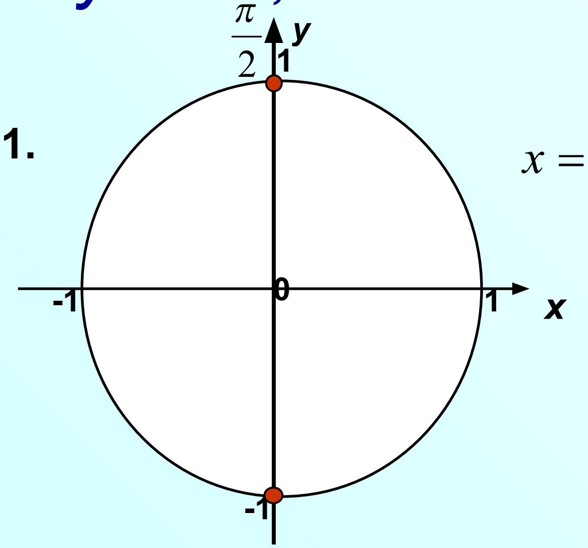
Решение простейших тригонометрических уравнений с помощью единичной окружности

10 класс

Разминка

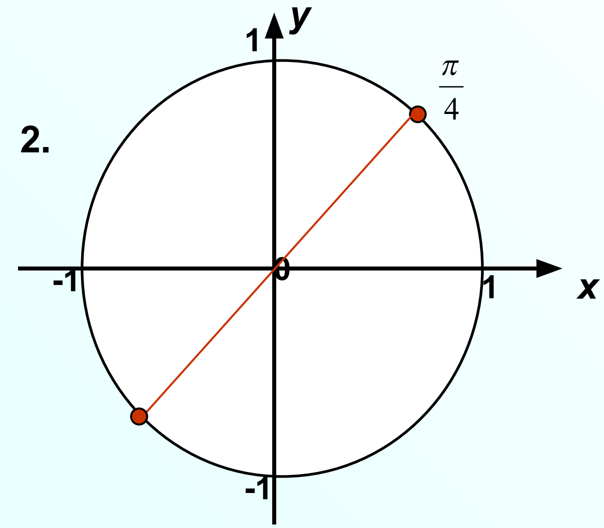
Записать с помощью формулы множество углов, соответствующих данным точкам

1.



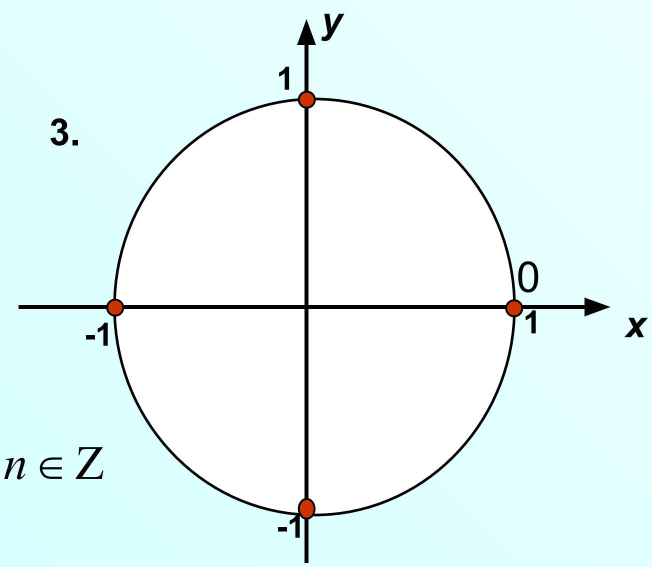
$$x = \frac{\pi}{2} + \pi n,\ n \in Z$$

2.

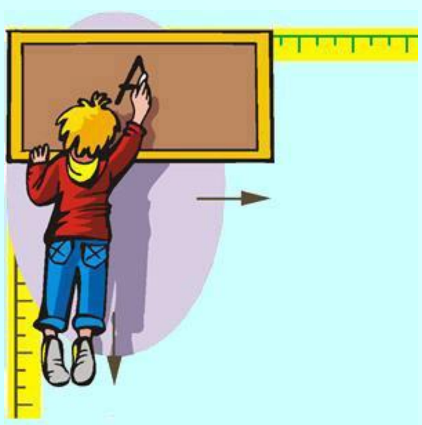


$$x = \frac{\pi}{4} + \pi n,\ n \in Z$$

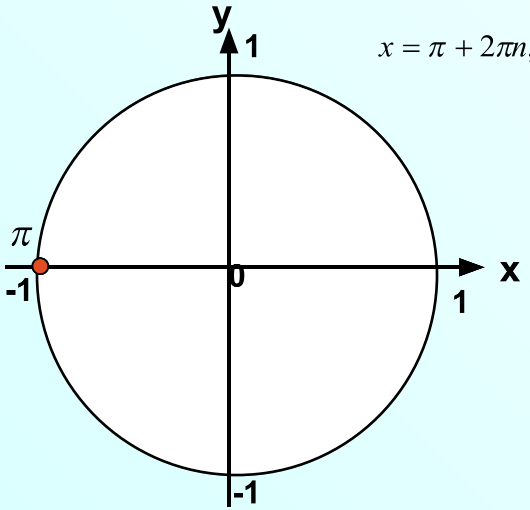
3.



$$x = \frac{\pi}{2} n,\ n \in Z$$

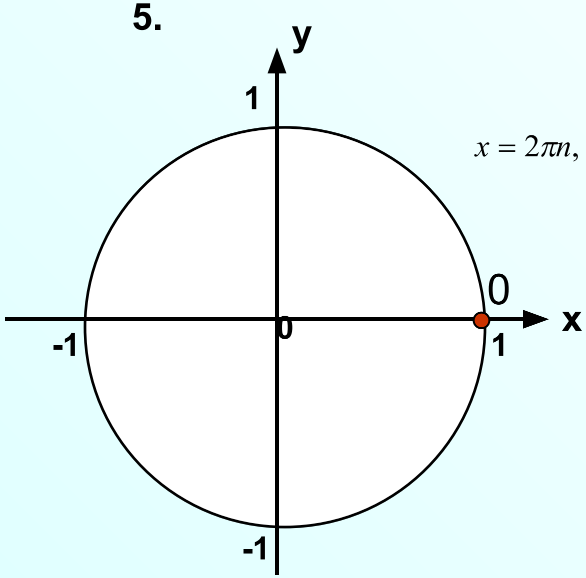


4.



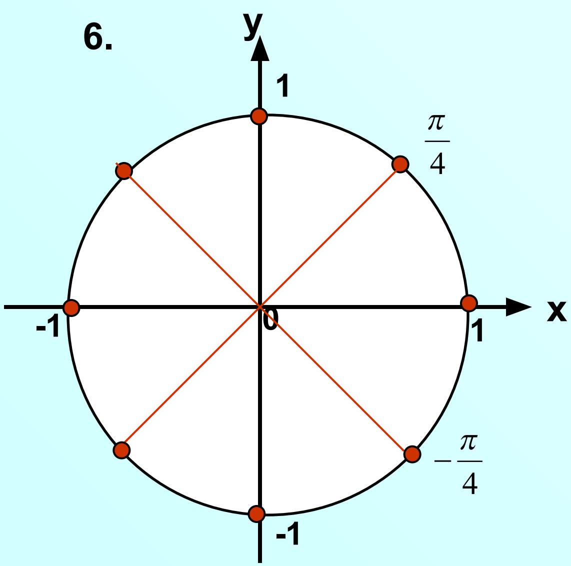
$x = \pi + 2\pi n,\ n \in Z$

5.



$x = 2\pi n,\ n \in Z$

6.

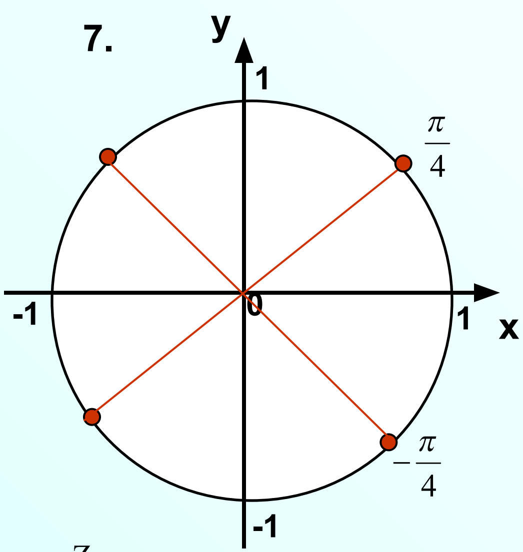


$x = \frac{\pi}{4} n,\ n \in Z$

7.

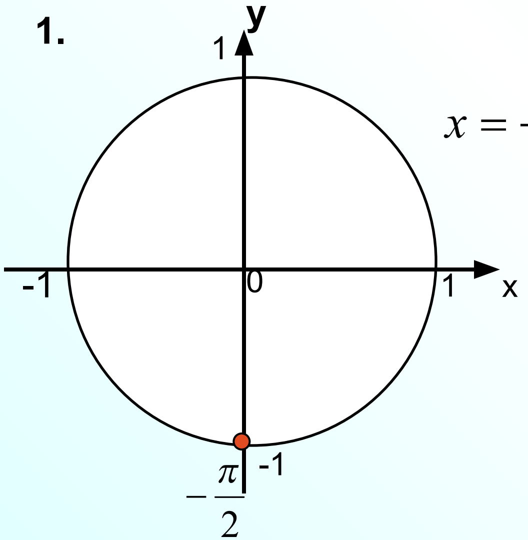
$x = \frac{\pi}{4} + \frac{\pi}{2} n,\ n \in Z$

или $x = \pm\frac{\pi}{4} + \pi n,\ n \in Z$



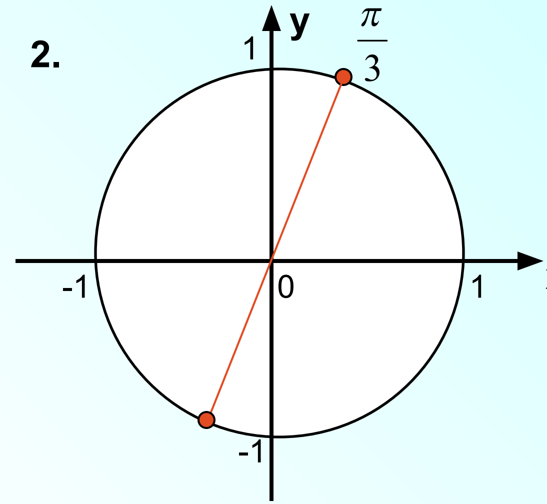
Обратная задача: *отметить точками углы, соответствующие данным формулам*

1.



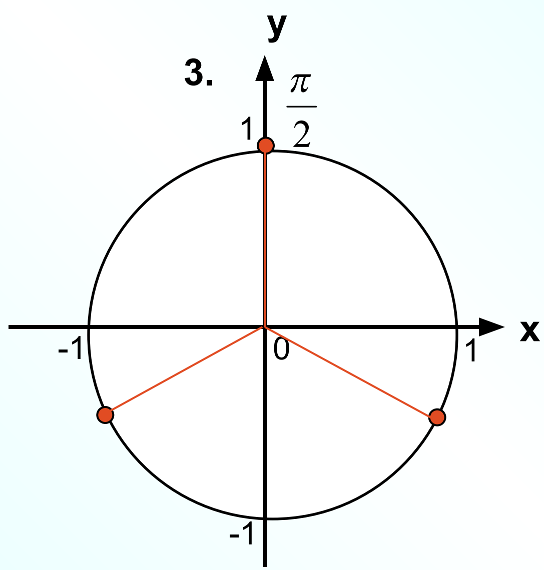
$$x = -\frac{\pi}{2} + 2\pi n, n \in Z$$

2.



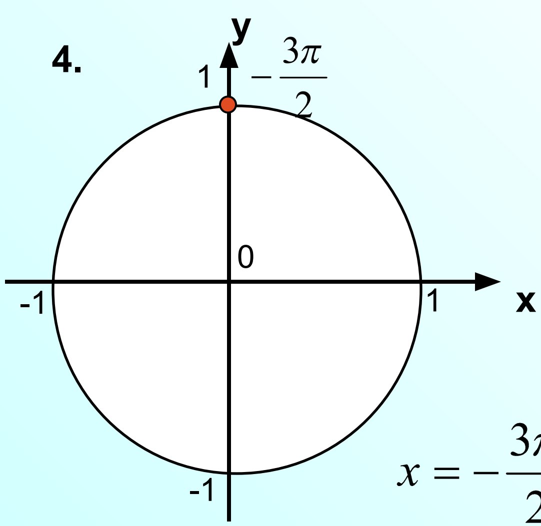
$$x = \frac{\pi}{3} + \pi n, n \in Z$$

3.



$$x = \frac{\pi}{2} + \frac{2\pi}{3} n, n \in Z$$

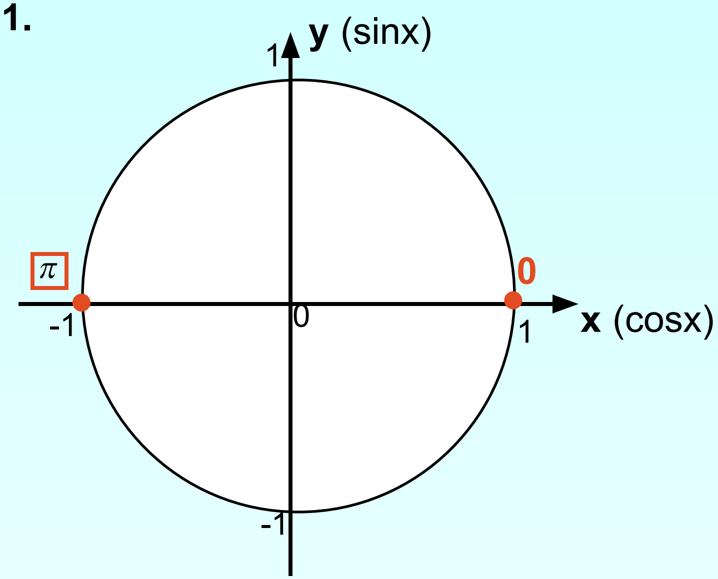
4.



$$x = -\frac{3\pi}{2} + 2\pi n, n \in Z$$

Решить уравнения

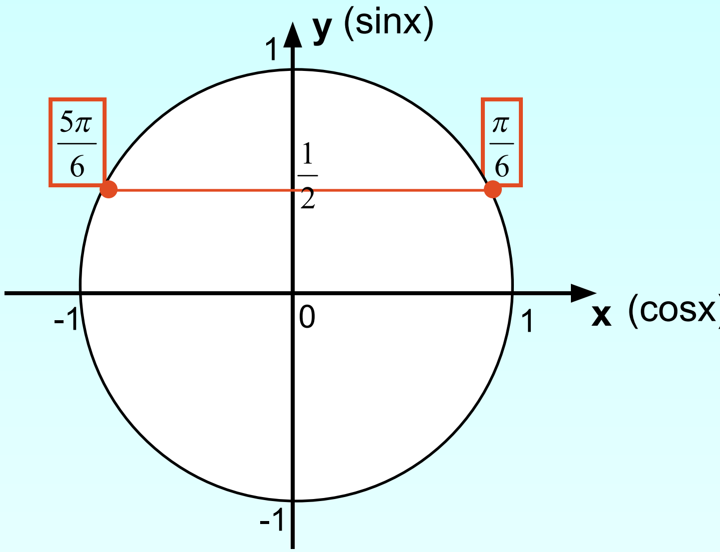
1.



$$\sin x = 0$$

$$x = \pi n,\ n \in Z$$

2.

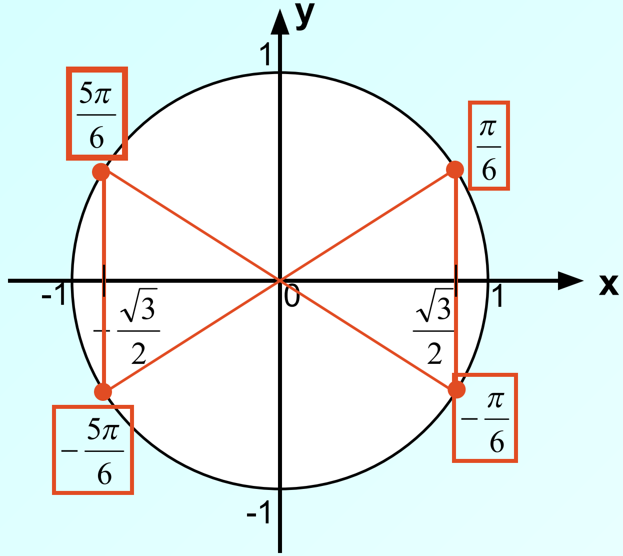


$$\sin x = \frac{1}{2}$$

$$\left[\begin{aligned} x &= \frac{\pi}{6} + 2\pi n,\ n \in Z \\ x &= \frac{5\pi}{6} + 2\pi n,\ n \in Z \end{aligned}\right.$$

3. $\cos^2 x = \dfrac{3}{4}$

$$\cos x = \pm\frac{\sqrt{3}}{2}$$



$$\left[\begin{array}{l} x = \pm\dfrac{\pi}{6} + 2\pi n,\ n \in Z; \\ x = \pm\dfrac{5\pi}{6} + 2\pi n,\ n \in Z. \end{array}\right.$$

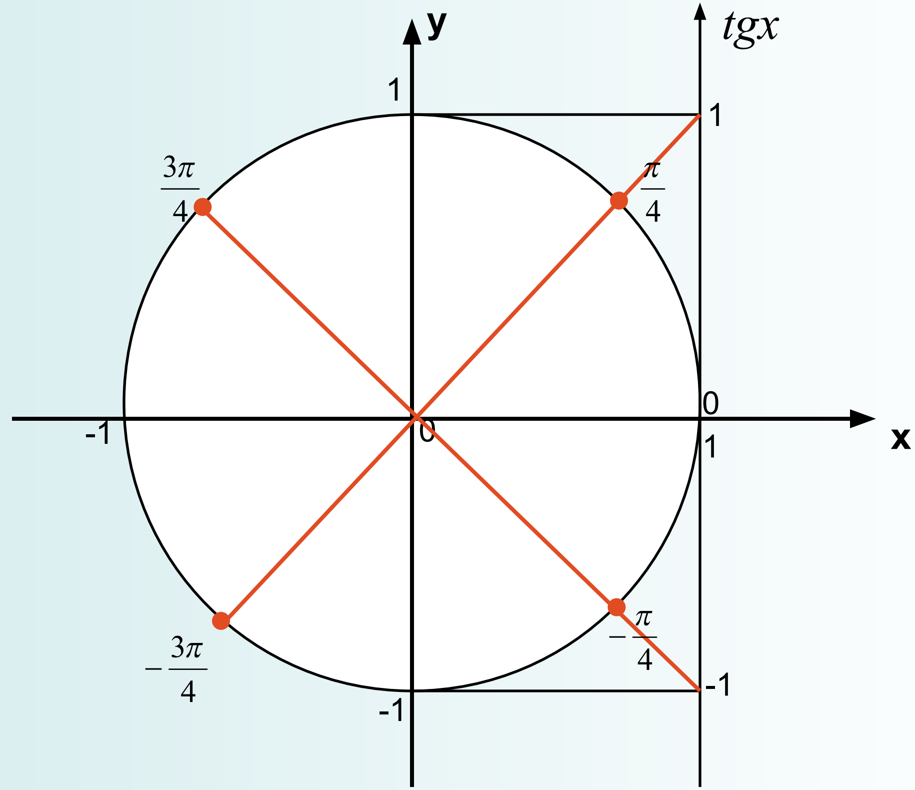
$$\left[\begin{array}{l} x = \dfrac{\pi}{6} + \pi n,\ n \in Z; \\ x = -\dfrac{\pi}{6} + \pi n,\ n \in Z. \end{array}\right.$$

$$x = \pm\frac{\pi}{6} + \pi n,\ n \in Z$$

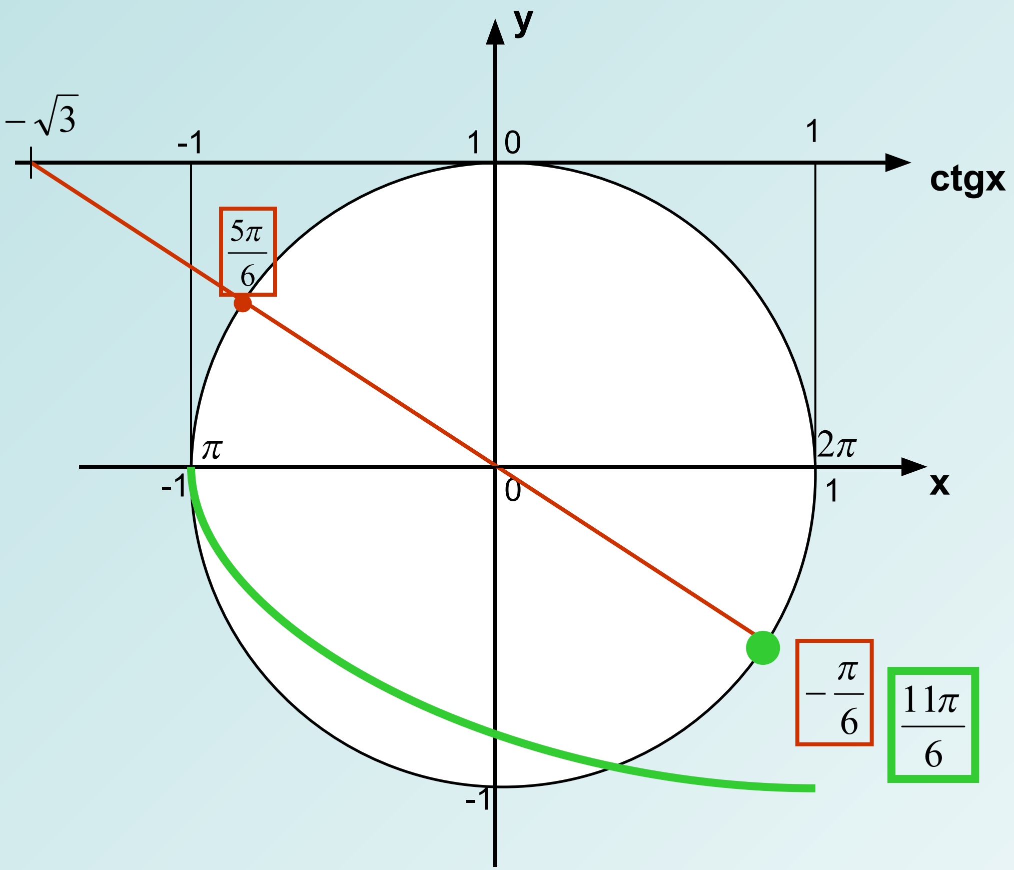
$$tg^2 x = 1$$

$$tgx = \pm 1$$

$$x = \frac{\pi}{4} + \frac{\pi}{2} n, n \in Z$$

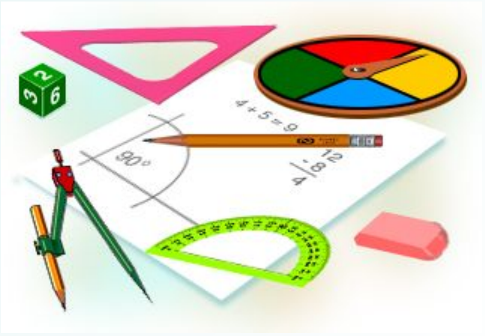


$$ctgx = -\sqrt{3},\ \text{где } x \in [\pi; 2\pi]$$



$$x = -\frac{\pi}{6} + \pi n,\ n \in Z$$

$$\frac{5\pi}{6} + \pi = \frac{11\pi}{6}$$



Аналитический способ отбора корней из промежутка

$$x = -\frac{\pi}{6} + \pi n,\ n \in Z$$

$$\pi \le -\frac{\pi}{6} + \pi n \le 2\pi,\ n \in Z;$$

$$1 \le -\frac{1}{6} + n \le 2,\ n \in Z;$$

$$6 \le -1 + 6n \le 12,\ n \in Z;$$

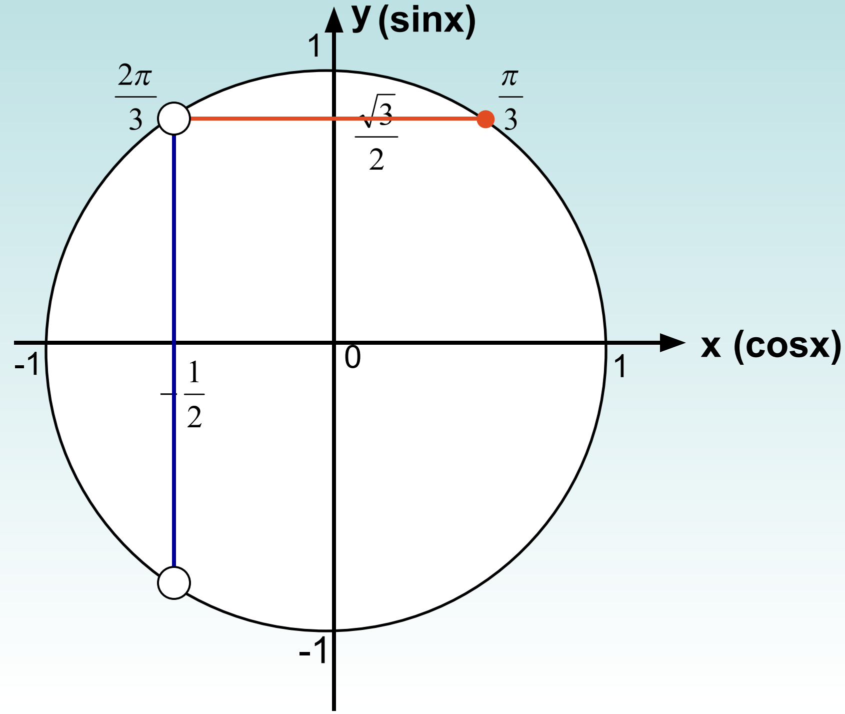
$$7 \le 6n \le 13,\ n \in Z;$$

$$1\frac{1}{6} \le n \le 2\frac{1}{6},\ n \in Z;$$

$$n = 2; \quad x = -\frac{\pi}{6} + 2\pi = \frac{11\pi}{6}$$

ОТВЕТ: $\frac{11\pi}{6}$

$$\frac{\sin x - \frac{\sqrt{3}}{2}}{\cos x + \frac{1}{2}} = 0$$



$$\begin{cases} \sin x = \frac{\sqrt{3}}{2}; \\ \cos x \neq -\frac{1}{2}. \end{cases}$$

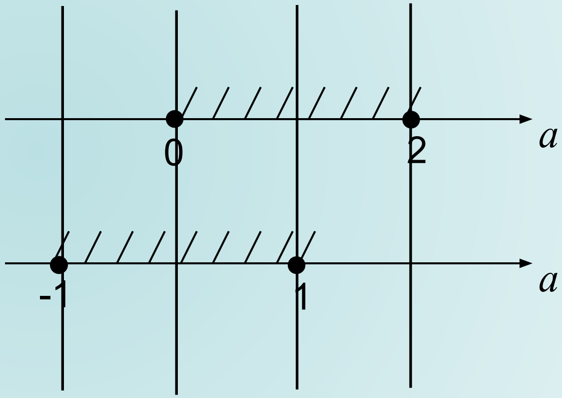
$$x = \frac{\pi}{3} + 2\pi n, \in Z$$

При каких значениях параметра a уравнение $\sin^2 x-(2a-1)\cdot\sin x+a^2-a=0$ имеет решение?

$$D=\left(-(2a-1)\right)^2-4\cdot1\cdot\left(a^2-a\right)=4a^2-4a+1-4a^2+4a=1$$

$$\left[\begin{array}{l}\sin x=a-1;\\ \sin x=a.\end{array}\right.$$

$$\left[\begin{array}{l}-1\le a-1\le 1,\\ -1\le a\le 1;\end{array}\right.$$



$$\left[\begin{array}{l}0\le a\le 2,\\ -1\le a\le 1.\end{array}\right.$$

$$a\in[-1;\,2]$$

Самостоятельная работа

ВАРИАНТ 1

Решите уравнения

$$\cos x = 0$$

$$\sin x = -\frac{1}{2}$$

$$ctgx = \sqrt{3}$$

$$tgx \cdot \cos x = 1$$

ВАРИАНТ2

$$\sin x = 1$$

$$\cos x = -\frac{\sqrt{2}}{2}$$

$$tgx = \frac{\sqrt{3}}{3}$$

$$ctgx \cdot \sin x = 1$$

ДОПОЛНИТЕЛЬНОЕ ЗАДАНИЕ

$$\frac{\sin x}{\sqrt{\pi - x^2}} = 0$$

$$\cos x \cdot \left(\sin x + \sqrt{2}\right) = 0$$

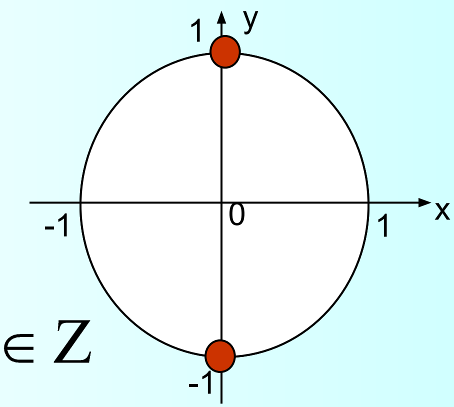
$$x \in [1;\ 3]$$

Проверяем решения

ВАРИАНТ 1

1.

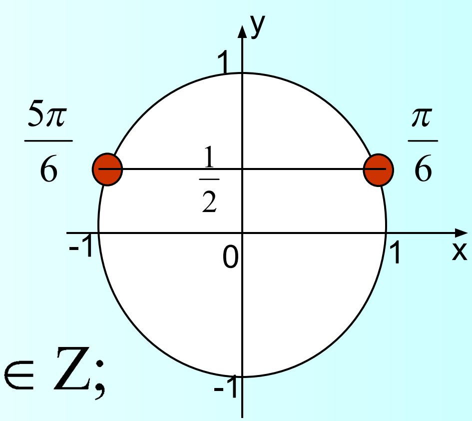
$$\cos x = 0$$



$$x = \frac{\pi}{2} + \pi n,\ n \in Z$$

2.

$$\sin x = \frac{1}{2}$$

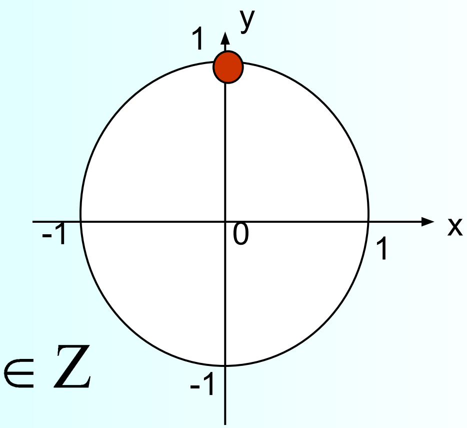


$$\left[\begin{array}{l} x = \frac{\pi}{6} = 2\pi n,\ n \in Z; \\ x = \frac{5\pi}{6} + 2\pi n,\ n \in Z. \end{array}\right.$$

ВАРИАНТ 2

1.

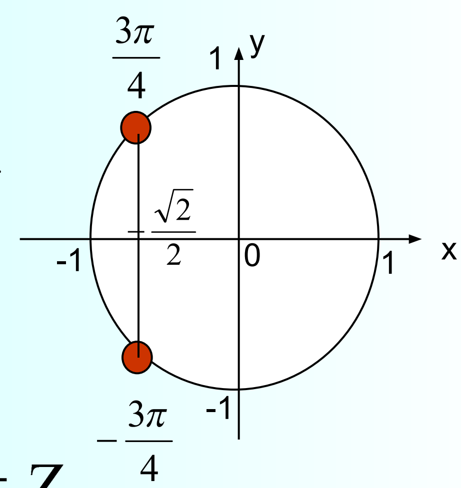
$$\sin x = 1$$



$$x = \frac{\pi}{2} + 2\pi n,\ n \in Z$$

2.

$$\cos x = -\frac{\sqrt{2}}{2}$$

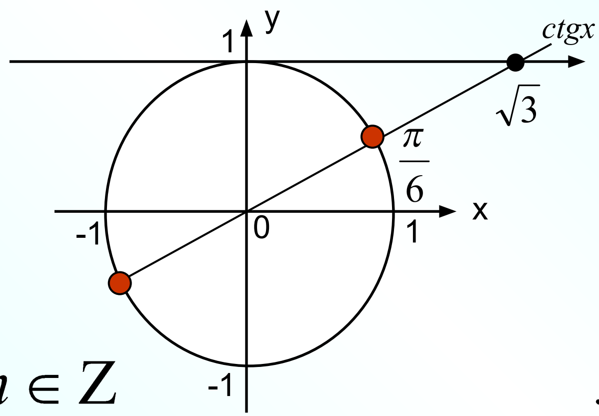


$$x = \pm\frac{3\pi}{4} + 2\pi n,\ n \in Z$$

ВАРИАНТ 1

3.

$$ctgx = \sqrt{3}$$

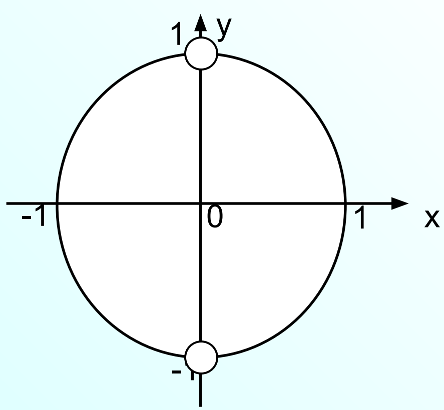


$$x = \frac{\pi}{6} + \pi n,\ n \in Z$$

4.

$$tgx \cdot \cos x = 1$$

$$\begin{cases} \cos x \neq 0; \\ \sin x = 1. \end{cases}$$

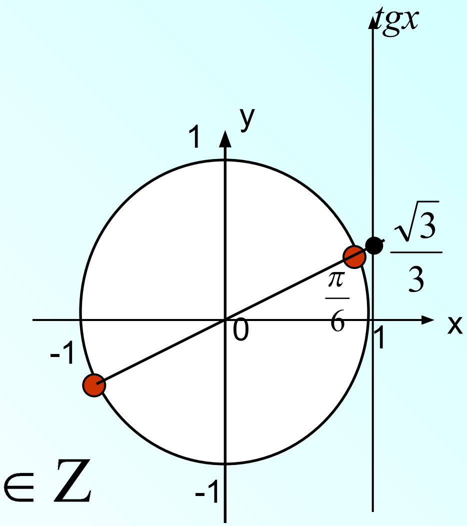


Нет решений

ВАРИАНТ 2

3.

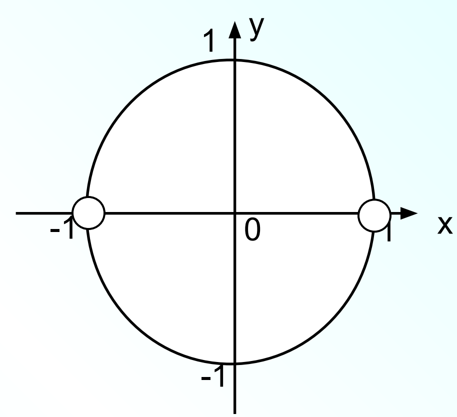
$$tgx = \frac{\sqrt{3}}{3}$$



$$x = \frac{\pi}{6} + \pi n,\ n \in Z$$

4.

$$ctgx \cdot \sin x = 1$$



$$\begin{cases} \sin x \neq 0; \\ \cos x = 1. \end{cases}$$

Нет решений

Домашняя работа

1. Дополнительные задания из самостоятельной работы.
2. № 13.28, 13.29
3. При каких значениях параметра уравнение $\sin x = \dfrac{a^2}{a-2}$ имеет решения?
4. Найти наибольший отрицательный корень уравнения $\cos^2 x - 2\sin x + 2 = 0.$

Спасибо за урок!