

МБОУ СОШ № 6

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Электронный справочник по тригонометрическим формулам

Тригонометрические формулы

1. Тригонометрические тождества.
2. Формулы сложения.
3. Формулы двойного аргумента.
4. Формулы половинного аргумента.
5. Формулы приведения.
6. Формулы преобразования суммы и разности в произведение.
7. Формулы преобразования произведения в сумму.

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\cos a = \pm \sqrt{1 - \sin^2 a}$$

$$\sin a = \pm \sqrt{1 - \cos^2 a}$$

$$\operatorname{tga} \cdot \operatorname{ctga} = 1$$

$$\operatorname{tga} = \frac{1}{\operatorname{ctga}}$$

$$1 + \operatorname{tg}^2 a = \frac{1}{\cos^2 a}$$

$$\operatorname{ctga} = \frac{1}{\operatorname{tga}}$$

$$1 + \operatorname{ctg}^2 a = \frac{1}{\sin^2 a}$$

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1) Доказать, что при $a \neq \pi k, k \in \mathbb{Z}$ справедливо равенство $1 + \operatorname{ctg}^2 a = \frac{1}{\sin^2 a}$

$$1 + \operatorname{ctg}^2 a = 1 + \frac{\cos^2 a}{\sin^2 a} = \frac{\sin^2 a + \cos^2 a}{\sin^2 a} = \frac{1}{\sin^2 a}$$

2) Вычислить $\sin a$, если $\cos a = -\frac{3}{5}$ и $\pi < a < \frac{3\pi}{2}$

Воспользуемся формулой $\sin a = \pm \sqrt{1 - \cos^2 a}$

Т.к. $\pi < a < \frac{3\pi}{2}$ – III квадрант, то $\sin a < 0$

$$\sin a = -\sqrt{1 - \cos^2 a} = -\sqrt{1 - \frac{9}{25}} = -\frac{4}{5}$$

3) Вычислить $\operatorname{ctg} a$, если $\operatorname{tga} = 13$

По формуле $\operatorname{ctga} = \frac{1}{\operatorname{tga}}$ находим:

$$\operatorname{ctga} = \frac{1}{\operatorname{tga}} = \frac{1}{13}$$

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$$\sin(a + \beta) = \sin a \cdot \cos \beta + \cos a \cdot \sin \beta$$

$$\sin(a - \beta) = \sin a \cdot \cos \beta - \cos a \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \cdot \operatorname{tg}\beta}$$

$$\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha \cdot \operatorname{tg}\beta}$$

$$\operatorname{ctg}(\alpha + \beta) = \frac{\operatorname{ctg}\alpha \cdot \operatorname{ctg}\beta - 1}{\operatorname{ctg}\alpha + \operatorname{ctg}\beta}$$

$$\operatorname{ctg}(\alpha - \beta) = \frac{\operatorname{ctg}\alpha \cdot \operatorname{ctg}\beta + 1}{\operatorname{ctg}\beta - \operatorname{ctg}\alpha}$$

$$1) \sin 585^{\circ} = \sin(360^{\circ} + 225^{\circ}) = \sin 225^{\circ} = \sin(180^{\circ} + 45^{\circ}) = \\ = -\sin 45^{\circ} = -\sqrt{2} / 2$$

$$2) \cos 75^{\circ} = \cos(45^{\circ} - 30^{\circ}) = \cos 45^{\circ} \cdot \cos 30^{\circ} - \sin 45^{\circ} \cdot \sin 30^{\circ} = \\ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$3) \operatorname{tg} 225^{\circ} = \operatorname{tg}(180^{\circ} + 45^{\circ}) = \frac{\operatorname{tg} 180^{\circ} + \operatorname{tg} 45^{\circ}}{1 - \operatorname{tg} 180^{\circ} \cdot \operatorname{tg} 45^{\circ}} = \frac{0 + 1}{1 - 0 \cdot 1} = 1$$

$$4) \operatorname{ctg} 780^{\circ} = \operatorname{ctg}(2 \cdot 360^{\circ} + 60^{\circ}) = \operatorname{ctg} 60^{\circ} = \sqrt{3} / 3$$

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$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2\operatorname{ctg} \alpha}$$

$$1) \sin 120^{\circ} = \sin(2 \cdot 60^{\circ}) = 2 \cdot \sin 60^{\circ} \cdot \cos 60^{\circ} = 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$2) \cos 240^{\circ} = \cos^2 120^{\circ} - \sin^2 120^{\circ} = (\cos^2 60^{\circ} - \sin^2 60^{\circ})^2 - (2 \sin 60^{\circ} \cos 60^{\circ})^2 = \\ = \left(\frac{3}{4} - \frac{1}{4}\right)^2 - \left(2 \cdot \frac{\sqrt{3}}{4}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

$$3) \operatorname{tg} 720^{\circ} = \operatorname{tg}(2 \cdot 360^{\circ} + 0^{\circ}) = \operatorname{tg} 0^{\circ} = 0$$

$$4) \text{ если } \operatorname{ctg} \alpha = \frac{\sqrt{3}}{2}, \text{ то } \operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2 \operatorname{ctg} \alpha} = \frac{\left(\frac{\sqrt{3}}{2}\right)^2 - 1}{2 \cdot \frac{\sqrt{3}}{2}} = \frac{-\frac{1}{4}}{\sqrt{3}} = -\frac{\sqrt{3}}{12}$$

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$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

$$\operatorname{tg}^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

$$\operatorname{ctg}^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{1 - \cos \alpha}$$

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$$1) \sin^2 15^\circ = \frac{1 - \cos 30^\circ}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2} = \frac{2 - \sqrt{3}}{4}$$

$$2) 2 \cos^2 \frac{\pi}{8} - 1 = 2 \cdot \frac{1 + \cos \frac{\pi}{4}}{2} - 1 = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$3) \text{если } \cos \alpha = 0,6 \text{ и } 0 < \alpha < \frac{\pi}{2}, \text{ то } \operatorname{tg} \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \sqrt{\frac{0,4}{1,6}} =$$
$$= \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$4) \text{если } \cos \alpha = -0,02 \text{ и } 0 < \alpha < \pi, \text{ то } \cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}} = \sqrt{\frac{1 - 0,02}{2}} = 0,7$$

$$\sin(a + 2\pi k) = \sin a \quad \cos(a + 2\pi k) = \cos a \quad k \in \mathbb{Z}$$

$$\begin{aligned} \cos\left(\frac{\pi}{2} - a\right) &= \sin a & \cos\left(\frac{\pi}{2} + a\right) &= -\sin a \\ \cos(\pi - a) &= -\cos a & \cos(\pi + a) &= -\cos a \\ \cos\left(\frac{3\pi}{2} - a\right) &= -\sin a & \cos\left(\frac{3\pi}{2} + a\right) &= \sin a \end{aligned}$$

$$\begin{aligned} \sin\left(\frac{\pi}{2} - a\right) &= \cos a & \sin\left(\frac{\pi}{2} + a\right) &= \cos a \\ \sin(\pi - a) &= \sin a & \sin(\pi + a) &= -\sin a \\ \sin\left(\frac{3\pi}{2} - a\right) &= -\cos a & \sin\left(\frac{3\pi}{2} + a\right) &= -\cos a \end{aligned}$$

$$\operatorname{tg}(a + \pi k) = \operatorname{tga} \quad \operatorname{ctg}(a + \pi k) = \operatorname{ctga}$$

$$\begin{aligned} \operatorname{tg}\left(\frac{\pi}{2} - a\right) &= \operatorname{ctga} & \operatorname{tg}\left(\frac{\pi}{2} + a\right) &= -\operatorname{ctga} \\ \operatorname{ctg}\left(\frac{\pi}{2} - a\right) &= \operatorname{tga} & \operatorname{ctg}\left(\frac{\pi}{2} + a\right) &= -\operatorname{tga} \end{aligned}$$

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1) *Вычислить* $\sin 930^\circ$

$$\sin 930^\circ = \sin(3 \cdot 360^\circ - 150^\circ) = -\sin 150^\circ = -\sin(180^\circ - 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$$

2) *Вычислить* $\operatorname{tg} \frac{11\pi}{3}$

$$\operatorname{tg} \frac{11\pi}{3} = \operatorname{tg} \left(4\pi - \frac{\pi}{3} \right) = \operatorname{tg} \left(-\frac{\pi}{3} \right) = -\operatorname{tg} \frac{\pi}{3} = -\sqrt{3}$$

3) *Вычислить* $\operatorname{tg} \frac{13\pi}{4}$

$$\operatorname{tg} \frac{13\pi}{4} = \operatorname{tg} \left(3\pi + \frac{\pi}{4} \right) = \operatorname{tg} \frac{\pi}{4} = 1$$

4) *Вычислить* $\cos \frac{15\pi}{4}$

$$\cos \frac{15\pi}{4} = \cos \left(4\pi - \frac{\pi}{4} \right) = \cos \left(-\frac{\pi}{4} \right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

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$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$$

$$\operatorname{tg} \alpha - \operatorname{tg} \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta}$$

$$1) \cos 105^\circ + \cos 75^\circ = 2 \cos \frac{105^\circ + 75^\circ}{2} \cos \frac{105^\circ - 75^\circ}{2} = 2 \cos 90^\circ \cos 15^\circ = 0;$$

$$2) \cos \frac{11\pi}{12} - \cos \frac{5\pi}{12} = -2 \sin \frac{\frac{11\pi}{12} + \frac{5\pi}{12}}{2} \sin \frac{\frac{11\pi}{12} - \frac{5\pi}{12}}{2} = -2 \sin \frac{2\pi}{3} \sin \frac{\pi}{4} =$$
$$= -2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{6}}{2};$$

$$3) \sin 105^\circ + \sin 165^\circ = 2 \sin \frac{105^\circ + 165^\circ}{2} \cos \frac{105^\circ - 165^\circ}{2} = 2 \sin 135^\circ \cos(-30)^\circ =$$
$$= 2 \cdot \sin(180^\circ - 45^\circ) \cdot \cos 30^\circ = 2 \cdot \sin 45^\circ \cdot \cos 30^\circ = 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2};$$

$$4) \sin 10^\circ + \sin 50^\circ = 2 \sin \left(\frac{10^\circ + 50^\circ}{2} \right) \cos \left(\frac{10^\circ - 50^\circ}{2} \right) = 2 \sin 30^\circ \cos(-20^\circ) =$$
$$= 2 \cdot \frac{1}{2} \cos 20^\circ = \cos 20^\circ$$

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$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

1) Преобразовать произведение в сумму

$$\begin{aligned}\cos x \cos 6x &= \frac{1}{2} [\cos(x - 6x) + \cos(x + 6x)] \\ &= \frac{1}{2} \cos 5x + \frac{1}{2} \cos 7x\end{aligned}$$

2) Решить уравнение: $\cos x \cos 2x = \sin x \sin 2x$

$$\begin{aligned}\frac{1}{2} [\cos(x - 2x) + \cos(x + 2x)] \\ &= \frac{1}{2} [\cos(x - 2x) - \cos(x + 2x)] \\ \cos(-x) + \cos 3x &= \cos(-x) - \cos 3x\end{aligned}$$

$$2 \cos 3x = 0$$

$$\cos 3x = 0$$

$$x = \frac{\pi}{6} + \frac{\pi n}{3}, n \in Z$$

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