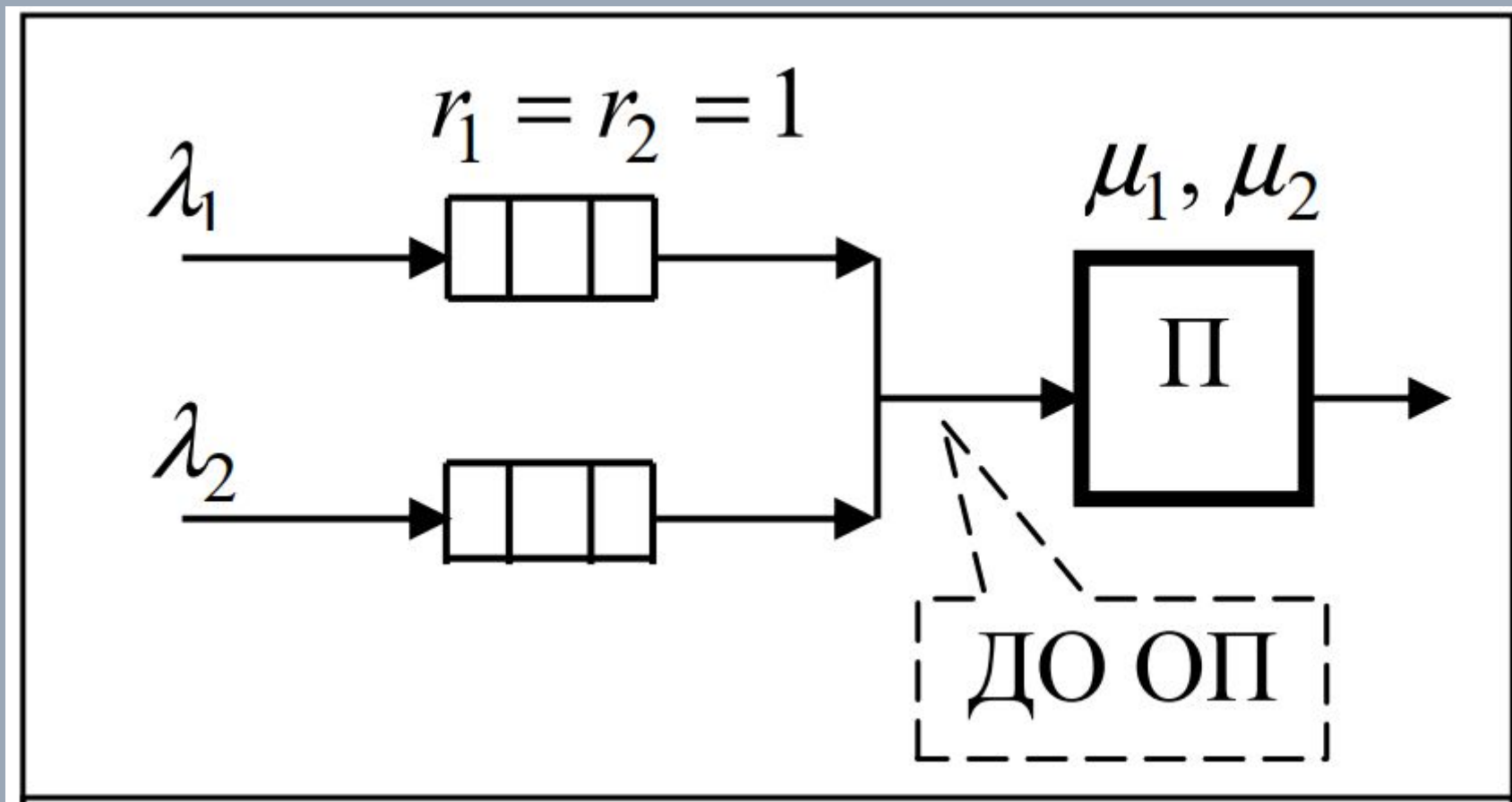




non-uniform
flow
requests and
relative
priorities



SINGLE-CHANNEL QS WITH NON-UNIFORM FLOW
REQUESTS AND RELATIVE PRIORITIES

. Description of the system

11.1. The system is single-channel.

1.2. Incoming stream of applications - heterogeneous: the system enters two classes of applications.

1.3. Accumulators for orders each class - limited capacity: $r_1 = r_2 = 1$.

1.4. Buffering discipline - without displacing orders: if at receipt of any application in the system

class the corresponding drive is full, then the application is lost

1.4. Service discipline - with relative priorities: first class applications have priority over second class applications class.

Assumptions

2.1. Entries of two classes entering the system form the simplest flows with intensities λ_1 and λ_2 , respectively.

2.2. The service times for customers of each class are distributed exponentially with intensities

$\mu_1 = 1 / b_1$ and $\mu_2 = 1 / b_2$, where b_1 and b_2 are the average durations of servicing requests of the class 1 and 2 respectively.

A stationary regime always exists in the QS, since it cannot be endless queues.

3. Coding the states of a random process

E0: (0 / 0,0) - there are no orders in the system;

E1: (1 / 0,0) - a class 1 request is being serviced in the device;

E2: (2 / 0,0) - a class 2 request is being serviced in the device; E4: (1 / 0,1) - there is a class 1 request and one a class 2 claim is awaiting service in the second drive;

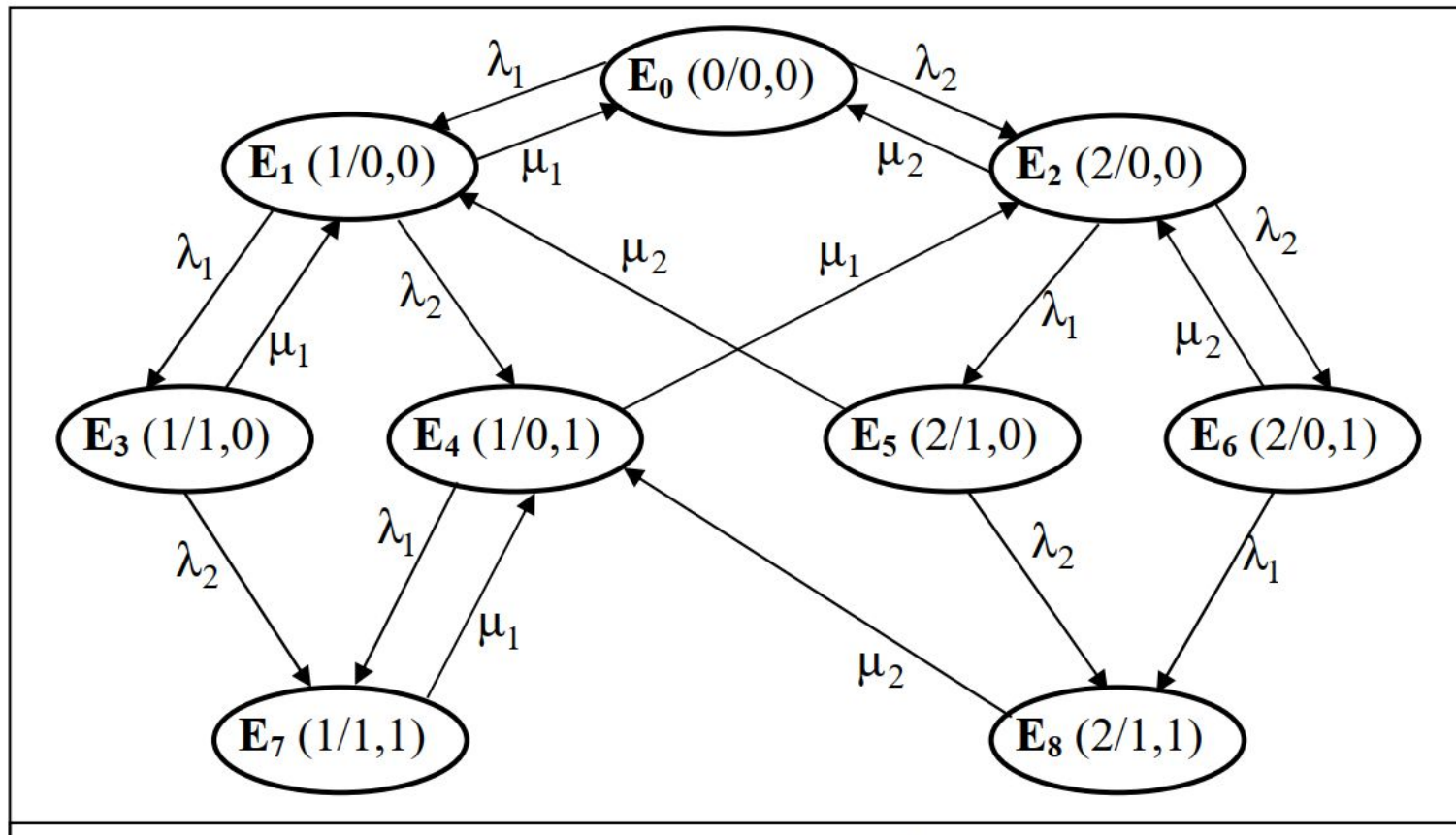
E5: (2 / 1,0) - there is a class 2 request and one a class 1 claim is awaiting service in the first drive;

E6: (2 / 0,1) - there is a class 2 request and one a class 2 claim is awaiting service in the second drive;

E7: (1 / 1,1) - a class 1 request is being serviced, and one service of each class is expected in the corresponding drives;

E8: (2 / 1,1) - a class 2 request is being serviced, and one service of each class is expected to be served in the corresponding drives.

4. Labeled transition graph of a random process



system of balance equations

$$\left\{ \begin{array}{l} (\lambda_1 + \lambda_2) p_0 = \mu_1 p_1 + \mu_2 p_2 \\ (\lambda_1 + \lambda_2 + \mu_1) p_1 = \lambda_1 p_0 + \mu_1 p_3 + \mu_2 p_5 \\ (\lambda_1 + \lambda_2 + \mu_2) p_2 = \lambda_2 p_0 + \mu_1 p_4 + \mu_2 p_6 \\ (\lambda_2 + \mu_1) p_3 = \lambda_1 p_1 \\ (\lambda_1 + \mu_1) p_4 = \lambda_2 p_1 + \mu_1 p_7 + \mu_2 p_8 \\ (\lambda_2 + \mu_2) p_5 = \lambda_1 p_2 \\ (\lambda_1 + \mu_2) p_6 = \lambda_2 p_2 \\ \mu_1 p_7 = \lambda_2 p_8 + \lambda_1 p_4 \\ \mu_2 p_8 = \lambda_2 p_5 + \lambda_1 p_6 \\ p_0 + p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 = 1 \end{array} \right.$$

5. Calculation of the characteristics of the system

1) нагрузка: $y_1 = \lambda_1 / \mu_1 = \lambda_1 b_1$; $y_2 = \lambda_2 / \mu_2 = \lambda_2 b_2$;

2) загрузка, создаваемая заявками, которая может трактоваться как вероятность того, что на обслуживании в приборе находится заявка класса 1 и 2 соответственно: $\rho_1 = p_1 + p_3 + p_4 + p_7$; $\rho_2 = p_2 + p_5 + p_6 + p_8$;

3) среднее число заявок в очереди:

$$l_1 = p_3 + p_5 + p_7 + p_8; \quad l_2 = p_4 + p_6 + p_7 + p_8;$$

4) среднее число заявок в системе:

$$m_1 = p_1 + 2p_3 + p_4 + p_5 + 2p_7 + p_8 = l_1 + \rho_1;$$

$$m_2 = p_2 + p_4 + p_5 + 2p_6 + p_7 + 2p_8 = l_2 + \rho_2;$$

5) вероятность потери заявок:

$$\pi_1 = p_3 + p_5 + p_7 + p_8; \quad \pi_2 = p_4 + p_6 + p_7 + p_8;$$

6) производительность по каждому классу заявок (интенсивность непотерянных заявок):

$$\lambda'_1 = \lambda_1(1 - \pi_1); \quad \lambda'_2 = \lambda_2(1 - \pi_2);$$

7) среднее время ожидания заявок:

$$w_1 = l_1 / \lambda'_1; \quad w_2 = l_2 / \lambda'_2$$

8) среднее время пребывания заявок

$$u_1 = m_1 / \lambda'_1 = w_1 + b; \quad u_2 = m_2 / \lambda'_2 = w_2 + b$$

Calculation of total flow characteristics service requests

- 1) total system load: $Y = y_1 + y_2$;
- 2) system load: $R = \rho_1 + \rho_2$;
- 3) system downtime ratio: $\eta = 1 - R$;
- 4) the total number of requests in all queues: $l = l_1 + l_2$;
- 5) the total number of requests in the system: $m = m_1 + m_2 = l + R$
- 6) the probability of losing orders: $\pi = \pi_1 + \pi_2$;
- 7) system performance (intensity of the total flow of serviced requests): $\lambda' = \lambda_1' + \lambda_2' = \lambda (1 - \pi)$;
- 8) the average waiting time for requests of the total flow: $w = (\lambda w + \lambda w) / \lambda = l / \lambda$;
- 9) the average sojourn time of claims of the total flow: $u = u + u' = m' = w + b$