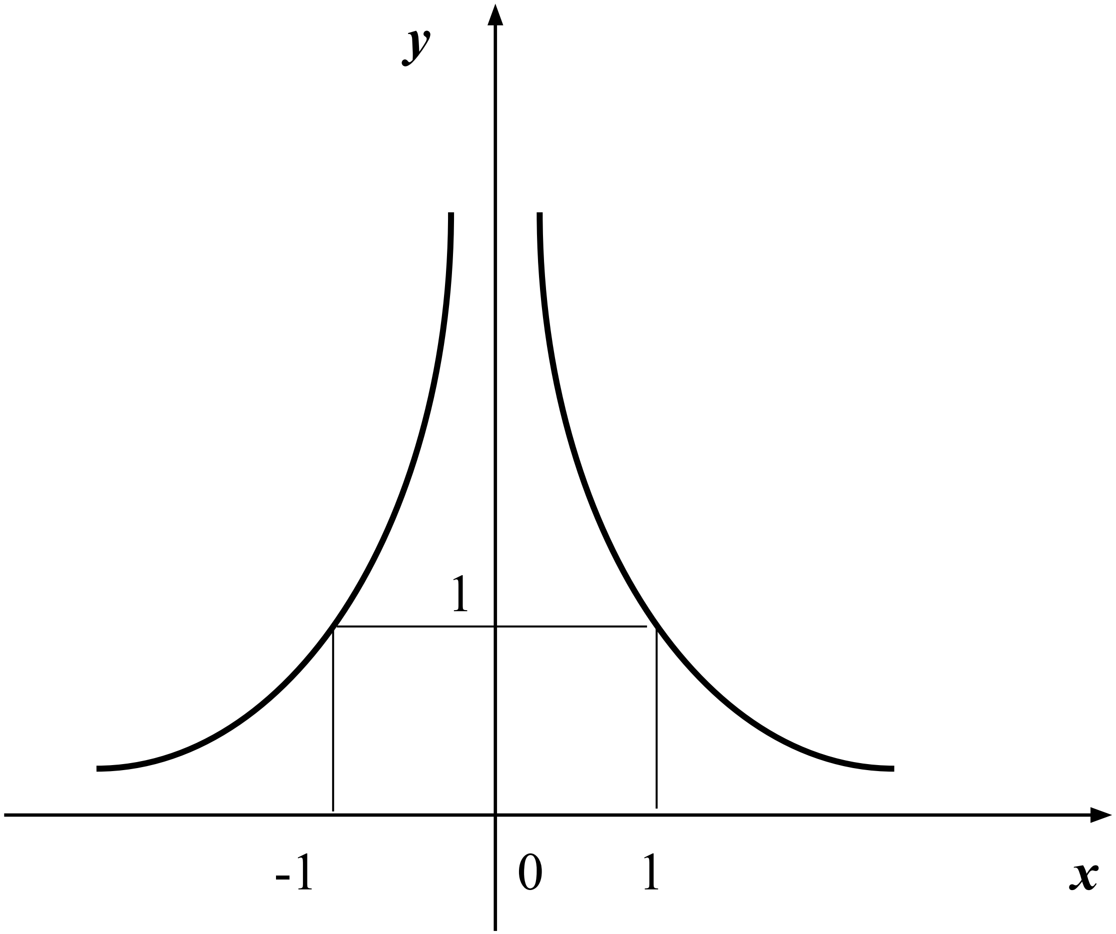
$D(f) = R, \quad E(f) = R$

,

.

- odd;
- increasing on $D(f) = R$

$$c)\ \alpha = -2n, \quad n \in N: \ \ y = \frac{1}{x^{2n}}$$



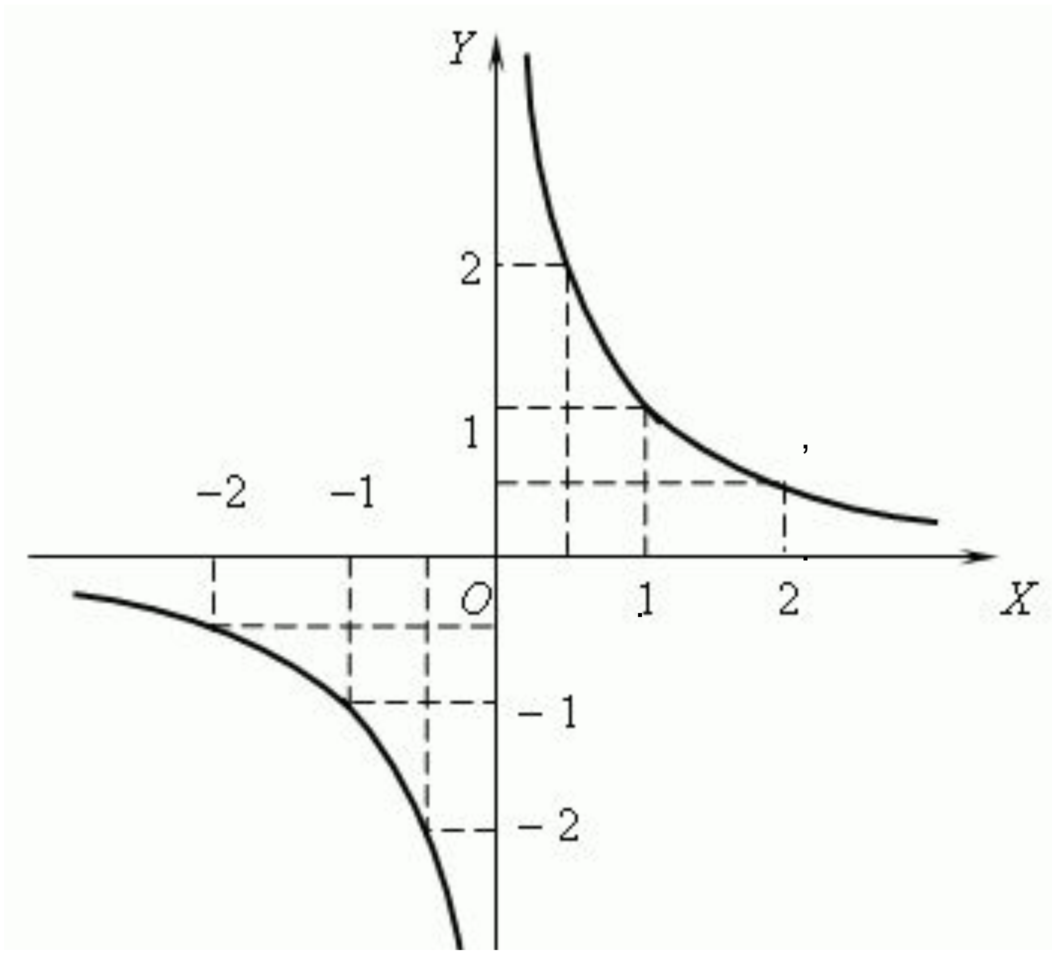
:

$$D(f) = R \setminus \{0\}$$

$$E(f) =]0; +\infty[$$

- even;
- increasing on (-∞; 0),
- decreasing on (0; +∞);

$$d)\ \alpha = -2n+1, \quad n \in N: \qquad y = \frac{1}{x^{2n-1}}$$

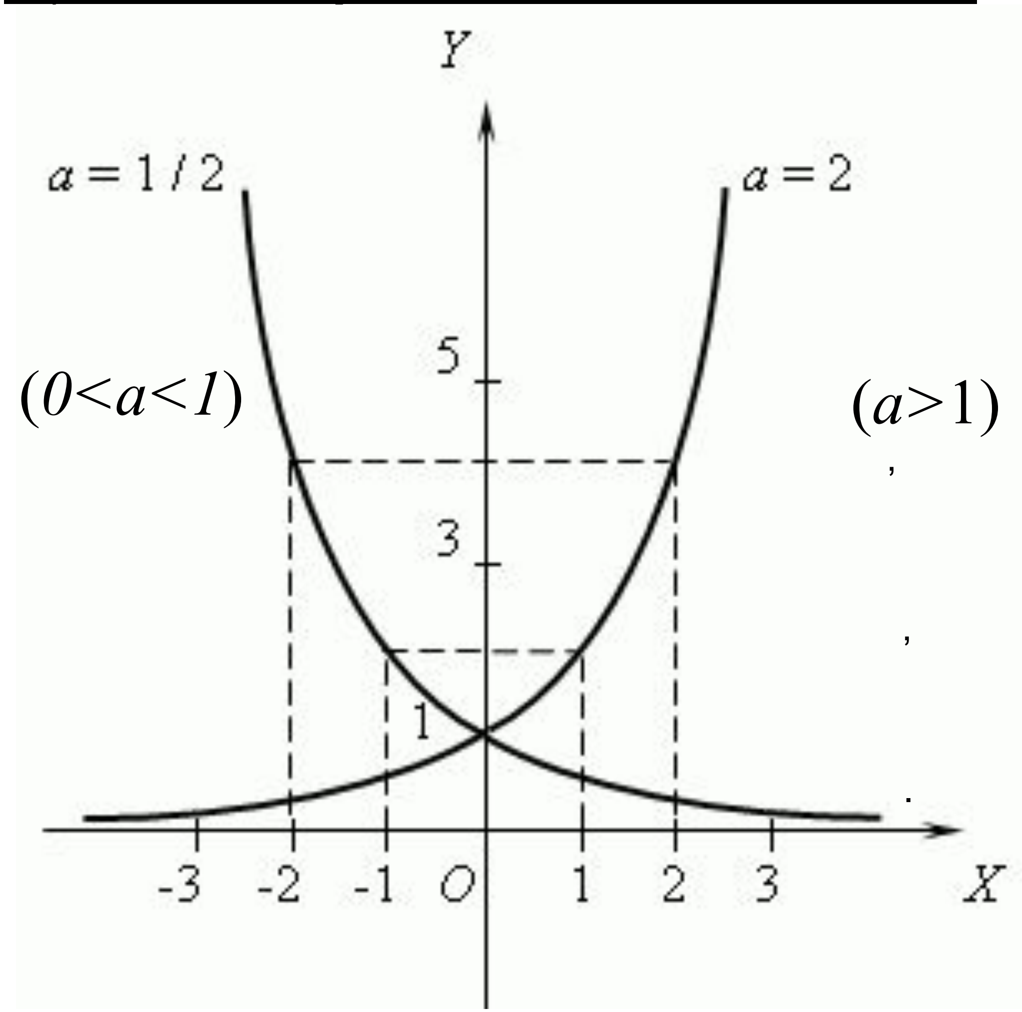


$D(f) = R \setminus \{0\}$

$E(f) = R \setminus \{0\}$

- odd;
- decreasing on $]-\infty; 0[$ and $]0; +\infty[$

2) The exponential function: $y = a^x \left(a > 0; \quad a \neq 1\right)$

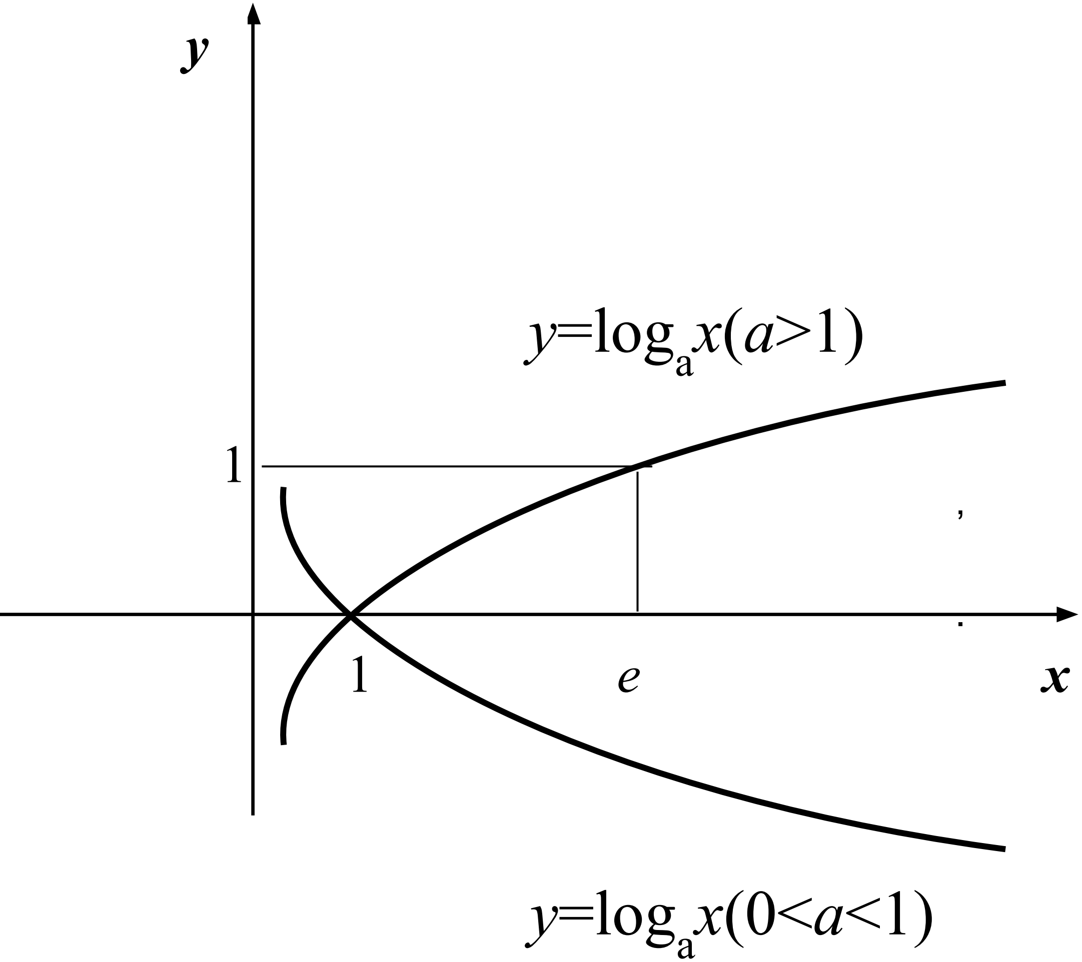


$D(f) = R$

$E(f) =]0; +\infty[$

If 0<*a*<1 then the function is decreasing,
if *a*>1 then the function is increasing.

<u>3) The logarithmic function:</u> $y=log_a x \ (a>0; a \neq 1)$



$D(f)=\left]0;+\infty\right[$

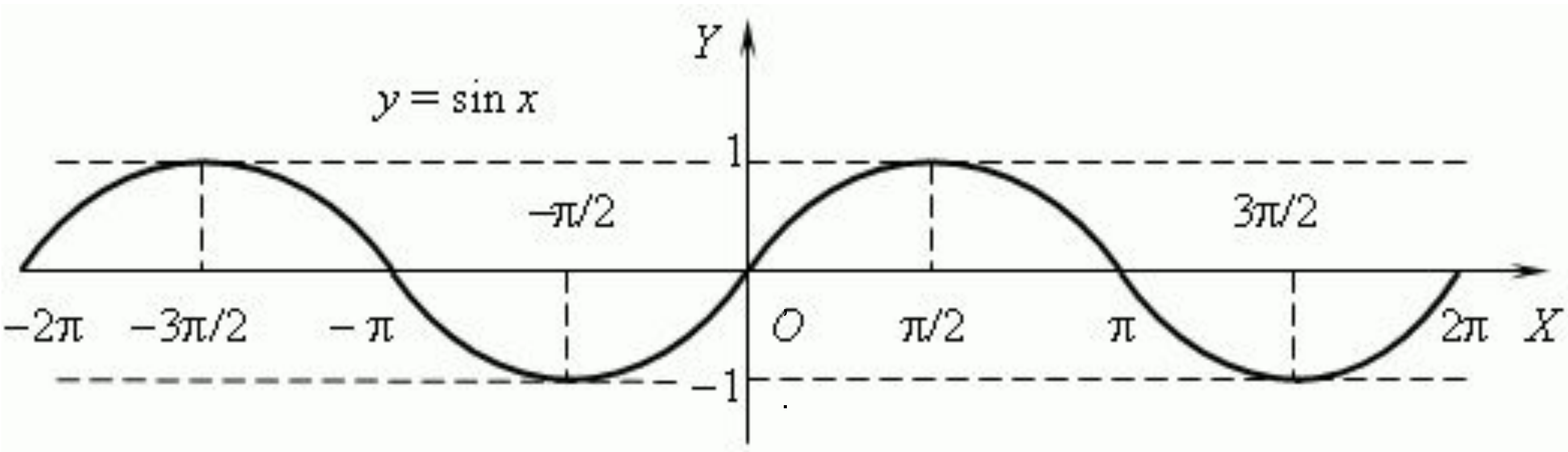
$E(f)=R$

If $0<a<1$ then the function is decreasing,

if $a>1$ then the function is increasing.

4) The trigonometric functions:

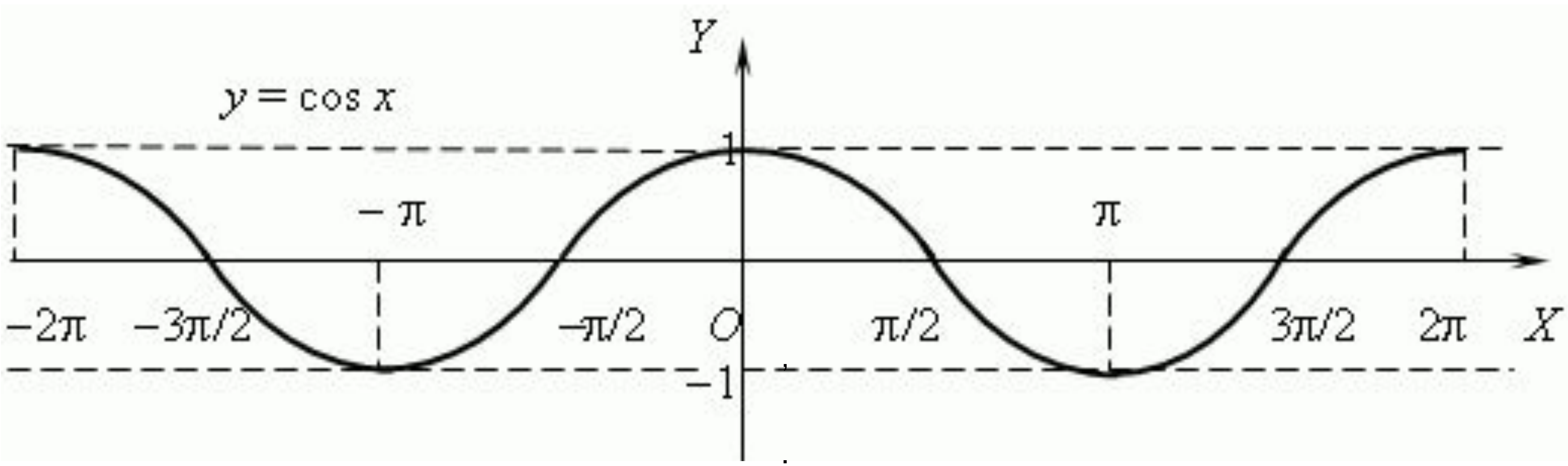
$a)\ y = \sin x$



$$D(f) = R, \quad E(f) = [-1; 1]$$

The function is odd and periodic, period T=2π

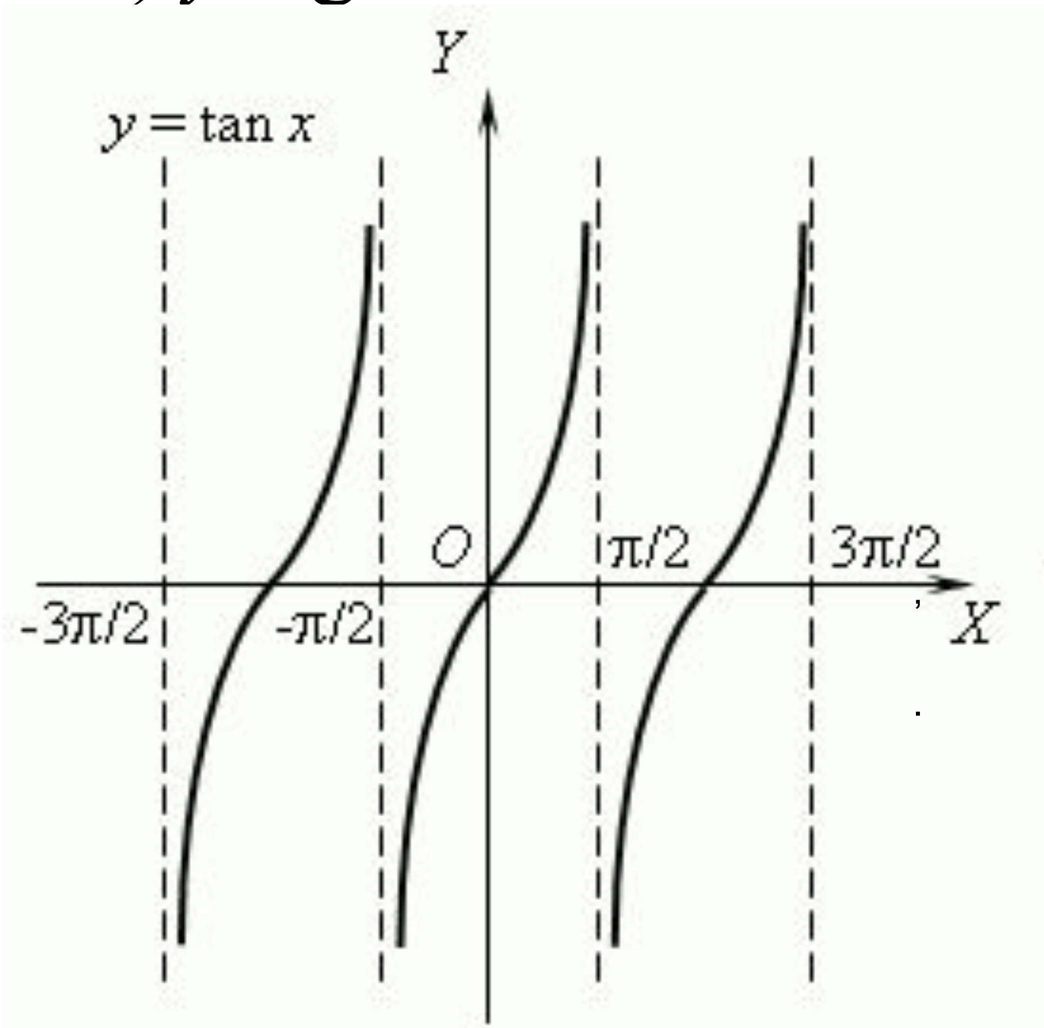
b) y=cos$x$



$$D(f) = R, \qquad E(f) = [-1;1]$$

The function is even and periodic, period T=2π

c) $y=\text{tg}x$

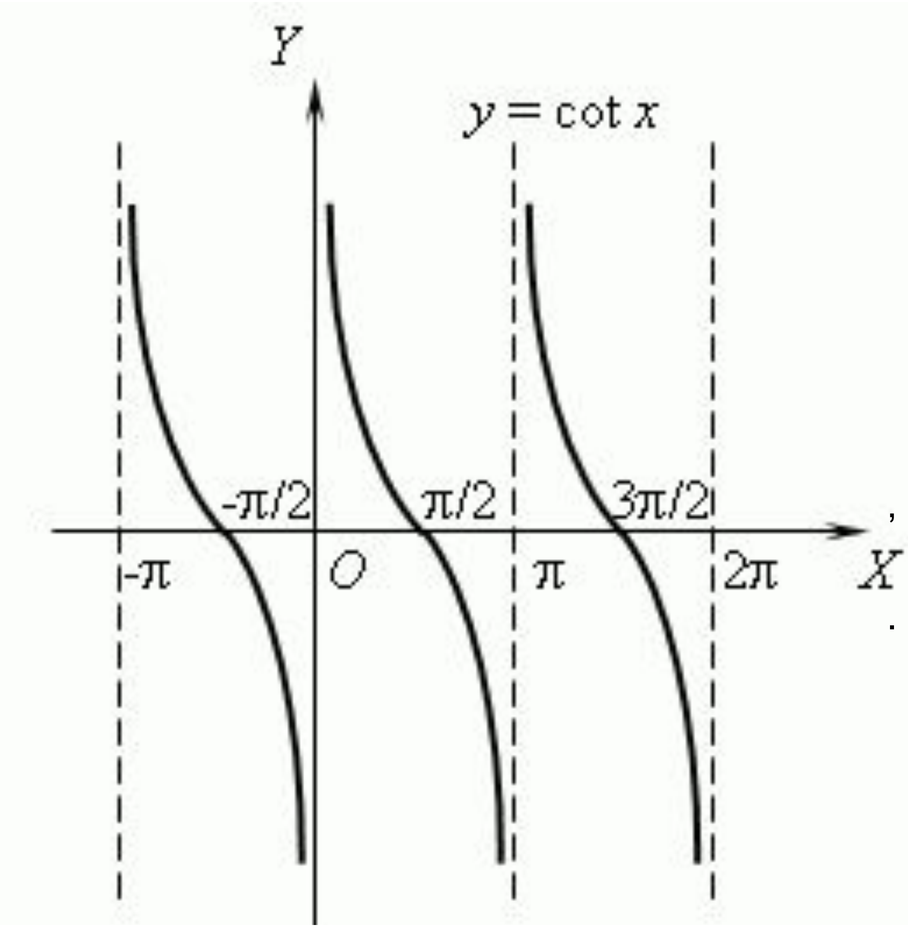


$$D(f) = R \setminus \left\{ \frac{\pi}{2} + \pi k \;\middle|\; k \in Z \right\}$$

$$E(f) = R$$

The function is odd and periodic, period $T=\pi$

d) y=ctg$x$



$$D(f) = R \setminus \{\pi k \mid k \in Z\}$$

$$E(f) = R$$

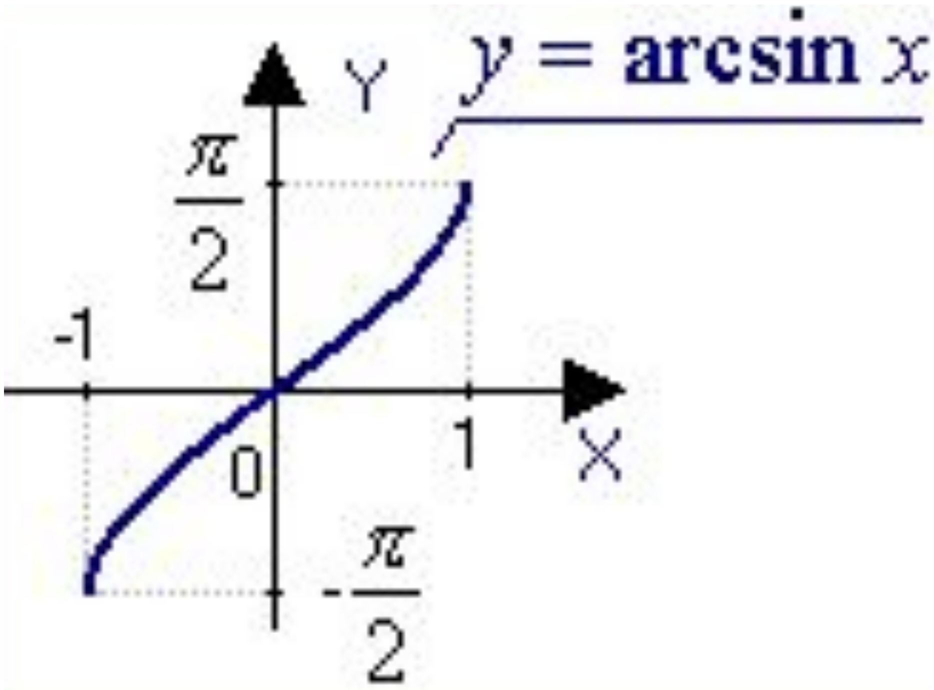
The function is odd and periodic, period $T=\pi$

5) The inverse trigonometric functions:

a) $y=\arcsin x$

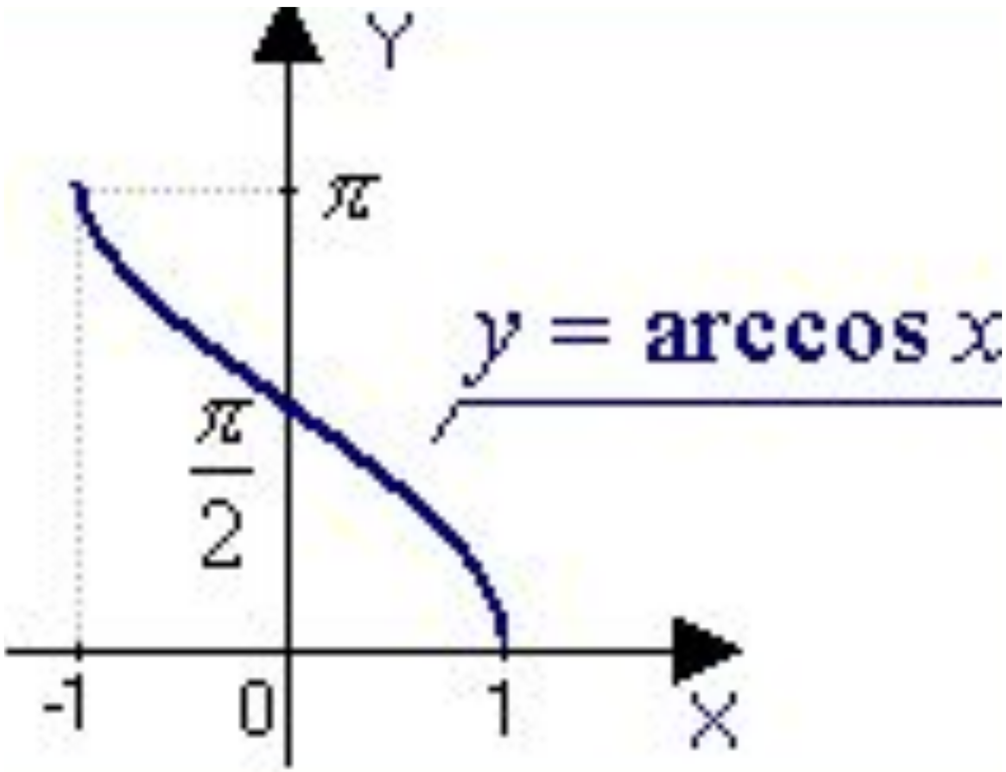
$$D(f)=[-1;1]$$

$$E(f)=\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$$



The function is odd and increasing

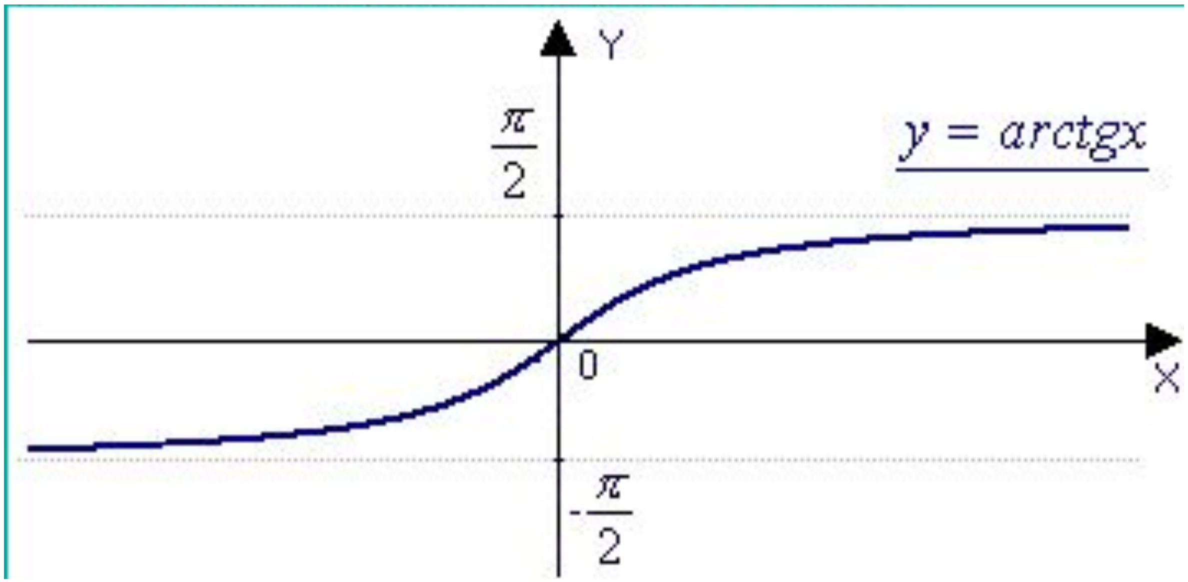
b) y=arccos$x$



$$D(f) = [-1; 1]$$
$$E(f) = [0; \pi]$$

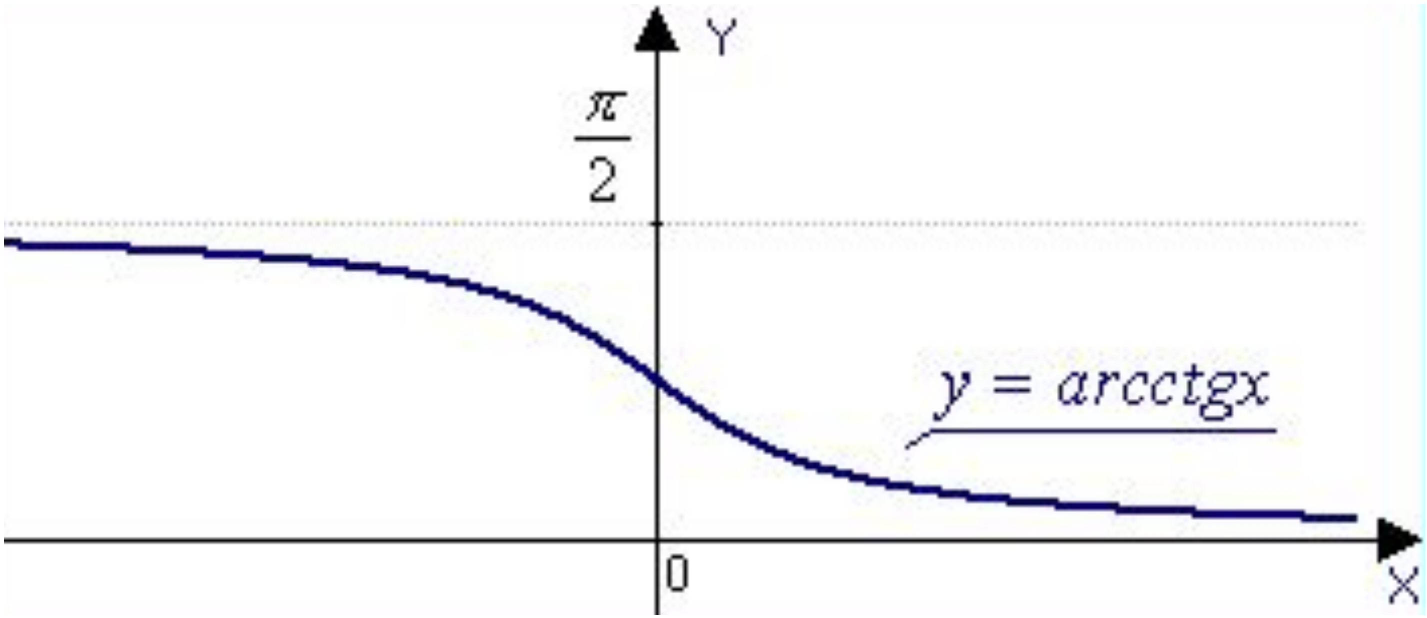
The function is decreasing

c) y=arctg$x$, $E(f) = R,$ $E(f) = \left]-\frac{\pi}{2};\ \frac{\pi}{2}\right[$



The function is odd and increasing

d) y=arcctg$x$, $D(f) =]-\infty; +\infty[$, $E(f) =]0; \pi[$



The function is decreasing

Examples

1) $y = \dfrac{e^{\cos x} - 3arctgx}{\ln^2 x + 2}$,

2) $y = |x|$,

3) $y = 1 + x + x^2 + x^3 + \ldots$,

4) $y = \begin{cases} x + 1, & x \le 0, \\ x^2, & x > 0 \end{cases}$

$y=f(u)$, $u=g(x)$

$y=f(g(x))$ – the <u>composite function</u>

u – the intermediate variable,

$f(u)$ - external function,

$g(x)$ - internal function

An <u>elementary function</u> is a function of one variable built from a finite number of the basic elementary function and constants through composition and combinations using the four elementary operations (+ – × ÷).

<u>Examples:</u>

$$\frac{e^{\tan(x)}}{1+x^2}\sin\left(\sqrt{1+\ln^2 x}\right)$$

$$y=|x|,$$

$$y=1+x+x^2+x^3+\ldots$$

$$y=\begin{cases} x^2+1, & x<2, \\ x+4, & x\geq 2 \end{cases}$$

The home work:

Sketch the graph:

a) $y=x^2-4x+5$;

b) $y=3+2ln(x-1)$

c) $y = 2\cos\frac{x}{2}$