KAZAKH BRITISH TECHNICAL UNIVERSITY

Physics 1

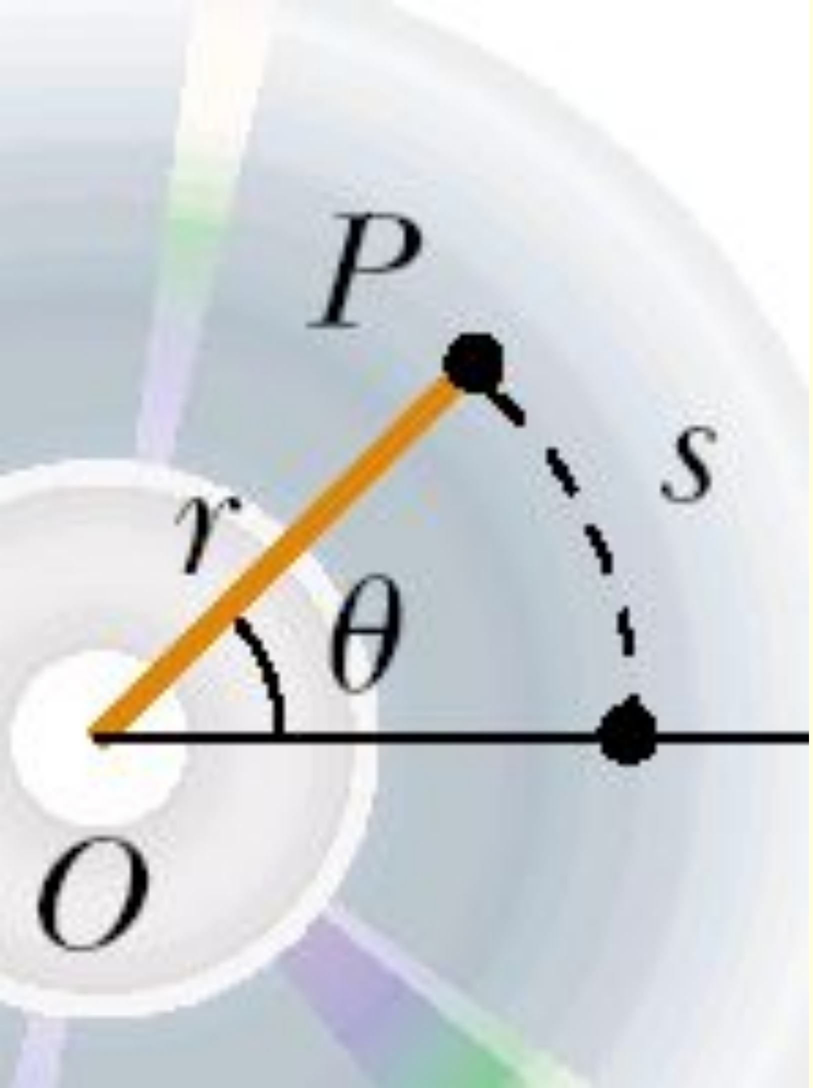
Lecture 4

- Rotation of rigid bodies.
- Angular momentum and torque.
- Properties of fluids.

Rotation of Rigid Bodies in General case

- When a rigid object is rotating about a *fixed* axis, every particle of the object rotates through the same angle in a given time interval and has the same angular speed and the same angular acceleration. So the rotational motion of the entire rigid object as well as individual particles in the object can be described by three angles. Using these three angles we can greatly simplify the analysis of rigid-object rotation.

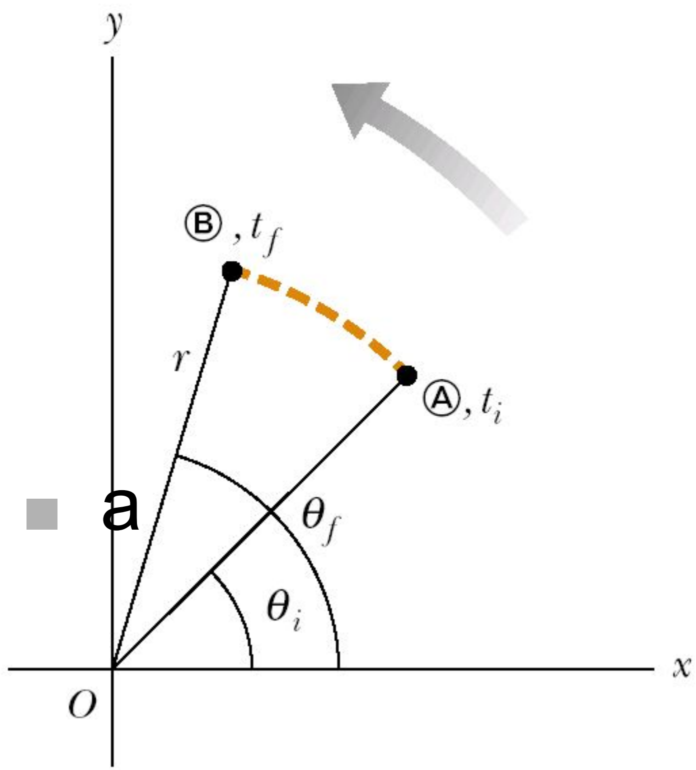
Radians



Angle in radians equals the ratio of the arc length *s* and the radius *r*:

$$\theta = \frac{s}{r}$$

Angular kinematics



- a

$$\alpha = \frac{d^2\theta}{dt^2}$$

- Angular displacement:

$$\Delta\theta \equiv \theta_f - \theta_i$$

- Instantaneous angular speed:

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- Instantaneous angular acceleration:

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

- Average angular speed:

$$\langle \omega \rangle \equiv \overline{\omega} \equiv \frac{\Delta \theta}{\Delta t}$$

- Average angular acceleration:

$$\langle \alpha \rangle \equiv \overline{\alpha} \equiv \frac{\Delta \omega}{\Delta t} \equiv \frac{\Delta^2 \theta}{\Delta t^2}$$

Angular and linear quantities

- Every particle of the object moves in a circle whose center is the axis of rotation.
- Linear velocity: $$v = r\omega$$
- Tangential acceleration: $$a_t = r\alpha$$
- Centripetal acceleration: $$a_c = \frac{v^2}{r} = r\omega^2$$

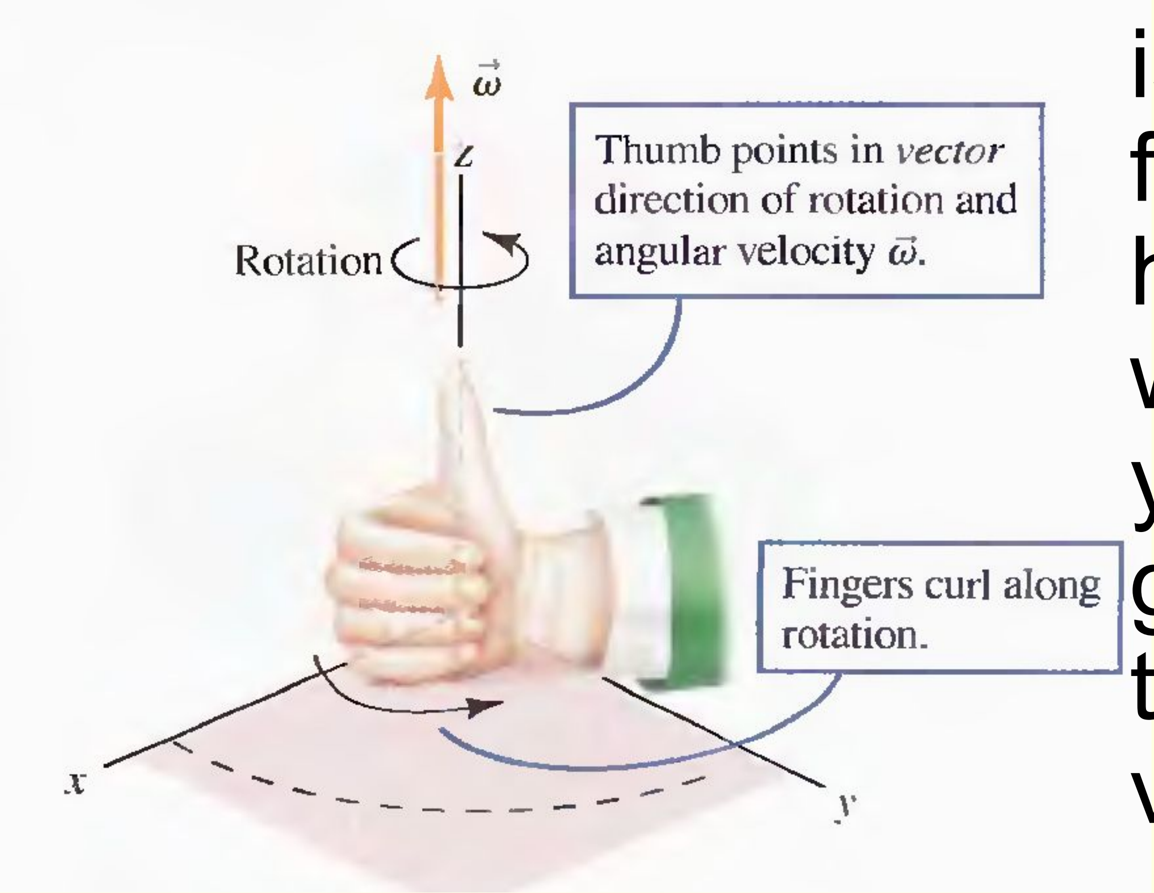
Total linear acceleration

- Tangential acceleration is perpendicular to the centripetal one, so the magnitude of total linear acceleration is

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4}$$

Angular velocity

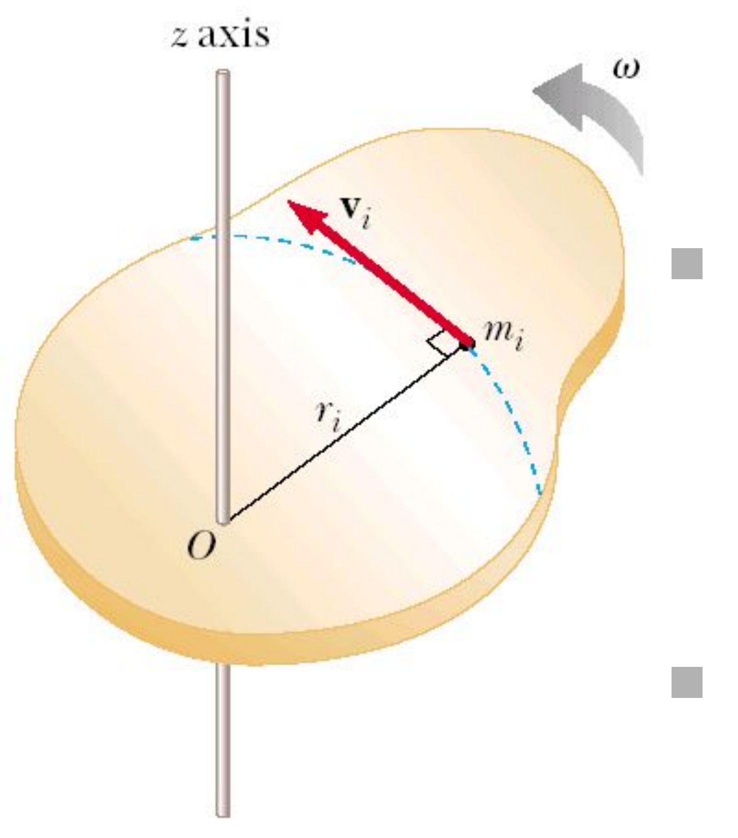
- Angular velocity is a vector.



The right hand rule is applied: If the fingers of your righ hand curl along with the rotation your thumb will give the direction c the angular velocity.

Rotational Kinetic Energy

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$



- $$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$
- Moment of rotational inertia

$$I \equiv \sum_i m_i r_i^2$$

- Rotational kinetic energy

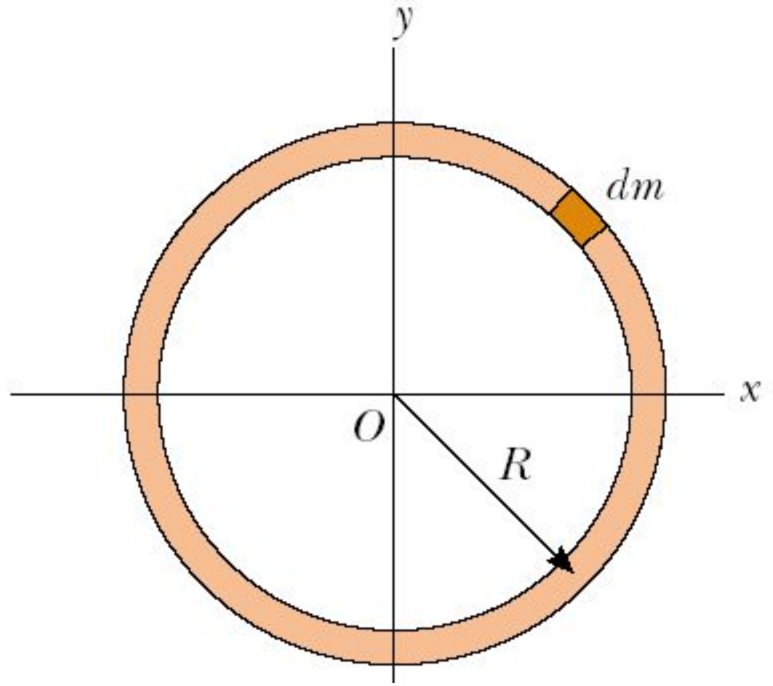
$$K_R = \frac{1}{2} I\omega^2$$

Calculations of Moments of Inertia

$$I = \lim_{\Delta m_i \to 0} \sum_i {r_i}^2 \Delta m_i = \int r^2 \, dm$$

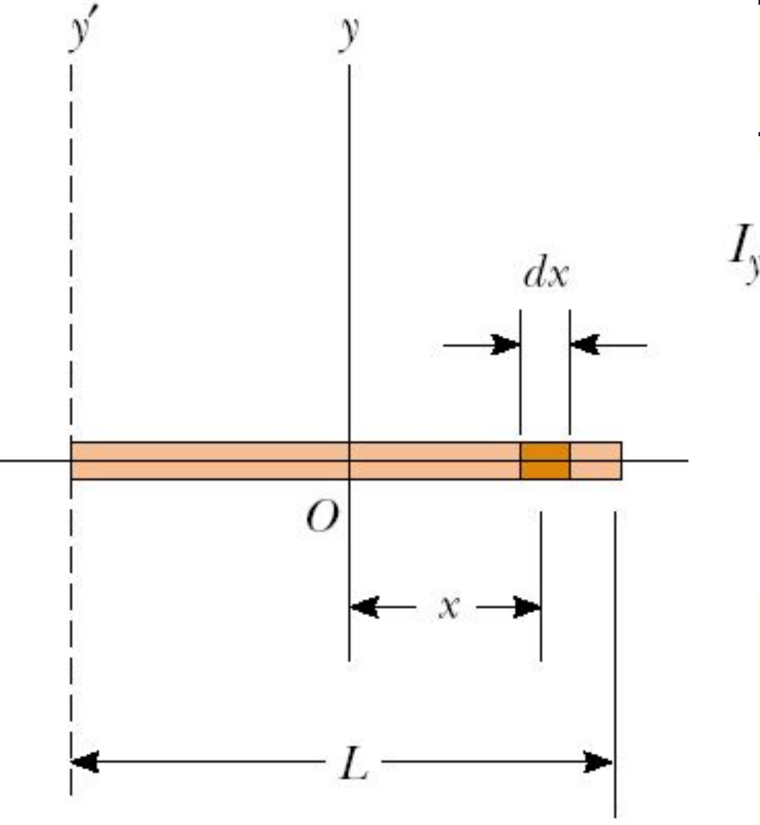
$$I = \int \rho r^2 \, dV$$

Uniform Thin Hoop



$$I_z = \int r^2\, dm = R^2 \int dm = MR^2$$

Uniform Rigid Rod



$$dm = \lambda\, dx = \frac{M}{L}\, dx$$

$$I_y = \int r^2\, dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L}\, dx = \frac{M}{L}\int_{-L/2}^{L/2} x^2\, d$$

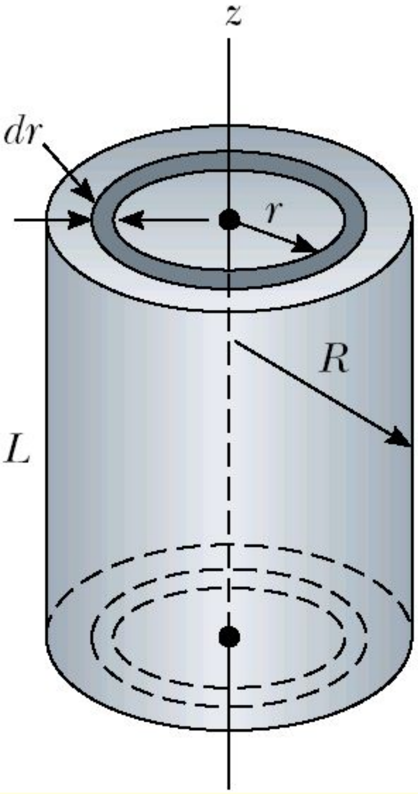
$$= \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{1}{12} ML^2$$

Uniform Solid Cylinder

$$dV = L\,dA = L(2\pi r)\,dr$$

$$dm = \rho\,dV = 2\pi\rho L r\,dr.$$

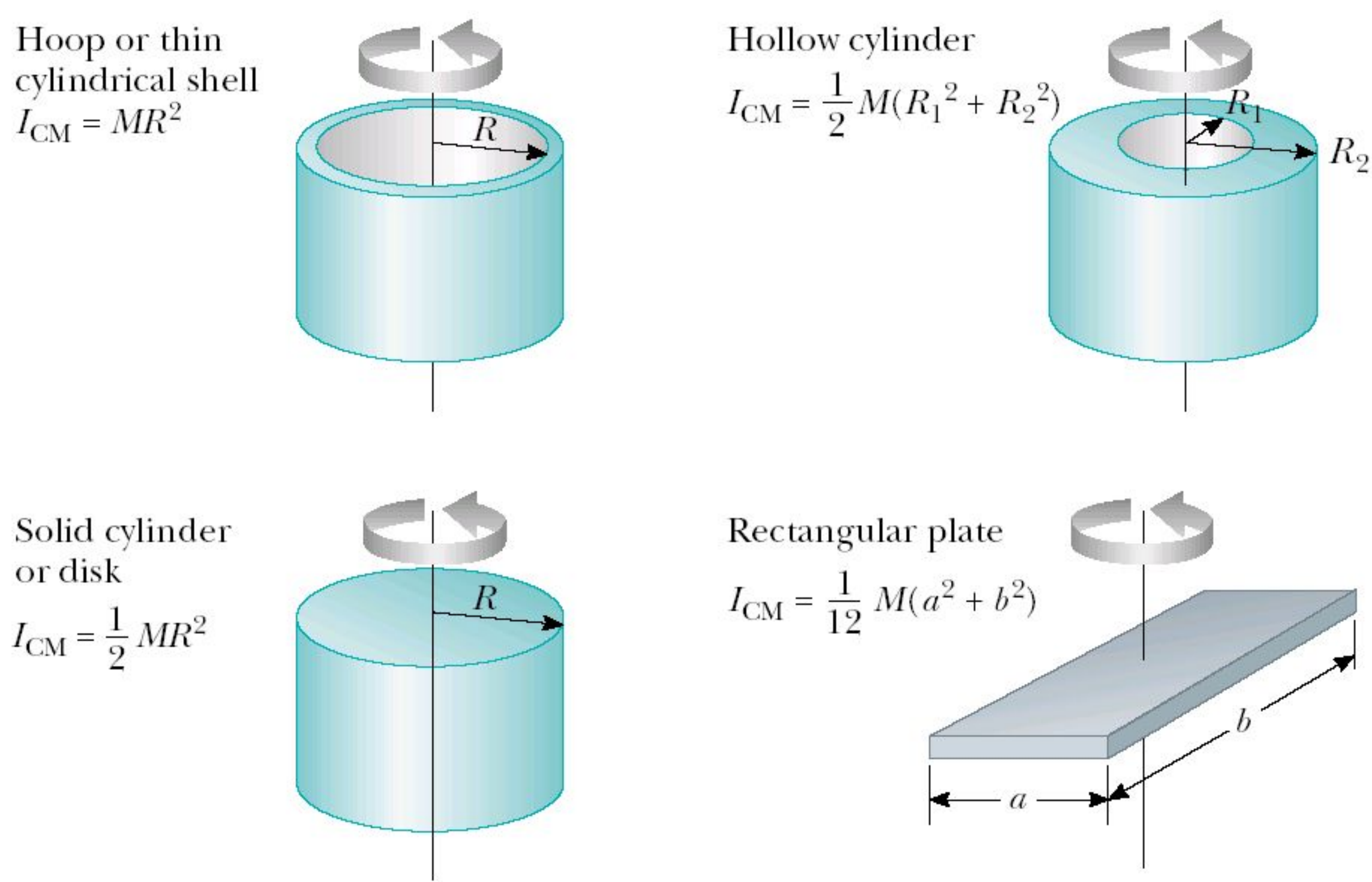
$$I_z = \int r^2\,dm = \int r^2(2\pi\rho L r\,dr) = 2\pi\rho L\int_0^R r^3 dr = \tfrac{1}{2}\pi\rho L R^4$$



$$\rho = M/V = M/\pi R^2 L.$$

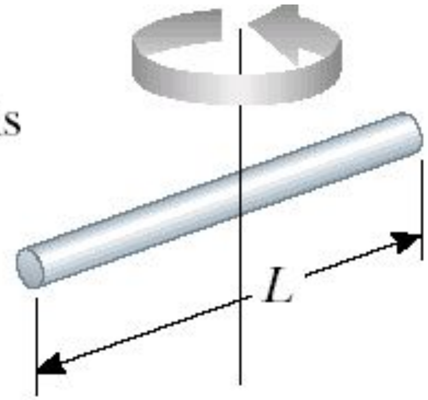
$$I_z = \tfrac{1}{2}MR^2$$

Moments of Inertia of Homogeneous Rigid Objects with Different Geometries



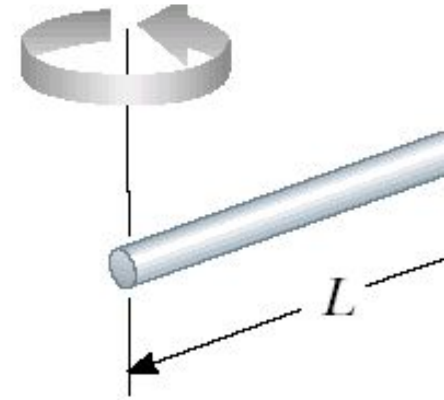
Long thin rod with rotation axis through center

$$I_{CM} = \frac{1}{12} ML^2$$



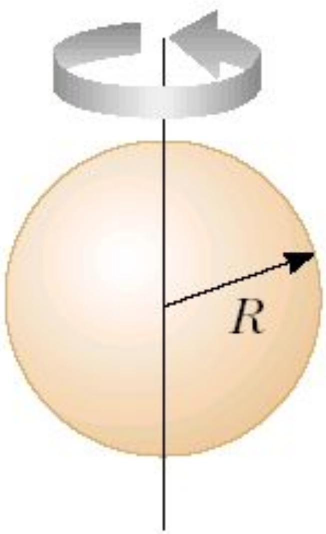
Long thin rod with rotation axis through end

$$I = \frac{1}{3} ML^2$$



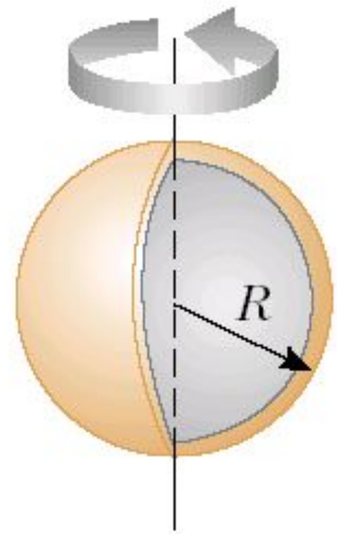
Solid sphere

$$I_{CM} = \frac{2}{5} MR^2$$



Thin spherical shell

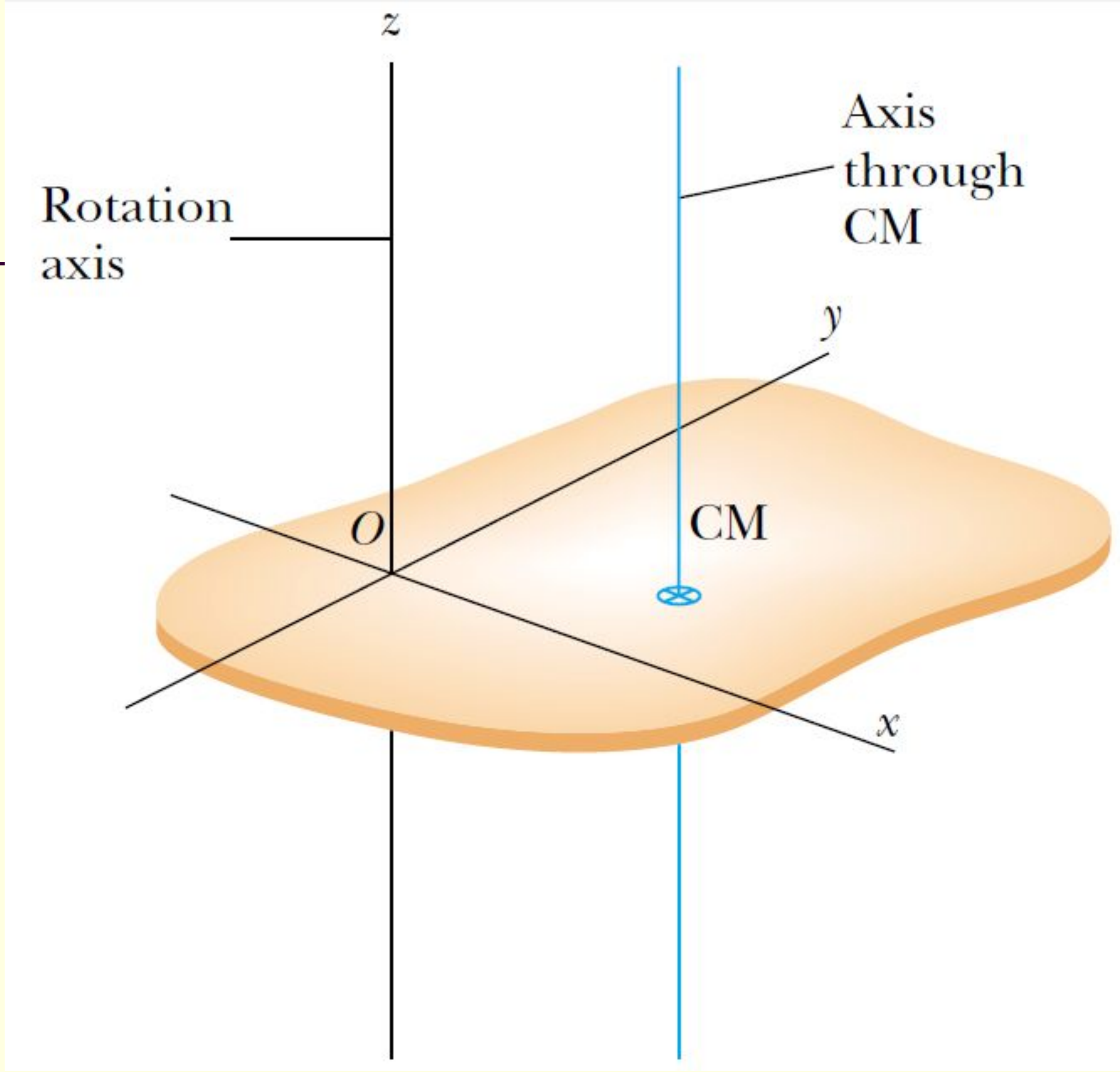
$$I_{CM} = \frac{2}{3} MR^2$$



Parallel-axis theorem

- Suppose the moment of inertia about an axis through the center of mass of an object is I_{CM}. Then the moment of inertia about any axis parallel to and a distance *D* away from this axis is

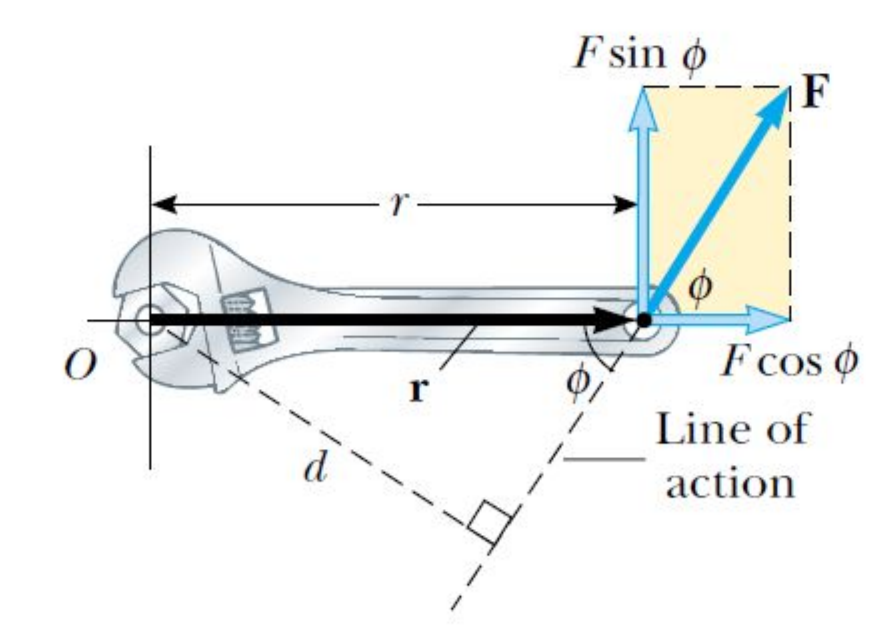
$$I = I_{\mathrm{CM}} + MD^2$$


z
Rotation
axis
Axis
through
CM
y
O
CM
x

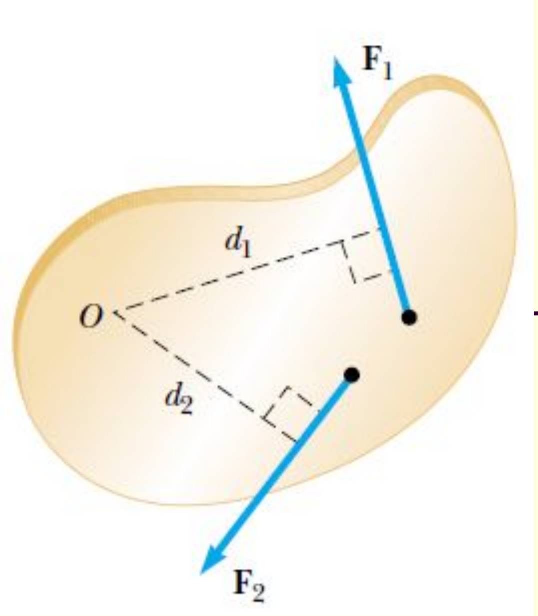
Torque

- When a force is exerted on a rigid object pivoted about an axis, the object tends to rotate about that axis. The tendency of a force to rotate an object about some axis is measured by a vector quantity called torque τ (Greek tau).

$$\boldsymbol{\tau} \equiv \mathbf{r} \times \mathbf{F}$$



The force F has a greater rotating tendency about axis *O* as *F* increases and as the moment arm *d* increases. The component *F* sinφ tends to rotate the wrench about axis *O*.



The force F_1 tends to rotate the object counterclockwise about *O*, and F_2 tends to rotate it clockwise.

We use the convention that the sign of the torque resulting from a force is positive if the turning tendency of the force is counterclockwise and is negative if the turning tendency is clockwise. Then

$$\sum \tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$$

Torque is not Force
Torque is not Work

Torque should not be confused with force. Forces can cause a change in linear motion, as described by Newton’s second law. Forces can also cause a change in rotational motion, but the effectiveness of the forces in causing this change depends on both the forces and the moment arms of the forces, in the combination that we call *torque*. Torque has units of force times length—newton · meters in SI units—and should be reported in these units.

Do not confuse torque and work, which have the same units but are very different concepts.

Rotational Dynamics

$$\Sigma \mathbf{F} = d\mathbf{p}/dt$$

$$\mathbf{r} \times \sum \mathbf{F} = \sum \boldsymbol{\tau} = \mathbf{r} \times \frac{d\mathbf{p}}{dt}$$

Let's add $\frac{d\mathbf{r}}{dt} \times \mathbf{p}$ which equals zero, as $d\mathbf{r}/dt = \mathbf{v}$ and $\mathbf{v}$ and $\mathbf{p}$ are parallel.

Then: $$\sum \boldsymbol{\tau} = \mathbf{r} \times \frac{d\mathbf{p}}{dt} + \frac{d\mathbf{r}}{dt} \times \mathbf{p}$$

$$\sum \boldsymbol{\tau} = \frac{d(\mathbf{r} \times \mathbf{p})}{dt}$$

$$\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$$

Rotational analogue of Newton's second law

- Quantity ***L*** is an instantaneous angular momentum.

$$\sum \boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$$

- The torque acting on a particle is equal to the time rate of change of the particle's angular momentum.

Net External Torque

- The net external torque acting on a system about some axis passing through an origin in an inertial frame equals the time rate of change of the total angular momentum of the system about that origin:

$$\sum \boldsymbol{\tau}_{\text{ext}} = \frac{d\mathbf{L}_{\text{tot}}}{dt}$$

Angular Momentum of a Rotating Rigid Object

- Angular momentum for each particle of an object:

$$L_i = m_i r_i^2 \omega$$

- Angular momentum for the whole object:

$$L_z = \sum_i L_i = \sum_i m_i r_i^2 \omega = \left(\sum_i m_i r_i^2 \right) \omega$$

$$L_z = I\omega$$

Angular acceleration

$$\frac{dL_z}{dt} = I\frac{d\omega}{dt} = I\alpha$$

$$\sum \tau_{\text{ext}} = I\alpha$$

The Law of Angular Momentum Conservation

- The total angular momentum of a system is constant if the resultant external torque acting on the system is zero, that is, if the system is isolated.

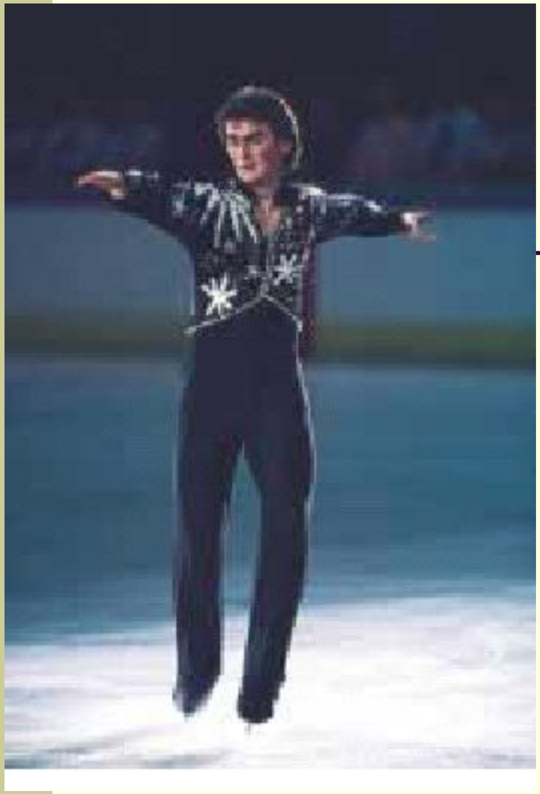
$$\sum \boldsymbol{\tau}_{\text{ext}} = \frac{d\mathbf{L}_{\text{tot}}}{dt} = 0$$

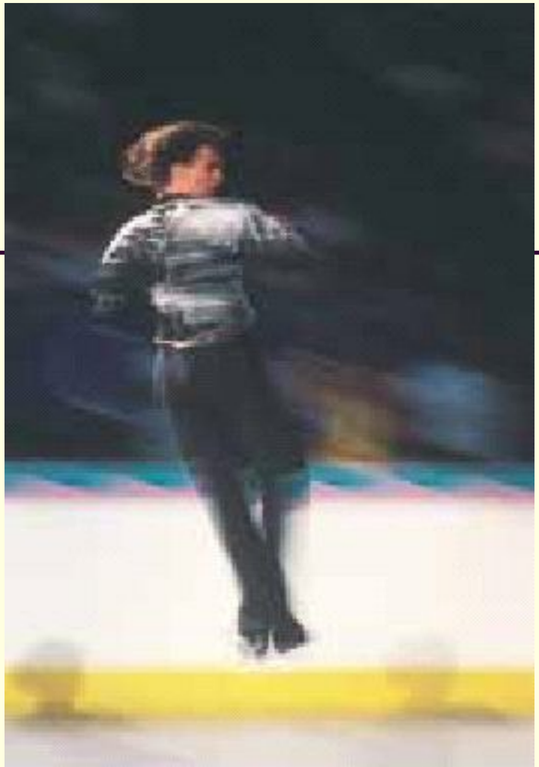
$$\mathbf{L}_{\text{tot}} = \text{constant}$$

$$\mathbf{L}_{\text{tot}} = \text{constant}$$

$$I_i \omega_i = I_f \omega_f = \text{constant}$$

- Change in internal structure of a rotating body can result in change of its angular velocity.





- When a rotating skater pulls his hands towards his body he spins faster.

Three Laws of Conservation for an Isolated System

$$\left.\begin{aligned} E_i &= E_f \\ \mathbf{p}_i &= \mathbf{p}_f \\ \mathbf{L}_i &= \mathbf{L}_f \end{aligned}\right\}$$

Full mechanical energy, linear momentum and angular momentum of an isolated system remain constant.

Work-Kinetic Theory for Rotations

- Similarly to linear motion:

$$dW \equiv \vec{\tau} \cdot d\vec{\theta}.$$

$$W = \int_{\theta_0}^{\theta} \tau \, d\theta = \int_0^t I \frac{d\omega}{dt} \omega dt = \int_0^t I \frac{1}{2} \frac{d\omega^2}{dt} dt$$

$$= \frac{1}{2} I \int_{\omega_0^2}^{\omega^2} d\omega^2 = \frac{1}{2} I (\omega^2 - \omega_0^2) = K - K_0.$$

- The net work done by external forces in rotating a symmetric rigid object about a fixed axis equals the change in the object's rotational energy.

Equations for Rotational and Linear Motions

Rotational Motion About a Fixed Axis	Linear Motion
Angular speed $\omega = d\theta/dt$	Linear speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Linear acceleration $a = dv/dt$
Net torque $\Sigma\tau = I\alpha$	Net force $\Sigma F = ma$
If $\alpha =$ constant $\begin{cases} \omega_f = \omega_i + \alpha_t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If $a =$ constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$	Work $W = \int_{x_i}^{x_f} F_x \, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $\mathcal{P} = \tau\omega$	Power $\mathcal{P} = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

Independent Study for IHW2

1. Vector multiplication (through their components ***i,j,k***).Right-hand rule of Vector multiplication.
2. Elasticity
   1. Demonstrate by example and discussion your understanding of *elasticity, elastic limit, stress, strain,* and *ultimate strength*.
   2. Write and apply formulas for calculating Young's modulus, shear modulus, and bulk modulus. Units of stress.

3. Fluids

1. Define *absolute pressure, gauge pressure*, and *atmospheric pressure*, and demonstrate by examples your understanding of the relationships between these terms.
2. Pascal’s law.
3. Archimedes’s law.
4. Rate of flow of a fluid.
5. Bernoulli’s equation.
6. Torricelli’s theorem.

Literature to Independent Study

1. Lecture on Physics Summary by Umarov. (Intranet)
2. Fishbane Physics for Scientists… (Intranet)
3. Serway Physics for Scientists… (Intranet)

Problems

1. A solid sphere and a hollow sphere have the same mass and radius. Which momentum of rotational inertia is higher if it is? Prove your answer with formulae.
2. What are the units for, are these quantities vectors or scalars:
   1. Angular momentum
   2. Angular kinetic energy
   3. Angular displacement
   4. Tangential acceleration
   5. Angular acceleration
   6. Torque