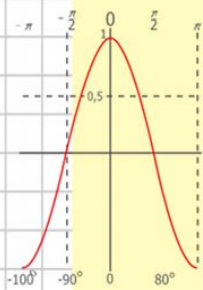
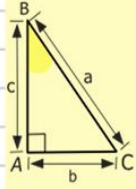
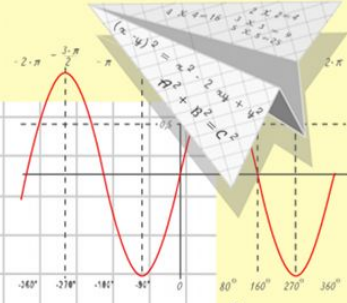


Математик

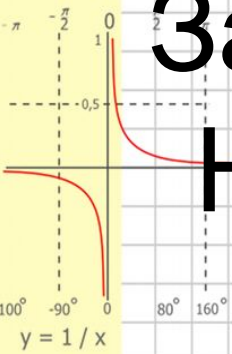
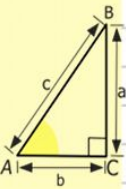
а



- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$

Занятие 105. Первообразная. Неопределенный интеграл.

1. Первообразная
2. Неопределенный интеграл



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

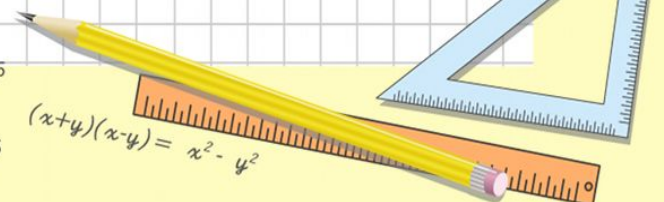
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



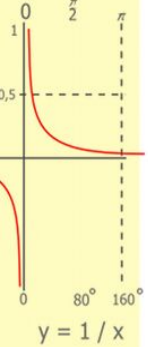
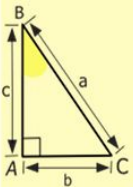
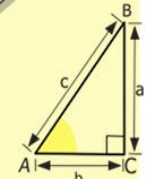
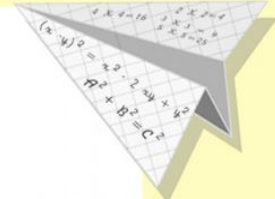
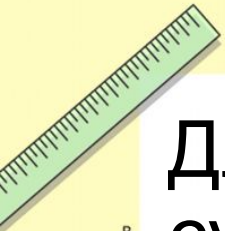
Первообразная

Для каждой математической операции существует обратная операция.

Например, для сложения – это вычитание, для умножения – деление, для возведения в степень – извлечение корня, и т.д.

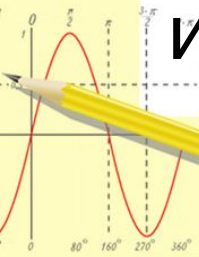
Для обратной операции дается определение, причем через исходную операцию, а также присваивается соответствующее обозначение.

Существует обратная операция и для дифференцирования. Называется она - интегрирование.



$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 4\ 2 \\ \hline 21\ 0 \\ + 84\ 0 \\ \hline 105\ 0\ 00 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

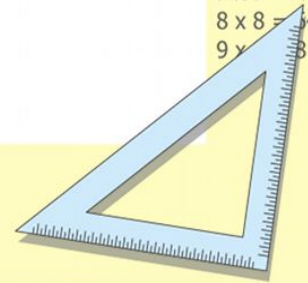


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

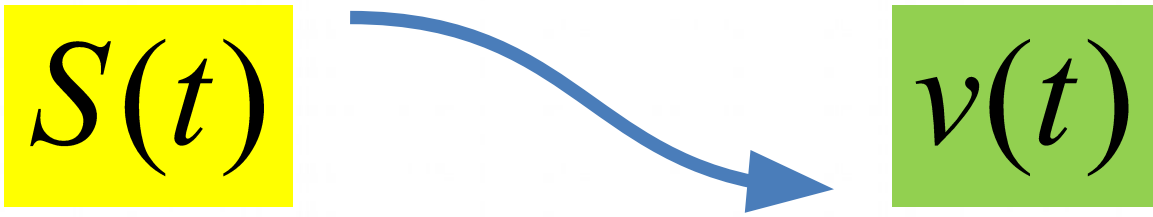
$$(x+y)(x-y) = x^2 - y^2$$



Первообразная

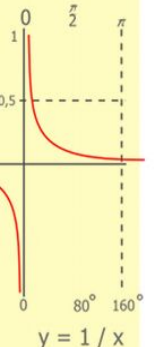
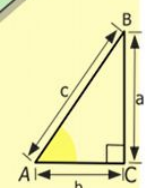
Вспомним одну из основных задач физики, решение которой способствовало появлению операции дифференцирования:

по данному закону движения найти мгновенную скорость.



дифференцирование

$$S'(t) = v(t)$$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

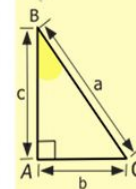
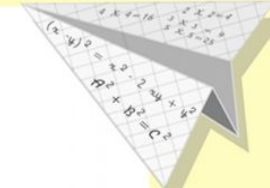
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



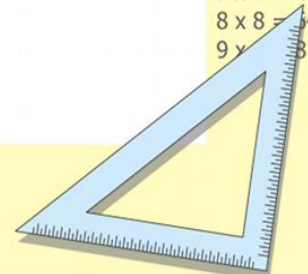
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

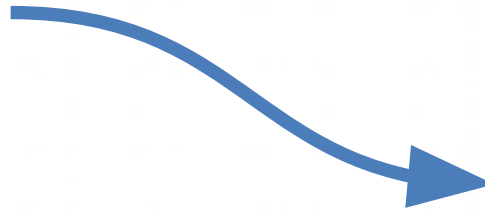
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



Первообразная

Сформулируем обратную задачу:
по данной скорости найти закон движения
тела

$$v(t)$$



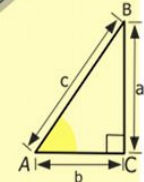
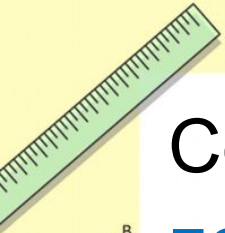
$$S(t)$$

интегрирование

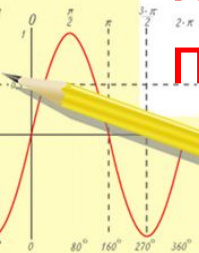
Суть задачи:

по данной **производной** восстановить формулу
функции

Новой операции дали название –
интегрирование, а результату операции –
первообразная.



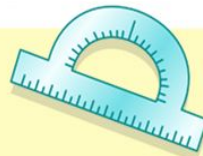
$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

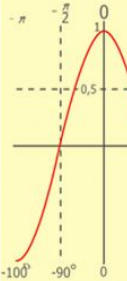
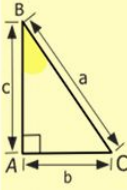
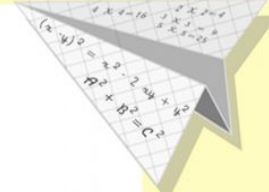


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

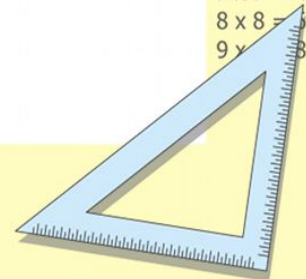
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



Первообразная

Первообразные принято обозначать той же буквой, что и функцию, только заглавной.

Например:

функция $f(x)$, ее первообразная $F(x)$

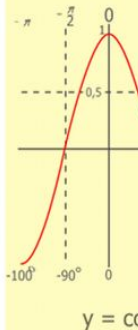
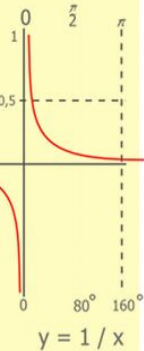
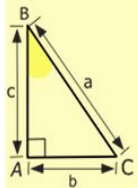
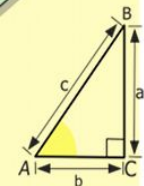
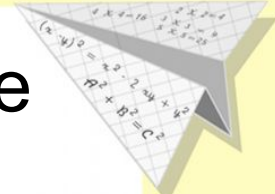
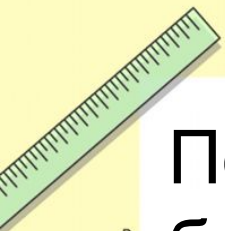
функция $g(x)$, ее первообразная $G(x)$

функция $h(x)$, ее первообразная $H(x)$

Определение

Функция $F(x)$ является первообразной функции $f(x)$, если:

$$F'(x) = f(x)$$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

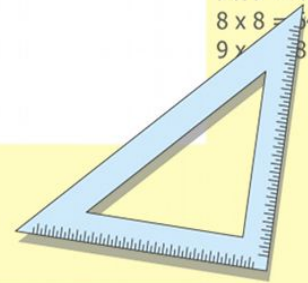


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



Первообразная

Задача. Является ли функция $y=g(x)$ первообразной для функции $y=f(x)$?

$$f(x) = x^3, \quad g(x) = \frac{1}{4}x^4$$

$$g'(x) = \frac{1}{4}(x^4)' = \frac{1}{4} * 4x^3 = x^3 = f(x)$$

$$F(x) = g(x)$$

$$f(x) = \sin x - \cos x, \quad g(x) = \sin x + \cos x$$

$$g'(x) = (\sin x)' + (\cos x)' = \cos x - \sin x \neq f(x)$$

$$F(x) \neq g(x)$$

$$f(x) = \frac{1}{\sqrt{x}}, \quad g(x) = 2\sqrt{x}$$

$$g'(x) = 2(\sqrt{x})' = 2 * \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}} = f(x)$$

$$F(x) = g(x)$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

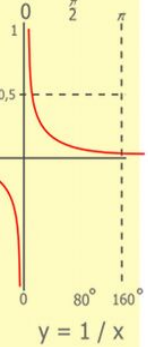
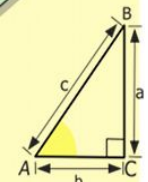
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

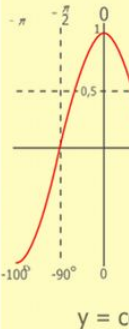
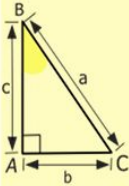
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

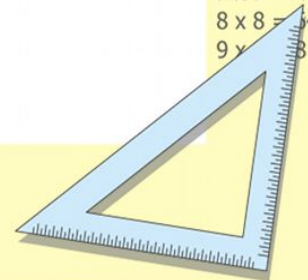
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 2100 \\ \hline 105000 \end{array}$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



ОСНОВНОЕ СВОЙСТВО

первообразных

Проблема: единственность первообразной?

$$f(x) = x^3, \quad g_1(x) = \frac{1}{4}x^4, \quad g_2(x) = \frac{1}{4}x^4 + 7, \quad g_3(x) = \frac{1}{4}x^4 - 12$$

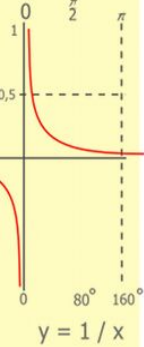
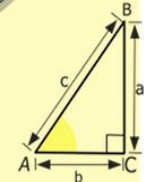
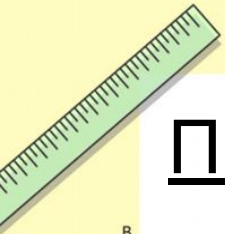
$$g_1'(x) = \frac{1}{4}(x^4)' = \frac{1}{4} * 4x^3 = x^3 = f(x)$$

$$g_2'(x) = \frac{1}{4}(x^4)' + (7)' = \frac{1}{4} * 4x^3 + 0 = x^3 = f(x)$$

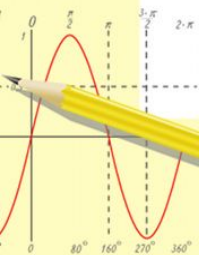
$$g_3'(x) = \frac{1}{4}(x^4)' - (12)' = \frac{1}{4} * 4x^3 - 0 = x^3 = f(x)$$

$$g'(x) = \frac{1}{4}(x^4)' + (C)' = \frac{1}{4} * 4x^3 + 0 = x^3 = f(x)$$

$$F(x) = \frac{1}{4}x^4 + C$$



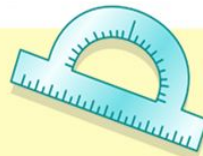
$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 2100 \\ + 840 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

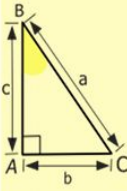
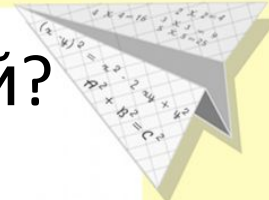


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

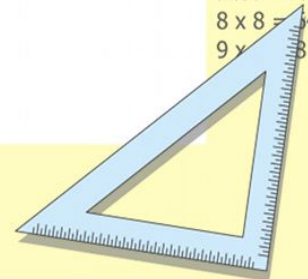
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



ОСНОВНОЕ СВОЙСТВО

первообразных

Если $F(x)$ – одна из первообразных для функции $f(x)$, то множество всех первообразных для функции $f(x)$ имеет вид:

$$F(x) + C, \text{ где } C = const$$

$$f(x) = \frac{1}{\sqrt{x}}$$

$$F(x) = 2\sqrt{x} + C$$

$$f(x) = 2^x$$

$$F(x) = \frac{2^x}{\ln 2} + C$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

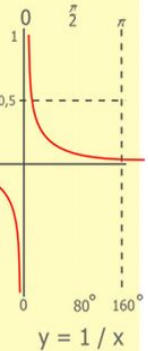
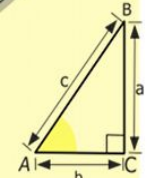
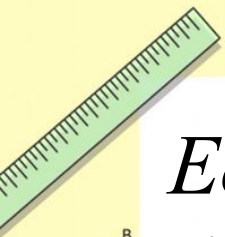
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

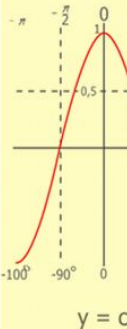
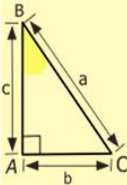
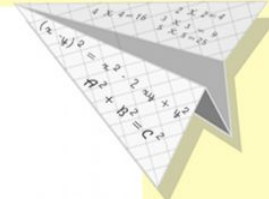
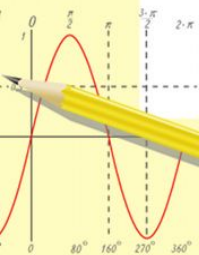
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

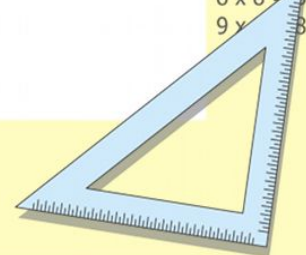
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 210 \\ \hline 105000 \end{array}$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



Неопределенный интеграл

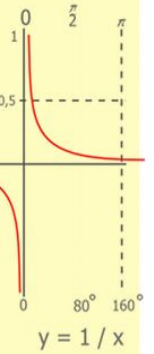
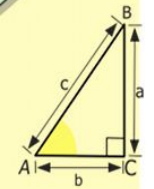
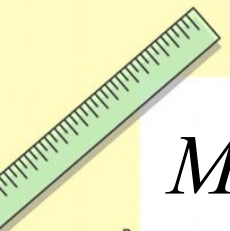
Множество всех первообразных для функции $f(x)$ назвали **неопределенным интегралом** данной функции:

$$\int f(x) dx = F(x) + C$$

знак
интеграла

формула
функции

дифференциал
функции



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 2100 \\ + 8400 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

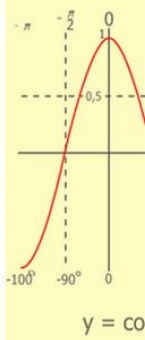
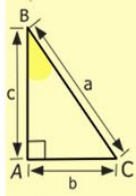
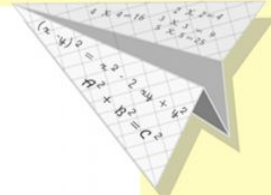
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

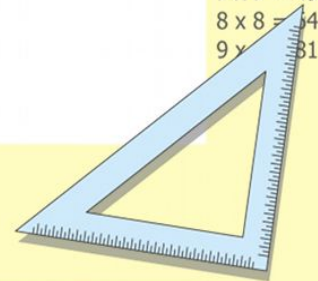


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



Неопределенный интеграл

$$f(x) = \frac{1}{\sqrt{x}}$$

$$F(x) = 2\sqrt{x} + C$$

$$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

$$f(x) = 2^x$$

$$F(x) = \frac{2^x}{\ln 2} + C$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + C$$

$$f(x) = x^3$$

$$F(x) = \frac{1}{4}x^4 + C$$

$$\int x^3 dx = \frac{1}{4}x^4 + C$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

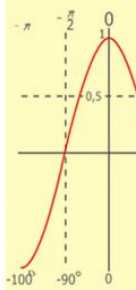
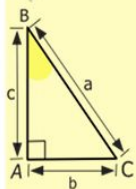
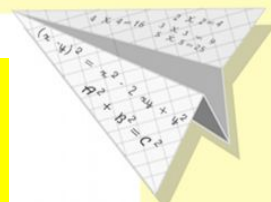
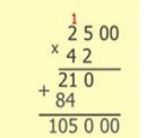
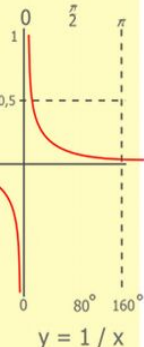
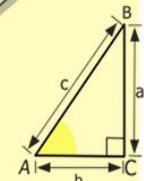
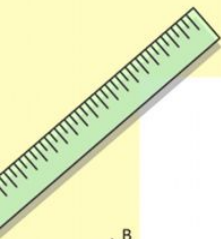
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



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