Отбор корней в тригонометрических уравнениях.

Уравнения, имеющие ограничения в области определения.

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Ограничения в области определения:

1. Знаменатель.

$$\frac{P(x)}{Q(x)}; \qquad Q(x) \neq 0;$$

2. Корень четной степени.

$$\sqrt{P(x)}; \quad P(x) \geq 0$$

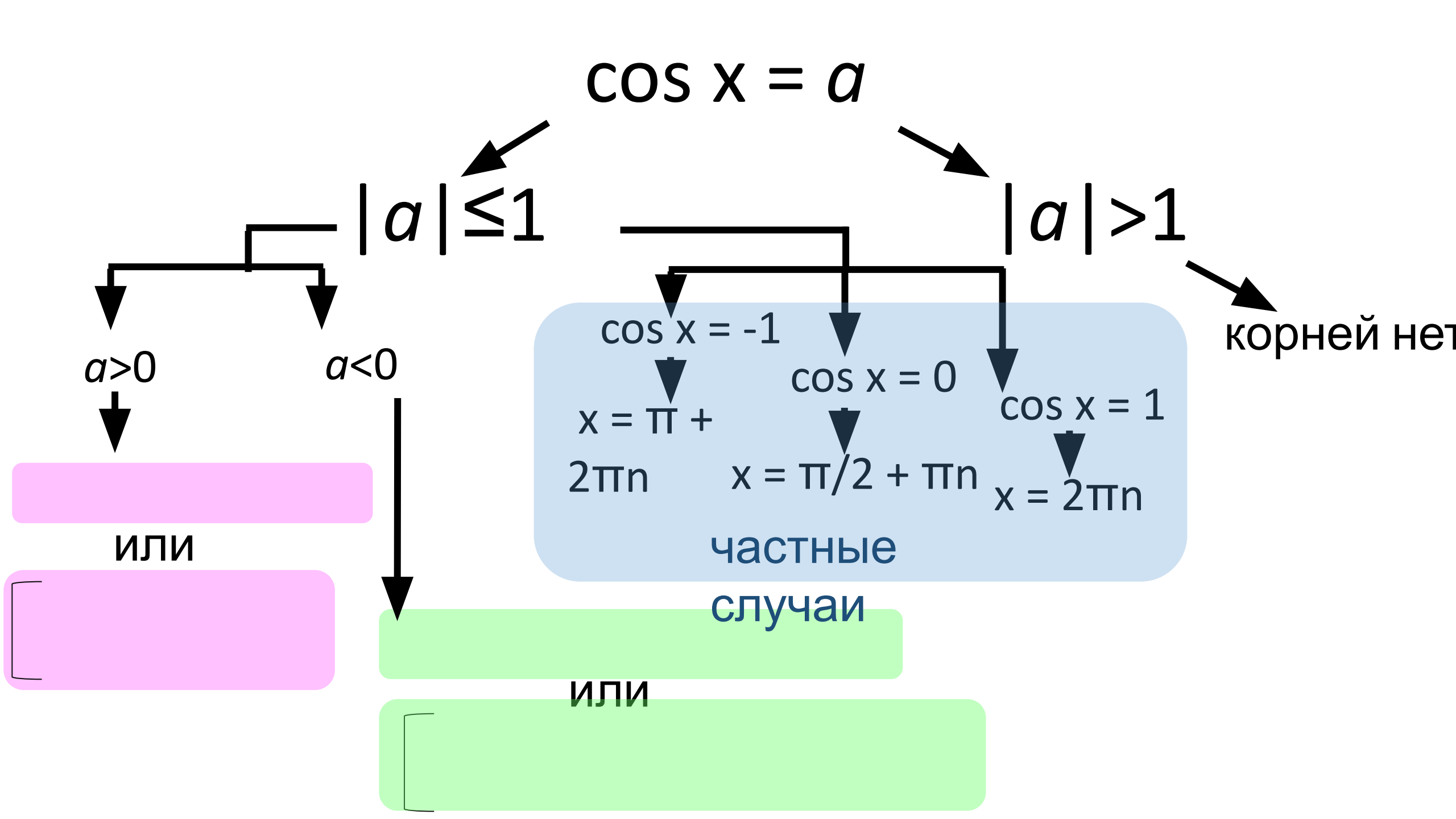
3. Тангенс, котангенс.

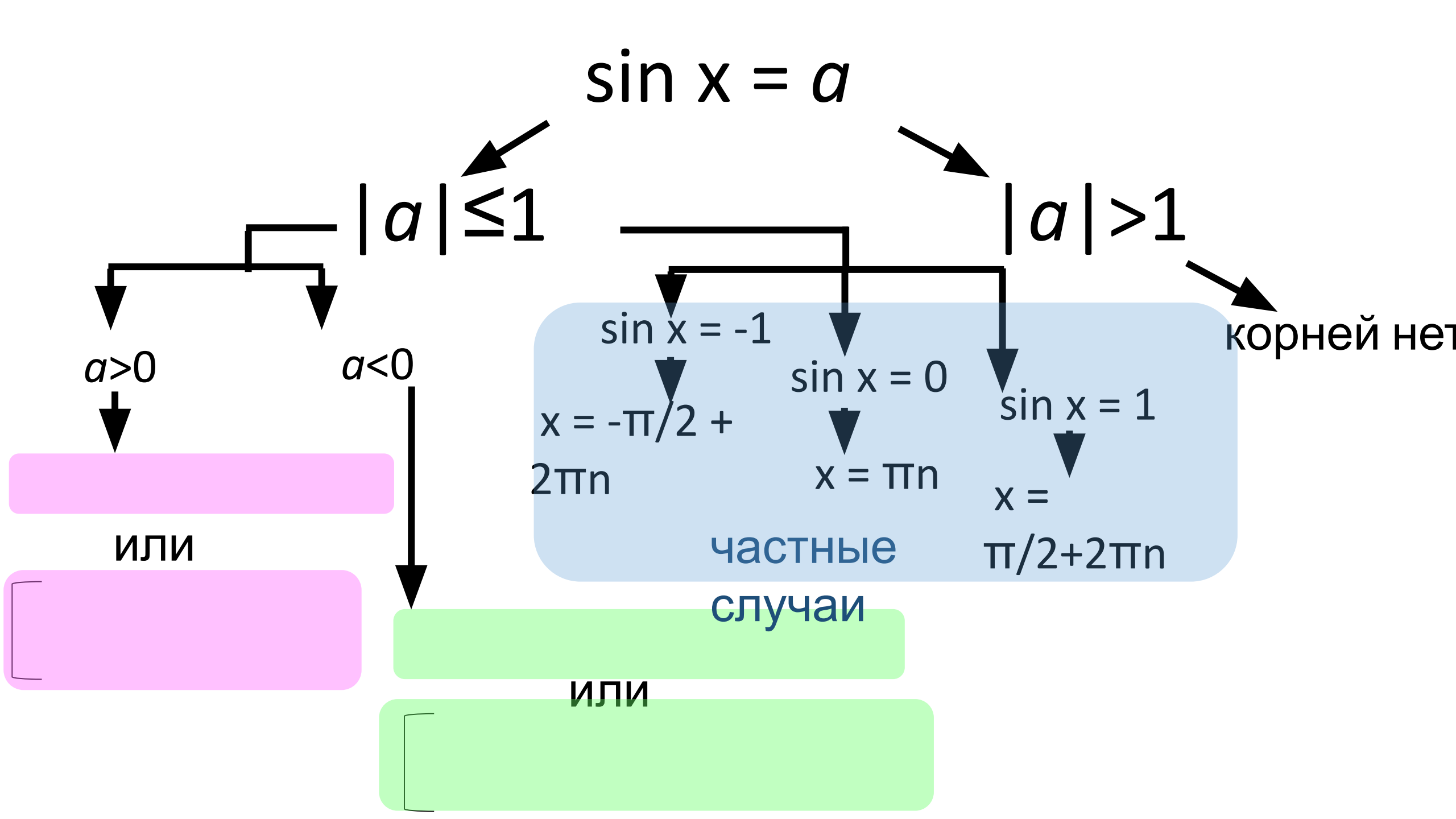
$$tgx; \qquad cosx \neq 0;$$

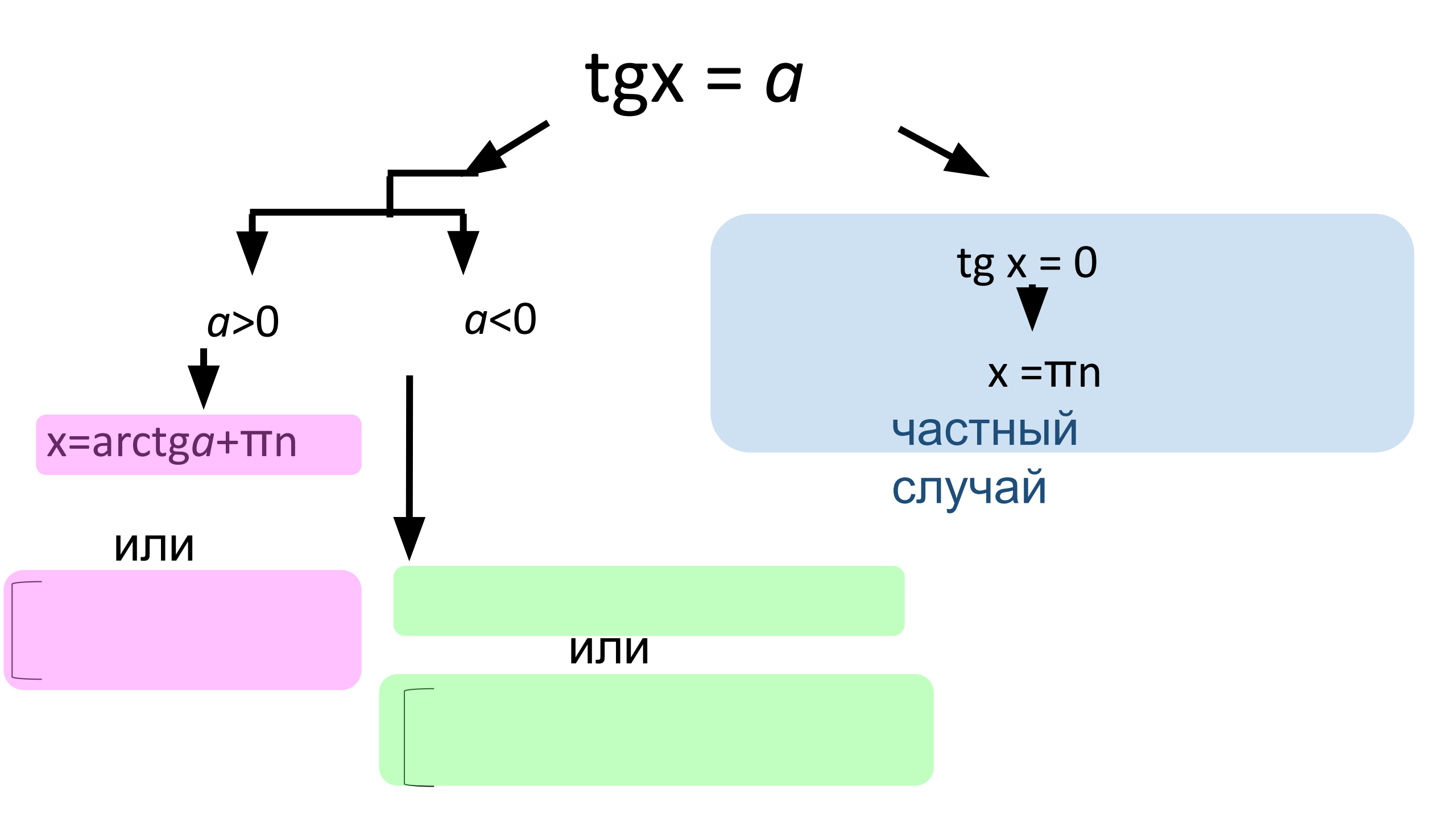
$$ctgx; \qquad sinx \neq 0;$$

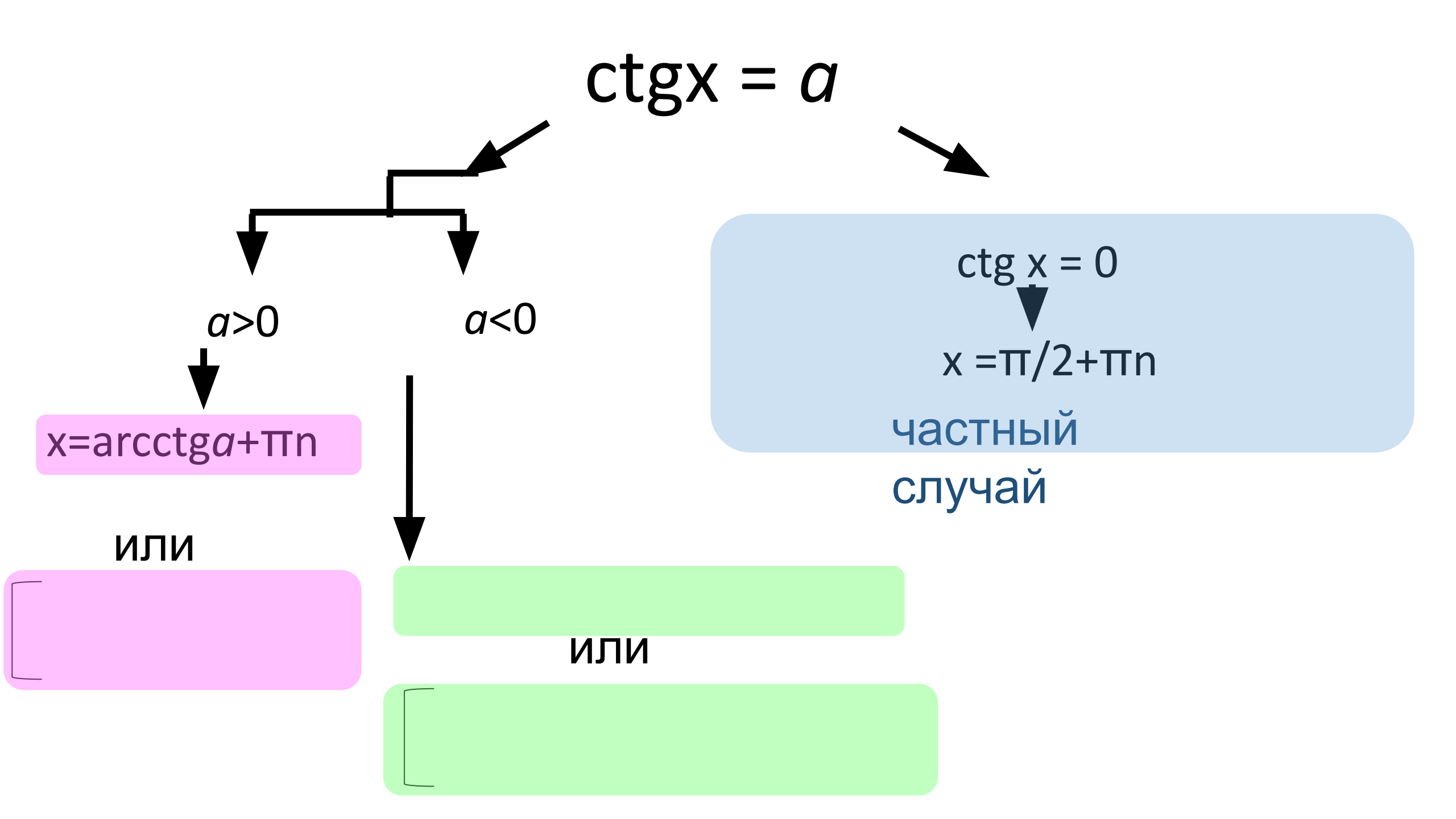
4. Логарифм.

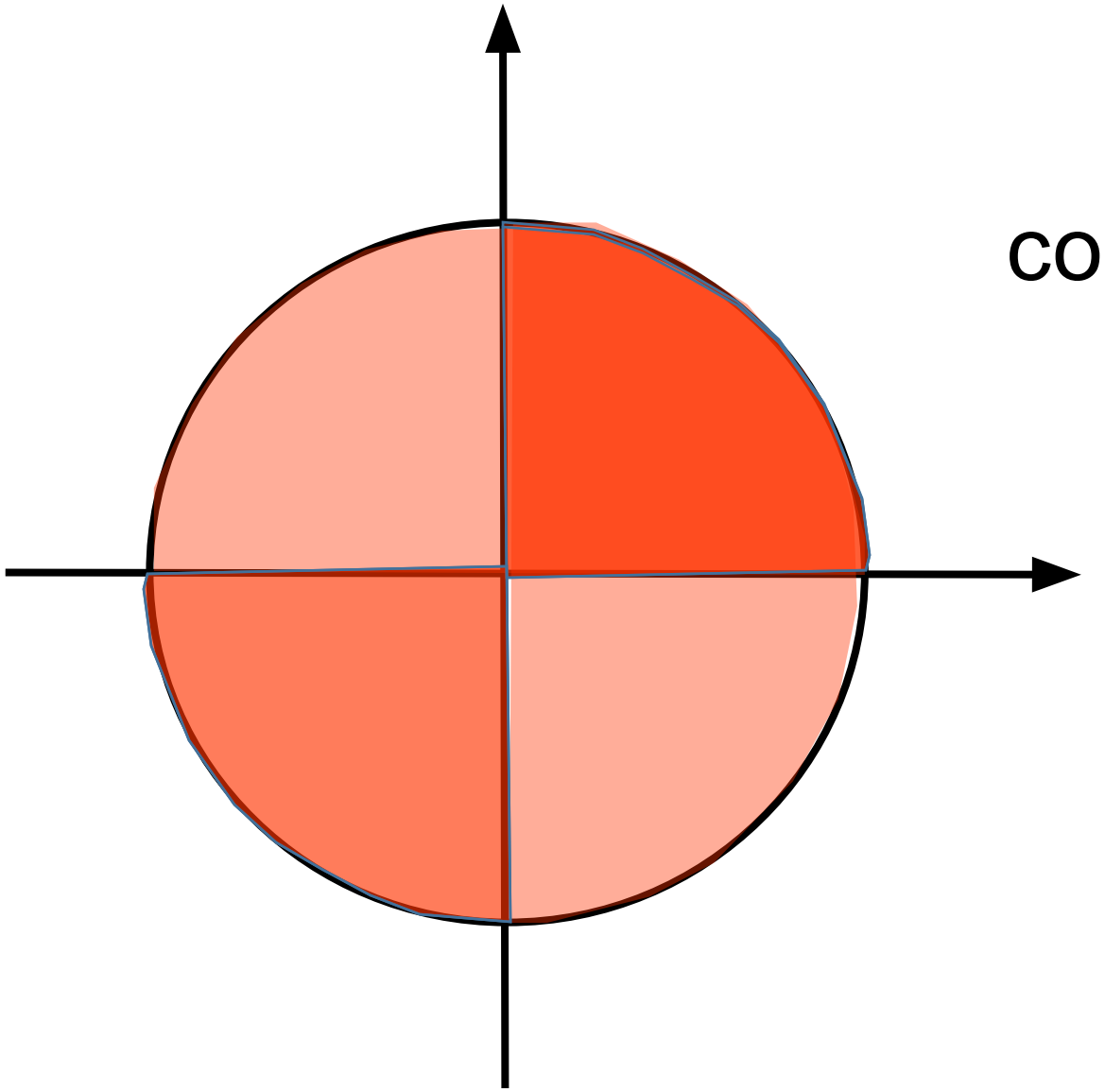
$$log_a b; \qquad b > 0; \quad a > 0; \quad a \neq 1$$


cos x = a
|a|≤1
|a|>1
a>0
a<0
или
cos x = -1
cos x = 0
cos x = 1
x = π + 2πn
x = π/2 + πn
x = 2πn
частные случаи
корней нет
или


sin x = a
|a|≤1
|a|>1
a>0
a<0
sin x = -1
sin x = 0
sin x = 1
корней нет
x = -π/2 + 2πn
x = πn
x = π/2+2πn
частные случаи
или
или

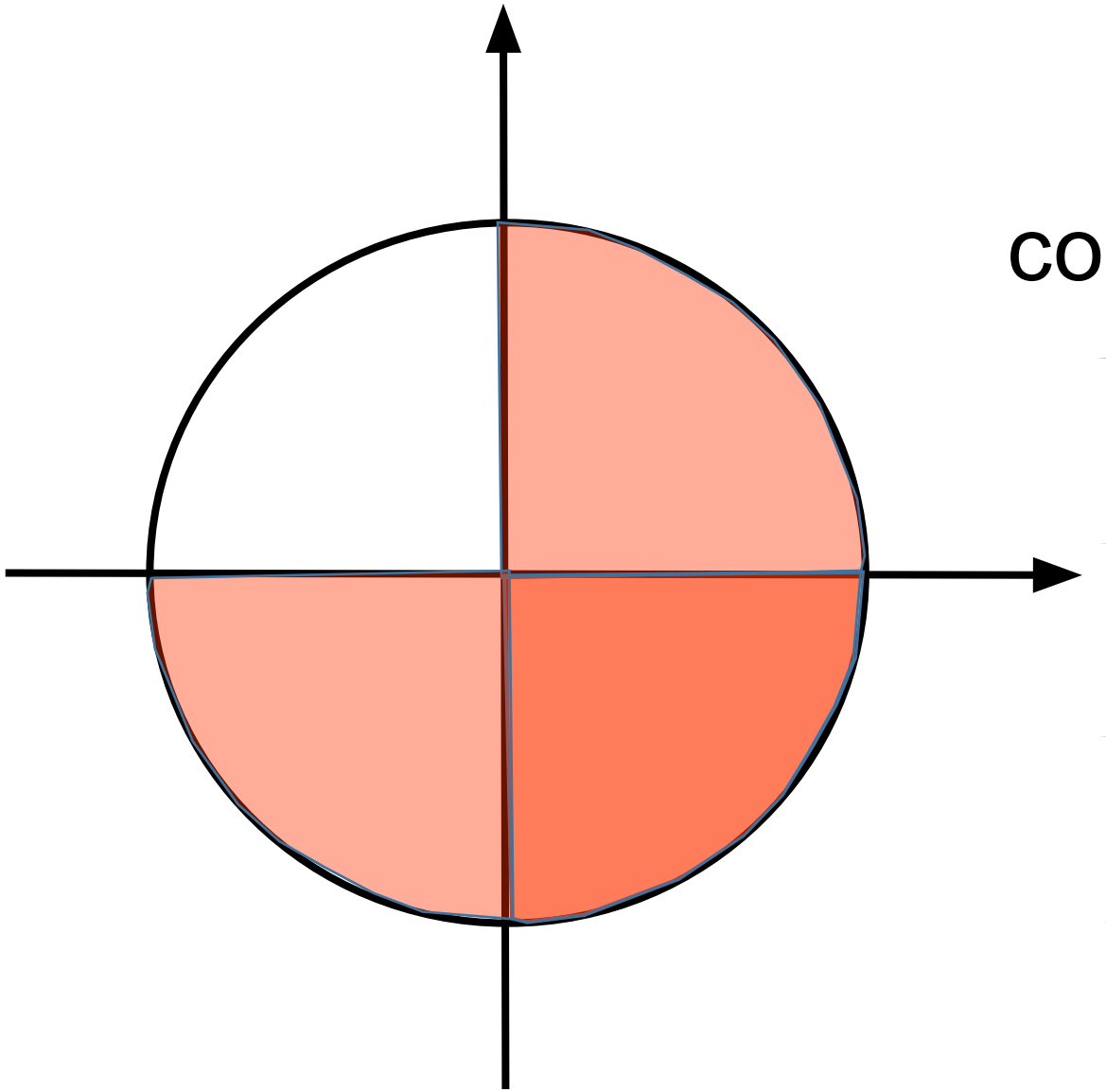

tgx = a
a>0
a<0
x=arctgа+πn
или
tg x = 0
x =πn
частный
случай
или


ctgx = a
a>0
a<0
x=arcctga+πn
или
или
ctg x = 0
x =π/2+πn
частный
случай



Отметить на тригонометрической окружности области, соответствующие условиям:

Отметить на тригонометрической окружности области, соответствующие условиям:



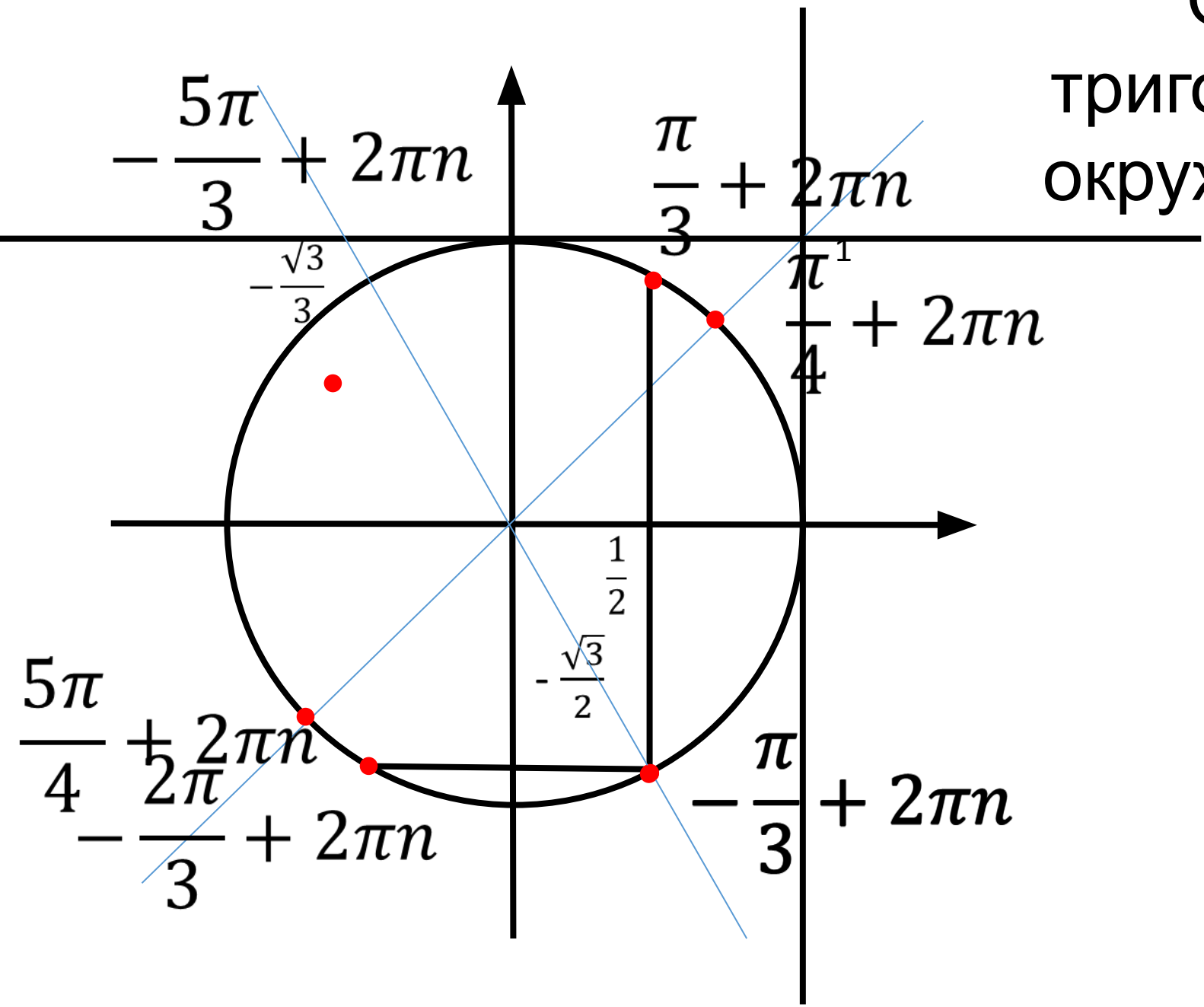
Отметить на тригонометрической окружности решение уравнений:

$\sin x = -\frac{\sqrt{3}}{2}$

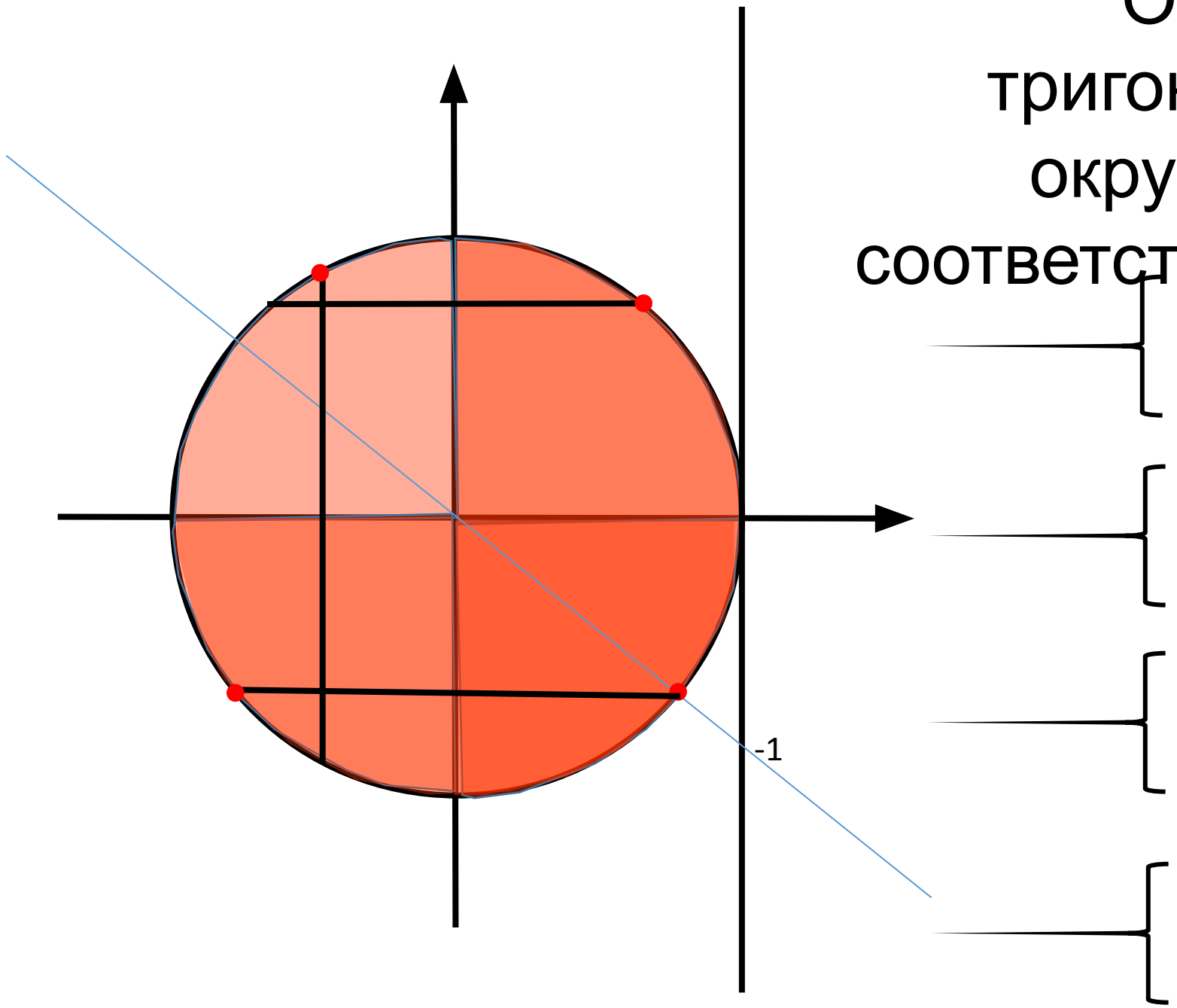
$\cos x = \frac{1}{2}$

$\operatorname{tg} x = 1$

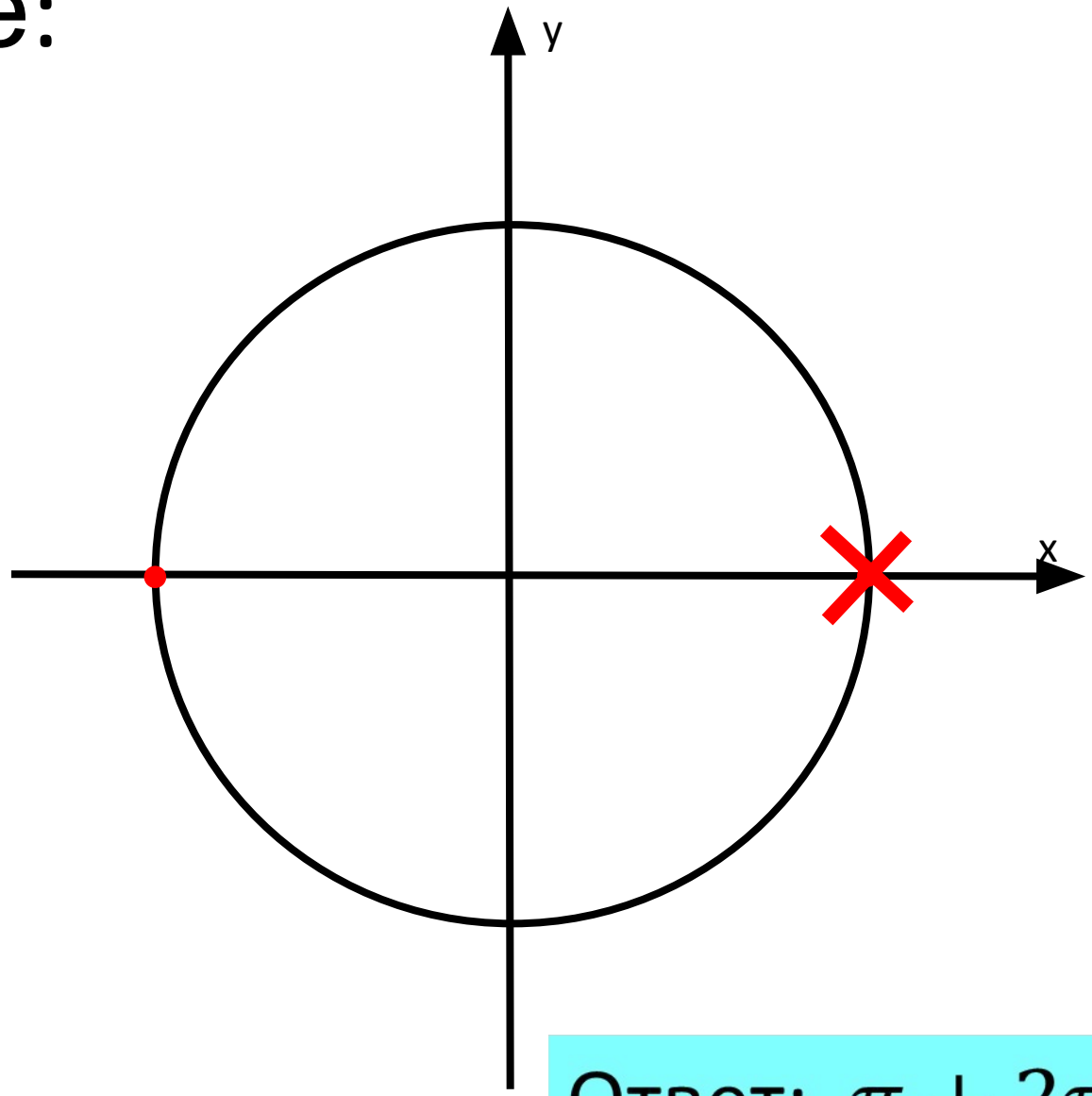
$\operatorname{ctg} = -\frac{\sqrt{3}}{3}$



Отметить на тригонометрической окружности точки, соответствующие условиям:



Решите уравнение:



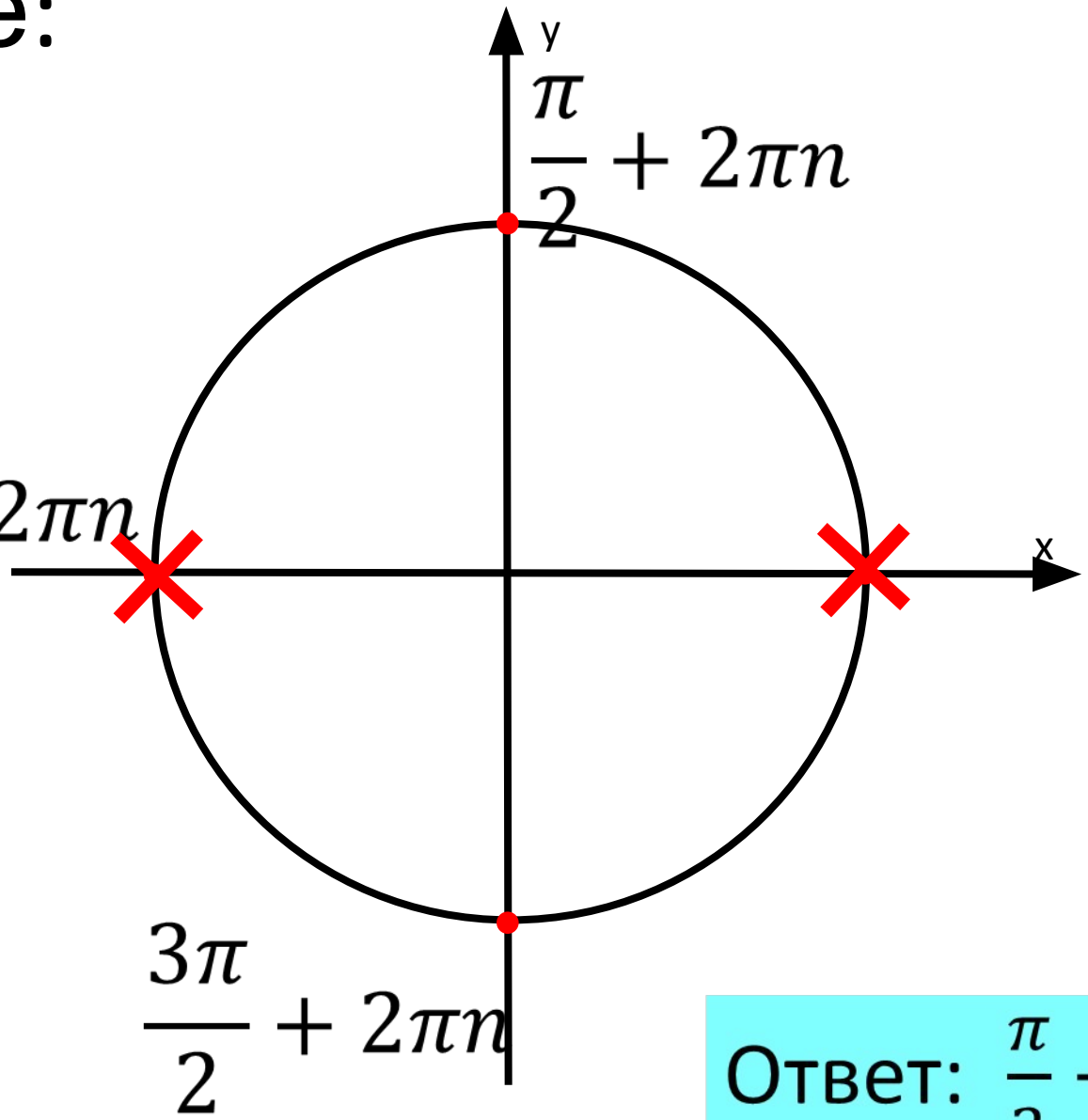
Ответ: $\pi + 2\pi n,$
$n \epsilon Z$

Решите уравнение:

$$\frac{sin\,x + sin3x}{1 - cos2x} = 0;$$

$$\begin{cases} \left[\begin{array}{l} sin2x = 0, \\ \cos(-x) = 0 \end{array}\right. \\ cos2x \neq 1; \end{cases}$$

$$\begin{cases} \left[\begin{array}{l} x = \dfrac{\pi n}{2}, \\ x = \dfrac{\pi}{2} + \pi \end{array}\right. \\ x \neq \pi n. \end{cases}$$



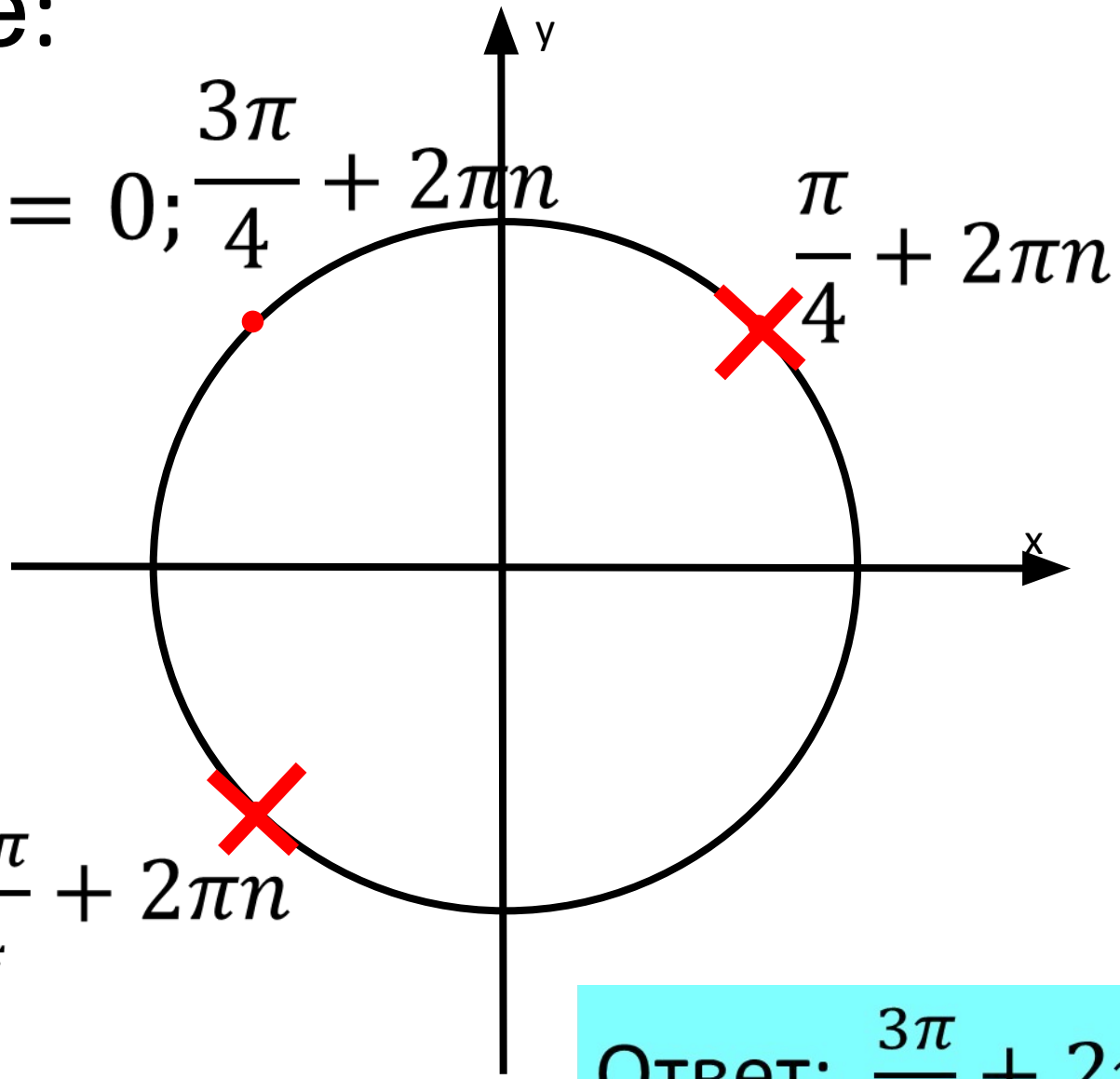
Ответ: $\frac{\pi}{2} + \pi n$, $n \in Z$

Решите уравнение:

$$\frac{cos2x + \sqrt{2}\,cosx + 1}{tgx - 1} = 0;$$

$$\begin{cases} \left[\begin{array}{l} cosx = 0, \\ \cos x = -\frac{\sqrt{2}}{2} \end{array}\right. \\ tgx - 1 \neq 0, \\ cosx \neq 0 \end{cases}$$

$$\begin{cases} \left[\begin{array}{l} x = \frac{3\pi}{4} + 2\pi n, \\ x = -\frac{3\pi}{4} + 2\pi n \end{array}\right. \\ x \neq \frac{\pi}{4} + \pi n, \end{cases}$$



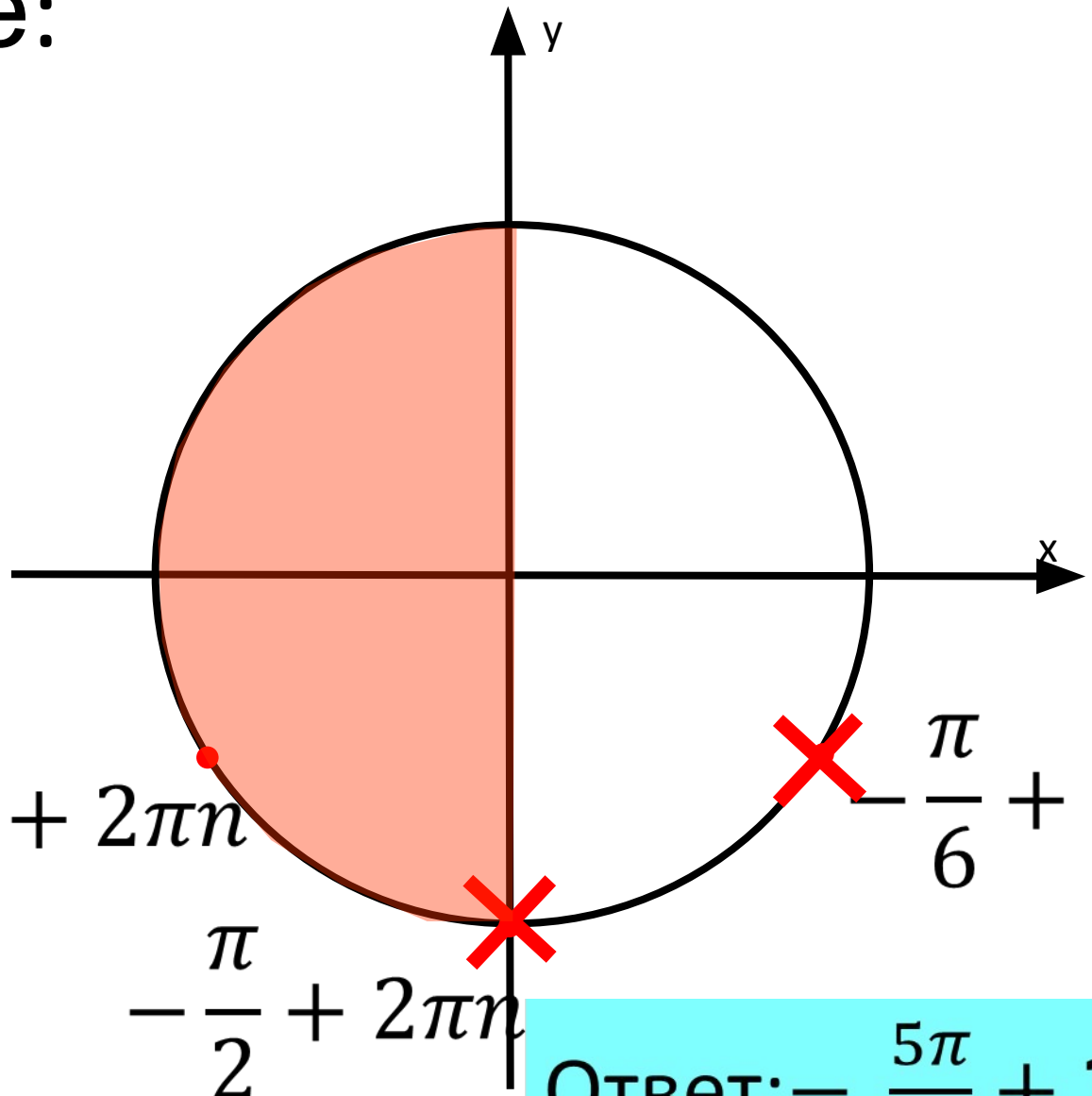
Ответ: $\frac{3\pi}{4} + 2\pi n$, $n \in Z$

Решите уравнение:

$$\frac{2sin^2x+3sinx+1}{\sqrt{-cosx}}=0;$$

$$\begin{cases}\left[\begin{array}{l} sinx=-1, \\ sin\,x=-\frac{1}{2}, \end{array}\right. \\ cosx<0; \end{cases}$$

$$\begin{cases}\left[\begin{array}{l} x=-\frac{\pi}{2}+2\pi n, \\ x=-\frac{\pi}{6}+2\pi n, \\ x=-\frac{5\pi}{6}+2\pi n, \end{array}\right. \\ cosx<0; \end{cases}$$



$$-\frac{5\pi}{6}+2\pi n$$

$$-\frac{\pi}{6}+2\pi n$$

$$-\frac{\pi}{2}+2\pi n$$

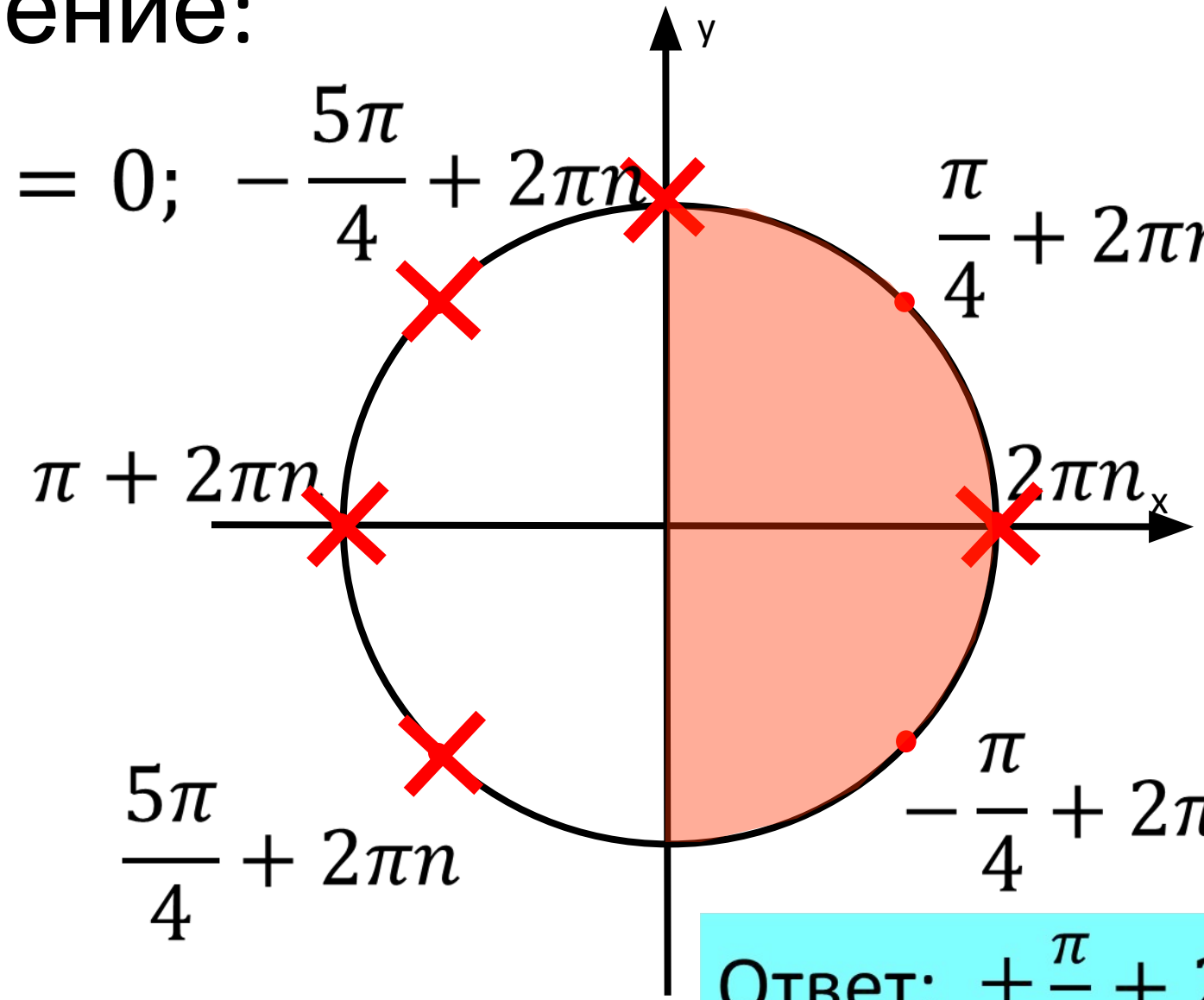
Ответ: $-\frac{5\pi}{6}+2\pi n,\ n\in Z$

Решите уравнение:

$$\frac{tg^3x - tgx}{\ln(sinx)} = 0;$$

$$\begin{cases} \left[\begin{array}{l} tgx = 0, \\ tgx = 1 \\ \operatorname{tg} x = -1 \end{array}\right. \\ \ln(sinx) \neq 0, \\ sinx > 0 \end{cases}$$

$$\begin{cases} \left[\begin{array}{l} x = \pi n, \\ x = \dfrac{\pi}{4} + \pi n, \\ x = -\dfrac{\pi}{4} + \pi n, \end{array}\right. \\ x \neq \dfrac{\pi}{2} + 2\pi n, \\ \underline{sinx > 0} \end{cases}$$



Ответ: $\pm\frac{\pi}{4} + 2\pi n,$ $n \in Z$

Самостоятельная работа:

1. $\frac{cosx}{sinx-1}=0;$

2. $\frac{sinx+\frac{\sqrt{2}}{2}}{1-tgx}=0;$

3. $\frac{cosx-cos5x}{1-sinx}=0;$

4. $\frac{cos3x+cosx}{sin\frac{x}{2}}=0;$

5. $\frac{2sin^2x-sinx}{2cosx+\sqrt{3}}=0;$

6. $\frac{cos2x+\sqrt{3}sinx-1}{tgx-\sqrt{3}}=0;$

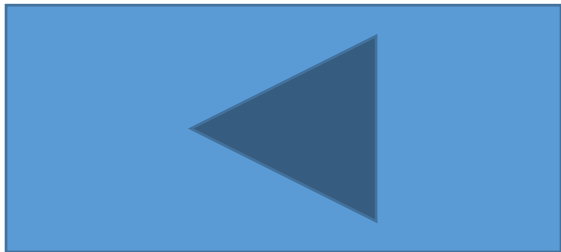
7. $\frac{5sinxcosx-4cosx}{\sqrt{ctgx}}=0;$

8. $\frac{7sin^2x+8cosx-8}{\sqrt{-19tgx}}=0;$

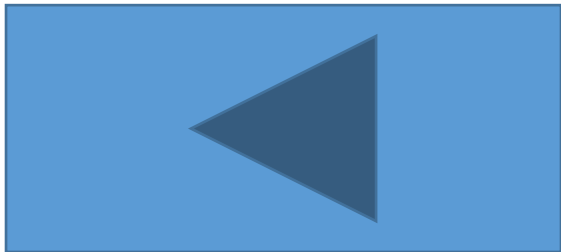
9. $\frac{2cos^3x+3cos^2x+cosx}{\lg(ctgx)}=0;$

10. $\frac{16^{sinx}-6\cdot4^{sinx}+8}{log_\pi(1-2cosx)}=0;$

$$1.\,x = -\frac{\pi}{2} + 2\pi n$$



$$2.\,x = -\frac{\pi}{4} + 2\pi n$$



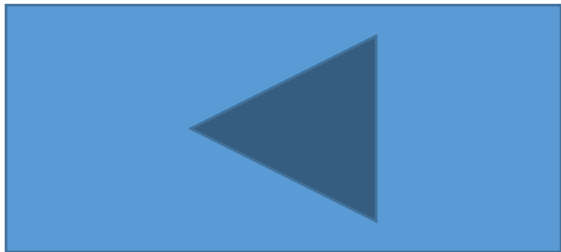
$$3.\ x = -\frac{\pi}{2} + 2\pi n;$$

$$x = \frac{\pi n}{3};$$

$$x = \pi n.$$

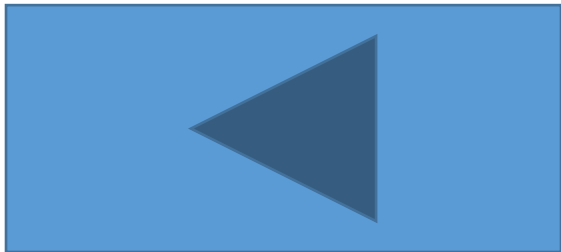
$$4.\ x = \frac{\pi}{2} + \pi n;$$

$$x = \pi + 2\pi n.$$



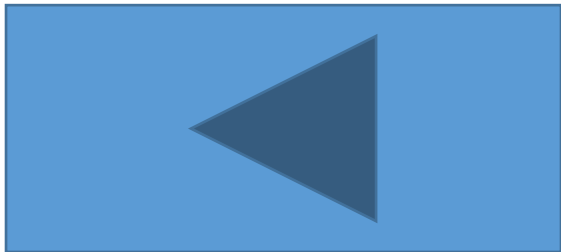
$$5.\ x = \frac{\pi}{6} + \pi n;$$

$$x = \pi n.$$

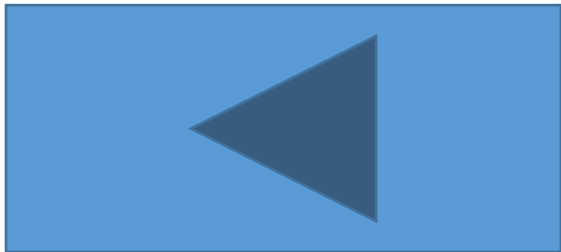


$$6.\ x = \frac{2\pi}{3} + 2\pi n;$$

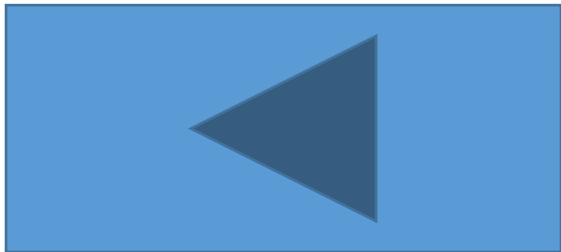
$$x = \pi n.$$



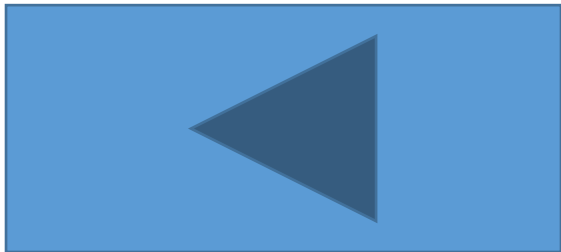
$$7.\, x = arcsin\frac{4}{5} + 2\pi n$$



$$8.\, x = -arcsin\frac{1}{7} + 2\pi n.$$



$$9.\, x = -\frac{2\pi}{3} + 2\pi n$$



$$10.\, x = \frac{5\pi}{6} + 2\pi n$$