

# Решение треугольников

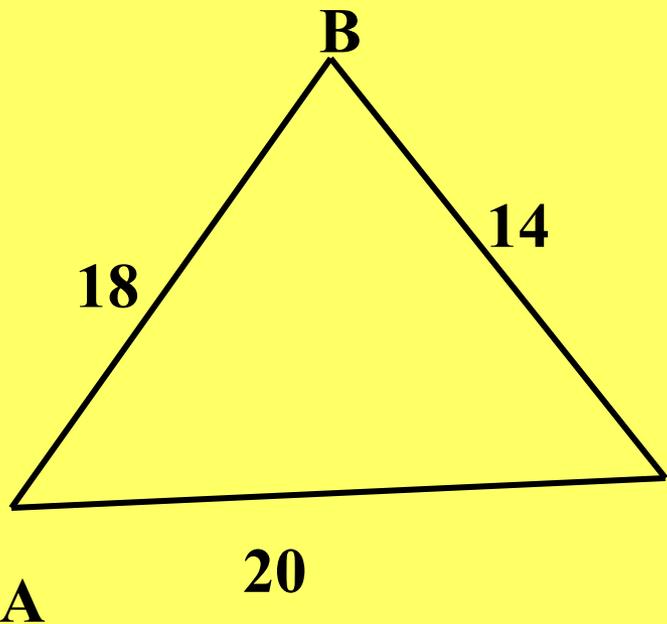




В науке нет широкой столбовой  
дороги и только тот достигнет ее  
сияющих вершин, кто не страшась  
усталости карабкается по ее  
каменистым тропам.

К. Маркс

№ 1025 (3)



Дано:  $\triangle ABC$ ,  $AB=18$ ,  $BC=14$ ,  
 $AC=20$

Найти:  $\angle A$ ,  $\angle B$ ,  $\angle C$

Решение:

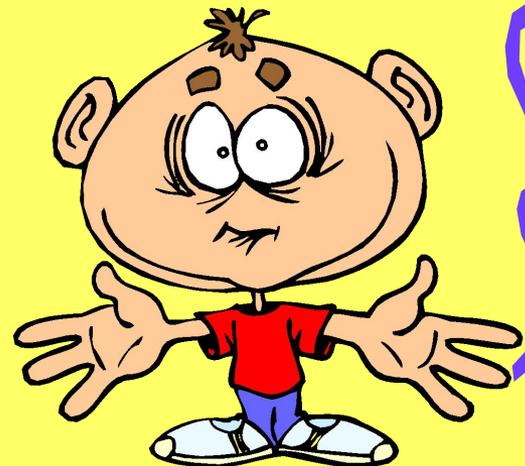
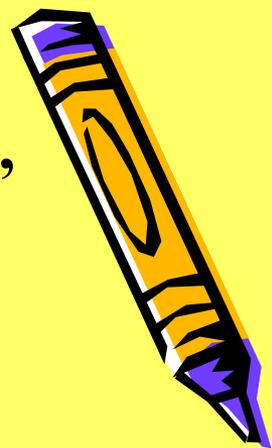
1) По теореме косинусов

$$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos B$$

$$20^2 = 18^2 + 14^2 - 2 \cdot 18 \cdot 14$$

$$\cos B = \frac{20^2 - 18^2 - 14^2}{-2 \cdot 18 \cdot 14} = \frac{400 - 324 - 196}{-504} = \frac{5}{21}$$

$$\angle B \approx 76^\circ$$



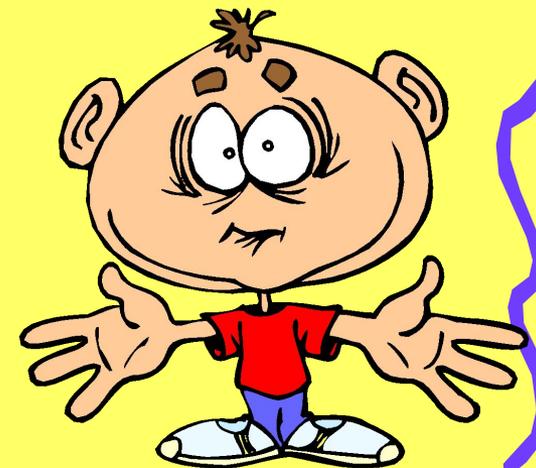
**2) по теореме синусов**

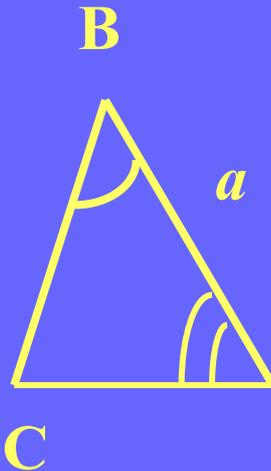
$$\frac{AC}{\sin B} = \frac{BC}{\sin A}$$

$$\sin A = \frac{BC \cdot \sin B}{AC} = \frac{14 \cdot \sin 76^\circ}{20} = 0,6792$$

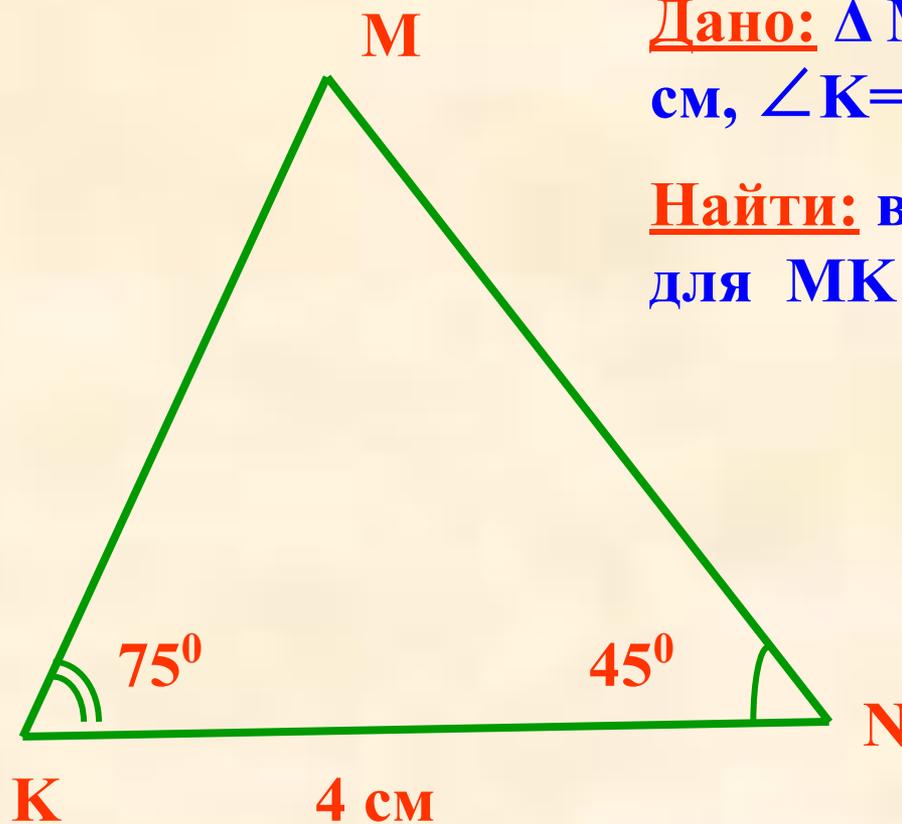
$$\angle A \approx 43^\circ$$

$$\angle C = 180^\circ - 76^\circ - 43^\circ = 61^\circ$$



	Дано	Найти	Решение
	$a, \angle B, \angle C$	$\angle A, b, c$	$\angle A = 180^\circ - (\angle B + \angle C)$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $b = \frac{a \sin B}{\sin A}; c = \frac{a \sin C}{\sin A}$

## Задача 1



Дано:  $\triangle MKN$ ,  $KN = 4$   
см,  $\angle K = 75^\circ$ ,  $\angle N = 45^\circ$ .

Найти: выражение  
для МК



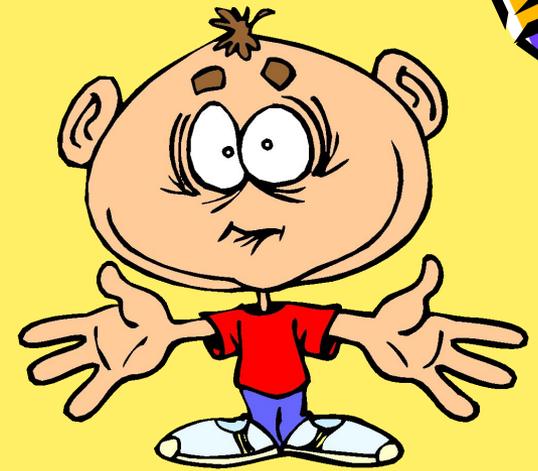
**Ответ:** 1)  $\angle M = 180^{\circ} - 120^{\circ} = 60^{\circ}$

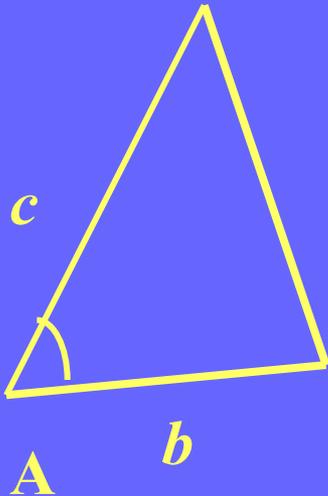
2) по теореме синусов

$$\frac{MK}{\sin N} = \frac{KN}{\sin M}$$

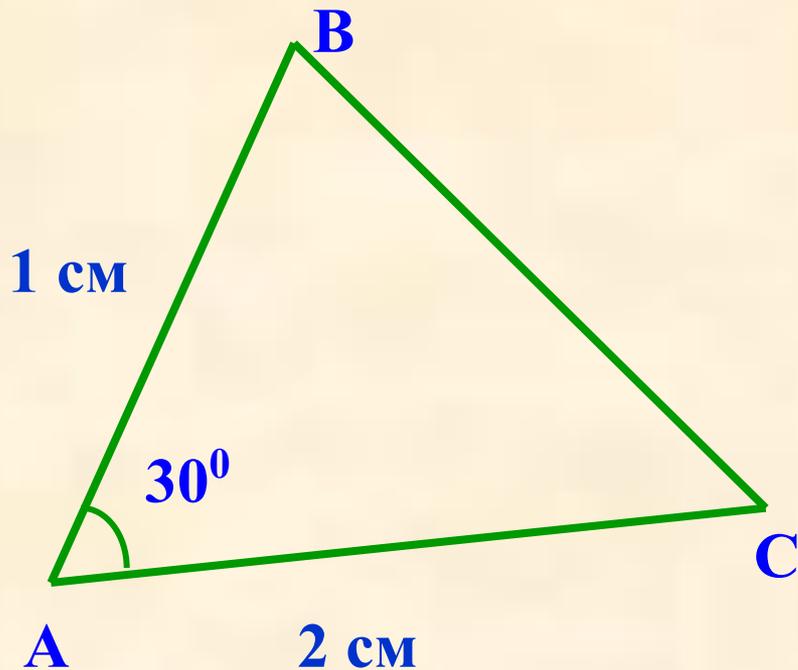
$$\frac{MK}{\sin 45^{\circ}} = \frac{4}{\sin 60^{\circ}}$$

$$MK = \frac{4 \cdot \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{2} \cdot 2}{\sqrt{3}} = \frac{12\sqrt{2}}{3}$$



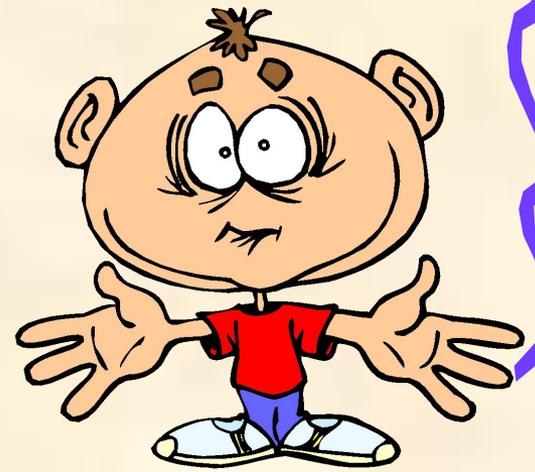
	Дано	Найти	Решение
 <p>A triangle with vertices at the top, bottom-left, and bottom-right. The side opposite the top vertex is labeled <math>a</math>. The side opposite the bottom-left vertex is labeled <math>b</math>. The side opposite the bottom-right vertex is labeled <math>c</math>. The angle at the bottom-left vertex is marked with an arc and labeled <math>A</math>.</p>	$b, c, \angle A$	$a, \angle B, \angle C$	$a = \sqrt{c^2 + b^2 - 2bc \cos A}$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\sin B = \frac{b \sin A}{a}; \sin C = \frac{c \sin A}{a}$ <p><math>\angle B</math> и <math>\angle C</math> — по таблицам</p>

## Задача 2



Дано:  $\triangle ABC$ ,  $AB = 1$  см,  
 $AC = 2$  см,  $\angle A = 30^\circ$ .

Найти: выражение для  
 $BC$

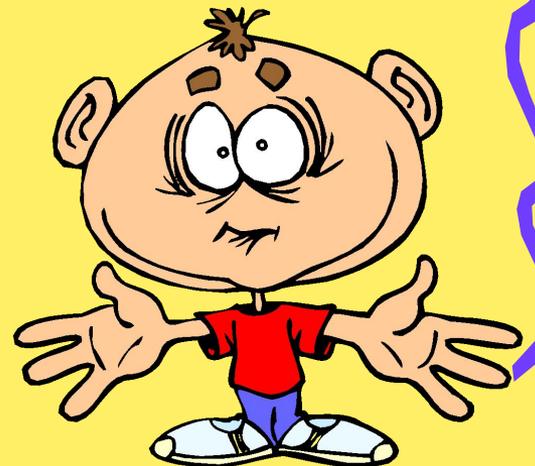


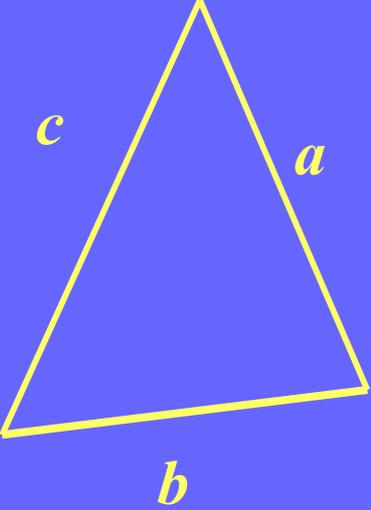
**Ответ:** по теореме косинусов

$$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos 30^\circ$$

$$BC^2 = 1 + 4 - 4 \cdot \frac{\sqrt{3}}{2}$$

$$BC = \sqrt{5 - 2\sqrt{3}}$$

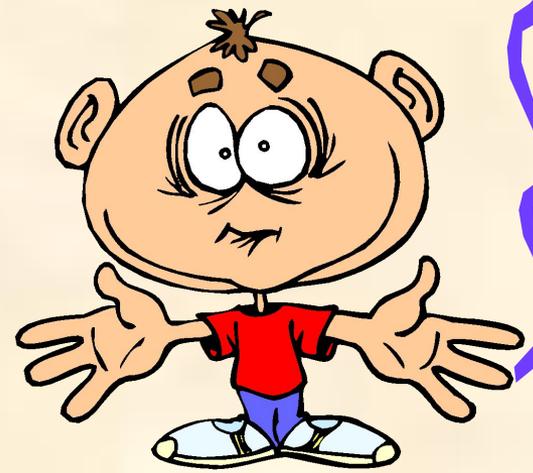
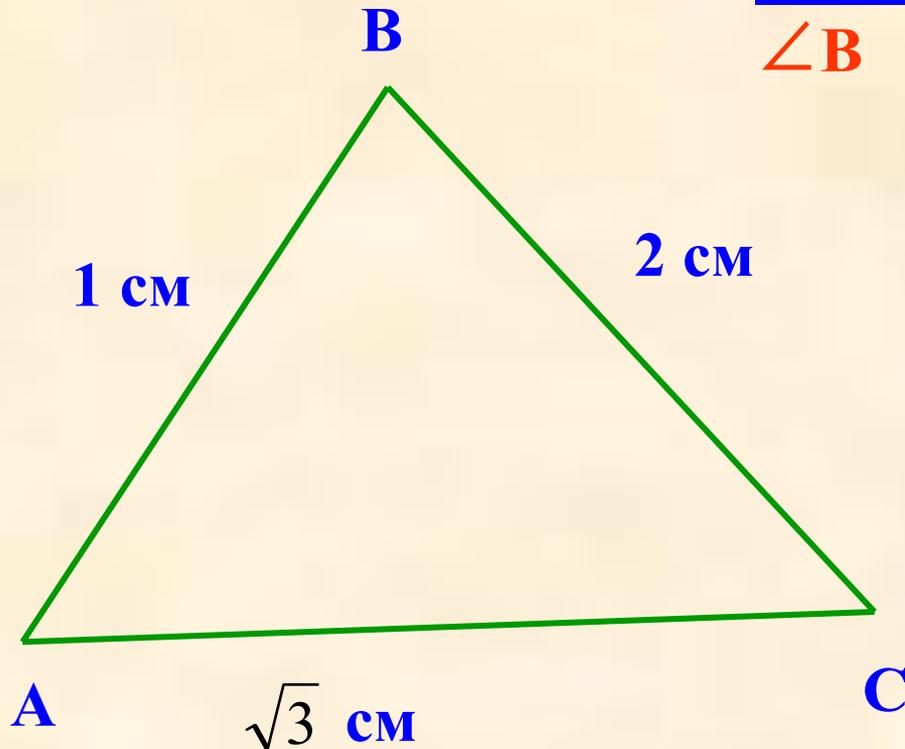


	Дано	Найти	Решение
	$a, b, c,$	$\angle A, \angle B,$ $\angle C$	$a^2 = c^2 + b^2 - 2bc \cos A$ $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\sin B = \frac{b \sin A}{a};$ $\sin C = \frac{c \sin A}{a}$ $\angle A, \angle B, \angle C$ — по таблицам или $\angle C = 180^\circ - (\angle A + \angle B)$

### Задача 3

Дано:  $\triangle ABC$ ,  $AB = 1$  см,  
 $BC = 2$  см,  $AC = \sqrt{3}$  см

Найти: выражение для  
 $\angle B$



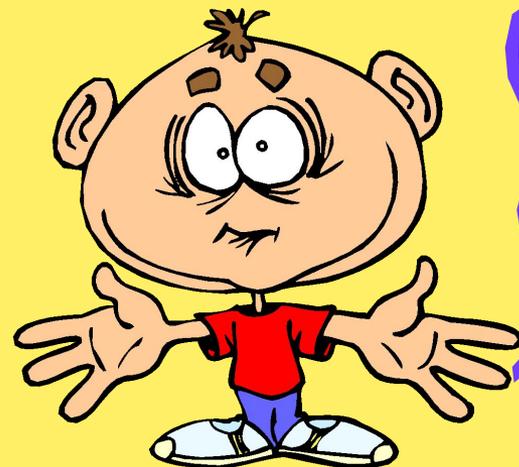
**Ответ:** по теореме косинусов

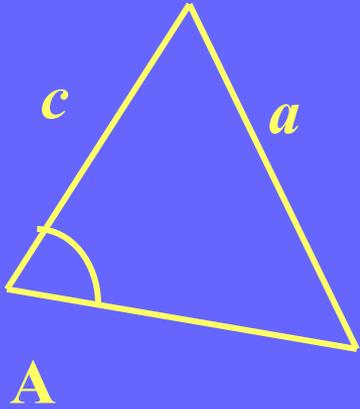
$$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos B$$

$$\cos B = \frac{AB^2 + BC^2 - AC^2}{2AB \cdot BC}$$

$$\cos B = \frac{1 + 4 - 3}{4} = \frac{1}{2}$$

$$\angle B = 60^\circ$$

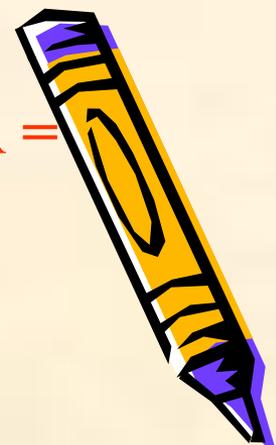
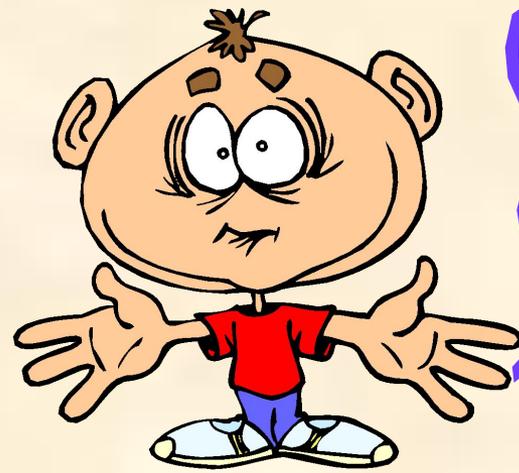
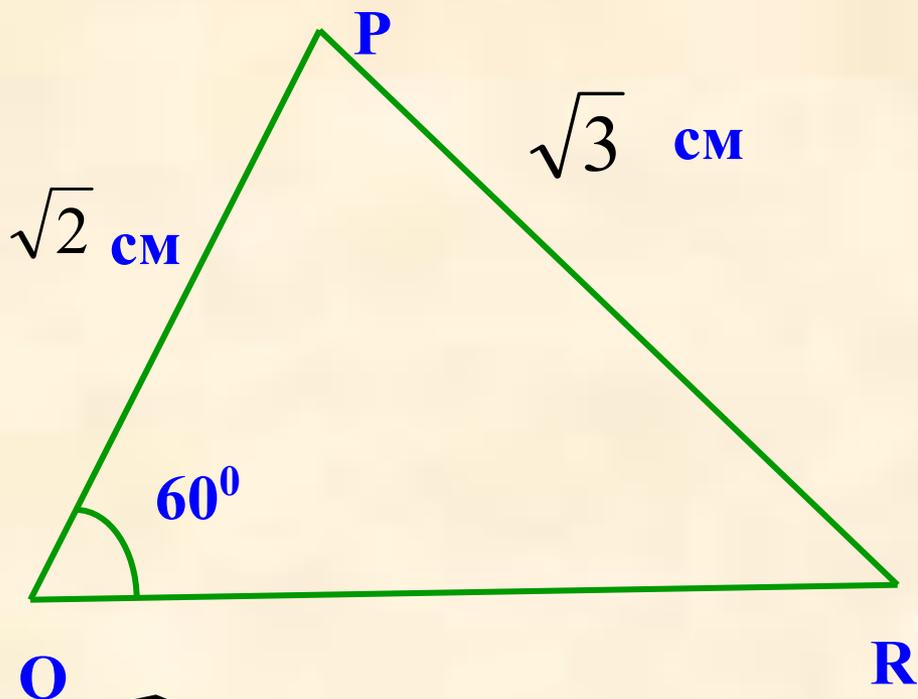


	Дано	Найти	Решение
 <p>A triangle with vertices at the top, bottom-left, and bottom-right. The side opposite the top vertex is labeled <math>a</math>. The side opposite the bottom-left vertex is labeled <math>c</math>. The side opposite the bottom-right vertex is labeled <math>b</math>. The angle at the bottom-left vertex is labeled <math>A</math>.</p>	$a, c,$ $\angle A$	$b, \angle B,$ $\angle C$	$\frac{c}{\sin C} = \frac{a}{\sin A}; \sin C = \frac{c \sin A}{a}$ $\angle B = 180^\circ - (\angle A + \angle C)$ $\frac{a}{\sin A} = \frac{b}{\sin B}; b = \frac{a \sin B}{\sin A}$

## Задача 4

Дано:  $\triangle OPR$ ,  $OP = \sqrt{2}$  см,  $PR = \sqrt{3}$  см

Найти: выражение для  $\angle R$



# Ответ: по теореме синусов

$$\frac{PR}{\sin O} = \frac{OP}{\sin R}$$

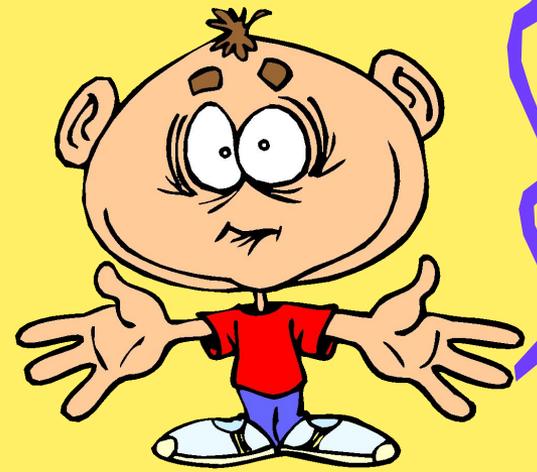
$$\frac{\sqrt{3}}{\sin 60^{\circ}} = \frac{\sqrt{2}}{\sin R}$$

$$\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{2}}{\sin R}$$

2

$$\sin R = 1$$

$$\angle R = 90^{\circ}$$



№ 1037

