

Лекция 4. Плоские электромагнитные волны в диспергирующих средах

1. Поперечные электромагнитные волны в газе осцилляторов

- Дисперсионное уравнение
- Поглощение волн

2. Электромагнитные волны в изотропной плазме:

- Поперечные электромагнитные волны
- Продольные волны

3. Плоские волны в анизотропных средах

- Дисперсионное уравнение для волн в анизотропных средах
- Двойное лучепреломление
- Волны в плазме в магнитном поле
- Вращение плоскости поляризации

$$\text{div } \mathbf{D} = 0,$$

$$(\mathbf{k} \cdot \mathbf{D}(\omega, \mathbf{k})) = 0,$$

$$\text{rot rot } \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{D}}{\partial t^2}.$$

$$[\mathbf{k} \cdot [\mathbf{k} \cdot \mathbf{E}(\omega, \mathbf{k})]] = -\frac{\omega^2}{c^2} \mathbf{D}(\omega, \mathbf{k}).$$

$$\varepsilon^l(\omega, \mathbf{k})(\mathbf{k} \cdot \mathbf{E}^l) = 0,$$

$$\varepsilon^l(\omega, \mathbf{k}) = 0$$

$$\left(k^2 - \frac{\omega^2}{c^2} \varepsilon^{tr}(\omega, \mathbf{k}) \right) \mathbf{E}^{tr} = \frac{\omega^2}{c^2} \varepsilon^l(\omega, \mathbf{k}) \mathbf{E}^l.$$

$$k^2 - \frac{\omega^2}{c^2} \varepsilon^{tr}(\omega, \mathbf{k}) = 0$$

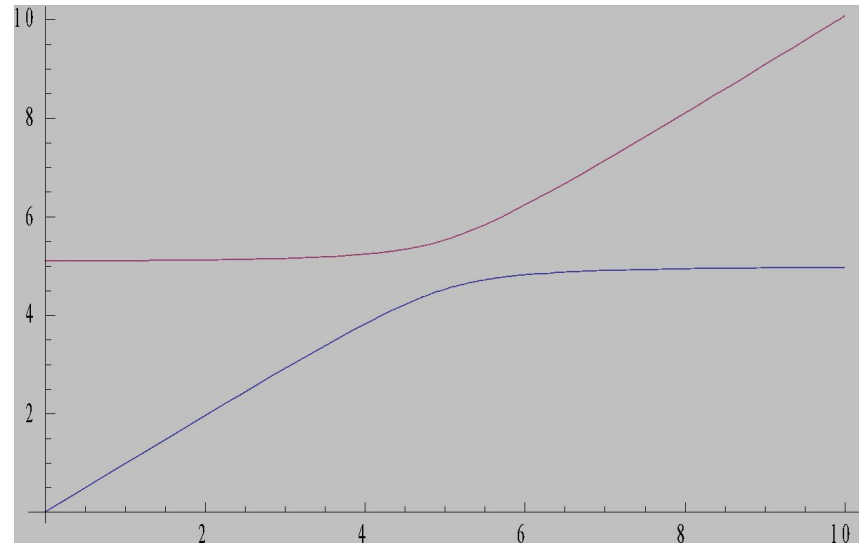
$$k^2 - \frac{\omega^2}{c^2} \left(1 - \frac{\omega_0^2}{\omega^2 - \omega_s^2 + i\nu\omega} \right) = 0$$

$$\omega_{1,2} = \sqrt{\frac{1}{2}(\omega_s^2 + k^2 c^2 + \omega_0^2)} \mp \frac{1}{2} \sqrt{(\omega_s^2 + k^2 c^2 + \omega_0^2)^2 - 4\omega_s^2 k^2 c^2}$$

$$\omega_0^2 \omega_s^2 \ll (\omega_s^2 - k^2 c^2)^2$$

$$\omega_1 = kc - \frac{1}{2} \frac{\omega_0^2}{\omega_s^2 - k^2 c^2} kc,$$

$$\omega_2 = \omega_s + \frac{\omega_0^2}{\omega_s^2 - k^2 c^2} \omega_s.$$

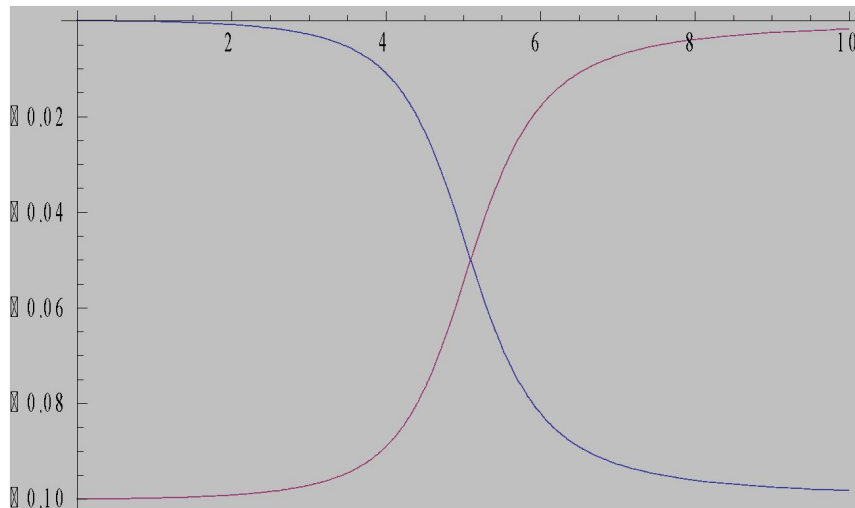
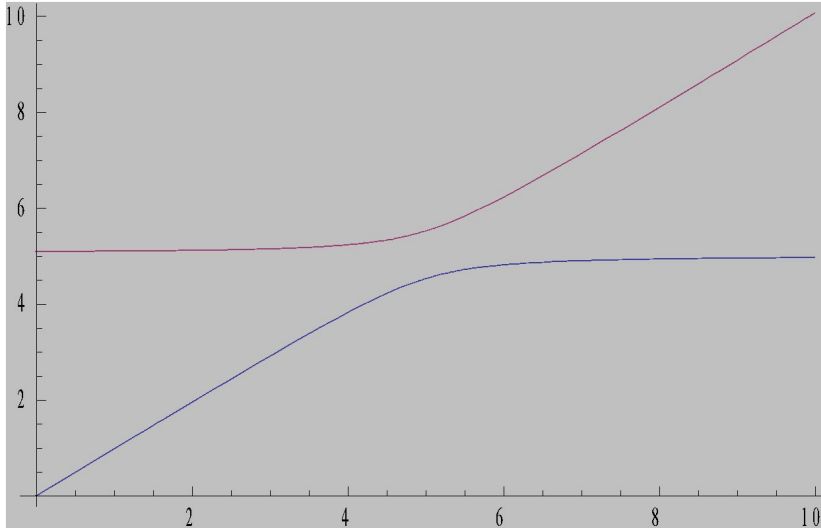


$$vkc \ll |\omega_s^2 - k^2c^2|$$

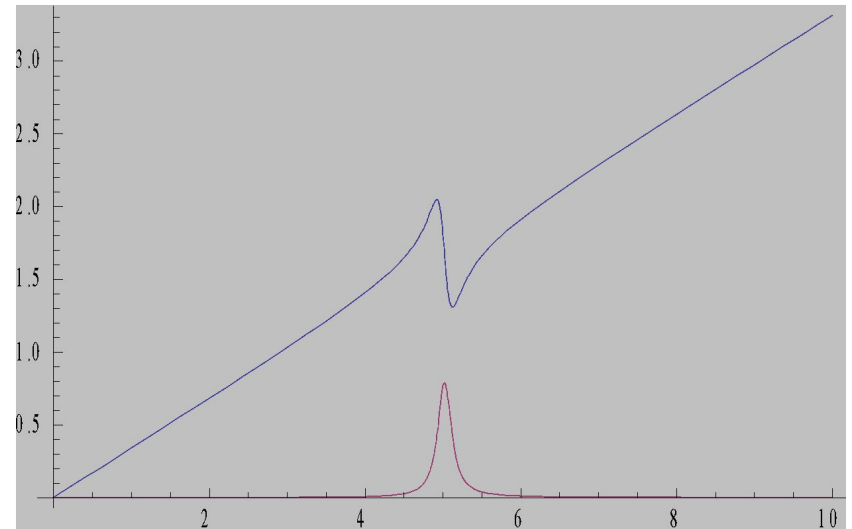
$$\omega_1 = kc - \frac{1}{2} \frac{\omega_0^2}{\omega_s^2 - k^2c^2} kc - iv \frac{\omega_0^2 k^2 c^2}{(\omega_s^2 - k^2c^2)^2}$$

$$v \ll \omega_s$$

$$\omega_2 = \omega_s + \frac{\omega_0^2}{\omega_s^2 - k^2c^2} \omega_s - \frac{1}{2} iv$$



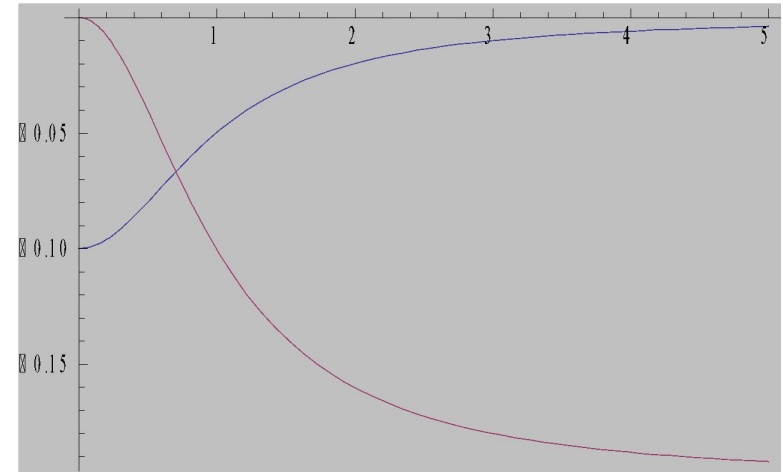
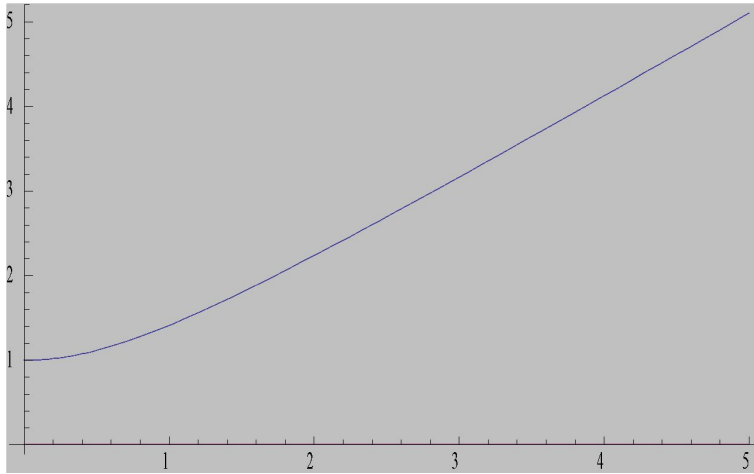
$$k(\omega) = \frac{\omega}{c} \sqrt{1 - \frac{\omega_0^2}{\omega^2 - \omega_s^2 + iv\omega}}$$



$$k(\omega_s) = \frac{\omega_s}{c} \sqrt{1 + i \frac{\omega_0^2}{v\omega_s}}$$

$$\omega_{Li} \ll \omega_{Le} : \quad k^2 = \frac{\omega^2}{c^2} \left(1 - \frac{\omega_{Le}^2}{\omega(\omega + i\nu_e)} \right)$$

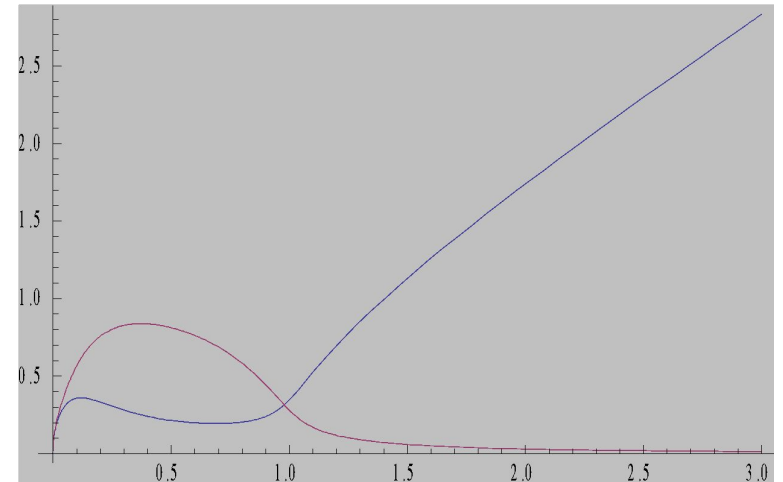
$$\nu_e \ll \omega_{Le} : \quad \omega_{1,2} = \sqrt{k^2 c^2 + \omega_{Le}^2} - i \frac{\nu_e}{2} \frac{\omega_{Le}^2}{k^2 c^2 + \omega_{Le}^2}; \quad \omega_3 = -i \nu_e \frac{k^2 c^2}{k^2 c^2 + \omega_{Le}^2}$$



$$k = \frac{\omega}{c} \sqrt{1 - \frac{\omega_{Le}^2}{\omega(\omega + i\nu_e)}}$$

$$\omega^2 < \omega_{Le}^2 : \quad \exp[-(\omega/c) \sqrt{\omega_{Le}^2/\omega^2 - 1} z], \quad z > 0$$

$$|\omega| \ll \nu_e : \quad k = \frac{i-1}{\sqrt{2}} \frac{\omega_{Le}}{c} \sqrt{\frac{\omega}{\nu_e}} = (i-1) \sqrt{\frac{2\pi\sigma(0)\omega}{c^2}}$$



$$1 - \frac{\omega_{Li}^2}{\omega(\omega + iv_i) - k^2 V_{\gamma i}^2} - \frac{\omega_{Le}^2}{\omega(\omega + iv_e) - k^2 V_{\gamma e}^2}$$

$$|\omega| > \omega_{Le} : \omega_{1,2} = \pm \sqrt{\omega_{Le}^2 + k^2 V_{\gamma e}^2 - \frac{v_e^2}{4} - i \frac{v_e}{2}} \quad \left| \quad v_e^2 \gg \omega_{Le}^2 + k^2 V_{\gamma e}^2 : \omega_1 = -iv_e^{-1}(k^2 V_{\gamma e}^2 + \omega_{Le}^2), \omega_2 = -iv_e \right.$$

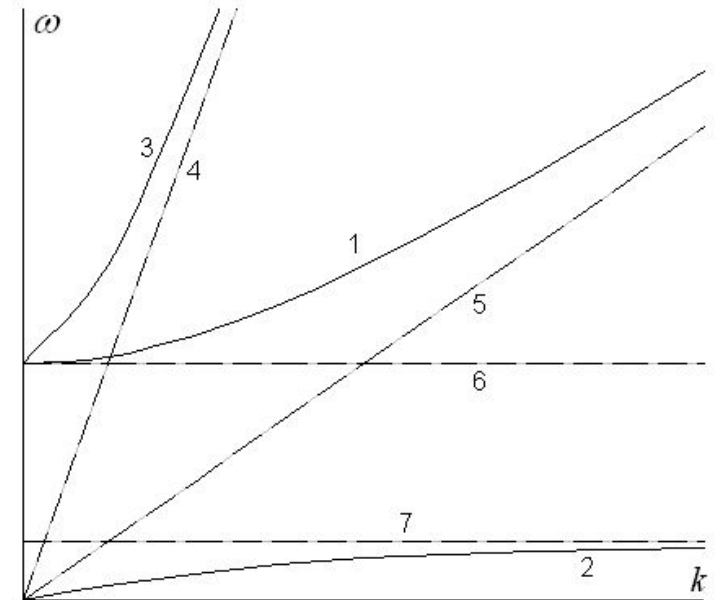
$$kV_{\gamma e} \gg |\omega| \gg kV_{\gamma i} : 1 + \frac{\omega_{Le}^2}{k^2 V_{\gamma e}^2} - \frac{\omega_{Li}^2}{\omega(\omega + iv_i)} = 0$$

$$\omega_{1,2} = \pm \sqrt{\omega_{is}^2(k) - \frac{v_i^2}{4} - i \frac{v_i}{2}}$$

$$\omega_{is}(k) = \omega_{Li} \frac{kV_{\gamma e}}{\sqrt{\omega_{Le}^2 + k^2 V_{\gamma e}^2}}$$

$$v_i^2 \gg \omega_{is}^2(k) : \omega_1 = -iv_i^{-1} \omega_{is}^2(k), \omega_2 = -iv_i$$

$$\omega \rightarrow 0 : k^2 + 1/r_D^2 = 0, \quad r_D = \left(\frac{\omega_{Li}^2}{V_{\gamma i}^2} + \frac{\omega_{Le}^2}{V_{\gamma e}^2} \right)^{-1/2}$$



$$k_i \varepsilon_{ij}(\omega, \mathbf{k}) E_j(\omega, \mathbf{k}) = 0,$$

$$\left[k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \mathbf{k}) \right] E_j(\omega, \mathbf{k}) = 0.$$

$$\det \left[k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \mathbf{k}) \right]$$

$$\mathbf{k} = \{0, 0, k_z\}$$

$$\left(k_z^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} \right) E_x - \frac{\omega^2}{c^2} \varepsilon_{xy} E_y = \frac{\omega^2}{c^2} \varepsilon_{xz} E_z,$$

$$\left(k_z^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} \right) E_y - \frac{\omega^2}{c^2} \varepsilon_{yx} E_x = \frac{\omega^2}{c^2} \varepsilon_{yz} E_z,$$

$$\varepsilon_{zz} E_z = -\varepsilon_{zx} E_x - \varepsilon_{zy} E_y.$$

$$\varepsilon'_{ij} = \varepsilon'_{ji}, \quad \varepsilon''_{ij} = 0$$

$$D_x = \varepsilon_{xx} E_x, \quad D_y = \varepsilon_{yy} E_y, \quad D_z = \varepsilon_{zz} E_z$$

$$\left(k_z^2 + k_y^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} \right) \left(k_z^2 + k_x^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} \right) \left(k_x^2 + k_y^2 - \frac{\omega^2}{c^2} \varepsilon_{zz} \right) -$$

$$- k_z^2 k_y^2 \left(k_z^2 + k_y^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} \right) - k_z^2 k_x^2 \left(k_z^2 + k_x^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} \right) -$$

$$- k_x^2 k_y^2 \left(k_x^2 + k_y^2 - \frac{\omega^2}{c^2} \varepsilon_{zz} \right) - 2k_x^2 k_y^2 k_z^2 = 0.$$

$$k_y = 0 \quad \left(k_z^2 + k_x^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} \right) \left(\varepsilon_{zz} k_z^2 + \varepsilon_{xx} k_x^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} \varepsilon_{zz} \right) = 0.$$

$$k_{z1} = \sqrt{\frac{\omega^2}{c^2} \varepsilon_{yy} - k_x^2}, \quad k_{z2} = \sqrt{\frac{\omega^2}{c^2} \varepsilon_{xx} - k_x^2 \frac{\varepsilon_{xx}}{\varepsilon_{zz}}} \quad \tan \alpha = k_x c / \sqrt{\omega^2 - k_x^2 c^2}$$

$$A_1 \exp(-i\omega t + i\mathbf{k}_1 \cdot \mathbf{r}) + A_2 \exp(-i\omega t + i\mathbf{k}_2 \cdot \mathbf{r})$$

$$\tan \beta_{1,2} = k_x / k_{z1,2}$$

$$\mathbf{k} = \{0, 0, k_z\}$$

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \frac{\omega_{Le}^2}{\omega^2 - \Omega_e^2} \equiv \varepsilon_{\perp}(\omega),$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = -i \frac{\omega_{Le}^2 \Omega_e}{\omega (\omega^2 - \Omega_e^2)} \equiv ig(\omega),$$

$$\varepsilon_{zz} = 1 - \frac{\omega_{Le}^2}{\omega^2} \equiv \varepsilon_{\parallel}(\omega). \quad \varepsilon_{ij} = \varepsilon_{ji}^*$$

$$\left(k_z^2 - \frac{\omega^2}{c^2} \varepsilon_{\perp}(\omega) \right) E_x - i \frac{\omega^2}{c^2} g(\omega) E_y = 0,$$

$$\left(k_z^2 - \frac{\omega^2}{c^2} \varepsilon_{\perp}(\omega) \right) E_y + i \frac{\omega^2}{c^2} g(\omega) E_x = 0,$$

$$\varepsilon_{\parallel}(\omega) E_z = 0.$$

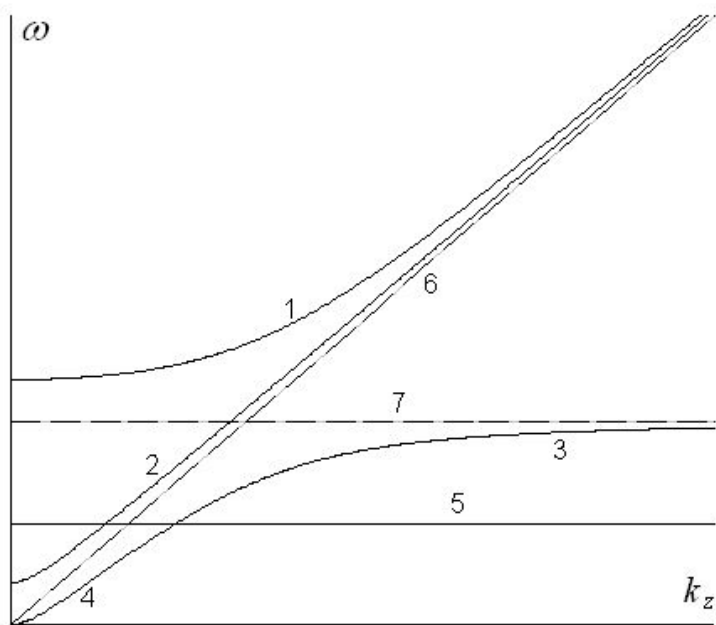
$$k_z^2 - \frac{\omega^2}{c^2} \varepsilon_{\perp}(\omega) = \pm \frac{\omega^2}{c^2} g(\omega) \rightarrow (\omega^2 - k_z^2 c^2)(\omega \mp \Omega_e) = \omega \omega_{Le}^2$$

$$E_x / E_y = \pm i \rightarrow E_x(t, z) = \pm A \sin(\omega t - k_z z + \phi), \quad E_y(t, z) = A \cos(\omega t - k_z z + \phi)$$

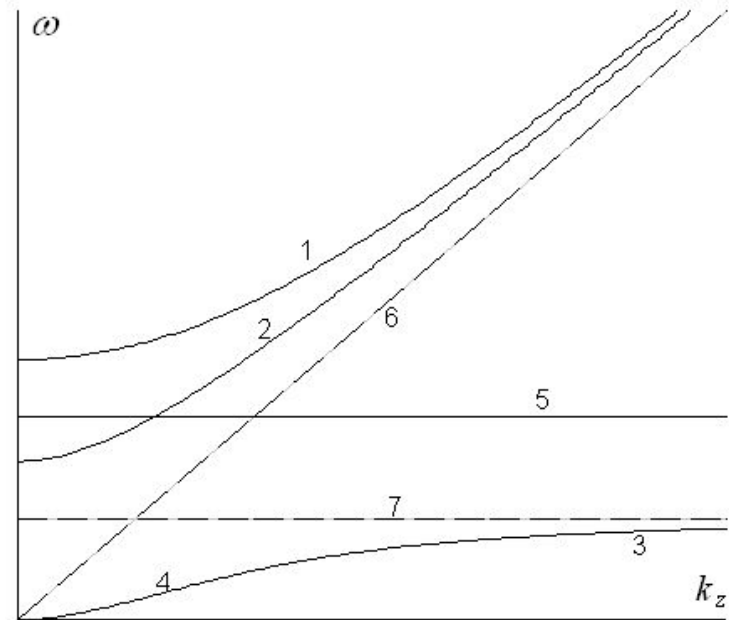
$$\omega = \begin{cases} \sqrt{k_z^2 c^2 + \omega_{Le}^2 \frac{k_z c}{k_z c \mp \Omega_e}}, & \omega_{Le}^2 \ll k_z c |k_z c \mp \Omega_e| \\ \sqrt{\omega_{Le}^2 + \Omega_e^2/4 \pm \Omega_e/2}, & k_z c \ll \omega \end{cases}$$

$$\omega = \Omega_e + \Omega_e \frac{\omega_{Le}^2}{\Omega_e^2 - k_z^2 c^2}, \quad \omega_{Le}^2 \ll |\Omega_e^2 - k_z^2 c^2|$$

$$\omega = k_z^2 c^2 \frac{\Omega_e}{\omega_{Le}^2}, \quad k_z^2 c^2 \ll \omega_{Le}^2$$



$$\Omega_e > \omega_{Le}$$



$$\Omega_e < \omega_{Le}$$

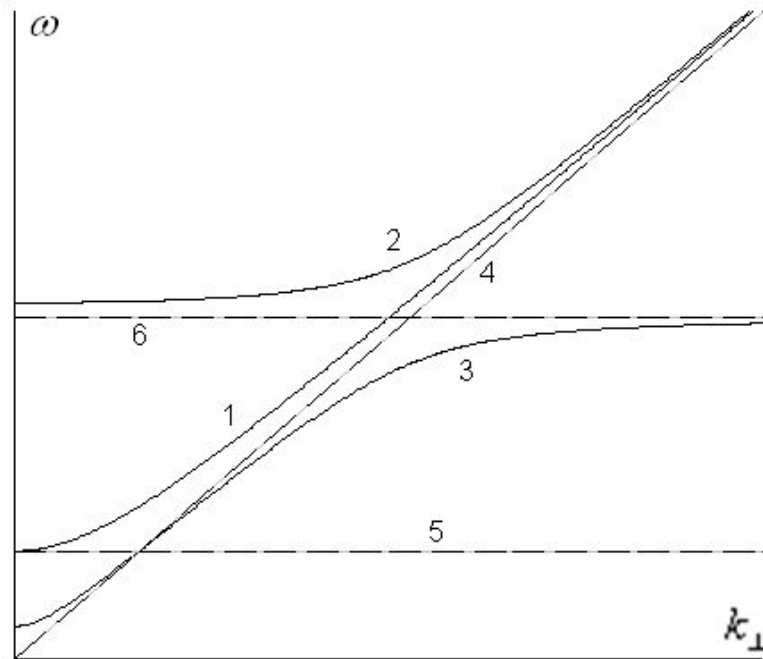
$$\mathbf{k} = \{k_x, 0, 0\}$$

$$\varepsilon_{\perp}(\omega)E_x + ig(\omega)E_y = 0,$$

$$\left(k_{\perp}^2 - \frac{\omega^2}{c^2} \varepsilon_{\perp}(\omega)\right)E_y = -i \frac{\omega^2}{c^2} g(\omega)E_x,$$

$$\left(k_{\perp}^2 - \frac{\omega^2}{c^2} \varepsilon_{\parallel}(\omega)\right)E_z = 0. \quad \longrightarrow \quad \omega = \sqrt{k_x^2 c^2 + \omega_{Le}^2}$$

$$k_x^2 - \frac{\omega^2}{c^2} \frac{\varepsilon_{\perp}^2(\omega) - g^2(\omega)}{\varepsilon_{\perp}(\omega)} = 0 \rightarrow (\omega^2 - \omega_{Le}^2)^2 - \omega^2 \Omega_e^2 = k_x^2 c^2 (\omega^2 - \Omega_g^2)$$



$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{\perp} & ig & 0 \\ -ig & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\parallel} \end{pmatrix}$$

$$k_z^2 - \frac{\omega^2}{c^2} (\varepsilon_{\perp} \pm g) = 0$$

$$E_x/E_y = \pm i$$

$$E_x(t, z) = \pm A \sin(\omega t - k_z z + \phi)$$

$$E_y(t, z) = A \cos(\omega t - k_z z + \phi)$$

$$k_{z1}(\omega) = \frac{\omega}{c} \sqrt{\varepsilon_{\perp} + g},$$

$$k_{z2}(\omega) = \frac{\omega}{c} \sqrt{\varepsilon_{\perp} - g}.$$

$$E_{x1}(t, z) = A_1 \sin(\omega t - k_{z1} z + \phi_1),$$

$$E_{y1}(t, z) = A_1 \cos(\omega t - k_{z1} z + \phi_1),$$

$$E_{x2}(t, z) = -A_2 \sin(\omega t - k_{z2} z + \phi_2),$$

$$E_{y2}(t, z) = A_2 \cos(\omega t - k_{z2} z + \phi_2).$$

$$E_{x1}(t, 0) + E_{x2}(t, 0) = 0,$$

$$E_{y1}(t, 0) + E_{y2}(t, 0) = A_0 \cos \omega t,$$

$$E_x(t, z) = E_{x1}(t, z) + E_{x2}(t, z) = A_0 \cos\left(\omega t - \frac{k_{z1} + k_{z2}}{2} z\right) \sin\left(\frac{k_{z1} - k_{z2}}{2} z\right),$$

$$E_y(t, z) = E_{y1}(t, z) + E_{y2}(t, z) = A_0 \cos\left(\omega t - \frac{k_{z1} + k_{z2}}{2} z\right) \cos\left(\frac{k_{z1} - k_{z2}}{2} z\right).$$

$$\theta = \frac{1}{2} (k_{z1} - k_{z2}) z = \frac{\omega}{c} g \left(\sqrt{\varepsilon_{\perp} + g} + \sqrt{\varepsilon_{\perp} - g} \right)^{-1} z$$

$$\theta = \frac{1}{2} \frac{\omega}{c} g z$$

$$\theta = -\frac{\Omega_e}{c} \frac{\omega_{Le}^2}{\omega^2 - \Omega_e^2} \left(\sqrt{1 - \frac{\omega_{Le}^2}{\omega(\omega - \Omega_e)}} + \sqrt{1 - \frac{\omega_{Le}^2}{\omega(\omega + \Omega_e)}} \right)^{-1} z$$

$$\theta = -\frac{1}{2} \frac{\Omega_e}{c} \frac{\omega_{Le}^2}{\omega^2} z$$

