KAZAKH BRITISH TECHNICAL UNIVERSITY

Physics 2

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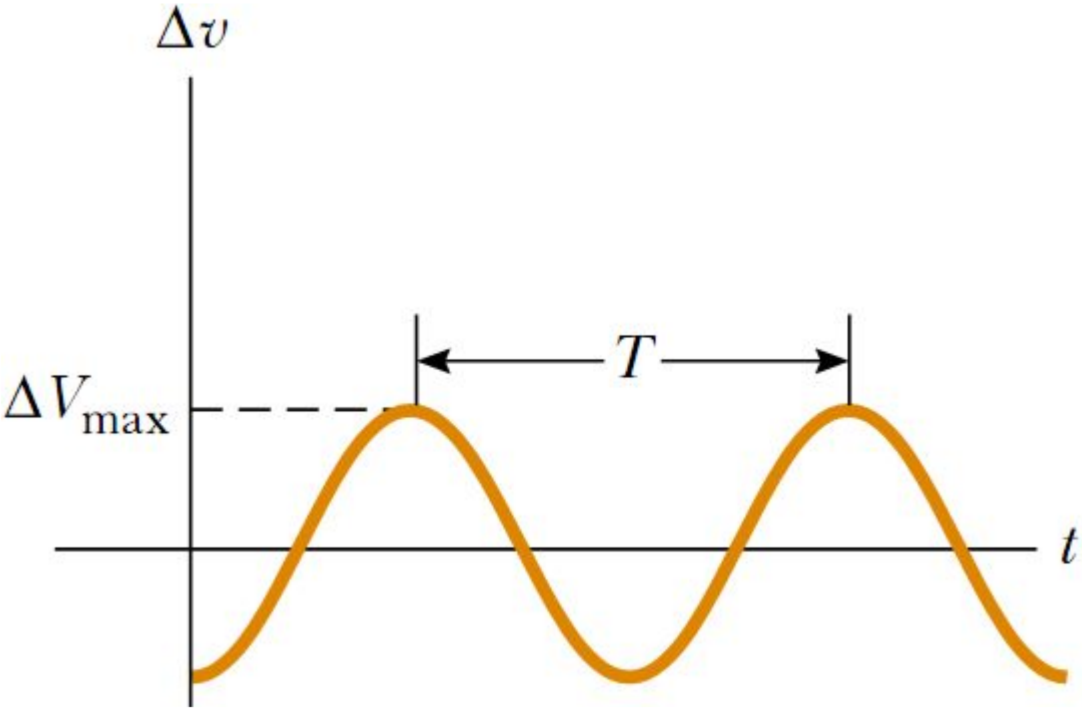
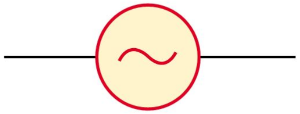
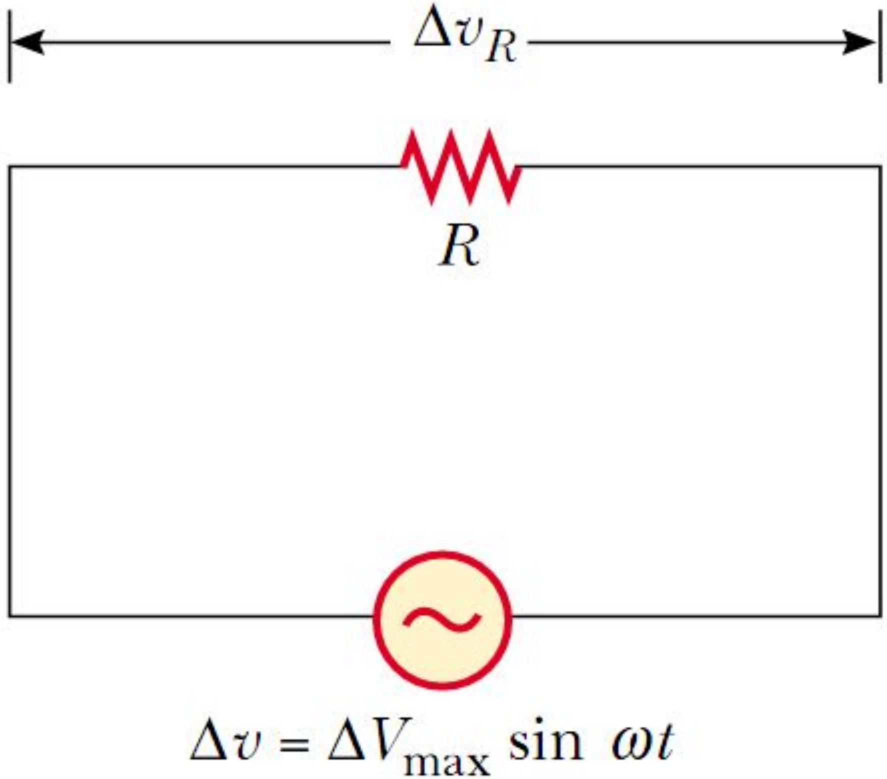
Lecture 3

- Alternating Current (AC)
- Inductors in AC Circuits
- Capacitors in AC Circuits
- Series RLC Circuit
- Impedance

Alternating Current (AC)



- The voltage supplied by an AC source is harmonic (sinusoidal) with a period *T*.
- AC source is designated by 



Applying Kirchhoff's loop, at any instant:

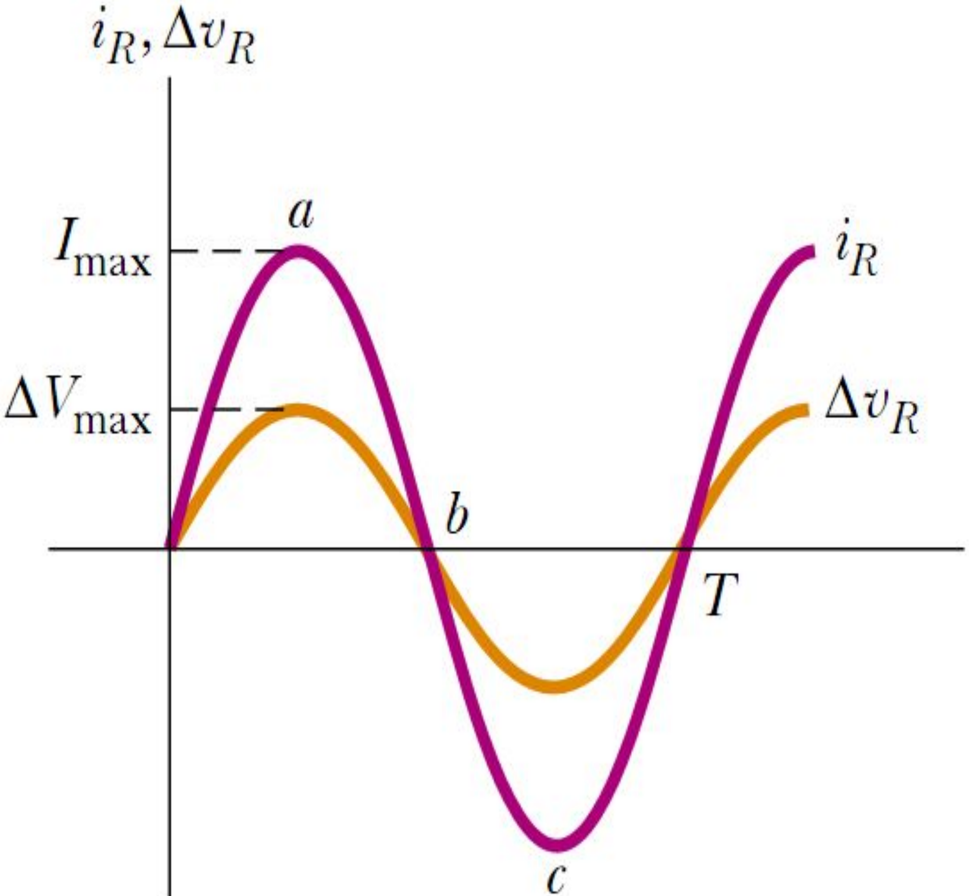
$$\Delta v = \Delta v_R = \Delta V_{max} \sin \omega t$$

The instantaneous current in the resistor is:

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

Where I_{max} is the maximum current: $I_{max} = \dfrac{\Delta V_{max}}{R}$

And eventually: $\Delta v_R = I_{max} R \sin \omega t$



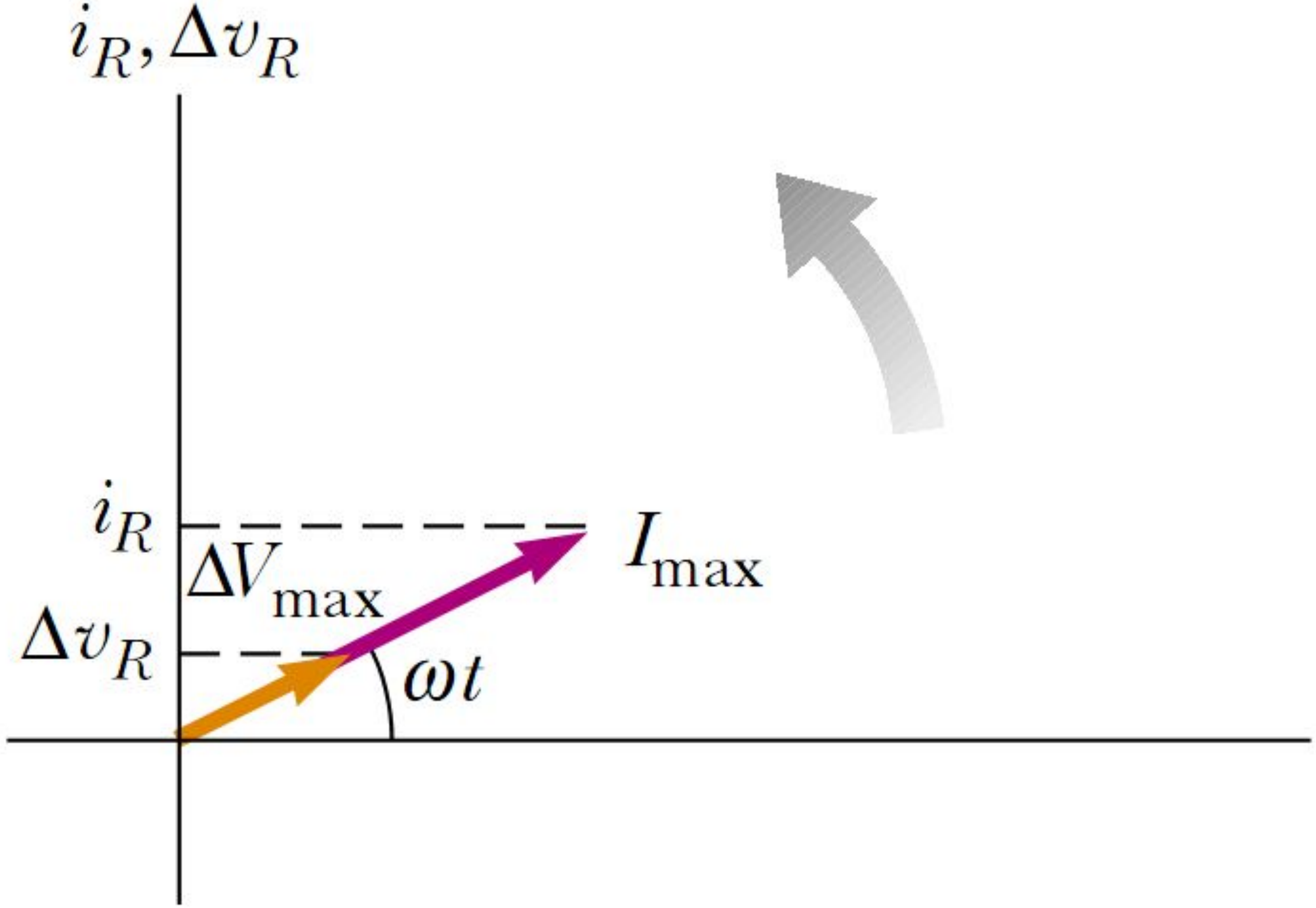
Plots of the instantaneous current i_R and instantaneous voltage Δv_R across a resistor as functions of time. The current is in phase with the voltage, which means that the current is zero when the voltage is zero, maximum when the voltage is maximum, and minimum when the voltage is minimum. At time t = T, one cycle of the time-varying voltage and current has been completed.

So, for a sinusoidal applied voltage, the current in a resistor is always **in phase** with the voltage across the resistor.

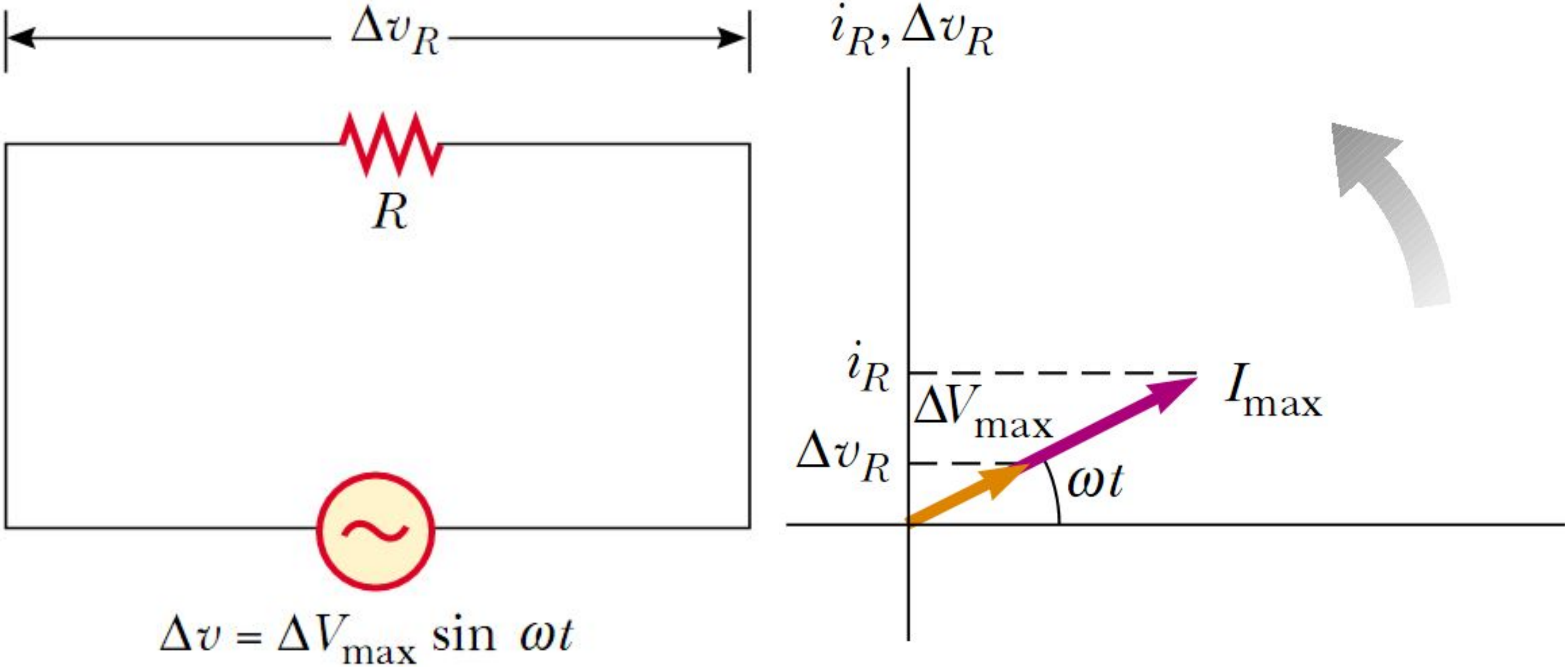
Phasor Diagrams

A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents (V_{max} for voltage and I_{max} for current in the present discussion) and which **rotates counterclockwise** at an angular speed equal to the angular frequency associated with the variable. The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.

Phasor diagram for a circuit with a resistor is:

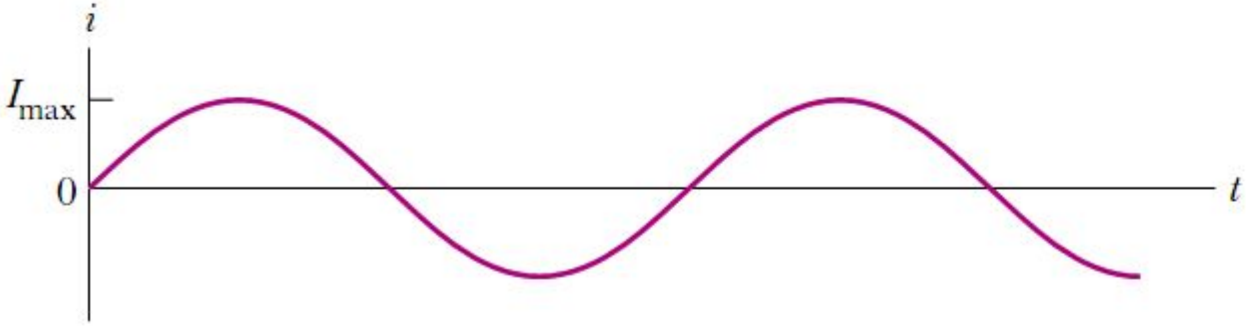


The phasor diagram for the resistive circuit shows that the current is in phase with the voltage.



The projections of the phasor arrows onto the vertical axis are determined by a sine function of the angle of the phasor with respect to the horizontal axis. We can use the projections of phasors to represent values of current or voltage that vary sinusoidally in time.

RMS



$$I_{rms} = \sqrt{\overline{i^2}}$$

- Because I^2 varies as $\sin^2 \omega t$ and because the average value of I^2 is $I_{max}/2$, the rms current is

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max}$$

- Thus, the average power delivered to a resistor that carries an alternating current is

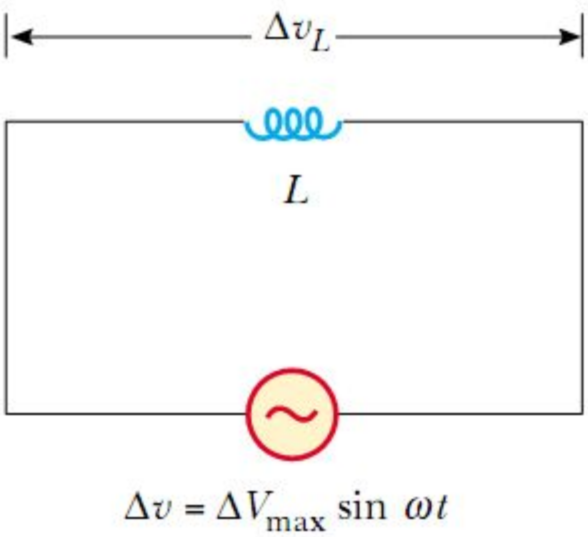
$$\mathcal{P}_{av} = I_{rms}^2 R$$

- Alternating voltage is also best discussed in terms of rms voltage, and the relationship is identical to that for current:

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max}$$

One reason we use rms values when discussing alternating currents and voltages in this chapter is that AC ammeters and voltmeters are designed to read rms values. Furthermore, with rms values, many of the equations we use have the same form as their direct current counterparts.

Inductors in AC Circuits



Kirchhoff's rule for AC circuit with an inductor is:

$$\Delta v - L\frac{di}{dt} = 0$$

After derivation we get:

$$i_L = \frac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

i_L is the current through the inductor L.

For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° (one-quarter cycle in time).

The maximal current in the inductor is

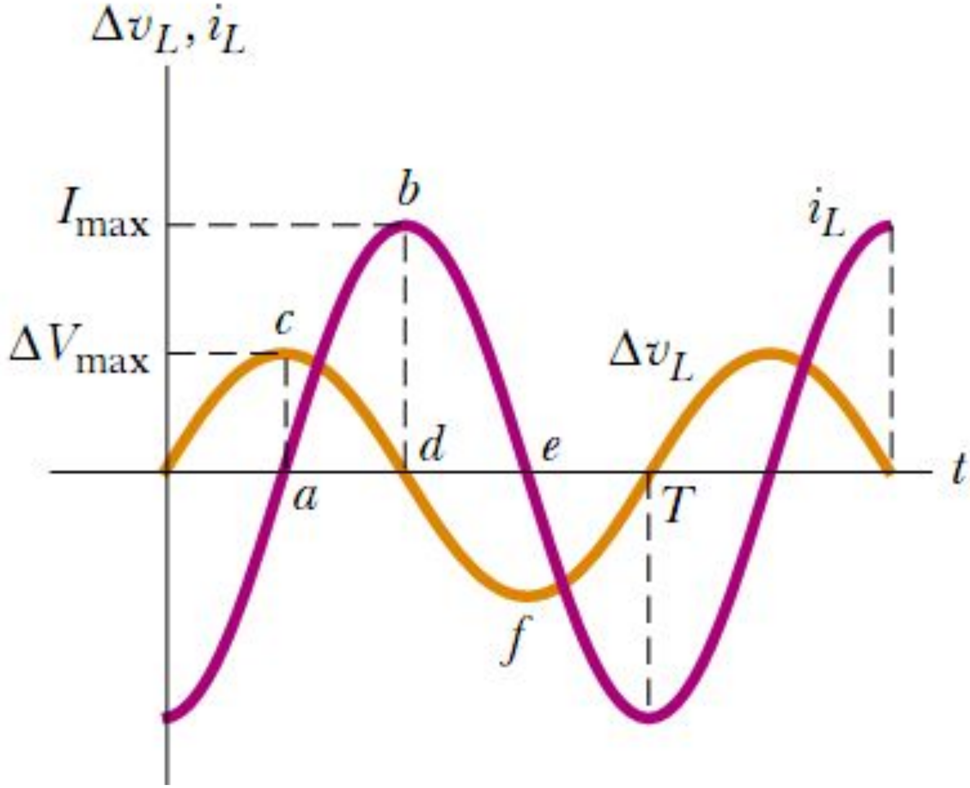
$$I_{\max} = \frac{\Delta V_{\max}}{\omega L}$$

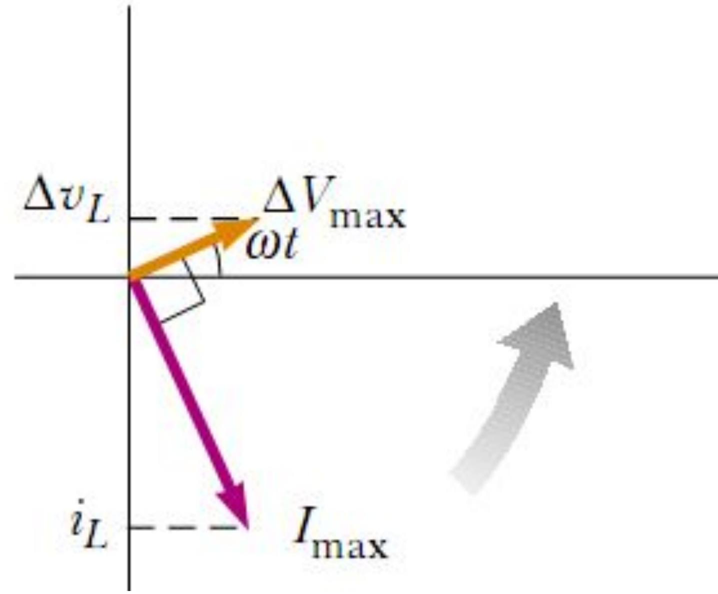
We can define the **inductive reactance** as resistance of an inductor to the harmonic current:

$$X_L \equiv \omega L$$

The instantaneous voltage across the inductor is:

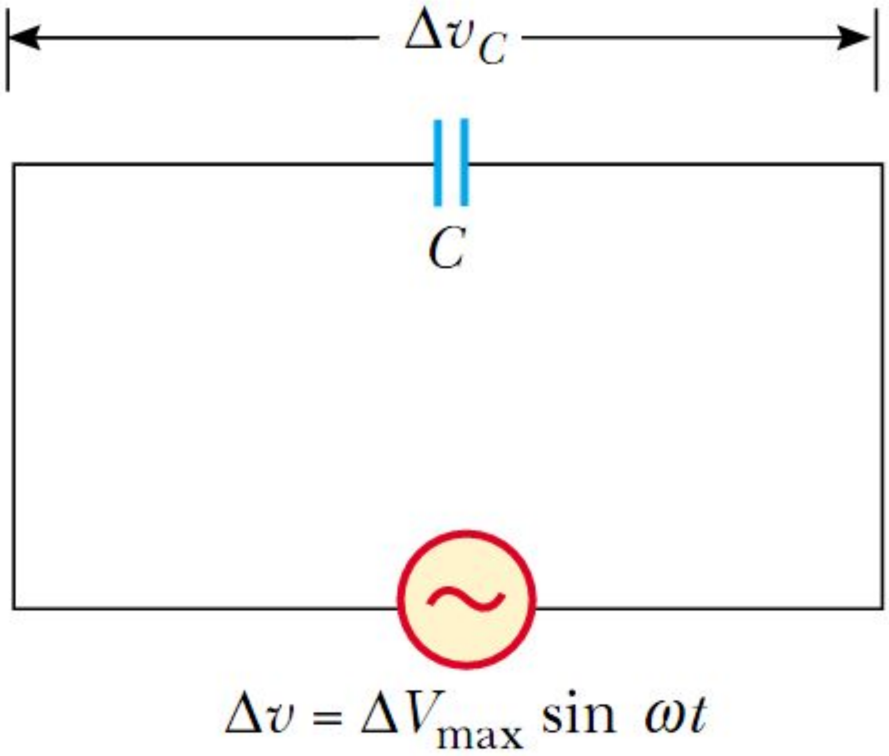
$$\Delta v_L = -L\frac{di}{dt} = -\Delta V_{\max}\sin\omega t = -I_{\max}X_L\sin\omega t$$





Phasor diagram for the inductive circuit, showing that the current lags behind the voltage by 90°.

Capacitors in AC



The current is π/2 rad = 90° out of phase with the voltage across the capacitor:

$$i_C = \omega C \Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

For a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90°.

The maximal current is:

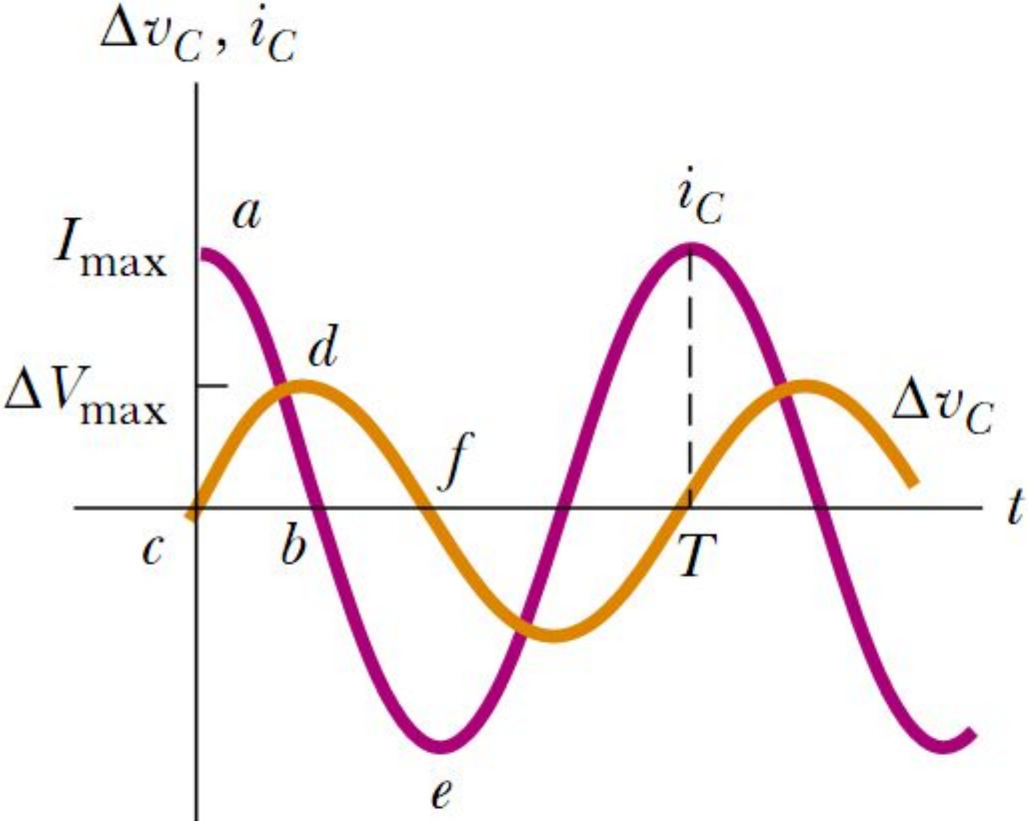
$$I_{\max} = \omega C \, \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)}$$

The **capacitive reactance** of the capacitor to the sinusoidal current is:

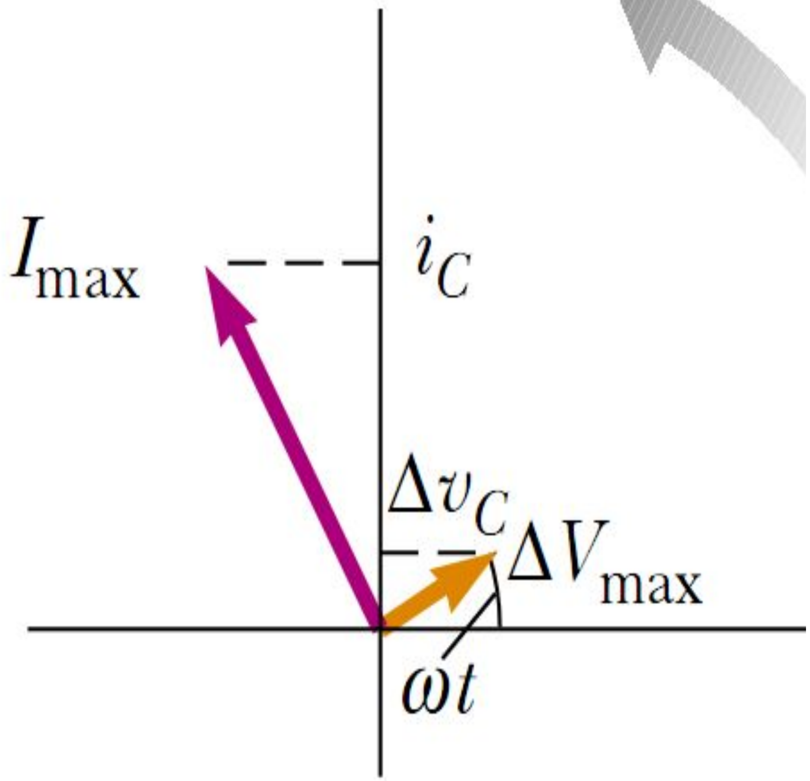
$$X_C \equiv \frac{1}{\omega C}$$

Then the instantaneous voltage across the capacitor is:

$$\Delta v_C = \Delta V_{\max} \sin \omega t = I_{\max} X_C \sin \omega t$$

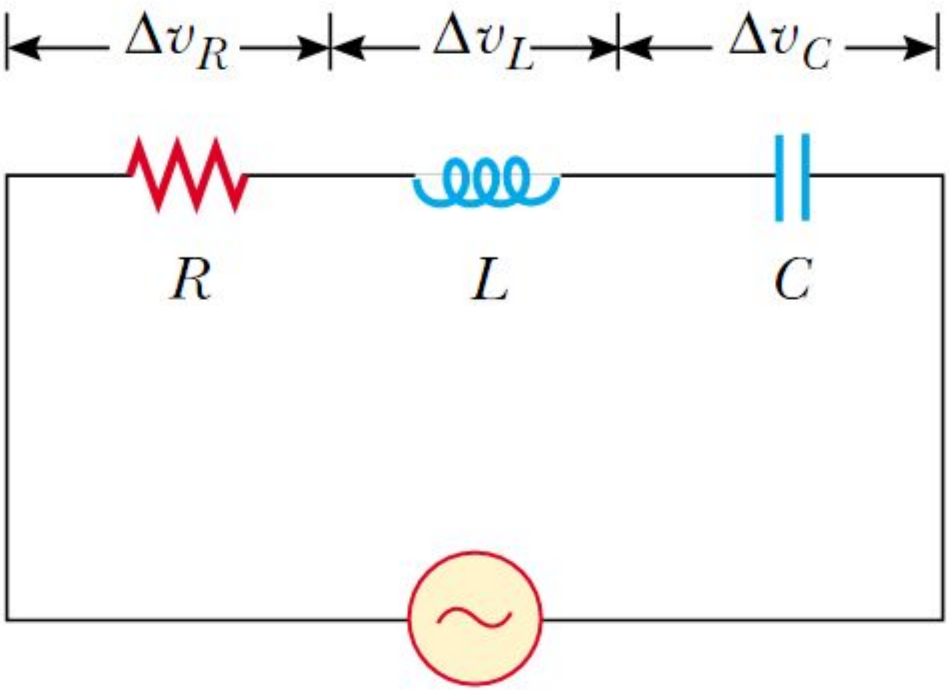


Plot of the instantaneous current i_C and instantaneous voltage ΔV_C across a capacitor as functions of time. The voltage lags behind the current by 90°.



Phasor diagram for the capacitive circuit, showing that the current leads the voltage by 90°.

The RLC Series Circuit



For convenience, and not losing generalization, we can assume that the applied voltage is

$$\Delta v = \Delta V_{\max} \sin \omega t$$

and the current is

$$i = I_{\max} \sin(\omega t - \phi)$$

- Where $\varphi = const$ is some phase angle between the current and the applied voltage. Because the elements are in series, the current everywhere in the circuit must be the same at any instant. That is, the current at all points in a series AC circuit has the same amplitude and phase.

The voltage across each element has a different amplitude and phase:

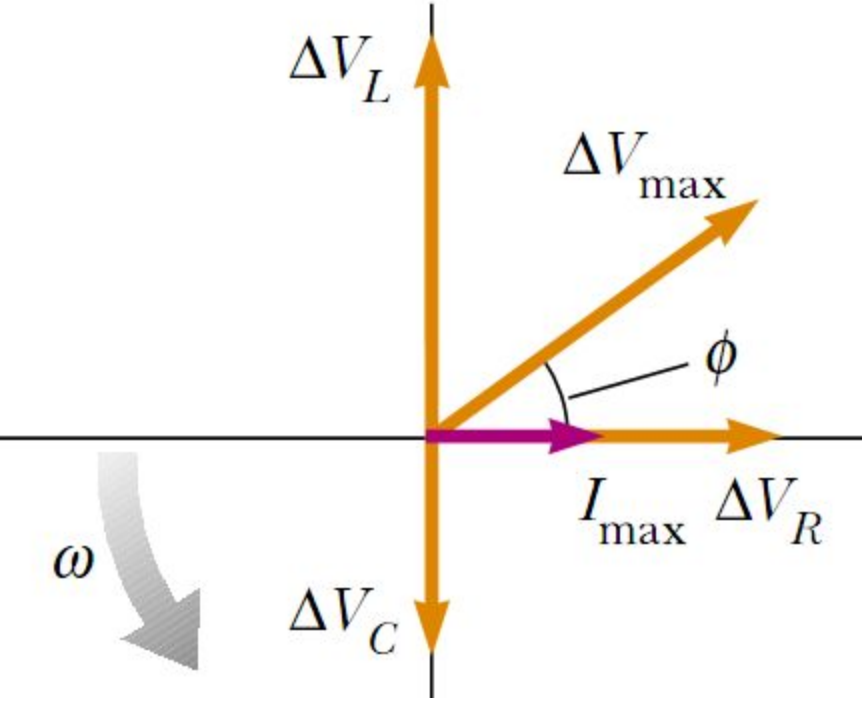
$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin \left(\omega t + \frac{\pi}{2} \right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin \left(\omega t - \frac{\pi}{2} \right) = -\Delta V_C \cos \omega t$$

$$\Delta V_R = I_{\max} R \qquad \Delta V_L = I_{\max} X_L \qquad \Delta V_C = I_{\max} X_C$$

$$\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$$



$$\Delta V_{\max} = \sqrt{\Delta V_R{}^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{\max} R)^2 + (I_{\max} X_L - I_{\max} X_C)^2}$$

$$\Delta V_{\max} = I_{\max} \sqrt{R^2 + (X_L - X_C)^2}$$

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Impedance

Using the previous calculations we can define a new parameter **impedance**:

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

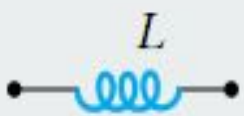
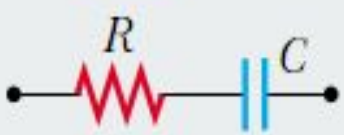
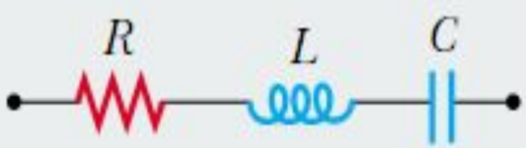
So, the amplitudes of voltage and current are related as

$$\Delta V_{\max} = I_{\max} Z$$

Using the phasor diagram:

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C{}^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L{}^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

[a] In each case, an AC voltage (not shown) is applied across the elements.

Power in AC Circuit

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R$$

- The average power delivered by the source is converted to internal energy in the resistor.
- No power losses are associated with pure capacitors and pure inductors in an AC circuit.

Series RLC Circuit Resonance

A series RLC circuit is in resonance when the current has its maximum value.

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z}$$

So resonance is at $X_L = X_C$, the frequency ω_0 when $X_L = X_C$ is called the resonance frequency:

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

This frequency corresponds to the natural frequency of oscillation of an LC circuit

The average power dissipating in the resistor is

$$\mathcal{P}_{\text{av}} = \frac{(\Delta V_{\text{rms}})^2 \; R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0{}^2)^2}$$

Then at resonance the average power is a maximum and equals $(\Delta V_{\text{rms}})^2/R$ **.**

Units in Si

- voltage (potential difference) V V (Volt)
- current (electric current) I A (Ampere)
- inductance L H (Henry)
- inductive reactance X_L Ω (Ohm)
- capacitive reactance X_C Ω (Ohm)
- Impedance Z Ω (Ohm)
- Power P W (Watt)