Karmarkar Algorithm

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Contents

LP Solutions

- Simplex
  - Dantzig 1947
- Ellipsoid
  - Kachian 1979
- Interior Point
  - Affine Method 1967
  - Log Barrier Method 1977
  - Projective Method 1984
    - Narendra Karmarkar form AT&T Bell Labs

Simplex vs Interior Point

	Simplex method	Interior-point method
Trial solutions	CPF (Corner Point Feasible) solutions	Interior points (points inside the boundary of the feasible region)
complexity	worst case:# iterations can increase exponentially in the number of variables n:	Polynomial time

Linear Programming

General LP

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Karmarkar's Canonical Form

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Minimize $q^T x$

Subject to

- $Sx \geq t$

Karmarkar’s Algorithm

Minimize $q^T x$

Subject to

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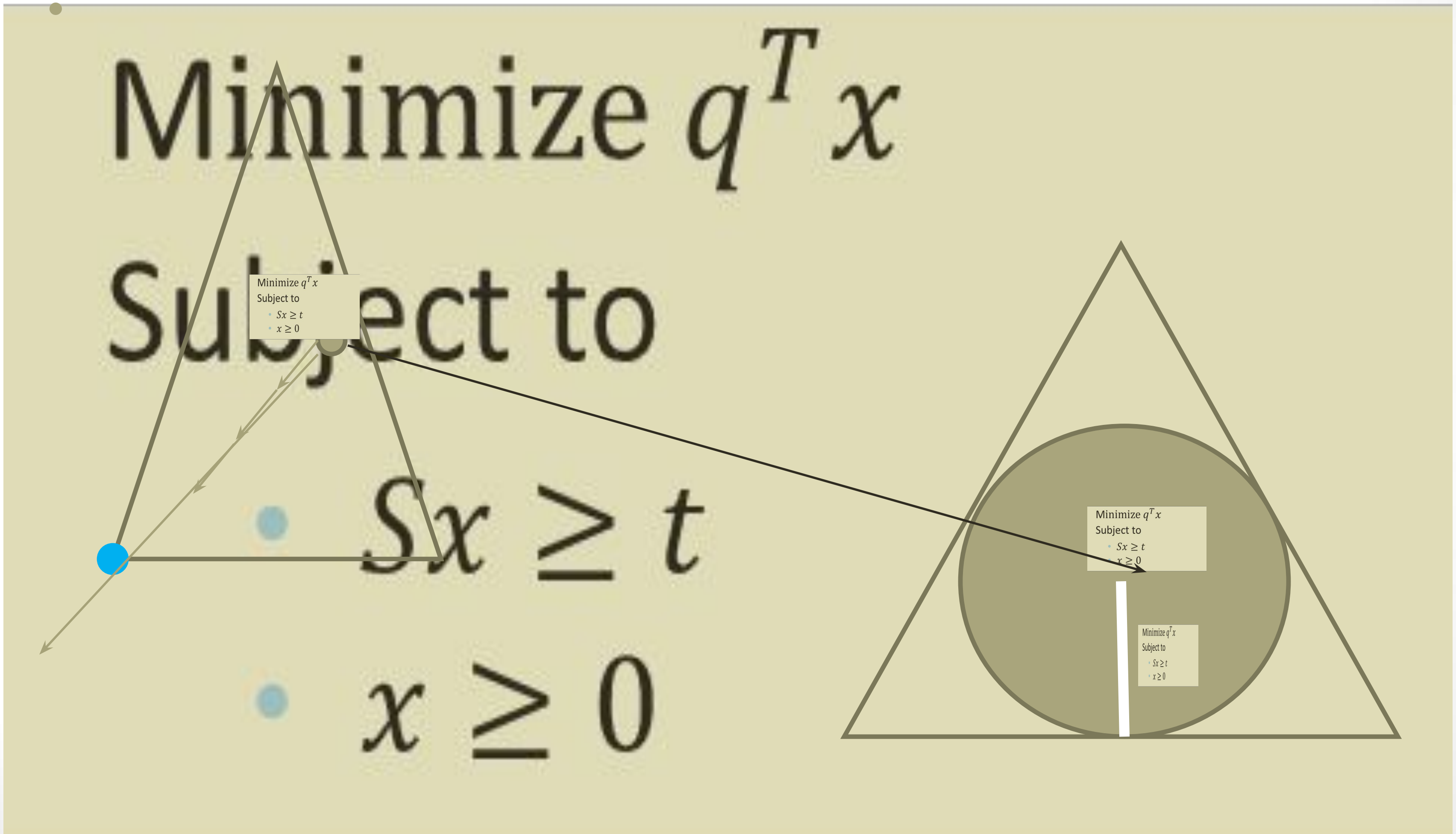
Step 2.1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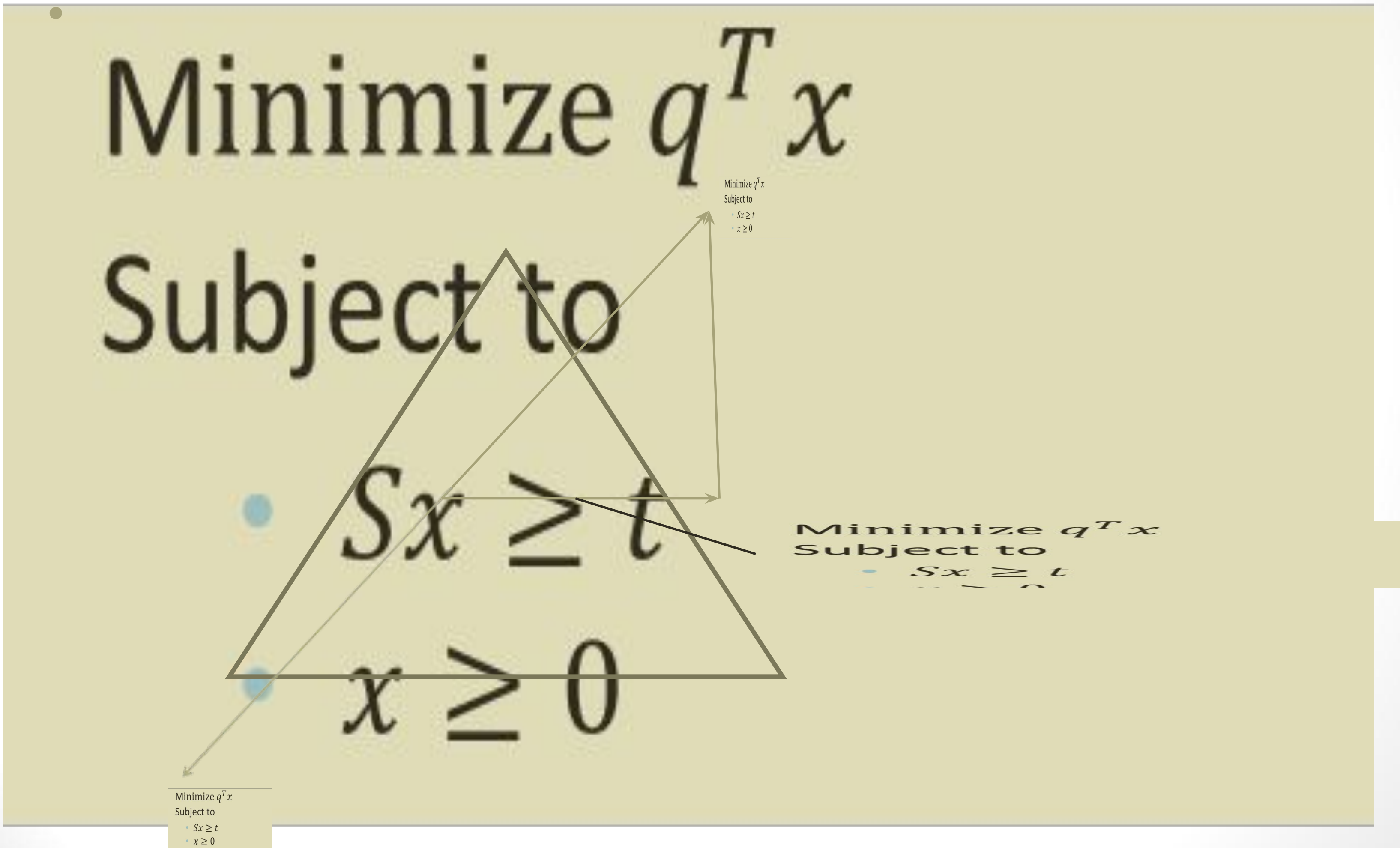
Step 2.2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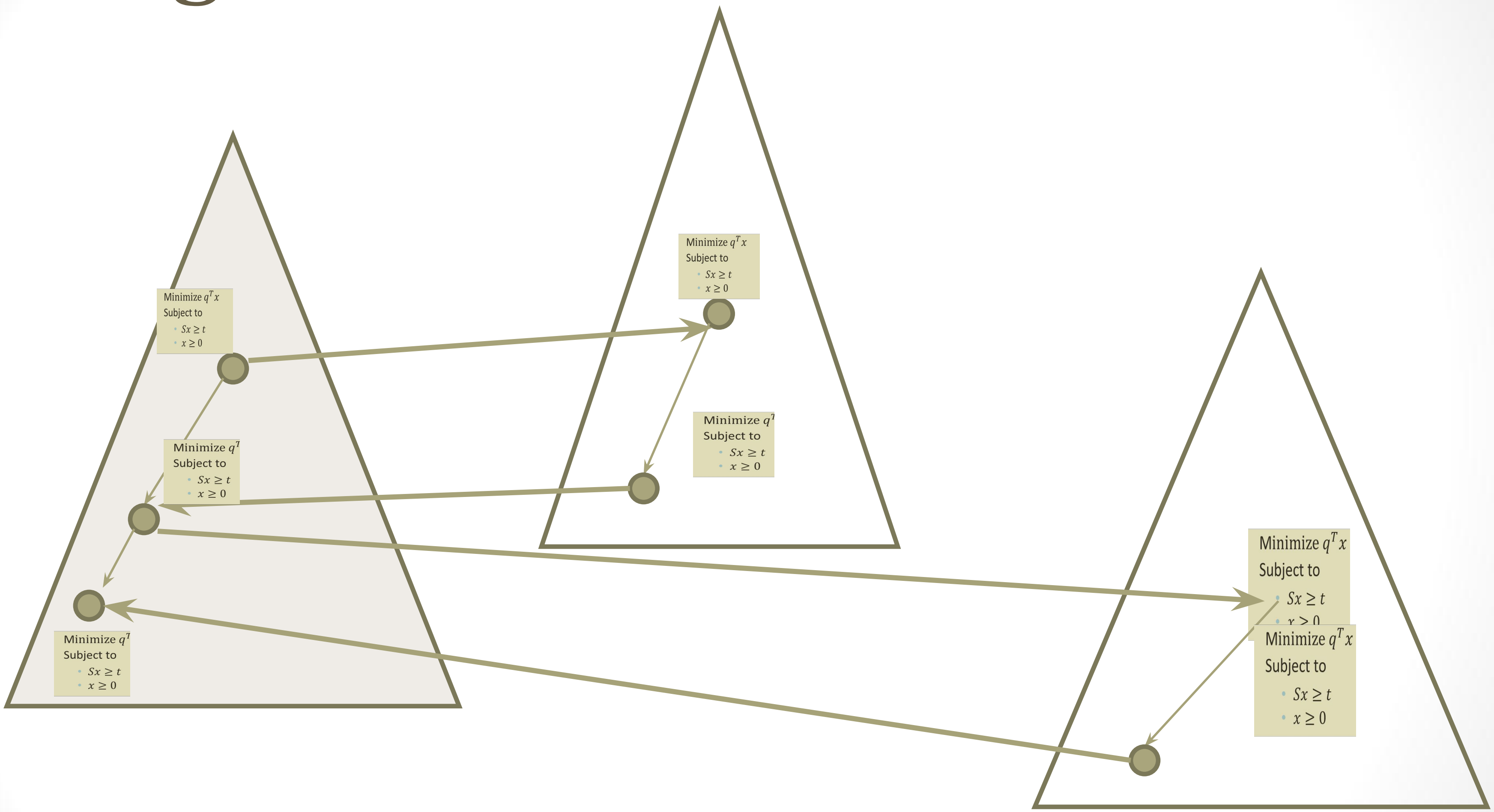
Big Picture



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- Transformation to Karmarkar's canonical form
- Projective transformation
- Orthogonal projection
- Complexity analysis

Karmarkar's Algorithm

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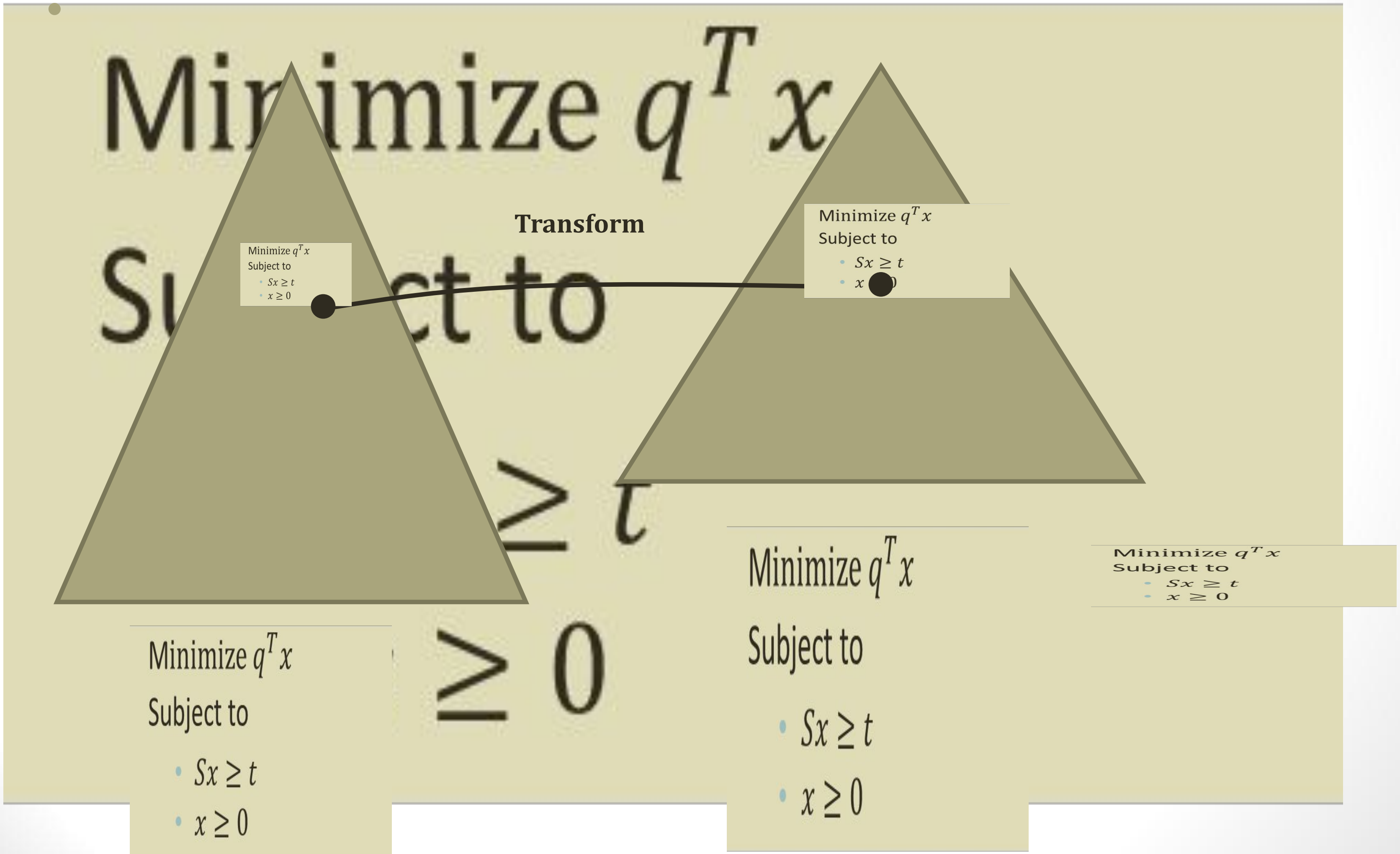
Projective Transformation



Projective transformation

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $\vec{x} \geq \vec{0}$

Problem transformation:

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Transform →

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Problem transformation:

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Convert →

Minimize $q^T x$

Subject to

- $Sx \geq t$
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Karmarkar's Algorithm

Minimize $q^T x$

Subject to

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Orthogonal Projection

Minimize $q^T x$

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- $x \geq 0$

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t to

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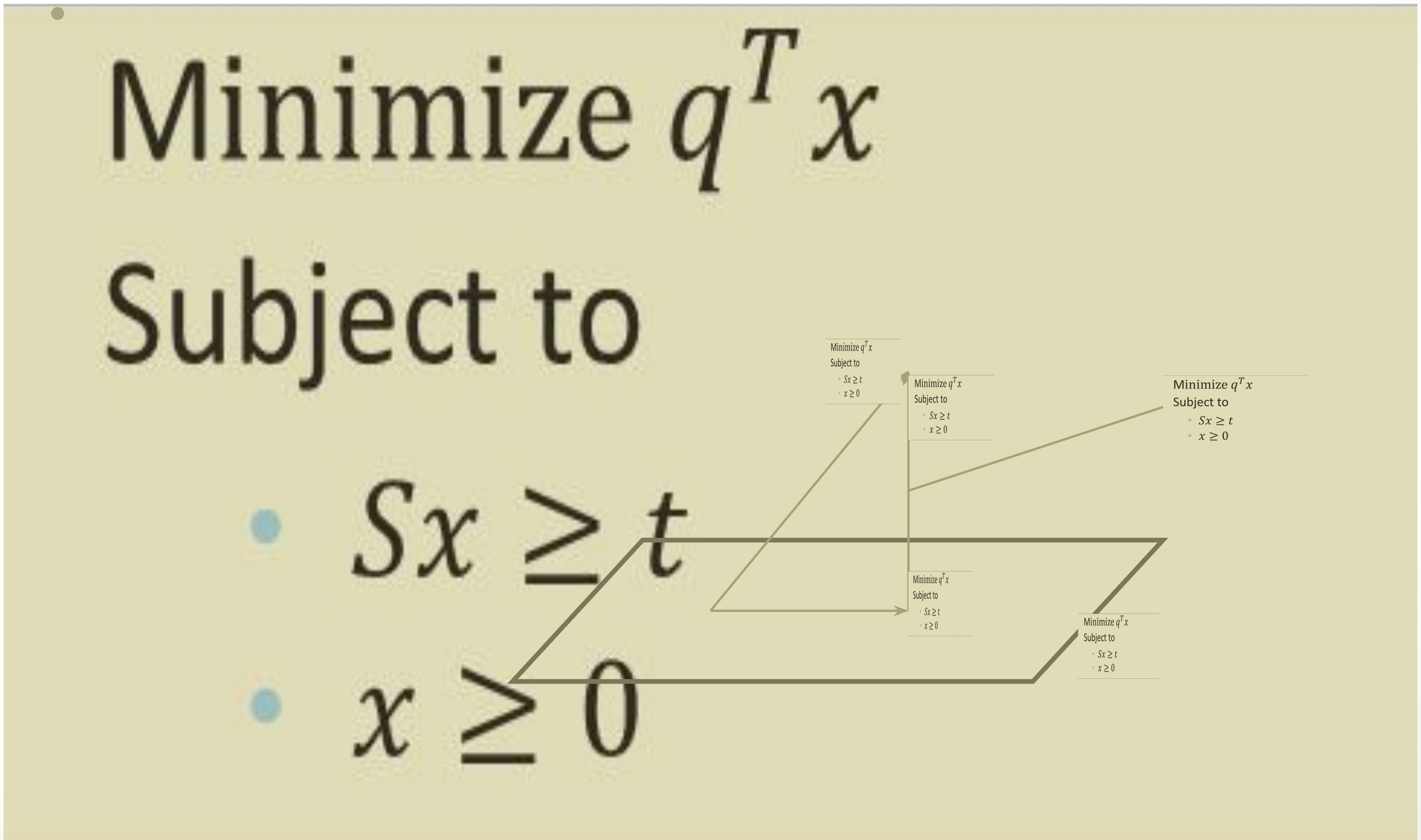
- $x \geq 0$

Orthogonal Projection



Orthogonal Projection

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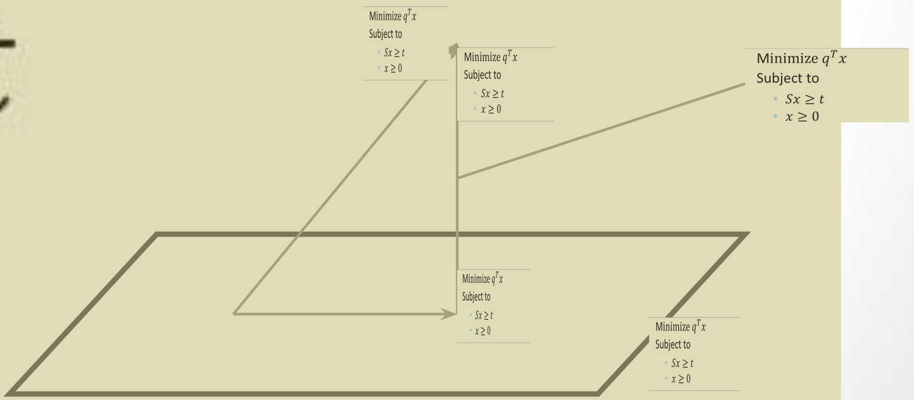

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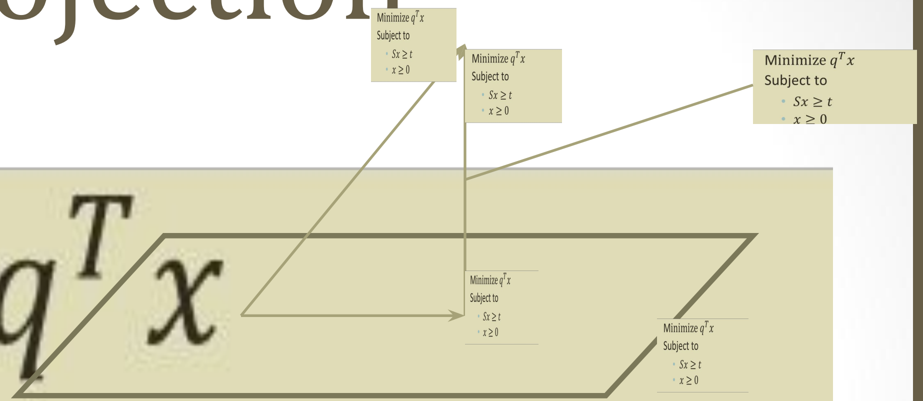

Orthogonal Projection

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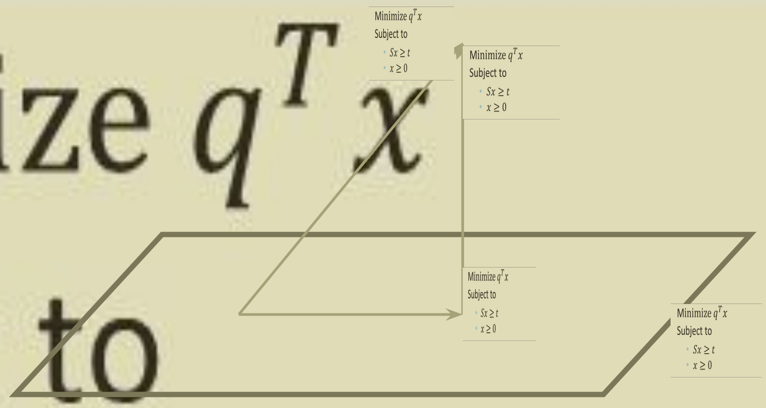


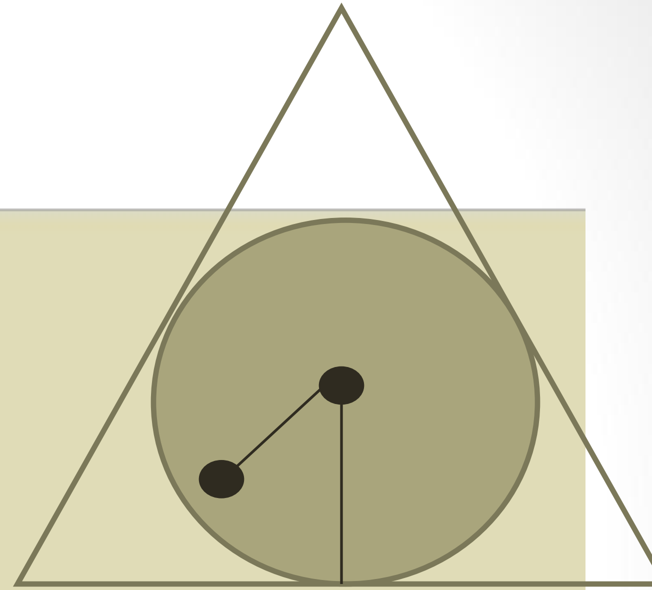
Calculate step size

Minimize $q^T x$

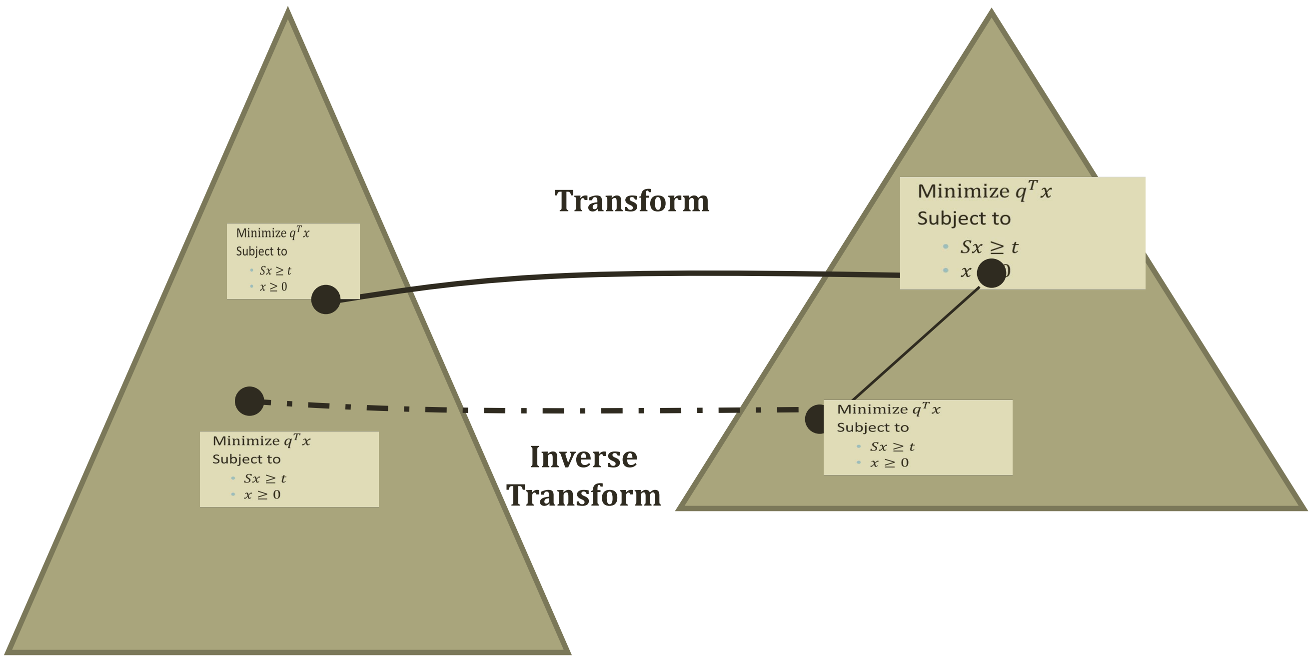
Subject to

- $Sx \geq t$
- $x \geq 0$

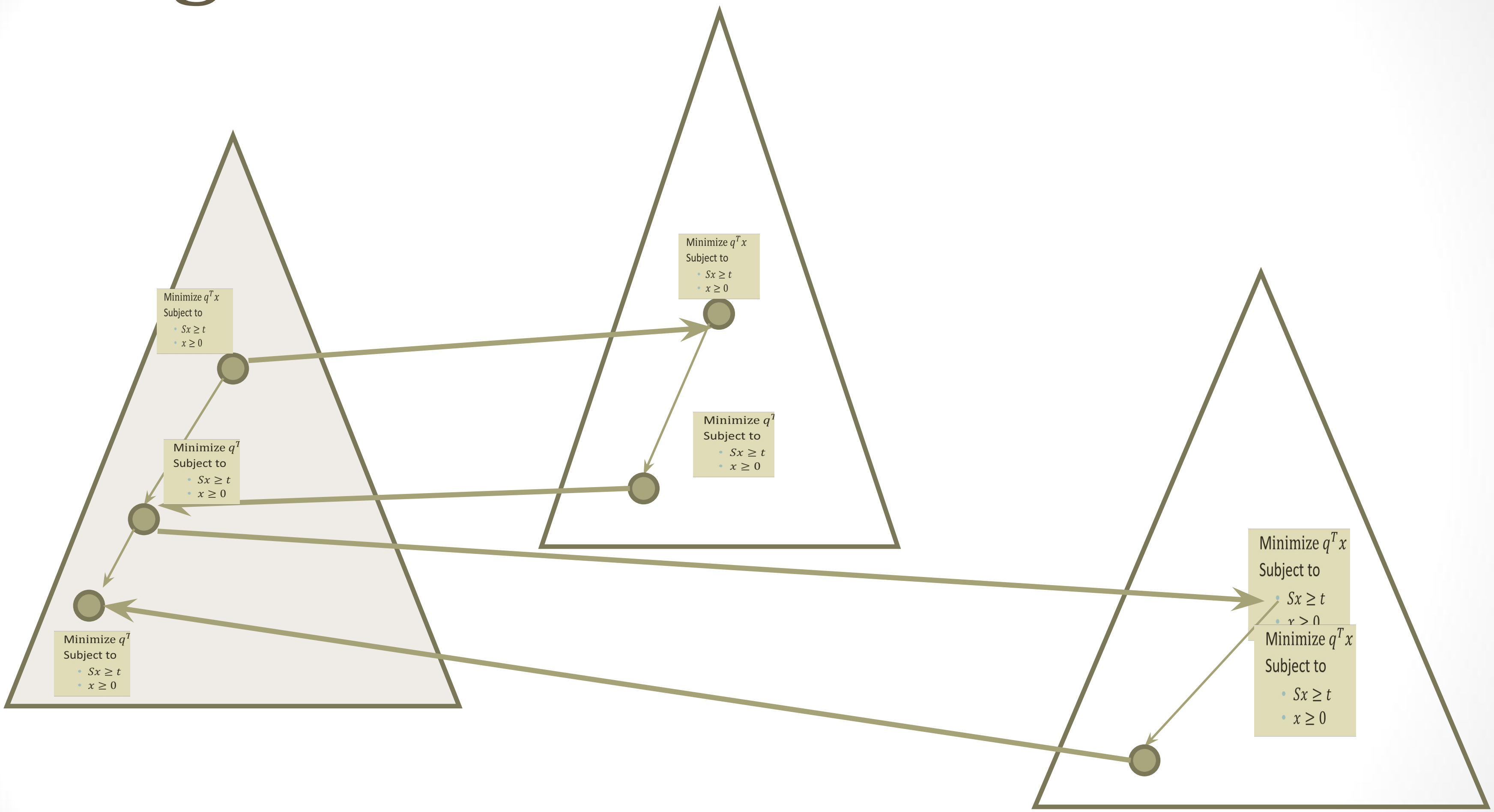
- $Sx \geq t$
- $x \geq 0$



Movement and inverse transformation



Big Picture



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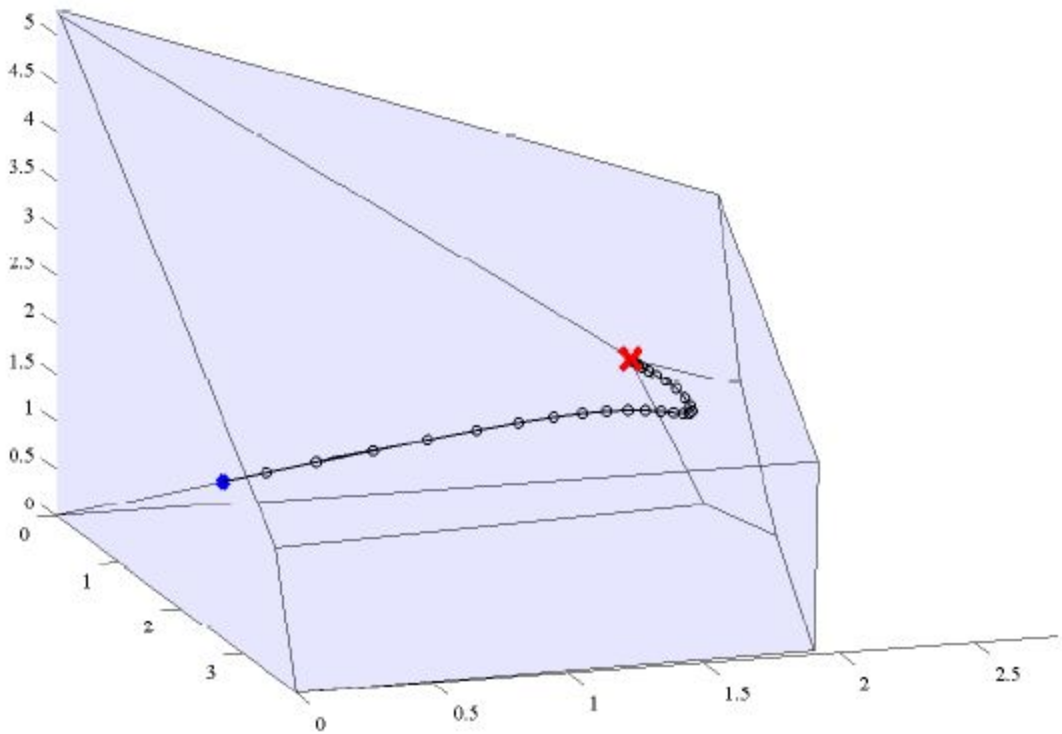
Matlab Demo



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Running Time

- Total complexity of iterative algorithm =
  (# of iterations) x (operations in each iteration)

- We will prove that the # of iterations = $O(nL)$

- Operations in each iteration = $O(n^{2.5})$

- Therefore running time of Karmarkar's algorithm = $O(n^{3.5}L)$

of iterations

Minimize $q^T x$

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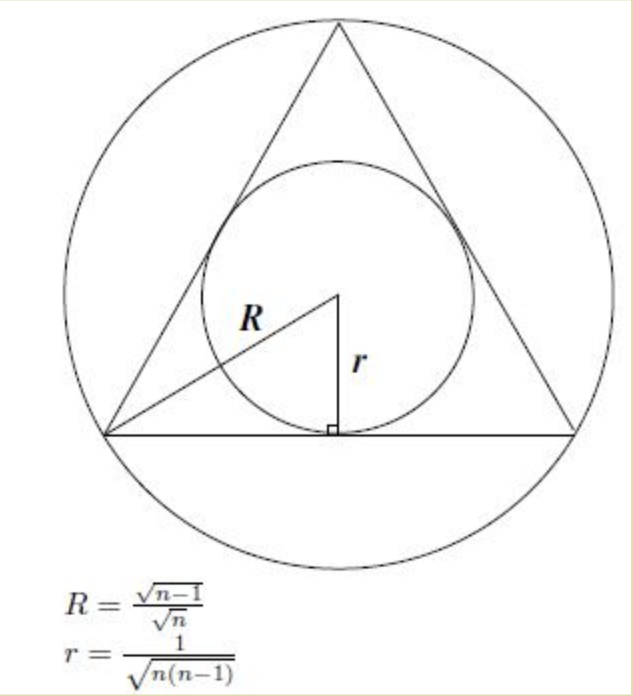

of iterations

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Subject to

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of iterations

Minimize $q^T x$

Subject to

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Rank-one modification

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Method

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Rank-one modification (cont)

- Minimize $q^T x$

  Subject to
  - $Sx \geq t$
  - $x \geq 0$

Performance Analysis

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Performance analysis - 2

Minimize $q^T x$

Subject to

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- $x \geq 0$

Performance Analysis - 3

Minimize $q^T x$

Subject to

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Performance Analysis - 4

Minimize $q^T x$

Subject to

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Performance Analysis - 5

Minimize $q^T x$

Subject to

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Transform to canonical form

General LP

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Karmarkar's Canonical Form

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Step 1:Convert LP to a feasibility problem

- Combine primal and dual problems

Primal

Minimize $q^T x$

Subject to

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- $x \geq 0$

Dual

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Combined

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

- LP becomes a feasibility problem

Step 2: Convert inequality to equality

- Introduce slack and surplus variable

Minimize $q^T x$

Subject to

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- $x \geq 0$

Step 3: Convert feasibility problem to LP

Minimize $q^T x$

Subject to

Minimize $q^T x$

Subject to

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Step 3: Convert feasibility problem to LP

Minimize $q^T x$

Subject to

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- Change of notation

Minimize $q^T x$

Subject to

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- $x \geq 0$

Minimize $q^T x$

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Minimize $q^T x$

Subject to

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- $x \geq 0$

Step 5: Get the minimum value of Canonical form

Minimize $q^T x$

Subject to

- $Sx \geq t$
- $x \geq 0$

Step 5: Get the minimum value of Canonical form

- The transformed problem is

Minimize $q^T x$

Subject to

- $Sx \geq t$

References

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Q&A